Programazioaren Metodologia

Baldintzadun egiturak:

$\{\emptyset\} \equiv zenbatzero = N_k(1 \le k \le i \wedge A(k)=0) \wedge 1 \le i \le n$

if $A(i+1)=0$ then
    $zenbatzero := zenbatzero + 1;$
end if;
$I := i+1;$

$\{\Psi\} \equiv \{ zenbatzero = N_k(1 \le k \le i \wedge A(k)=0) \wedge 1 \le i \le n\}$

Frogapenaren egitura:

A

$\{\emptyset \wedge A(i+1)=0\}$ $zenbatzero := zenbatzero + 1;$ $\{\emptyset_1\}$

A.2

$(\emptyset \wedge A(i+1) \ne 0) \to \emptyset_1$

A.3

$\emptyset \to def(A(i+1)=0)$

$\emptyset \equiv \{ zenbatzero = N_k(1 \le k \le i \wedge A(k)=0) \wedge 1 \le i \le n\}$

$\emptyset_1 \equiv \{ zenbatzero = N_k(1 \le k \le i+1 \wedge A(k)=0) \wedge 1 \le i+1 \le n\}$

$\emptyset\Psi \equiv \{ zenbatzero = N_k(1 \le k \le i \wedge A(k)=0) \wedge 1 \le i \le n\}$

Frogapena:

1. $\{ zenbatzero + 1 = N_k(1 \le k \le i \wedge A(k)=0) \wedge 1 \le i \le n\}$ $zenbatzero := zenbatzero + 1;$ $\{\emptyset_1\}$ (AA)
2. $(\emptyset \wedge A(i+1)=0) \to (zenbatzero + 1 = N_k(1 \le k \le i+1 \wedge A(k)=0) \wedge 1 \le i \le n)$
3. $\{\emptyset \wedge A(i+1)=0\}$ $zenbatzero := zenbatzero + 1;$ $\{\emptyset_1\}$ 2,1 eta ODE
4. $(\emptyset \wedge A(i+1) \ne 0) \to (zenbatzero = N_k(1 \le k \le i+1 \wedge A(k)=0) \wedge 1 \le i \le n)$
5. $\emptyset \to (1 \le i+1 \le n) \to def(A(i+1)=0)$
6. $\{\emptyset\}$ if $A(i+1)=0$ Then $zenbatzero := zenbatzero + 1;$ end if; $\{\emptyset_1\}$ 3,4,5, eta BDE
7. $\{ zenbatzero = N_k(1 \le k \le i+1 \wedge A(k)=0) \wedge 1 \le i+1 \le n\}$
8. $\{\emptyset_1\}$ $i := i+1;$ $\{\Psi\}$
9. $\{\emptyset\}$ if $A(i+1)=0$ then $zenbatzero := zenbatzero + 1;$ end if; $i := i+1;$ $\{\Psi\}$ 6,8 eta KPE

6. Izan bedi ondoko Hoare-ren hirukotea:

$\{\, true \,\}$

```
  if x = 0 then
    y := 3;
  end if;
```

$\{\, \psi \,\}$

Ondoko $\{\, \psi \,\}$-ren artean hirukotea egiazkoa egiten duena aukera ezazu:

$\{\, x = 0 \;\wedge\; y = 3 \,\}$

$\{\, x = 0 \;\leftrightarrow\; y = 3 \,\}$

$\{\, x = 0 \;\rightarrow\; y = 3 \,\}$

$\{\, y = 3 \;\rightarrow\; x = 0 \,\}$

7. Zuzena al da ondoko baieztapena? Zuzentasuna frogatu edo kontradibidea jarri, kasuan kasu.

7.1. $\{\, true \,\}$

```
  if x < y then
    lag := x; x := y; y := lag;
  elsif y < z then
    lag := y; y := z; z := lag;
  else
    lag := x; x := z; z := lag;
  end if;
```

$\{\, x \geq y \geq z \,\}$

7.2. $\{\, true \,\}$

```
  if x < y then
    lag := x; x := y; y := lag;
  end if;
  if y < z then
    lag := y; y := z; z := lag;
  end if;
  if x < y then
    lag := x; x := y; y := lag;
  end if;
```

$\{\, x \geq y \geq z \,\}$

```
else
   zut := zut+1;
end if;
```
$\{\ 1 \leq err, zut \leq n\ \wedge\ (err-1)\times n + zut = b+1\ \}$ 8, 12, 13 eta **(BDE)**

4. X aldagaia eta t terminoa izanik, ondoko baieztapenak egiazkoak ala faltsuak diren esan. Egiazkoa bada frogatu, eta, bestela, kontradibide bat bilatu.

