Matrizecll

1

$$\begin{cases} 3A + 4B = \begin{pmatrix} 5 & 6 & 18 & 15 \\ 28 & 19 & 26 & -19 \\ -3 & -8 & 13 & 16 \end{pmatrix} \\ 5A - 3B = \begin{pmatrix} 18 & 10 & 30 & -4 \\ 8 & 22 & 24 & -22 \\ -5 & 6 & 12 & 17 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} 9A + 12B = \begin{pmatrix} 15 & 18 & 54 & 45 \\ 84 & 57 & 78 & -57 \\ -9 & -24 & 39 & 48 \end{pmatrix} \\ 20A - 12B = \begin{pmatrix} 72 & 40 & 120 & -16 \\ 32 & 88 & 96 & -88 \\ -20 & 24 & 48 & 68 \end{pmatrix} \end{cases}$$

$$\begin{cases} \frac{3}{29}\begin{pmatrix} 87 & 58 & 174 & 29 \\ 116 & 145 & 174 & -145 \\ -29 & 0 & 87 & 116 \end{pmatrix} + 4B = \begin{pmatrix} 5 & 6 & 18 & 15 \\ 28 & 19 & 26 & -19 \\ -3 & -8 & 13 & 16 \end{pmatrix} \\ 29A = \begin{pmatrix} 87 & 58 & 174 & 29 \\ 116 & 145 & 174 & -145 \\ -29 & 0 & 87 & 116 \end{pmatrix} \Rightarrow A = \frac{1}{29}\begin{pmatrix} 87 & 58 & 174 & 29 \\ 116 & 145 & 174 & -145 \\ -29 & 0 & 87 & 116 \end{pmatrix} \end{cases}$$

$$\begin{cases} \begin{pmatrix} 9 & 6 & 18 & 3 \\ 12 & 15 & 18 & -15 \\ -3 & 0 & 9 & 12 \end{pmatrix} + 4B = \begin{pmatrix} 5 & 6 & 18 & 15 \\ 28 & 19 & 26 & -19 \\ -3 & -8 & 13 & 16 \end{pmatrix} \Rightarrow 4B = \begin{pmatrix} -4 & 0 & 0 & 12 \\ 16 & 4 & 8 & -4 \\ 0 & -8 & 4 & 4 \end{pmatrix} \Rightarrow \end{cases}$$

$$\Rightarrow B = \begin{pmatrix} -1 & 0 & 0 & 3 \\ 4 & 1 & 2 & -1 \\ 0 & -4 & 1 & 1 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 3 & 2 & 6 & 1 \\ 4 & 5 & 6 & -5 \\ -1 & 0 & 3 & 4 \end{pmatrix}$$

2)

$$\left.\begin{aligned} 5x + 3y &= A \\ 3x + 2y &= B \end{aligned}\right\} \left.\begin{aligned} -15x - 9y &= -3A \\ +15x + 10y &= 5B \end{aligned}\right\} y = 5B - 3A \Rightarrow$$

$$\Rightarrow y = \begin{pmatrix} 5 & -5 \\ -10 & 45 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ -12 & 45 \end{pmatrix} = \begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix}$$

$$3x + 2\begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix} = B \Rightarrow 3x = \begin{pmatrix} 1 & -1 \\ -2 & 9 \end{pmatrix} - \begin{pmatrix} -2 & -10 \\ 4 & 0 \end{pmatrix} \Rightarrow 3x = \begin{pmatrix} 3 & 9 \\ -6 & 9 \end{pmatrix} \Rightarrow$$

$$x = \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix}$$

$$x^2 = \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -5 & 12 \\ -8 & 3 \end{pmatrix}$$

$$y^2 = \begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} -9 & 5 \\ -2 & -10 \end{pmatrix}$$

$$x^2 + y^2 = \begin{pmatrix} -5 & 12 \\ -8 & 3 \end{pmatrix} \begin{pmatrix} -9 & 5 \\ -2 & -10 \end{pmatrix} = \begin{pmatrix} -14 & 17 \\ -10 & -7 \end{pmatrix}$$

4)

$$M = \begin{pmatrix} 0 & bd & -cd & bc \\ -ae & 0 & ce & ac \\ af & -bf & 0 & ab \\ -ef & -df & -de & 0 \end{pmatrix}$$

$$M^2 = \begin{pmatrix} 0 & bd & -cd & bc \\ -ae & 0 & ce & ac \\ af & -bf & 0 & ab \\ -ef & -df & -de & 0 \end{pmatrix} \begin{pmatrix} 0 & bd & -cd & bc \\ -ae & 0 & ce & ac \\ af & -bf & 0 & ab \\ -ef & -df & -de & 0 \end{pmatrix} = \begin{pmatrix} -aebd + acdf - bcef & 0 & bdce - bdce & bdac - cdab \\ ceaf - acef & -aebd - bfce - dfac & aecd - acde & -aebc + ceab \\ 0 & 0 & -afcd - bfce - abde & 0 \\ 0 & 0 & 0 & -efbc - dfac - deab \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

5)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^2 = A \Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \begin{pmatrix} a^2+bc & ab+bd \\ ac+dc & cb+d^2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} a^2+bc = a \Rightarrow bc = a-a^2 \\ ab+bd = b \Rightarrow a+d = 1 \\ ac+dc = c \Rightarrow c(a+d) = c \Rightarrow a+d = 1 \\ cb+d^2 = d \Rightarrow \cancel{a-a^2+(1-a)^2 = 1-a = 1 \Rightarrow a-a^2+1-2a+a^2 = 1-a \Rightarrow 1 = 1} \end{cases}$$

$$\left.\begin{matrix} cb = d-d^2 \\ cb = a-a^2 \end{matrix}\right\} \left.\begin{matrix} cb = d-d^2 \\ cb = 1-d-(1-d)^2 \end{matrix}\right\} \begin{matrix} cb = d-d^2 \Rightarrow c = \dfrac{d-d^2}{b} \\ \cancel{cb} = 0 \end{matrix}$$

$a = 1-d$

$$A = \begin{pmatrix} 1-d & b \\ \dfrac{d-d^2}{b} & d \end{pmatrix}$$

6)

$A^2 = A$ bete

$B = 2A - I \rightarrow B^2 = I$ Frogatillo dugu.

$B = 2A-I \Rightarrow \cancel{2A = B+I \Rightarrow 2A^2 = B+I \Rightarrow 2B+I = B+I}$
$A^2=A$

$$\Rightarrow B^2 = (2A-I)^2 \Rightarrow B^2 = 4A^2 - 4A + I \underset{A^2=A}{\Rightarrow} B^2 = 4A - 4A + I \Rightarrow \boxed{B^2 = I}$$

1)

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} \xrightarrow[\mathcal{E}_{21}(-1)]{\mathcal{E}_{31}(-1)} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} \xrightarrow[\mathcal{E}_{32}(-3)]{} \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 6 \end{vmatrix} = \cancel{24}\ 12$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{vmatrix} \xrightarrow[\mathcal{E}_{51}(-5)\ \mathcal{E}_{41}(-4)\ \mathcal{E}_{31}(-3)\ \mathcal{E}_{21}(-2)]{} \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -1 & -2 & -3 & -9 \\ 0 & -2 & -4 & & \end{vmatrix}$$

$$\xrightarrow[\mathcal{E}_{54}(-1)\ \mathcal{E}_{43}(-1)\ \mathcal{E}_{32}(-1)\ \mathcal{E}_{21}(-1)]{} \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & -4 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \end{vmatrix} \xrightarrow[\mathcal{E}_{32}\ \mathcal{E}_{23}(-1)]{} \begin{vmatrix} 0 & 1 & 2 & 3 & 9 \\ 0 & 0 & 0 & 5 & -5 \\ 0 & 0 & 5 & -5 & 0 \\ 0 & 5 & -5 & 0 & 0 \\ 1 & -4 & 1 & 1 & 1 \end{vmatrix} \xrightarrow[P_{45}\ P]{} \begin{vmatrix} 1 & 2 & 3 & 9 \\ 0 & 0 & 5 & -5 \\ 0 & 5 & -5 & 0 \\ 5 & -5 & 0 & 0 \end{vmatrix} \rightarrow$$

$$\xrightarrow[\mathcal{E}_{41}(-5)]{} \begin{vmatrix} 1 & 2 & 3 & 9 \\ 0 & 0 & 5 & -5 \\ 0 & 5 & -5 & 0 \\ 0 & -15 & 15 & -45 \end{vmatrix} \rightarrow 15 \begin{vmatrix} 1 & 2 & 3 & 9 \\ 0 & 0 & 5 & -5 \\ 0 & 5 & -5 & 0 \\ 0 & -1 & 1 & -3 \end{vmatrix} \rightarrow 15 \begin{vmatrix} 0 & 5 & -5 \\ 5 & -5 & 0 \\ -1 & 1 & -3 \end{vmatrix} \xrightarrow[P_{31}\ P_{23}]{}$$

$$\rightarrow 15 \begin{vmatrix} -1 & 1 & -3 \\ 0 & 5 & -5 \\ 5 & -5 & 0 \end{vmatrix} \xrightarrow[\mathcal{E}_{31}(-5)]{} 15\cdot5\cdot3 \begin{vmatrix} -1 & 1 & -3 \\ 0 & 5 & -5 \\ 0 & 0 & 15 \end{vmatrix} \rightarrow \begin{vmatrix} -1 & 1 & -3 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{vmatrix} \rightarrow$$