4.1.

$\{\ False\ \}$
```
x := t;
```
$\{\ \varphi\ \}$

4.2.

$\{\ True\ \}$
```
x := t;
```
$\{\ \varphi\ \}$

5. Egiaztatu ondoko baieztapenak:

5.1.

$\{\ True\ \}$
```
if i <= j then
   if j < k then
      m := k;
   else
      m := j;
   end if;
else
   if i < k then
      m := k;
   else
      m := i;
   end if;
end if;
```
$\{\ m = max(i,j,k)\ \}$

5.2.

$\{\ True\ \}$
```
if bakoitia(x) then
   x := x+1;
end if;
```
$\{\ bikoitia(x)\ \}$

5.3.

$\{\ \neg berdin\ \}$
```
if x = y then
   berdin := true;
end if;
```
$\{\ berdin \leftrightarrow x = y\ \}$

Αποδείξεις

1. $\{\emptyset \land A(i)=3\} \rightarrow (w+1 = N_j(1 \le j \le i \land A(j)=3) \land v = n-w \land 1 \le i \le n\}$

2. $\{w+1 = N_j(1 \le j \le i \land A(j)=3) \land v = n-w \land 1 \le i \le n\}$

$w := w+1;$

$\{w = N_j(1 \le j \le i \land A(j)=3) \land v = n-(w-1) \land 1 \le i \le n\}$

3. $\{\emptyset \land A(i)=3\}$

$w := w+1$

$\{w = N_j(1 \le j \le i \land A(j)=3) \land v = n-w+1 \land 1 \le i \le n\}$ 1,2 είς ~~KPE~~ ΟΡΕ

4. $\{\emptyset_1\}$

$v := n-w;$

$\{w = N_j(1 \le j \le i \land A(j)=3) \land v = v+1 \land 1 \le i \le n\}$

5. $\{\emptyset \land A(i)=3\}$

$w := w+1;$

$v := n-w;$

$\{w = N_j(1 \le j \le i \land A(j)=3) \land v = v+1 \land 1 \le i \le n\}$ 3,4 είς ~~ΟΡΕ~~ KPE

6. $(\emptyset \land A(i) \ne 3) \rightarrow (\emptyset_1)$

7. $\emptyset_1 \rightarrow (1 \le i+1 \le n+1) \land (\mathrm{dot}(A(i)=3)$

8. $\{\emptyset_1\}\ i := i+1\ \{\psi\}$

9. $\{\emptyset\}$ if $A(i)=3$ then $w := w+1;\ v := n-w;$ end if; $i := i+1;$ $\{\psi\}$ 7,8 ΙΡΕ

$\{x \geq 0\}$

$r := 0$

$\{S \wedge B\} \equiv \{0 \leq r \wedge (0 \leq r^2 \leq x\}$

while $(r+1)^2 <= x$ loop

$\{0 \leq r \wedge 0 \leq r^2 \leq x\}$

$r := r+1$

$\{0 \leq r-1 \wedge 0 \leq (r-1)^2 \leq x\} \to \{0 \leq r \wedge 0 \leq (r-1)^2 \leq x$

end loop

$\{0 \leq r^2 \leq x < (r+1)^2\}$

3

$A \to \{\emptyset \wedge A(i) = 3\}\ w := w+1;\ v := n-v;\ \{\emptyset_1\}$

$A_1 \to \{\emptyset \wedge A(i) \neq 3\} \to (\emptyset_1)$

$A_2 \to \emptyset \to \text{def}(A(i) = 3)$

$\{\emptyset\} \equiv \{w = N_j(1 \leq j < i \wedge A(j) = 3) \wedge v = n - w \wedge 1 \leq i \leq n\}$

$\{\emptyset_1\} \equiv \{w = N_j(1 \leq j < i+1 \wedge A(j) = 3) \wedge v = n-w \wedge 1 \leq i+1 \leq n+1\}$

$\{\varphi\} \equiv \{w = N_j(1 \leq j < i \wedge A(j) = 3 \wedge v = n-w \wedge 1 \leq i+1 \leq n+1\}$

Froggepere:

1. $\{\cancel{w=N_j}\ \emptyset \wedge A(i) = 3\}\ w := w+1;\ v := n-v\ \{\emptyset_1\}$ (AA)

2. $(\emptyset \wedge A(i) \neq 3) \to (w = N_j(1 \leq j < i \wedge A(j) = 3 \wedge v = n-w \wedge 1 \leq i \leq n) \to$

$\to (w = N_j(1 \leq j < i+1 \wedge A(j) = 3 \wedge v = n-w \wedge 1 \leq i+1 \leq n+1)$

3.