1)

$$\begin{vmatrix} 0 & 3 & 7 & 2 & 3 \\ 1 & 4 & 1 & 4 & 4 \\ 3 & 4 & 0 & 3 & 4 \\ 9 & 8 & 3 & 7 & 9 \\ 8 & 4 & 2 & 0 & 1 \end{vmatrix} \xrightarrow{P_{21}} \begin{vmatrix} 1 & 4 & 1 & 4 & 4 \\ 0 & 3 & 7 & 2 & 3 \\ 3 & 4 & 0 & 3 & 4 \\ 9 & 8 & 3 & 7 & 9 \\ 8 & 4 & 2 & 0 & 1 \end{vmatrix} \xrightarrow{\mathcal{E}_{43}(-3)} \begin{vmatrix} 1 & 4 & 1 & 4 & 4 \\ 0 & 3 & 7 & 2 & 3 \\ 3 & 4 & 0 & 3 & 4 \\ 0 & -4 & 3 & -2 & 3 \\ 8 & 4 & 2 & 0 & 1 \end{vmatrix} \rightarrow$$

$$\xrightarrow[\mathcal{E}_{31}(-3)]{\mathcal{E}_{51}(-8)} \begin{vmatrix} 1 & \cancel{4} & \cancel{1} & \cancel{4} & \cancel{4} \\ 0 & 3 & 7 & 2 & 3 \\ 0 & -8 & -3 & -9 & -8 \\ 0 & -4 & 3 & -2 & 3 \\ 0 & -28 & -6 & -32 & -31 \end{vmatrix} \rightarrow \begin{vmatrix} 3 & 7 & 2 & 3 \\ -8 & -3 & -9 & -8 \\ -4 & 3 & -2 & 3 \\ -28 & -6 & -32 & -31 \end{vmatrix} \xrightarrow{\mathcal{E}_{42}(-4)} \begin{vmatrix} 3 & 7 & 2 & 3 \\ -8 & -3 & -9 & -8 \\ -4 & 3 & -2 & 3 \\ 0 & 6 & 4 & 1 \end{vmatrix} \rightarrow$$

$$\xrightarrow{\mathcal{E}_{23}(-2)} \begin{vmatrix} 3 & 7 & 2 & 3 \\ 0 & -15 & -1 & -20 \\ -4 & 3 & -2 & 3 \\ 0 & 6 & 4 & 1 \end{vmatrix} \xrightarrow[D_3(3)]{D_1(4)} \begin{vmatrix} 12 & 28 & 8 & 12 \\ 0 & -15 & -1 & -20 \\ -12 & 9 & -6 & 9 \\ 0 & 6 & 4 & 1 \end{vmatrix} \xrightarrow{\mathcal{E}_{31}(1)} \begin{vmatrix} 12 & \cancel{28} & \cancel{8} & \cancel{12} \\ 0 & -15 & -1 & -20 \\ 0 & 37 & 2 & 21 \\ 0 & 6 & 4 & 1 \end{vmatrix} \rightarrow$$

$$\rightarrow 12 \begin{vmatrix} -15 & -1 & -20 \\ 37 & -1 & -20 \\ 6 & 4 & 1 \end{vmatrix} = 12\left((15+120-2960) - (120-32+30)\right)$$

$$\xrightarrow{\mathcal{E}_{23}(1)} \Big|$$

4/

$$\begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 0 & 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 5 & 6 & 8 \\ 4 & 9 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 1 & 9 & 5 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 0 & 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 5 & 4 \\ 1 & 6 & 9 \\ 1 & 8 & 2 \end{vmatrix}^{T} \Rightarrow \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 0 & 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 1 & 5 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 1 & 9 & 5 \end{vmatrix}$$

$$A = A^{T}$$

3/

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} \rightarrow \frac{1}{xyz} \begin{vmatrix} xyz & x^2 & x^3 \\ xyz & y^2 & y^3 \\ xyz & z^2 & z^3 \end{vmatrix} \rightarrow$$

$$\rightarrow \frac{1}{xyz} (xyz) \begin{vmatrix} yz & x & x^2 \\ xz & y & y^2 \\ \cancel{xy} & z & z^2 \end{vmatrix}$$

1)

$$\begin{pmatrix} 2 & 4 & 0 \\ 0 & 2 & 1 \\ 3 & 0 & 2 \end{pmatrix} \Rightarrow \begin{vmatrix} 2 & 4 & 0 \\ 0 & 2 & 1 \\ 3 & 0 & 2 \end{vmatrix} = (8+12) - ( \; = 20$$

$$A^{-1} = \frac{1}{|A|} \, Adg^{T}(A) = \frac{1}{20} \begin{pmatrix} 2 & 0 & 3 \\ 4 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{10} & 0 & \frac{3}{20} \\ \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & \frac{1}{20} & \frac{1}{10} \end{pmatrix}$$

$3x + 4y + z + 2t = 3$
$6x + 8y + 2z + 5t = 7$
$9x + 12y + 3z + 10t = 13$

$$\overset{A}{\begin{pmatrix} 3 & 4 & 1 & 2 \\ 6 & 8 & 2 & 5 \\ 9 & 12 & 3 & 10 \end{pmatrix}} \overset{\bar{x}}{\begin{pmatrix} x \\ y \\ t \\ z \end{pmatrix}} = \overset{\bar{b}}{\begin{pmatrix} 3 \\ 7 \\ 13 \end{pmatrix}}$$

$$\left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 6 & 8 & 2 & 5 & 7 \\ 9 & 12 & 3 & 10 & 13 \end{array}\right) \xrightarrow[\substack{\mathcal{E}_{31}(-3) \\ \mathcal{E}_{21}(-2)}]{} \left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 4 & 4 \end{array}\right) \xrightarrow[\mathcal{E}_{32}(-4)]{} \left(\begin{array}{cccc|c} 3 & 4 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} 3x + 4y + z + 2t = 3 \\ t = 1 \end{cases} \Rightarrow r(A) = 2 = rg(A|b) = 2 < 4 = n \ \boxed{\text{indeterminata}}$$

Aldegai aslleclt: $(y, z)$
Gincrriollo aldegaiche: $(x, t)$

$$\bar{S}_o = \bar{S}_p + \bar{S}_H$$

$$\bar{S}_p \Rightarrow \begin{cases} 3x + 4y + z + 2t = 0 \\ t = 0 \end{cases} \Rightarrow \begin{cases} 3x = -4y - z \Rightarrow x = -\frac{4y + z}{3} \end{cases}$$

$$\bar{S}_p = y \begin{pmatrix} -\frac{4y}{3} \\ 1 \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} -\frac{1}{3} \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\bar{S}_H \Rightarrow \begin{cases} 3x + 2t = 3 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3} \\ t = 1 \end{cases}$$

$$\bar{S}_H = \begin{pmatrix} \frac{1}{3} \\ 0 \\ 0 \\ t \end{pmatrix} \Rightarrow \bar{S}_o = \bar{S}_p + \bar{S}_H = y \begin{pmatrix} -\frac{4y}{3} \\ \cancel{1} \\ 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} -\frac{1}{3} \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{3} \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

1.1

a)

$$a \neq 0 \begin{cases} au + v = 1 \\ 4u + av = 2 \end{cases} \Rightarrow \overset{A}{\begin{pmatrix} a & 1 \\ 4 & a \end{pmatrix}} \overset{\bar{x}}{\begin{pmatrix} u \\ v \end{pmatrix}} = \overset{b}{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

Matrize zabaldua:

$$\left(\begin{array}{cc|c} a & 1 & 1 \\ 4 & a & 2 \end{array}\right) \xrightarrow[E_{21}(-4/a)]{} \left(\begin{array}{cc|c} a & 1 & 1 \\ 0 & a-\frac{4}{a} & 2-\frac{4}{a} \end{array}\right) \Rightarrow \left(a - \frac{4}{a}\right)v = 2 - \frac{4}{a} \Rightarrow a = 2$$

$a=2$ izan ezkero sistema bateragarri determinatua izango da, non,

$v=1$

$u=0$

b)

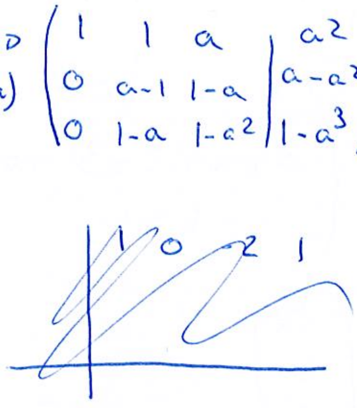
$$\begin{aligned} ax + y + z &= 1 \\ x + ay + z &= a \\ x + y + az &= a^2 \end{aligned} \Rightarrow \quad A\bar{x} = b$$

$$\left(\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & a^2 \end{array}\right) \xrightarrow[P_{21}]{} \left(\begin{array}{ccc|c} 1 & a & 1 & 1 \\ a & 1 & 1 & a \\ 1 & 1 & a & a^2 \end{array}\right) \rightarrow$$

$$\xrightarrow[E_{21}(-a)]{} \left(\begin{array}{ccc|c} 1 & a & 1 & 1 \\ 0 & 1-a^2 & 1-a & 0 \\ 1 & 1 & a & a^2 \end{array}\right) \xrightarrow[E_{31}(-1)]{} \left(\begin{array}{ccc|c} 1 & a & 1 & 1 \\ 0 & 1-a^2 & 1-a & 0 \\ 0 & 1-a & a-1 & a^2-1 \end{array}\right) \xrightarrow[E_{23}(1)]{} \left(\begin{array}{ccc|c} 1 & a+1 & 1 & 1 \\ 0 & 2-a-a^2 & 1-a & 0 \\ 0 & 0 & a-1 & a^2-1 \end{array}\right)$$

$$\xrightarrow[P_{31}]{} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 1 & a & 1 & a \\ a & 1 & 1 & 1 \end{array}\right) \xrightarrow[E_{21}(-1)]{} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 0 & a-1 & 1-a & a-a^2 \\ a & 1 & 1 & 1 \end{array}\right) \xrightarrow[E_{31}(-a)]{} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 0 & a-1 & 1-a & a-a^2 \\ 0 & 1-a & 1-a^2 & 1-a^3 \end{array}\right) \rightarrow$$

$$\xrightarrow[E_{31}(1)]{} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 0 & a-1 & 1-a & a-a^2 \\ 0 & 0 & 2-a-a^2 & 1+a-a^2-a^3 \end{array}\right)$$




$2-a-\cancel{a^2} = 1+a-\cancel{a^2}-a^3 \Rightarrow a^3 - 2a + 1 = 0$

$a - 1 = 0 \Rightarrow a = 1$

$2-a-a^2 = 0 \Rightarrow a = -2$
$\qquad\qquad\qquad\quad a = 1$

$1+a-a^2-a^3=0 \Rightarrow (a-1)(a+1)(a+1) \Rightarrow \cancel{z=(a+1)^2(1-a)}$

$$\begin{array}{r|rrrr} & -1 & -1 & 1 & 1 \\ 1 & & -1 & -2 & -1 \\ \hline & -1 & -2 & -1 & \boxed{0} \\ -1 & & 1 & 1 & \\ \hline & -1 & -1 & \boxed{0} & \\ -1 & & 1 & & \\ \hline & -1 & \boxed{0} & & \end{array}$$

$\Rightarrow (a+2)(a+1)\cdot z = (a-1)(a+1)^2 \Rightarrow z = \frac{\cancel{(a-1)}(a+1)^2}{\cancel{(a-1)}(a+2)} = \frac{(a+1)^2}{(a+2)}$

$(a-1)y+(1-a)z = a-a^2 \Rightarrow (a-1)y + \frac{(1-a)(a+1)^2}{(a+2)} = a(1-a) \Rightarrow$

$\Rightarrow (a-1)y = \frac{a(1-a)(a+2)}{(1-a)(a+1)^2} \Rightarrow y = \frac{a(a+2)}{(a-1)\cancel{(a+1)}(a+1)^2} = \frac{a(a+2)}{(a-1)(a+1)^2}$

$x+y+az = a^2 \Rightarrow x + \frac{a(a+2)}{(a-1)(a+1)^2} + \frac{a(a+1)^2(1-a)}{(a+2)} = a^2 \Rightarrow$

$\Rightarrow x = a^2 - \frac{a(a+2)^2 + a(a+1)^4(1-a)(a-1)}{(a-1)(a+1)^2(a+2)}$

## 1.2

a) $\begin{cases} u+4v+2w=-2 \\ -2u-8v+3w=32 \\ v+w=1 \end{cases} \Rightarrow A\bar{x}=\bar{b}$

$$\left(\begin{array}{ccc|c} 1 & 4 & 2 & -2 \\ -2 & -8 & 3 & 32 \\ 0 & 1 & 1 & 1 \end{array}\right) \xrightarrow{P_{31}} \left(\begin{array}{ccc|c} 1 & 4 & 2 & -2 \\ 0 & 1 & 1 & 1 \\ -2 & -8 & 3 & 32 \end{array}\right) \longrightarrow$$

$$P_{31} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{E_{31}(2)} \left(\begin{array}{ccc|c} 1 & 4 & 2 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 7 & 28 \end{array}\right) \xrightarrow{D_3(1/7)} \left(\begin{array}{ccc|c} 1 & 4 & 2 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$\begin{cases} u+4v+2w=-2 \\ v+w=1 \\ w=4 \end{cases}$ $\begin{cases} u-12+8=-2 \\ v+4=1 \Rightarrow v=-3 \\ w=4 \end{cases}$ $\begin{cases} u=2 \\ v=-3 \\ w=4 \end{cases}$

1.3

a)

$$\begin{cases} \quad v - w = 2 \\ u - v \quad = 2 \\ u \quad - w = 2 \end{cases} \Rightarrow \underset{A\bar{x}=\bar{b}}{} \left(\begin{array}{ccc|c} 0 & 1 & -1 & 2 \\ 1 & -1 & 0 & 2 \\ 1 & 0 & -1 & 2 \end{array}\right) \xrightarrow{P_{21}} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & -1 & 2 \end{array}\right) \rightarrow$$

$$\xrightarrow{\varepsilon_{31}(-1)} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & -1 & 0 \end{array}\right) \xrightarrow{\varepsilon_{32}(-1)} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & -2 \end{array}\right)$$

Sistema bateraezina da, beraz ez du soluziorik.

b)

$$\begin{cases} \quad v - w = 0 \\ u - v \quad = 0 \\ u \quad - w = 0 \end{cases} \Rightarrow A\bar{x} = \bar{b} \Rightarrow \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Matrize zabaldua:

$$\left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right) \xrightarrow{P_{21}} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right) \xrightarrow{\varepsilon_{31}(-1)} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \xrightarrow{\varepsilon_{32}(-1)} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$r(A) = 2 \quad n = 3 \quad r(A) < n \Rightarrow$ Ondorioz, Sistema bateragarri indeterminatua da eta infinitu soluzio ditu.

Aldagai askeak: $w$

$$\begin{cases} u - v = 0 \\ \quad v - w = 0 \end{cases} \Rightarrow \begin{cases} u - v = 0 \\ \quad v = w \end{cases} \quad \begin{cases} u = w \\ v = w \end{cases}$$

c)

$$\begin{cases} \phantom{u+{}} v + w = 1 \\ u + v \phantom{{}+w} = 1 \\ u \phantom{{}+v} + w = 1 \end{cases} \Longrightarrow A\bar{x} = \bar{b} \quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Matrize Zabaldua:

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right) \xrightarrow[P_{21}]{} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right) \xrightarrow[\mathcal{E}_{31}(-1)]{} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow[\mathcal{E}_{32}(1)]{} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{array}\right)$$

$$\begin{cases} u + v = 1 \\ v + w = 1 \\ 2w = 1 \end{cases} \quad \begin{cases} u + \frac{1}{2} = 1 \Rightarrow u = \frac{1}{2} \\ v + \frac{1}{2} = 1 \Rightarrow v = \frac{1}{2} \\ w = \frac{1}{2} \end{cases}$$

Sistema Bateragarri determinatua: $r(A) = r(A|b) = 3 = n$

2.2

a)

$$\begin{cases} 2u - 3v \phantom{{}+w} = 3 \\ 4u - 5v + w = 7 \\ 2u - v - 3w = 5 \end{cases} \Longrightarrow A\bar{x} = \bar{b} \quad \overset{A}{\begin{pmatrix} 2 & -3 & 0 \\ 4 & -5 & 1 \\ 2 & -1 & -3 \end{pmatrix}} \overset{\bar{x}}{\begin{pmatrix} u \\ v \\ w \end{pmatrix}} = \overset{\bar{b}}{\begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix}}$$

Matrize Zabaldua:

$$\left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 4 & -5 & 1 & 7 \\ 2 & -1 & -3 & 5 \end{array}\right) \xrightarrow[P_{32}]{} \left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 2 & -1 & -3 & 5 \\ 4 & -5 & 1 & 7 \end{array}\right) \longrightarrow$$

$$\xrightarrow[\substack{\mathcal{E}_{21}(-1) \\ \mathcal{E}_{31}(-2)}]{} \left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 0 & 2 & -3 & 2 \\ 0 & 1 & 1 & 1 \end{array}\right) \xrightarrow[P_{32}]{} \left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 2 & -3 & 1 \end{array}\right) \xrightarrow[\mathcal{E}_{32}(-2)]{} \left(\begin{array}{ccc|c} 2 & -3 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -5 & -3 \end{array}\right)$$

$r(A) = r(A|b) = 3 = n \Rightarrow$ Sistema Bateragarri determinatua.

$$\begin{cases} 2u - 3v = 3 \\ v + w = 2 \\ -5w = -3 \end{cases} \qquad \begin{cases} 2u - \frac{21}{5} = 3 \Rightarrow 2u = \frac{36}{5} \Rightarrow u = \frac{18}{5} \\ v + \frac{3}{5} = 2 \Rightarrow v = \frac{7}{5} \\ w = \frac{3}{5} \end{cases}$$

f)

$$\begin{cases} x - y + z = 3 \\ 5x + 2y - z = 5 \\ -3x - 4y + 3z = 1 \end{cases} \Rightarrow A\bar{x} = b \begin{pmatrix} 1 & -1 & 1 \\ 5 & 2 & -1 \\ 3 & -4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$$

Matrize zabaldua:

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 5 & 2 & -1 & 5 \\ -3 & -4 & 3 & 1 \end{array}\right) \xrightarrow[\varepsilon_{31}(+3)]{\varepsilon_{21}(-5)} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 7 & -6 & -10 \\ 0 & -7 & 6 & \cancel{18}\ 10 \end{array}\right) \xrightarrow{\varepsilon_{32}(-1)} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 7 & -6 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$r(A) = r(A|b) = 2$, $n = 3 \Rightarrow r(A|b) < n \Rightarrow 2 < 3$

Sistema bateragarri indeterminatua.