②

```
{true}
  m := x;
  if m < y then
     m := y
  end if;
  if m < z then
     m := z;
  end if;
{m = max(x,y,z)}
```

①

A → $\{m < y\}$ $m := y$ $\{max(x, y, z)\}$

A.1 → $\{m \geq y\}$ → (ϕ_1)

A_2 ~~$\{m < y\}$ → def(~~

ϕ → def $(m < y)$

B

$\{m < z\}$ $m := z$ $\{max(x, y, z)\}$

B.1 $\{m \geq z)$ → φ

B.2 ϕ_1 → def $(m < z)$

Trogopem:

1. $\{true\}$ $m := x$ $\{m = x\}$
2. $\{$ ~~m = x~~ $m < y\}$ $m := y$ $\{m = x \wedge$

//

1

(1) $\{\phi \equiv \{n > 0 \wedge m \geq 0\}$

$r := 1$
$i := m$

(2) $\{Inb \equiv \{0 \leq i \leq m \wedge r = n^{m-i}\}$
while $0 < i$ loop
(3) $\{\phi_1 \equiv \{0 < i \leq m \wedge r = n^{m-i}\} \to r*n = n^{(m-i)+}$
$r := r*n;$
(4) $\{\phi_2 \equiv \{0 < i \leq m \wedge r = n^{m-i+1}\}$
$i := i-1;$
(5) $\{\phi_3 \equiv \{0 < i \leq m \wedge r = n^{m-i}\}$
end loop;
(6) $\{\psi \equiv r = n^m\}$

2

1 $\{\phi \equiv \{1 < n\}$

$i := 0$
txikiegoak := true;

2 $\{\phi_1 \equiv \{INB \equiv \{1 \leq 0 \leq n \wedge$ (txikiegoak $\vee$ not txikiegoak)
while $i < n$ and txikiegoak loop
$i := i+1;$
(3) $\{\phi_2 \equiv \{0 \leq i \leq n \wedge$ txikiegoak$\}$
if $B(i) > A(i)$ then
txikiegoak := false;
end if;
end loop;
(4) $\{\psi \equiv (i = n \wedge$ txikiegoak$) \vee (0 < i < n \wedge$ not txikiegoak$)$

function f(t, n : Integer) return r : Integer is

$\{t \geq 0 \wedge t \neq 1 \wedge n \geq 0\}$

begin

if n = 0 then

r := 1;

elsif n = 1 then

r := t + 1;

else

r := f(t, n-1);

r := r + t^n;

end if;

end f;

$\{r = \frac{t^{n+1} - 1}{t - 1}$

Kasu uberiak

① $\{t \geq 0 \wedge t \neq 1 \wedge n \geq 0 \wedge n = 0\}$ r := 1 $\{r = \frac{t^{n+1} - 1}{t - 1}\}$

② $t \geq 0 \wedge t \neq 1 \wedge n \geq 0 \wedge n \neq 0 \wedge n = 1\}$ r := t + 1 $\{r = \frac{t^{n+1} - 1}{t - 1}\}$

Frogapena

1. $(t \geq 0 \wedge t \neq 1 \wedge n \geq 0 \wedge n = 0) \rightarrow ((t > 1 \vee t = 0) \wedge n = 0) \rightarrow$

$\rightarrow \left(1 = \frac{0^{0+1} - 1}{0 - 1}\right) \rightarrow \left(1 = \frac{t^{n+1} - 1}{t - 1}\right)$ (AA)

2. $\left\{1 = \frac{t^{n+1} - 1}{t - 1}\right\}$ r := 1 $\left\{r = \frac{t^{n+1} - 1}{t - 1}\right\}$ (AA)

3. $\{t \geq 0 \wedge t \neq 1 \wedge n \geq 0 \wedge n = 0\}$ r := 1 $\{r = \frac{t^{n+1} - 1}{t - 1}\}$ 1, 2 eta ODE

4. $(t \geq 0 \wedge t \neq 1 \wedge n \geq 0 \wedge n \neq 1) \rightarrow ((t > 1 \vee t = 0) \wedge (n > 1 \vee n = 0)) \rightarrow$

$\rightarrow \left(t + 1 = \frac{(t+1)^{n+1} - 1}{t + 1}\right)$

(I.H) $\{t \geq 0 \wedge t \neq 0 \wedge n-1 \geq 0\}\ f(t, n-1)\ \{r = \frac{t^n - 1}{t-1}\}$

7. $\{t \geq 0 \wedge t \neq 0 \wedge n \geq 0 \wedge n \neq 0 \wedge n \neq 1\} \rightarrow (t \geq 0 \wedge t \neq 0 \wedge n-1 \geq 0)$

8. $\{t \geq 0 \wedge t \neq 0 \wedge n-1 \geq 0\}$

$r := f(t, n-1)$

$\{r = \frac{t^n - 1}{t-1}\}$

9. $\{t \geq 0 \wedge t \neq 0 \wedge n \geq 0 \wedge n \neq 0 \wedge n \neq 1\}$

$r := f(t, n-1);$

$\{r = \frac{t^n - 1}{t-1}\}$ 7,8 et. ODE

10. $\{r = \frac{t^n - 1}{t-1}\} \rightarrow (r + t^n = \frac{t^n - 1}{t-1} + t^n) \rightarrow (r + t^n = \frac{t^n - 1}{t-1} + \frac{t^n(t-1)}{t-1}) \rightarrow$

$\rightarrow (r + t^n = \frac{t^n - 1 + t^{n+1} - t^n}{t-1}) \rightarrow (r + t^n = \frac{t^{n+1} - 1}{t-1})$ (AA)

11. $\{r + t^n = \frac{t^{n+1} - 1}{t-1}\}$

$r := r + t^n;$

$\{r = \frac{t^{n+1} - 1}{t-1}\}$

①

(1) $\{\phi \equiv \{n>1\}$

$i := 2;$

(2) $\{INB \equiv \{ A(i-1) = A(i) \leftrightarrow \exists j (2 \le j \le i \wedge A(j-1) = A(j)) \wedge 2 \le i \le n \}$

while $i < n$ and $A(i-1) \neq A(i)$ loop

(3) $\{\phi_1 \equiv \{2 \le i < n \wedge A(i-1) \neq A(i)\}$

$i := i+1;$

end loop;

(4) $\{\phi_2 \equiv \{(A(i-1) = A(i)) \leftrightarrow \exists j (2 \le j \le n \wedge A(j-1) = A(j))\}$

$b := A(i-1) = A(i);$

(5) $\{\psi \equiv \{ b \leftrightarrow \exists j (2 \le j \le n \wedge A(j-1) = A(j)\}$

(1) $\{\phi_1\} \equiv \{n>1 \wedge m \geq 1\}$

$i := 1;$

(2) $\{INB\} \equiv \{ m > A(i) \leftrightarrow \neg\nexists j (1 \le j \le i \wedge m < A(j)) \wedge 1 \le i \le n)$

while $i < n$ and $m > A(i)$ loop

(3) $\{\phi_1 \equiv \{n \geq 1 \wedge \phi \le i < n \wedge m > A(i) = \text{true} \wedge \neg\exists j (1 \le j \le i \wedge m < A(j))\}$

$i := i+1;$

end loop;

(4) $\{\phi_2\} \equiv \{ m > A(i) \leftrightarrow \neg\nexists j (1 \le j \le n \wedge m < A(j))\}$

$b := (m > A(i)$

(5) $\{\psi\} \equiv \{ b \leftrightarrow \neg\nexists j (1 \le j \le n \wedge m < A(j))\}$

2. $\{a \bmod a = 0 \wedge b \bmod a = 0 \wedge$
$\forall z((a \bmod z \wedge b \bmod z = 0) \rightarrow a \geq z\}$
$g := a;$
$\{Q_1\}$ (AA)

3. $a > 0 \wedge b \geq 0 \wedge a = b \rightarrow Q_1$

4. $\{a > 0 \wedge b \geq 0\}$ $g := a$; $m := g;$ $\{\varphi\}$ 1,2,3 eta 4 eta (ODE) eta (KPE)