Sistema homogeneoaren soluzioa:

$S_h$;

$$\begin{cases} x - y + z = 0 \\ 7y - 6z = 0 \end{cases} \qquad \begin{cases} x - \frac{6z}{7} + z = 0 \Rightarrow x = \frac{-z}{7} \\ 7y = 6z \Rightarrow y = \frac{6z}{7} \end{cases}$$

$$S_h = z \begin{pmatrix} -\frac{1}{7} \\ \frac{6}{7} \\ 1 \end{pmatrix}$$

Sistema partikularraren soluzioa

$$\begin{cases} x - y = 3 \\ 7y = -10 \end{cases} \qquad \begin{cases} x + \frac{10}{7} = 3 \Rightarrow x = \frac{11}{7} \\ y = -\frac{10}{7} \end{cases} \qquad S_p = \begin{pmatrix} \frac{11}{7} \\ -\frac{10}{7} \\ 0 \end{pmatrix} \qquad S_o = S_p + S_h = \begin{pmatrix} \frac{11}{7} \\ \frac{-10}{7} \\ 0 \end{pmatrix} + z \begin{pmatrix} \frac{-1}{7} \\ \frac{6}{7} \\ 1 \end{pmatrix}$$

## 3.1

$$A\bar{x}=\bar{b}\quad\begin{pmatrix}2&3&3\\0&5&7\\6&9&8\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}2\\2\\5\end{pmatrix}\Rightarrow\begin{pmatrix}2&3&3\\0&5&7\\6&9&8\end{pmatrix}\xrightarrow[E_{31}(-3)]{}\begin{pmatrix}2&3&3\\0&5&7\\0&0&-1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0\\0&1&0\\-3&0&1\end{pmatrix}$$

$$A=LU=\begin{pmatrix}1&0&0\\0&1&0\\+3&0&1\end{pmatrix}\begin{pmatrix}2&3&3\\0&5&7\\0&0&-1\end{pmatrix}$$

$$\bar{b}=\underset{E_{31}(-3)}{L}\cdot\bar{b}=\begin{pmatrix}1&0&0\\0&1&0\\-3&0&1\end{pmatrix}\cancel{\begin{pmatrix}2&3&3\\0&5&7\\6&9&8\end{pmatrix}}\begin{pmatrix}2\\2\\5\end{pmatrix}=\begin{pmatrix}2\\2\\-1\end{pmatrix}$$

## 3.8

a)

$$A=\begin{pmatrix}1&1&1\\2&-5&13\\-3&4&1\end{pmatrix}\quad\bar{b}=\begin{pmatrix}3\\10\\2\end{pmatrix}$$

$A=LU$ lortu,

Matrize bidez

$$\begin{pmatrix}1&1&1\\2&-5&13\\-3&4&1\end{pmatrix}\xrightarrow[E_{21}(-2)\ E_{31}(3)]{}\begin{pmatrix}1&1&1\\0&-7&11\\0&7&4\end{pmatrix}\xrightarrow[E_{32}(1)]{}\begin{pmatrix}1&1&1\\0&-7&11\\0&0&15\end{pmatrix}=U$$

$$E_{21}(-2)\cdot E_{31}(3)\cdot E_{32}(1)\Rightarrow(E_{21}(-2)\cdot E_{31}(3)\cdot E_{32}(1))^{-1}\Rightarrow(E_{21}(-2))^{-1}\cdot(E_{31}(3))^{-1}\cdot(E_{32}(1))^{-1}\Rightarrow$$

$$\Rightarrow E_{21}(2)\cdot E_{31}(-3)\cdot E_{32}(-1)=\begin{pmatrix}1&0&0\\2&1&0\\-3&-1&1\end{pmatrix}=L\Rightarrow\underset{A}{\begin{pmatrix}1&1&1\\2&-5&13\\-3&4&1\end{pmatrix}}=\underset{L}{\begin{pmatrix}1&0&0\\2&1&0\\-3&-1&1\end{pmatrix}}\underset{U}{\begin{pmatrix}1&1&1\\0&-7&11\\0&0&15\end{pmatrix}}$$

$$A\bar{x}=\bar{b} \Rightarrow L^{-1}\cdot\bar{b} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 10 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 21 \end{pmatrix} = \bar{S}$$

$$D_n(P,K) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots K & \\ & & & \ddots 1 \end{pmatrix}$$ honen bidez, edozein matrizeko biderkatuko dugu erp errenkada

~~P~~ errenkada
K -rekin,

$$\mathcal{E}_{ij}(K) = \begin{pmatrix} 1 & & & \\ i \dots & K^{i} & & \\ & 1 & & \\ & \vdots & \ddots & \\ & & & 1 \end{pmatrix}.$$

$$P_n\, i\, j = \begin{pmatrix} i{:}00 & 010 \\ j{:}00 & 001 \end{pmatrix}$$ trukatu errenkadak

$$\begin{vmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t^3 & t^2 \\ t^2 & t & 1 & t^3 \\ t^3 & t^2 & t & 1 \end{vmatrix} \underset{4.e-3e(t)}{=} \begin{vmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t^3 & t^2 \\ t^2 & t & 1 & t^3 \\ 0 & 0 & 0 & -t^4 \end{vmatrix} \underset{3e-2e(t)}{=} \begin{vmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t^3 & t^2 \\ 0 & 0 & 1-t^4 & 0 \\ 0 & 0 & 0 & 1-t^4 \end{vmatrix} =$$

$$\underset{2e-1.e(t)}{=} \begin{vmatrix} 1 & t & t^2 & t^3 \\ 0 & 1-t & 0 & t^2-t^4 \\ 0 & 0 & 1-t^4 & 0 \\ 0 & 0 & 0 & 1-t^4 \end{vmatrix} = (1-t^2)(1-t^4)^2$$

# Determinantaall



5

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 \\ x_1^3 & x_2^3 & x_3^3 & x_4^3 \end{vmatrix} = (x_2-x_1)(x_3-x_1)(x_4-x_1)(x_3-x_2)(x_4-x_2)(x_4-x_3)$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 \\ x_1^3 & x_2^3 & x_3^3 & x_4^3 \end{vmatrix} \xrightarrow[4e-3e(x_1)\to 4e]{} \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 \\ 0 & x_2^3-x_2^2x_1 & x_3^3-x_3^2x_1 & x_4^3-x_4^2x_1 \end{vmatrix} \longrightarrow$$

$$\xrightarrow[3e-2e(x_1)\to 3e]{} \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ 0 & x_2^2-x_2 x_1 & x_3^2-x_3 x_1 & x_4^2-x_4x_1 \\ 0 & x_2^2(x_2-x_1) & x_3^2(x_3-x_1) & x_4^2(x_4-x_1) \end{vmatrix} \longrightarrow$$

$$\xrightarrow[2e-1e(x_1)\to 2e]{} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x_2-x_1 & x_3-x_1 & x_4-x_1 \\ 0 & x_2(x_2-x_1) & x_3(x_3-x_1) & x_4(x_4-x_1) \\ 0 & x_2^2(x_2-x_1) & x_3^2(x_3-x_1) & x_4^2(x_4-x_1) \end{vmatrix} = \begin{vmatrix} x_2-x_1 & x_3-x_1 & x_4-x_1 \\ x_2(x_2-x_1) & x_3(x_3-x_1) & x_4(x_4-x_1) \\ x_2^2(x_2-x_1) & x_3^2(x_3-x_1) & x_4^2(x_4-x_1) \end{vmatrix} =$$

$$= (x_2-x_1)(x_3-x_1)(x_4-x_1) \begin{vmatrix} 1 & 1 & 1 \\ x_2 & x_3 & x_4 \\ x_2^2 & x_3^2 & x_4^2 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 0 & x_3-x_2 & x_4-x_2 \\ 0 & x_3^2-x_3x_2 & x_4^2-x_4x_2 \end{vmatrix} =$$

$$= (x_2-x_1)(x_3-x_1)(x_4-x_1)(x_3-x_2)(x_4-x_2) \begin{vmatrix} 1 & 1 \\ x_3 & x_4 \end{vmatrix}$$

$$= (x_2-x_1)(x_3-x_1)(x_4-x_1)(x_3-x_2)(x_4-x_2)(x_4-x_3)$$

# Determinanteak

(1)

$$\begin{vmatrix} 1 & \cancel{4}2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = 4\cdot 27 + 18 + 24 - (12 + 54 + 72) = \boxed{12}$$

(2)

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{vmatrix} \underset{\substack{5e-4e \\ 4e-3e \\ 3e-2e \\ 2e-1e}}{=} \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & -4 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \end{vmatrix} \underset{\substack{4e-5e \\ 3e-4e \\ 2e-3e \\ 1e-2e}}{=} \begin{vmatrix} 0 & 1 & 2 & 3 & 9 \\ 0 & 0 & 0 & 5 & -5 \\ 0 & 0 & 5 & -5 & 0 \\ 0 & 5 & -5 & 0 & 0 \\ 1 & -4 & 1 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 2 & 3 & 9 \\ 0 & 0 & 5 & -5 \\ 0 & 5 & -5 & 0 \\ 5 & -5 & 0 & 0 \end{vmatrix} \underset{e_4 - 5e_1}{=} \begin{vmatrix} 1 & 2 & 3 & 9 \\ 0 & 0 & 5 & -5 \\ 0 & 5 & -5 & 0 \\ 0 & -15 & -15 & -45 \end{vmatrix} = -15 \begin{vmatrix} 0 & 5 & -5 \\ 5 & -5 & 0 \\ -1 & -1 & 3 \end{vmatrix} = -15\cdot 5\cdot 5 \begin{vmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & -1 & 3 \end{vmatrix} =$$

$$= -15\cdot 5\cdot 5\cdot (0 + 0 + 1 - (-1 + 3 \, 0)) = -15\cdot 5\cdot 5\cdot 5 = 1875$$