5. $\{a \bmod g = 0 \wedge b \bmod g = 0 \wedge$
$\forall z((a \bmod z = 0 \wedge b \bmod z = 0) \rightarrow g \geq z\}$
$m := g$
$\{\varphi\}$ (AA)

6. $\{a \bmod g = 0 \wedge b \bmod g = 0 \wedge$
$\forall z((a \bmod z = 0 \wedge b \bmod z = 0) \rightarrow g \geq z\}$
$g := a;$
$\{Q_2\}$ (AA)

7. $\{a > 0 \wedge b \geq 0 \wedge b = 0) \rightarrow (Q_2)$

8. $\{a > 0 \wedge b \geq 0 \wedge b = 0\}$ $g := a$; $m := g;$ $\{\varphi\}$

9. $\{a \bmod g = 0 \wedge b \bmod g = 0 \wedge$
$\forall z((a \bmod z = 0 \wedge b \bmod z = 0) \rightarrow g \geq z\}$
$m := g;$
$\{\varphi\}$ (AA)

10. $(b \bmod g = 0 \wedge a \bmod g = 0 \wedge$
$\forall z(b \bmod z = 0 \wedge a \bmod z = 0) \rightarrow g \geq z) \rightarrow$ $(a \bmod g = 0 \wedge b \bmod g = 0 \wedge$
$\forall z(a \bmod z = 0 \wedge b \bmod z) \rightarrow g \geq z$

11. $\{b > 0 \wedge a \geq 0\}$ $g := zkh(b, a)$ $\{$I.H. post$\}$

12. $\{a > 0 \wedge b > 0 \wedge a < b\} \rightarrow (b > 0 \wedge a \geq 0) \rightarrow$ I.H. aurrebaldintza.

13. $\{a > 0 \wedge b > 0 \wedge a < b\}$ $g := zkh(b, a)$; $m := g;$ $\{\varphi\}$ 9,10,11,12 eta ODE eta KPE

Programazioaren Metodologia

```
function zkh(a,b : in integer) return g: integer is
begin
   if a=b then
      g:=a;
   elsif b=0 then
      g:=a;
   elsif a<b then
      g:= zkh(b,a);
   else
      g:= zkh(a-b, b);
   end if;
```

Frogapenaren eskema:

Kasu nabariak:

1

$a=b \rightarrow (\{a>0 \wedge b\geq 0\}\ g:=a\ \{\psi\})$

2

$b=0 \rightarrow (\{a>0 \wedge b\geq 0 \wedge b=0\}\ g:=a\ \{\psi\})$

Kasu induktiboak:

1

$a\neq b \wedge b\neq 0 \wedge a<b \rightarrow \{a>0 \wedge b>0 \wedge a<b\}\ g:=zkh(b,a)\ \{\psi\}$

2

$a\neq b \wedge b\neq 0 \wedge a>b \rightarrow \{a>0 \wedge b>0 \wedge a>b\}\ g:=zkh(a-b,b)\ \{\psi\}$

Frogapena

1.

$\{a \bmod g=0 \wedge b \bmod g=0 \wedge \forall z((a \bmod z=0 \wedge b \bmod z=0) \rightarrow g\geq z)\}$

$m:=g;$

$\{\psi\}$

① Frogopene

①

$$\underbrace{(x+y=a+b \wedge b\geq y\geq 0)}_{INB} \wedge \underbrace{y=0}_{7B} \rightarrow \underbrace{x=a+b}_{\psi}$$

②

$$\underbrace{x=a \wedge y=b \wedge y\geq 0}_{\emptyset} \longrightarrow \underbrace{(x+y=a+b \wedge b\geq y\geq 0)}_{INB}$$

③

$$\{x+y=a+b \wedge b\geq y\geq 0\} \wedge y\neq 0\}\ x:=x+1;\ y:=y-1;\ \{x+y=a+b \wedge b\geq y\geq 0\}$$

② Frogopena

①

$$\underbrace{(0\leq i\leq n \wedge s=\sum_{j=1}^{i} A(j)\}}_{INB} \wedge \underbrace{i=n)}_{7B} \longrightarrow \underbrace{s=\sum_{j=1}^{n} A(j)}_{\psi}$$

②

$$\underbrace{i=0 \wedge s=0 \wedge n\geq 1}_{\emptyset} \rightarrow \underbrace{0\leq i\leq n \wedge s=\sum_{j=1}^{i} A(j)}_{INB}$$