(3)

$$\begin{vmatrix} 0 & 3 & 7 & 2 & 3 \\ 1 & 4 & 1 & 4 & 4 \\ 3 & 4 & 0 & 3 & 4 \\ 9 & 8 & 3 & 7 & 9 \\ 8 & 4 & 2 & 0 & 1 \end{vmatrix} \xrightarrow{5z-2z} \begin{vmatrix} 0 & 2 & 7 & 2 & 0 \\ 1 & 4 & 1 & 4 & 0 \\ 3 & 4 & 0 & 3 & 0 \\ 9 & 8 & 3 & 7 & 1 \\ 8 & 4 & 2 & 0 & -3 \end{vmatrix} \xrightarrow{5z+3(e_4)} \begin{vmatrix} 0 & 2 & 7 & 2 & 0 \\ 1 & 4 & 1 & 4 & 0 \\ 3 & 4 & 0 & 3 & 0 \\ \cancel{9} & \cancel{8} & \cancel{3} & \cancel{7} & 1 \\ 35 & 28 & 11 & 21 & 0 \end{vmatrix} =$$

$$= -1 \begin{vmatrix} 0 & 2 & 7 & 2 \\ 1 & 4 & 1 & 4 \\ 3 & 4 & 0 & 3 \\ 35 & 28 & 11 & 21 \end{vmatrix} \underset{z_4 - z_2}{=} \begin{vmatrix} 0 & 2 & 7 & 0 \\ 1 & 4 & 1 & 0 \\ 3 & 4 & 0 & -1 \\ 35 & 28 & 11 & -7 \end{vmatrix} \underset{e_4 - 7e_3}{=} \begin{vmatrix} 0 & 2 & 7 & 0 \\ 1 & 4 & 1 & 0 \\ \cancel{3} & \cancel{4} & \cancel{0} & -1 \\ 14 & 0 & 11 & 0 \end{vmatrix} = -1 \begin{vmatrix} 0 & 2 & 7 \\ 1 & 4 & 1 \\ 14 & 0 & 11 \end{vmatrix} =$$

$$= 342$$

(2)

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & b+a \end{vmatrix} = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = \boxed{0}$$

Berdinak dira

(3)

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = \begin{vmatrix} yz & x & x^2 \\ xz & y & y^2 \\ yx & z & z^2 \end{vmatrix}$$

$$\begin{vmatrix} yz & x & x^2 \\ xz & y & y^2 \\ yx & z & z^2 \end{vmatrix} \xrightarrow{\frac{1}{x}} \begin{vmatrix} yzx & x^2 & x^3 \\ xz\cancel{y} & y & y^2 \\ yx\cancel{z} & z & z^2 \end{vmatrix} \rightarrow \cancel{(xyz)\begin{vmatrix} 1 & x & x^2 \\ 1 & x & y^2 \\ 1 & z & z^2 \end{vmatrix}}$$

$$\rightarrow \frac{1}{x}\cdot\frac{1}{y}\cdot\frac{1}{z}\begin{vmatrix} xyz & x^2 & x^3 \\ xyz & y^2 & y^3 \\ xyz & z^2 & z^3 \end{vmatrix} \rightarrow \frac{x}{x}\cdot\frac{y}{y}\cdot\frac{z}{z}\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} \rightarrow \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

$|A| = |A|^t$ dela kontuan hartuta, betetzen da berdintza.

(4)

$$\begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 0 & 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 5 & 6 & 8 \\ 4 & 9 & 2 \end{vmatrix} = \cancel{\begin{vmatrix} 2 & 7 & 10 \\ 6 & 14 & 10 \\ 4 & 13 & 3 \end{vmatrix} = 2\cdot\begin{vmatrix} 2 & 7 & 10 \\ 3 & 7 & 5 \\ 4 & 13 & 3 \end{vmatrix}}$$

$$= \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 0 & 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 5 & 4 \\ 1 & 6 & 9 \\ 1 & 8 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 0 & 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 1 & 5 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 1 & 9 & 5 \end{vmatrix}$$

$|C| = |B| + |A|$

13

b)

$$\begin{aligned} v - w &= 0 \\ u - v &= 0 \\ u - w &= 0 \end{aligned} \rightarrow \left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right) \Rightarrow A\bar{x} = \bar{O}$$

Ekuazio sistema Homogeneoa

Matrize zabaldua

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right) \overset{E_{31}(-1)}{\Rightarrow} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \overset{E_{32}(-1)}{\Rightarrow} \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\underset{A}{\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix}} \underset{\bar{x}}{\begin{pmatrix} u \\ v \\ w \end{pmatrix}} = \underset{\bar{O}}{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$

$r(A) = r(Ab) = 2 < 3 = n$

Sistema Bateragarri determinatua baldin $r = n$

Sistema Bateragarri indeterminatua baldin $r < n$

$$\begin{array}{c} \begin{array}{cccc} & U & & \bar{c} \end{array} \\ \left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \end{array} \quad U\bar{x} = \bar{o}$$

$$\begin{cases} 1u - 1v = 0 \Rightarrow u = v \Rightarrow u = v = w \\ w - w = 0 \Rightarrow v = w \end{cases}$$

aldagai askeak

$$\bar{S} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = + w \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

1.1

b)

$$\left.\begin{aligned} a\cancel{x} + y + z &= 1 \\ x + ay + z &= a \\ x + y + az &= a^2 \end{aligned}\right\} \Rightarrow \left(\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & a^2 \end{array}\right) \xRightarrow{\mathcal{E}_{23}(-1)} \cancel{\left(\begin{array}{ccc|c} a & 1 & 1 & 1 \\ 0 & a-1 & 1-a & a-a^2 \\ 1 & 1 & a & a \end{array}\right)}$$

$$\xRightarrow{P_{13}} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 1 & a & 1 & a \\ a & 1 & 1 & 1 \end{array}\right) \xRightarrow{\mathcal{E}_{21}(-1)} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 0 & a-1 & 1-a & a-a^2 \\ a & 1 & 1 & 1 \end{array}\right) \xRightarrow{\mathcal{E}_{31}(-a)} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 0 & a-1 & 1-a & a-a^2 \\ 0 & 1-a & 1-a & 1-a^3 \end{array}\right) \Rightarrow$$

$$\xRightarrow{\mathcal{E}_{32}(1)} \left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 0 & a-1 & 1-a & a-a^2 \\ 0 & 0 & 2-a-a^2 & -a^3-a^2+a+1 \end{array}\right) \xRightarrow{D_3\left(\frac{1}{a}\right)} \cancel{\left(\begin{array}{ccc|c} 1 & 1 & a & a^2 \\ 0 & a-1 & -a & a-a^2 \\ 0 & 0 & -2 & -a^2-a+2a \end{array}\right)}$$

$2-a-a^2 = -a^3-a^2+a+1 \Rightarrow a^3-2a+1=0$

$1,\ a-1,\ 2-a-a^2$

$a-1=0 \Rightarrow a=1$

$2-a-a^2=0 \Rightarrow \dfrac{1\pm\sqrt{1^2-4\cdot(-1)\cdot 2}}{-2} = \dfrac{1\pm 3}{-2} = \begin{cases} -2 \\ 1 \end{cases}$

$1+a-a^2-a^3=0 \Rightarrow (-1)(a+2)(a-1)z = (a+1)^2(1-a) \Rightarrow z = \dfrac{(a+1)^2}{a+2}$

$-a^3-a^2+a+1 \quad \underline{|\,a-1}$

$-a^2 \quad 2a \quad 0 \quad 1-$

| | | | | |
|---|---|---|---|---|
| | -1 | -1 | 1 | 1 |
| 1 | | -1 | -2 | -1 |
| | -1 | -2 | -1 | $\boxed{0}$ |
| -1 | | | 1 | 1 |
| | -1 | -1 | $\boxed{0}$ | |
| -1 | | 1 | | |
| | -1 | $\boxed{0}$ | | |

$(a-1)y + (1-a)z = a-a^2$

$(a-1)y + \dfrac{(1-a)(1+a)^2}{a+2} = a-a^2 \Rightarrow$

$\Rightarrow y = \dfrac{a(\cancel{a^2+1-a})(a+2)}{(a-1)(1-a)(1+a)^2} \Rightarrow y = \dfrac{a}{4} - z$

$2-a-a^2 \neq 0 \Rightarrow a+2 \neq 0$ etc $a-1 \neq 0$

$\begin{cases} 2-1-1^2=0 \\ 2-(-2)-(2)^2=0 \end{cases}$

3.7

$$\begin{aligned} u+v+2w &= 2\\ 2u+3v-w &= 5\\ 3u+4v+w &= c \end{aligned} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2\\ 2 & 3 & -1 & 5\\ 3 & 4 & 1 & c \end{array}\right) \xrightarrow[\substack{E_{21}(-2)\\ E_{31}(-3)}]{\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2\\ 0 & 1 & -5 & 1\\ 0 & 1 & -5 & c-6 \end{array}\right) \xrightarrow{\cancel{E_{32}(-1)}}$$

$$\cancel{\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2\\ 0 & 1 & -3 & 1\\ 0 & 0 & -2 & c-7 \end{array}\right) \Rightarrow \left.\begin{aligned} u+v+2w &= 2\\ v-3w &= 1\\ -2w &= c-7 \end{aligned}\right\} \begin{aligned} &u+\frac{23-3c}{2}+7-c=2 \Rightarrow u+\frac{23-3c}{2}\\ &v-\left(\frac{21-3c}{2}\right)=1 \Rightarrow v=\frac{23-3c}{2}\\ &w=\frac{7-c}{2} \end{aligned}}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2\\ 0 & 1 & -5 & 1\\ 0 & 1 & -5 & c-6 \end{array}\right) \xrightarrow{E_{32}(-1)} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 2\\ 0 & 1 & -5 & 1\\ 0 & 0 & 0 & c-7 \end{array}\right) \quad 0=c-7 \Rightarrow \boxed{c=7}$$

$r(A)=2$

$c-7\neq 0 \rightarrow$ kein $r(A\bar b)=3 \neq r(A\bar b)=2 \Rightarrow \boxed{c=7}$

↯

$u+v+2w=2 \Rightarrow u+1+5w+2w=2 \Rightarrow u=1-7w$

$v-5w=1 \Rightarrow v=1+5w$

$$\left.\begin{aligned} u+v+2w&=0\\ v&=5w \end{aligned}\right\} \begin{aligned} u&=-7w\\ v&=5w \end{aligned} \qquad \bar S_h = w\begin{pmatrix}7\\5\\1\end{pmatrix}$$

$$\left.\begin{aligned} u+v&=2\\ v&=1 \end{aligned}\right\} \begin{aligned} u&=1\\ v&=1 \end{aligned} \qquad \bar S_p = \begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$\bar S_0 = \bar S_h + \bar S_p = \begin{pmatrix}1\\1\\0\end{pmatrix} + w\begin{pmatrix}7\\5\\1\end{pmatrix}$$

3.8

$$A=\begin{pmatrix}1&2&0&3\\2&4&0&7\end{pmatrix} = \begin{pmatrix}b_1\\b_2\end{pmatrix} \Rightarrow \begin{pmatrix}1&2&0&3\\2&4&0&7\end{pmatrix} \xrightarrow{E_{21}(-2)} \begin{pmatrix}1&2&0&3\\0&0&0&1\end{pmatrix}$$

$$\left.\begin{aligned} x+2y+t&=b_1\\ t&=b_2-2b_1 \end{aligned}\right\} \quad x+2y+b_2-2b_1=b_1 \Rightarrow \cancel{x+2y=b_1-b_2} \quad x+2y+b_2-2b_1=b_1$$

$$x+2y=b_2+3b_1$$

3.3

$$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} \xrightarrow{E_{31}(-1)} \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Sistema homogeneoaren soluzioa

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\left.\begin{aligned} x + 2y + t &= 0 \\ y + z &= 0 \end{aligned}\right\}$$

$x - 2z + t = 0 \Rightarrow x = 2z - t$

$y = -z$

$A = L \cdot U \Rightarrow A = E_{31}(1) \cdot U$

$x = 2z - t$

$y = -z$

$$S_H = z\begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

1)

$$\left.\begin{array}{l} x - y + z = 3 \\ 5x + 2y - z = 5 \\ -3x - 4y + 3z = 1 \end{array}\right\} \Rightarrow \overset{A}{\begin{pmatrix} 1 & -1 & 1 \\ 5 & 2 & -1 \\ -3 & -4 & 3 \end{pmatrix}} \overset{\bar{x}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} \overset{=}{=} \overset{b}{\begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}}$$

matrize zabaldua

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 5 & 2 & -1 & 5 \\ -3 & -4 & 3 & 1 \end{array}\right) \mathcal{E}_{21}(-5) \left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 7 & -6 & -10 \\ -3 & -4 & 3 & 1 \end{array}\right) \mathcal{E}_{31}(3) \left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 7 & -6 & -10 \\ 0 & -7 & 6 & \cancel{8} \; 10 \end{array}\right) \mathcal{E}_{32}(1)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 0 & 7 & -6 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left.\begin{array}{l} x - y + z = 3 \\ \cancel{8}y - 6z = -10 \end{array}\right\} \quad \left.\begin{array}{l} x - y + z = 3 \\ y = \cancel{10}\ \dfrac{6y - 10}{\cancel{7}} \end{array}\right\} \quad x - \frac{6z - 10}{\cancel{7}} + z = 3 \Rightarrow x - \frac{6z}{7} + \frac{10}{7} + \frac{7z}{7} = \frac{21}{7} \Rightarrow$$

$$y = \frac{6y - 10}{7}$$

$$x = -\frac{8}{7} + \frac{11}{7} \Rightarrow x = \frac{11 - z}{\cancel{7}}$$

$$A = LU \begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -3 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 1 \\ 0 & 7 & -6 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 5 & 2 & -1 \\ -3 & -4 & 3 \end{pmatrix}$$

Sistema homogeneoaren soluzioa $U\bar{x} = \bar{0}$

$x - y + z = 0$

$7y - 6z = 0$

x, y aldagaiak oinarrizko aldagaiak

z aldagai atzagai askea

$y = \dfrac{6z}{7}$

$x = -\dfrac{1}{7} z$

$$S_H = \begin{pmatrix} -\frac{1}{7} \\ \frac{6}{7} \\ 1 \end{pmatrix}$$

$$S_p = \begin{cases} x - y = 3 & \Rightarrow x = -\dfrac{11}{7} \\ 7y = -10 & \Rightarrow y = -\dfrac{10}{7} \end{cases}$$

$$\bar{S}_0 = \bar{S}_H + \bar{S}_p = \begin{pmatrix} \frac{11}{7} \\ -\frac{10}{7} \\ 0 \end{pmatrix} + z \begin{pmatrix} -\frac{1}{7} \\ \frac{6}{7} \\ 1 \end{pmatrix}$$

c)

$$\left.\begin{aligned} u+v+w+t&=0\\ u+v+w-t&=+4\\ u+v-w+t&=-4\\ u-v+w+t&=2 \end{aligned}\right\} \Rightarrow \overset{A}{\begin{pmatrix} 1&1&1&1\\ 1&1&1&-1\\ 1&1&-1&1\\ 1&-1&1&1 \end{pmatrix}} \overset{\bar{x}}{\begin{pmatrix} u\\ v\\ w\\ t \end{pmatrix}} = \overset{\bar{b}}{\begin{pmatrix} 0\\ 4\\ -4\\ 2 \end{pmatrix}}$$

$$\left(\begin{array}{cccc|c} 1&1&1&1&0\\ 1&1&1&-1&4\\ 1&1&-1&1&-4\\ 1&-1&1&1&2 \end{array}\right) \begin{array}{l} \mathcal{E}_{31}(-1)\\ \mathcal{E}_{21}(-1)\\ \mathcal{E}_{41}(-1) \end{array} \left(\begin{array}{cccc|c} 1&1&1&1&0\\ 0&0&0&-2&4\\ 0&0&-2&0&-4\\ 0&-2&0&0&2 \end{array}\right) \Rightarrow P_{24} \left(\begin{array}{cccc|c} 1&1&1&1&0\\ 0&-2&0&0&2\\ 0&0&-2&0&-4\\ 0&0&0&-2&4 \end{array}\right)$$

$\cancel{-2t=4 \Rightarrow t=-2}$

$\cancel{-2w=-4 \Rightarrow w=2}$

$-2t=4 \Rightarrow t=-2$

$-2w=-4 \Rightarrow w=2$

$-2v=2 \Rightarrow v=-1$

$u=1$

PLU faktorizazioa.

$$P_{24}\cdot A = \left(\mathcal{E}_{21}(-1)\cdot\mathcal{E}_{31}(-1)\cdot\mathcal{E}_{41}(-1)\right)\cdot U$$

$$L = \left(\mathcal{E}_{21}(1)\cdot\mathcal{E}_{31}(1)\cdot\mathcal{E}_{41}(1)\right) \Rightarrow \begin{pmatrix} 1&0&0&0\\ 1&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 1&0&1&0\\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 1&0&0&1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1&0&0&0\\ 1&1&0&0\\ 1&0&1&0\\ 1&0&0&1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1&1&1&1\\ 0&-2&0&0\\ 0&0&-2&0\\ 0&0&0&-2 \end{pmatrix}$$

$$L\cdot U = \begin{pmatrix} 1&0&0&0\\ 1&1&0&0\\ 1&0&1&0\\ 1&0&0&1 \end{pmatrix} \begin{pmatrix} 1&1&1&1\\ 0&-2&0&0\\ 0&0&-2&0\\ 0&0&0&-2 \end{pmatrix} = \begin{vmatrix} 1&1&1&1\\ 1&-1&1&1\\ 1&1&-1&1\\ 1&1&1&-1 \end{vmatrix}$$

~~3.5~~

$$\cancel{\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} =}$$

3.6

$$U\bar{x} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$$

$$U\bar{x} = \bar{0}$$

Oinarrizko aldagaiak: $x_1, x_3$

Aldagai askeak: $x_2, x_4$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \Rightarrow x_1 - 6x_4 + 4x_4 + 2x_2 = 0 \Rightarrow x_1 = 2x_4 - 2x_2$$

$$x_3 + 2x_4 = 0 \Rightarrow x_3 = -2x_4$$

$$\bar{S}_H = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = x_4\begin{pmatrix} 2 \\ 0 \\ -2 \\ 1 \end{pmatrix} + x_2\begin{pmatrix} \cancel{1}\,-2 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$U\bar{x} = \bar{c}$$

$$\left.\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= a \\ x_3 + 2x_4 &= b \end{aligned}\right\} \quad \left.\begin{aligned} x_1 + 3x_3 &= a \\ x_3 &= b \end{aligned}\right\} \quad \begin{aligned} x_1 &= a - 3b \\ x_3 &= b \end{aligned}$$

$$\bar{S}_p = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} a-3b \\ 0 \\ b \\ 0 \end{pmatrix}$$

$$\bar{S} = \bar{S}_H + \bar{S}_p = x_4\begin{pmatrix} 2 \\ 0 \\ -2 \\ 1 \end{pmatrix} + x_2\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} a-3b \\ 0 \\ b \\ 0 \end{pmatrix} =$$