③

$$\underbrace{\{0\leq i\leq n \wedge s=\sum_{j=1}^{i} A(j)}_{INB} \wedge \underbrace{i\neq n}_{B}\}\ \{i:=i+1;\ s:=s+A(i);\ \underbrace{\{0\leq i\leq n \wedge s=\sum_{j=1}^{i} A(j)\}}_{INB}$$

PM

Arikétak

④

1)

$\{\phi\} \equiv \{n \geq 1\}$

i := 0 ; Dago := false;

$\{\phi_1\} \equiv \{n \geq 1 \wedge i = 0 \wedge Dago = false\}$

while not dago and i < n loop — $\{(dago \leftrightarrow \exists j (1 \leq j \leq i \wedge A(j) = x)) \wedge 0 \leq i \leq n\}$

$\{\phi_2\} \equiv \{n \geq 1 \wedge 0 \leq i \leq n \wedge Dago = false \wedge \neg\exists j (1 \leq j \leq i \wedge A(j) = x\}$

i := i+1;

$\{\phi_3\} \equiv \{n \geq 1 \wedge 0 < i \leq n \wedge \neg dago \wedge \neg\exists j (1 \leq j < i \wedge A(j) = x\}$

if A(i) = x then

$\{\phi_4\} \equiv \{$

dago := true;

$\{\phi_5\} \equiv \{n \geq 1 \wedge 1 \leq i \leq n \wedge dago \leftrightarrow \exists j (1 \leq j \leq n \wedge A(j) = x)\}$

end if;

end loop;

$\{\psi\} \equiv \{dago \leftrightarrow Badago(A(1..n), x)\}$

Iterazioak. Inbariantea

① $\{\phi\} \equiv \{x = a \wedge y = b \wedge y \geq 0\}$

while y ≠ 0 loop — $INB \equiv \{x + y = a + b \wedge b \geq y \geq 0\}$

x := x+1;

y := y-1;

end loop;

$\{\psi\} \equiv \{x = a + b$

② $\{\phi\} \equiv \{s = 0 \wedge i = 0 \wedge n \geq 1\}$

while i ≠ n loop — $INB \equiv \{0 \leq i \leq n \wedge s = \sum_{j=1}^{i} A(j)\}$

i := i+1;

s := s + A(i);

end loop;

$\{\psi\} \equiv \{s = \sum_{j=1}^{n} A(j)\}$

2.2

Aurre $\equiv \{ n \geq 0 \}$

$i := 0;\ p := 1;\ b := 1;$

while $\{ 0 \leq i \leq n \wedge b = \sum_{k=0}^{i} 2^k \wedge p = 2^i$

too. $i < n$

loop

$\phi_1 \equiv \{ 0 \leq i < n \wedge p = 2^i \wedge b = \sum_{k=0}^{i} 2^k \}$

$i := i+1;$

$\phi_2 \equiv \{ 0 \leq i \leq n \wedge p = 2^{i-1} \wedge b = \sum_{k=0}^{i-1} 2^k \}$

$p := p * 2;$

$\phi_3 \equiv \{ 0 \leq i \leq n \wedge p = 2^i \wedge b = \sum_{k=0}^{i-1} 2^k \}$

$b := b + p;$

end loop; $\phi_1 \equiv \{ 0 \leq i \leq n \wedge p = 2^i \wedge b = \sum_{k=0}^{i} 2^k \}$

Post $\equiv \{ b = \sum_{k=0}^{n} 2^k \}$

2.1

INB $\{ 1 \leq k \leq n \wedge \cancel{\forall i (1 \leq i \leq k \to A(i) \geq m)} \}$ txikiena$(A(1..n), m) \}$

$\mathcal{E} = n - k$

4.6

Aurre $\equiv \{ n \geq 1 \}$

INB $\equiv \{ 1 \leq m \leq n$

Post $\equiv \{$ BaturaBiderketa seleziolureena$(A(1..n), k)$ non seleziolureena$(A(1..n), x)$

Iterazioak. Adibideak

① Frogapen formala

1. $(x = a \land y = b \geq 0) \rightarrow (x+y = a+b \land 0 \leq y \leq b)$

2. $(x+y = a+b \land 0 \leq y \leq b \land y \neq 0) \rightarrow ((x+1)+(y-1) = a+b \land 0 \leq y-1 \leq b)$