# 3. Ekuazio Sistemen ebazpenak

## 3.1

ondo

$$A\bar{x}=\bar{b} \Rightarrow \begin{pmatrix}2&3&3\\0&5&7\\6&9&8\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}2\\2\\5\end{pmatrix} \Rightarrow \begin{pmatrix}2&3&3\\0&5&7\\6&9&8\end{pmatrix} \xrightarrow{E_{31}(-3)} \begin{pmatrix}2&3&3\\0&5&7\\0&0&-1\end{pmatrix}$$

$$\cancel{\begin{pmatrix}1&0&0\\0&1&0\\-3&0&1\end{pmatrix}}$$

$$E_{31}(-3)A=U \Rightarrow A=E_{31}(3)U \Rightarrow A=\begin{pmatrix}1&0&0\\0&1&0\\3&0&1\end{pmatrix}\begin{pmatrix}2&3&3\\0&5&7\\0&0&-1\end{pmatrix}$$

$$A\bar{x}=\bar{b} \sim U\bar{x}=\bar{c} \Rightarrow \underbrace{E_{31}(-3)\cdot A}_{U}\,\bar{x}=\underbrace{E_{31}(-3)\bar{b}}_{\bar{c}}$$

$$\overset{E_{31}(-3)}{\begin{pmatrix}1&0&0\\0&1&0\\-3&0&1\end{pmatrix}}\overset{\bar{b}}{\begin{pmatrix}2\\2\\5\end{pmatrix}}=\overset{\bar{c}}{\begin{pmatrix}2\\2\\-1\end{pmatrix}} \Rightarrow \bar{s}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

## 3.2

$$LU\bar{x}=\begin{pmatrix}1&0&0\\1&1&0\\1&0&1\end{pmatrix}\begin{pmatrix}2&4&4\\0&1&2\\0&0&1\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}2\\0\\2\end{pmatrix}=\bar{b}$$

$$\begin{pmatrix}1&0&0\\-1&1&0\\-1&0&1\end{pmatrix}\begin{pmatrix}2\\0\\2\end{pmatrix}=\begin{pmatrix}2\\-2\\0\end{pmatrix}=\bar{c}$$

$$LU\bar{x}=\bar{b} \sim U\bar{x}=\bar{c} \Rightarrow \begin{pmatrix}2&4&4\\0&1&2\\0&0&1\end{pmatrix}\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}2\\-2\\0\end{pmatrix} \Rightarrow \begin{pmatrix}5\\-2\\0\end{pmatrix}$$

# DETERMINANTEAK. ARIKETAK.

1. Kalkula itzazu ondoko determinanteen balioak:

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \\ 4 & 5 & 1 & 2 & 3 \\ 5 & 1 & 2 & 3 & 4 \end{vmatrix} \quad \begin{vmatrix} 0 & 3 & 7 & 2 & 3 \\ 1 & 4 & 1 & 4 & 4 \\ 3 & 4 & 0 & 3 & 4 \\ 9 & 8 & 3 & 7 & 9 \\ 8 & 4 & 2 & 0 & 1 \end{vmatrix}$$

2. Frogatu ondoko berdintza determinanteen propietateak erabiliz:

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

3. Frogatu ondoko berdintza determinanteen propietateak erabiliz:

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = \begin{vmatrix} yz & x & x^2 \\ xz & y & y^2 \\ xy & z & z^2 \end{vmatrix}$$

4. Frogatu ondoko berdintza determinanteak kalkulatu gabe:

$$\begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 0 & 4 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 5 & 6 & 8 \\ 4 & 9 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 6 & 9 \\ 1 & 8 & 2 \\ 1 & 9 & 5 \end{vmatrix}$$

5. Frogatu ondoko berdintza:

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 \\ x_1^3 & x_2^3 & x_3^3 & x_4^3 \end{vmatrix} = (x_2-x_1)(x_3-x_1)(x_4-x_1)(x_3-x_2)(x_4-x_2)(x_4-x_3)$$

# Matrizeak. Determinanteak

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \underset{+}{Sig}(\alpha_1)\, a_{11} \cdot a_{22} + \underset{-}{Sig}(\alpha_2)\, a_{12}\, a_{21}$$

$$\{\overset{\alpha_1}{(1,2)}, \overset{\alpha_2}{(2,1)}\}$$

$$\begin{cases} 1 < 2 \quad I(\alpha_1) = 0 \\ \alpha_1(1) = 1 < \alpha(2) = 2 \end{cases}$$

$$I(\alpha_2) = 1 \; \{ \alpha_2(1) = 2 > \alpha_2(2) = 1$$

$$1 < 2$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$I\alpha_1 = 0 \qquad I(\alpha_4) = 2$$

$$= \underset{\alpha(1)}{a_{11}}\, \underset{\alpha(2)}{a_{22}}\, \underset{\alpha_3}{a_{33}} + (-1)\, a_{12}\, a_{23}\, a_{31}$$

$$2 < 3 \quad 3 \not< 1$$

$$2 \not< 1$$

## *Determinanteen propietateak V*

- Matrize baten errenkadari (zutabeari) beste errenkada (zutabe) baten osagaiak zenbaki batez biderkaturik batzen badizkiogu, lortutako matrizearen determinantea eta jatorrizko matrizearena berdinak dira.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix} \overrightarrow{{}_{2(2.\ell)+1.\ell\to 1.\ell}} \begin{pmatrix} 9 & 12 & 15 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} = \begin{vmatrix} 9 & 12 & 15 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} = 27$$



## *Determinanteen propietateak VI*

- Orden bereko bi matrize karratuen arteko biderkaduraren determinantea eta bi matrizeen determinanteen arteko biderkadura berdina da.

$$|A \cdot B| = |A| \cdot |B|$$



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## Determinanteen propietateak I

- $|A| = |A^t|$ Matrize karratu baten determinantea eta matrize horren irauliaren determinatea berdinak dira.
- Matrize batean bi errenkada edo zutabe elkartrukatzen badira determinantearen zeinua aldatzen da.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} = 27 \text{ eta } \begin{vmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 0 \end{vmatrix} = -27$$

- Bi errenkada edo zutabe berdinak dituen matrize baten determinantea nulua da.

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 7 & 8 & 0 \end{vmatrix} = 0$$



## Determinanteen propietateak II

- Errenkada edo zutabe bateko osagai guztiak zenbaki batez biderkatuz gero determinantearen balioa zenbaki horrek berorrek biderkaturik izaten da. Adibidean, 3. errenkada 2tik biderkatuta dago

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 14 & 16 & 0 \end{vmatrix} = 54 = 2 \cdot \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} = 2 \cdot 27 = 54$$

- Bi errenkada edo zutabe proportzionalak dituen matrize karratu baten determinantea nulua da.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 10 & 12 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \end{vmatrix} = 2 \cdot 0 = 0$$



## Determinanteen propietateak III

- Zutabe edo errenkada bateko osagaiak bi edo elementu gehiagoren arteko batuketak badira, determinantea batuketa horien beste elementutan deskonposa daiteke horrenbesteko determinante osatuz, gainerako errenkadek edo zutabek berdin geratu beharko dute:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 4+3 & 5+2 & 6+5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 4 & 5 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{vmatrix} =$$

$$|C| = |A| + |B| = 0 + (-12) = -12$$

Deskonposatu dugu $C$ determinantea beste bi determinanteen batuketa bezala kalkulua sinplifikatzeko.



## Determinanteen propietateak IV

- $n$ ordeneko matrize bat eskalar batez biderkatuta, biderkaduraren determinantea matrizearen determinantea $k^n$ aldiz biderkatuta izango da, $|k \cdot A| = k^n |A|$.

$$A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \text{ matrizearen determinantea: } |A| = -4$$

$$2{\cdot}A = \begin{pmatrix} 2 & 6 \\ 2 & -2 \end{pmatrix} \text{ matrizearen determinantea: } |2{\cdot}A| = 2^2{\cdot}(-4)$$

## *Elementu baten adjuntua edo kofaktorea. I*

*Definizioa*

$a_{ij}$ *elementuaren adjuntua edo kofaktorea:*

$$A_{ij} = (-1)^{i+j} M_{ij}$$

Aurreko adibidearekin jarraituz:

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} = (-1) \cdot (-4) = 4$$



## *Determinantearen kalkulua adjuntuen bidez I*

$n$ ordeneko $A$ matrize baten determinantea kalkula daiteke errenkada edo zutabe bateko adjuntuen garapena bezala:

$$|A| = \sum_{j=1}^{n} a_{ij} A_{ij} = \sum_{j=1}^{n} a_{ij} A_{ij} \quad i \text{ lerroa edozein aukera daiteke}$$

$$|A| = \sum_{i=1}^{n} a_{ij} A_{ij} = \sum_{i=1}^{n} a_{ij} A_{ij} \quad j \text{ zutabea edozein aukera daiteke}$$

Errenkada baten edo zutabe baten elementuak dagozkien adjuntuaz biderkatutako batura determinantearen balio dela.



## *Determinantearen kalkulua adjuntuen bidez II*

Aurreko kalkuluak errazteko aukeratuko da zeroak edo zenbaki txikiak dituen errenkada edo zutabea. Adibidean, lehenengo errenkadako adjuntuen garapena erabili da.

$$|A| = \begin{vmatrix} 1 & -3 & -2 & 0 \\ 2 & -2 & -3 & 1 \\ -1 & 1 & 2 & 2 \\ 1 & 3 & 4 & 2 \end{vmatrix} = 1 \cdot A_{11} + (-3) \cdot A_{12} + (-2) \cdot A_{13} + 0 \cdot A_{14}$$



## *Determinantearen kalkulua adjuntuen bidez III*

$$|A| = 1 \cdot A_{11} + (-3) \cdot A_{12} + (-2) \cdot A_{13} + 0 \cdot A_{14} =$$

$$= 1 \cdot (-1)^{1+1} \begin{vmatrix} -2 & -3 & 1 \\ 1 & 2 & 2 \\ 3 & 4 & 2 \end{vmatrix} + (-3) \cdot (-1)^{1+2} \begin{vmatrix} 2 & -3 & 1 \\ -1 & 2 & 2 \\ 1 & 4 & 2 \end{vmatrix} +$$

$$+(-2) \cdot (-1)^{1+3} \begin{vmatrix} 2 & -2 & 1 \\ -1 & 1 & 2 \\ 1 & 3 & 2 \end{vmatrix} + 0 =$$

$$= (-6) + (-3)(-1)(-26) + (-2)(-20) = -44$$

## $n$ ordeneko matrize baten determinantea IV

$S_3 = \{123, 132, 213, 231, 312, 321\}$ 3 mailako permutazioen multzoa eta dagozkien alderantzikatze kopurua:

- $\alpha_1 = (\alpha_1(1) = 1, \alpha_1(2) = 2, \alpha_1(3) = 3)$,
  $1 < 2 < 3$ eta $\alpha_1(1) < \alpha_1(2) < \alpha_1(3) \Rightarrow \mathcal{I}(\alpha_1) = 0$
- $\alpha_2 = (\alpha_2(1) = 1, \alpha_2(2) = 3, \alpha_2(3) = 2)$,
  $2 < 3$ baina $\alpha_2(2) > \alpha_2(3) \Rightarrow \mathcal{I}(\alpha_2) = 1$
- $\alpha_3 = (\alpha_3(1) = 2, \alpha_3(2) = 1, \alpha_3(3) = 3)$,
  $1 < 2$ baina $\alpha_3(1) > \alpha_3(2)$
- $\alpha_4 = (\alpha_4(1) = 2, \alpha_4(2) = 3, \alpha_4(3) = 1)$,
  $1 < 3$ baina $\alpha_4(1) > \alpha_4(3)$ eta $2 < 3$ baina $\alpha_4(2) > \alpha_4(3) \Rightarrow \mathcal{I}(\alpha_4) = 2$
- $\alpha_5 = (\alpha_5(1) = 3, \alpha_5(2) = 1, \alpha_5(3) = 2)$,
  $1 < 2$ baina $\alpha_5(1) > \alpha_5(2)$ eta $1 < 3$ baina $\alpha_5(1) > \alpha_5(3)$
- $\alpha_6 = (\alpha_6(1) = 3, \alpha_6(2) = 2, \alpha_6(3) = 1)$,
  $1 < 2$ baina $\alpha_6(1) > \alpha_6(2)$, $1 < 3$ baina $\alpha_6(1) > \alpha_6(3)$ eta $2 < 3$ baina $\alpha_6(2) > \alpha_6(3)$
  $\Rightarrow \mathcal{I}(\alpha_6) = 3$



## $n$ ordeneko matrize baten determinantea V

- Aurreko adibidean, $\alpha_1 = (\alpha_1(1) = 1, \alpha_1(2) = 2, \alpha_1(3) = 3)$ lehenengo permutazioan ez dago alderantzikatzerik, $\mathcal{I}(\alpha_1) = 0$, elementuak ordenatuta baitaude, $1 < 2 < 3$ eta $\alpha_1(1) < \alpha_1(2) < \alpha_1(3)$
- Baina 2.permutazioan, $\alpha_2 = (\alpha_2(1) = 1, \alpha_2(2) = 3, \alpha_2(3) = 2)$, alderantzikatze kopurua $\mathcal{I}(\alpha_2) = 1$ izango da, ordena behin bakarrik aldatzen delako, $2 < 3$ eta $\alpha_2(2) > \alpha_2(3)$
- Beraz, $\mathcal{I}(\alpha_1) = 0$, $\mathcal{I}(\alpha_2) = 1$ eta beste permutazioen alderantzikatze kopurua:

$$\mathcal{I}(\alpha_3) = 1, \mathcal{I}(\alpha_4) = \mathcal{I}(\alpha_5) = 2, \mathcal{I}(\alpha_6) = 3$$



## $n$ ordeneko matrize baten determinantea VI

Beraz, 3 ordeneko matrize baten determinantea kalulatu nahi badugu formularekin:

$$|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$(-1)^{\mathcal{I}(\alpha_1)} a_{11} \cdot a_{22} \cdot a_{33} + (-1)^{\mathcal{I}(\alpha_2)} a_{11} \cdot a_{23} \cdot a_{32} +$$
$$(-1)^{\mathcal{I}(\alpha_3)} a_{12} \cdot a_{21} \cdot a_{33} + (-1)^{\mathcal{I}(\alpha_4)} a_{12} \cdot a_{23} \cdot a_{31} +$$
$$(-1)^{\mathcal{I}(\alpha_5)} a_{13} \cdot a_{21} \cdot a_{32} + (-1)^{\mathcal{I}(\alpha_6)} a_{13} \cdot a_{22} \cdot a_{31} =$$
$$a_{11} \cdot a_{22} \cdot a_{33} - a_{11} \cdot a_{23} \cdot a_{32}$$
$$-a_{12} \cdot a_{21} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31}$$
$$+a_{13} \cdot a_{21} \cdot a_{32} - a_{13} \cdot a_{22} \cdot a_{31}$$



## Matrizearen elementu baten minore osagarria. I

*Definizioa*

*A matrizearen $a_{ij}$ elementuaren* **minore osagarria**, *$M_{ij}$, i-garren errenkada eta j-garren zutabea kenduz agertzen den $n-1$ ordeneko matrizearen determinantea.*

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 4 & 2 & -3 \\ 1 & -1 & -2 \end{pmatrix}$$ matrizeko $a_{23}$ elementuaren minorea:

$$\mathbf{M}_{23} = \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} = (-1) - 3 = -4$$

## *Kontzeptua eta Notazioa III*

**3 ordeneko matrizearen determinantea kalkulatzeko Sarrus-en erregela:**

$$|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$a_{11} \cdot a_{22} \cdot a_{33} + a_{12} \cdot a_{23} \cdot a_{31} + a_{13} \cdot a_{21} \cdot a_{32}$$

$$-a_{13} \cdot a_{22} \cdot a_{31} - a_{12} \cdot a_{21} \cdot a_{33} - a_{11} \cdot a_{23} \cdot a_{32}$$

$$|\mathbf{A}| = \begin{vmatrix} 1 & 3 & 5 \\ 4 & 2 & -3 \\ 1 & -1 & -2 \end{vmatrix} = \text{-}22$$



## *n ordeneko matrize baten determinantea I*

Matrize baten determinantea kalkulatzeko batuketaren batugaia guztiak osatzeko $j$ indizearen permutazio posible guztiak erabiltzen direla.

Adibidez, 2 ordeneko matrizearen determinantearen batuketan batugai posible guztiak $2! = 2$ dira, eta j indizearen permutazio posible guztiak kalkulatzen baditugu ondoko multzoa lortuko dugu: $\{\{1,2\},\{2,1\}\}$. Kalkulu horiek egin ondoren, batugaiak osatzeko elementuen biderketa posible guztiak ondokoak dira:
$a_{11} \cdot a_{22}, a_{12} \cdot a_{21}$



## *n ordeneko matrize baten determinantea II*

$A = (a_{ij}) \in M(n,n)$ matrize karratuaren determinantea $\det(A) = |A|$ notazioaz adieraziko dugu eta ondoko formularekin kalkulatuko da:

$$|A| = \sum_{\alpha \in S_n} sig(\alpha) a_{1\alpha(1)} \cdots a_{n\alpha(n)}$$

$\alpha$ permutazioaren zeinua honela definitzen da:

$$sig(\alpha) = (-1)^{\mathcal{I}(\alpha)} = \begin{cases} 1 & \text{baldin } \mathcal{I}(\alpha) \text{ bikoitia bada} \\ -1 & \text{baldin } \mathcal{I}(\alpha) \text{ bakoitia bada} \end{cases}$$



## *n ordeneko matrize baten determinantea III*

- $n$ mailako permutazioa $\{1,\ldots,n\}$ zenbakien ordenazio bat da: $\alpha_k = (\alpha_k(1),\ldots,\alpha_k(n))$, $k = 1,\cdots,n!$( $n$ mailako $n!$ permutazio dago).
  $S_n$ notazioaz $n$ mailako permutazioen multzoa adierazten da. Adibidez, 3 mailako permutazioen multzoan $3! = 6$ elementu daude:
  $$S_3 = \{\alpha_1, \alpha_2, \ldots, \alpha_6\}$$
- $\alpha(j)$ eta $\alpha(k)$ permutazioak alderantzikatutak daudela esaten da baldin $j < k$ izanik $\alpha(j) > \alpha(k)$ betetzen bada. $\alpha$-ren alderantzikatze kopurua adierazteko $\mathcal{I}(\alpha)$ notazioa erabiliko dugu.

# *DETERMINANTEAK*

Informatika Fakultatea

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## *Aurkibidea*





## *Kontzeptua eta Notazioa I*

*Definizioa*

*$A \in M(n,n)$ matrize baten* **determinantea**, $|A| = det(A)$, *$n$ ordenako matrize karratuari dagokien zenbaki bat da.*

$A = (a_{11}) \in M(1,1)$ matrizearen determinantea $|A| = det(A) = a_{11}$

**Adibidez**

$A = (2)$ matrizearen determinantea $|A| = det(A) = 2$



## *Kontzeptua eta Notazioa II*

2 **ordeneko matrizearen determinantea:** $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$A$ matrizearen determinantea kalkulatzeko formula:

$$|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

Diagonal nagusian dauden osagaien biderkadura ken bigarren diagonalean daudenen biderkadura.