3. $\{(x+1)+(y-1) = a+b \land 0 \leq y-1 \leq b)\}$

   $x := x+1; \quad y := y-1;$

   $\{x+y = a+b \land 0 \leq y \leq b\}$ (AA) eta (KPE)

4. $\{x+y = a+b \land 0 \leq x \leq b \land y \neq 0\}$

   $x := x+1; \quad y := y+1;$

   $\{x+y = a+b \land 0 \leq y \leq b\}$ 2,3 eta ODE

5. $(x+y = a+b \land 0 \leq y \leq b \land y = 0) \rightarrow (x = a+b)$

6. $\{x = a \land y = b \geq 0\}$

   while ... loop

   $\{x := a+b\}$ 1,4,5 (whel)

② Frogapen formala

$\{x \geq 1\}$

$z := 1$

while $2 \cdot z \leq x$ loop $\{\text{ber}2(z) \land z \leq x\}$

$z := 2 \cdot z;$

end loop;

$\{z = \max\{k \mid \text{ber}2(k) \land k \leq x\}$

4. Gaia, Iterazioak

Programaziozen Metodologia

2.1

Aurre $\equiv \{ n \geq 1 \}$

$i := 0;\ w := 0;\ v := 0$

while $\{0 \leq i \leq n \land (w = N_j (1 \leq j \leq i \to A(j) \bmod 2 = 0) \land (v = N_j (0 \leq j \leq i \to A(j) \bmod 2 \neq 0)\}$

$i < n$

loop

$i := i + 1$

$\phi_1 \equiv \{1 \leq i \leq n \land n \geq 1 \land v = N_j (1 \leq j \leq i-1 \to A(j) \bmod 2 \neq 0) \land w = N_j (1 \leq j \leq i-1 \to A(j) \bmod$

if $(A(i) \bmod 2 = 0)$ then

$\phi_2 \equiv \{1 \leq i \leq n \land A(i) \bmod 2 = 0 \land v = N_j (1 \leq j \leq i-1 \land \to A(j) \bmod 2 \neq 0) \land w+1 = N_j (1 \leq j \leq i-$

$w := w+1;$

else

$\phi_3 \equiv \{1 \leq i \leq n \land v+1 = N_j (1 \leq j \leq i \land A(j) \bmod 2 \neq 0) \land w = N_j (1 \leq j \leq i \land A(j) \bmod 2 =$

$v := v+1;$

end if;

end loop;

$\phi_4 \equiv \{1 \leq i \leq n \land v = N_j (1 \leq j \leq i \land A(j) \bmod 2 \neq 0) \land w = N_j (1 \leq j \leq i \land A(j) \bmod 2 = 0\}$

$e := (w = v);$

Post $\equiv \{ e \leftrightarrow (N_i (1 \leq v \leq n \to A(v) \bmod 2 \neq 0) \land N_j (1 \leq w \leq n \to A(w) \bmod 2 = 0) \land i = j) \}$

3. $\{m \geq n \geq 0 \wedge (m = n \vee n = 0)\}$

$r := 1;$

$\{r := \dfrac{m!}{n! * (m-n)!}\}$ 1, 2 eta ODE

Kasu induktiboa $m \neq n \wedge n \neq 0$

(I.H) $\{m-1 \geq n \geq 0\}$ Konb(m-1, n); $\{r1 = \dfrac{(m-1)!}{n!(m-1-n)!}\}$

I.H $\{m-1 \geq n-1 \geq 0\}$ Konb(m-1, n-1); $\{r2 = \dfrac{(m-1)!}{(n-1)!(\cancel{m-1}-\cancel{n-1})!}\}$

4.

$(m \geq n \geq 0 \wedge n \neq 0 \wedge m \neq n) \rightarrow (m > n > 0) \rightarrow (m-1 \geq n \geq 0)$

5. $\{m-1 \geq n \geq 0\}$

$r1 := \text{Konb}(m-1, n);$

$\{r1 = \dfrac{(m-1)!}{n! * (m-1-n)!}\}$

6. $\{m \geq n \geq 0 \wedge n \neq 0 \wedge m \neq n\} \rightarrow (m-1 \geq n-1 \geq 0)$

$r1 := \text{Konb}(m-1, n)$

$\{r1 = \dfrac{(m-1)!}{n! * (m-1-n)!}\}$ 4, 5 eta ODE

7. $\{m > n \wedge n > 0\}$

$r2 := \text{Konb}(m-1, n-1);$

$\{r2 = \dfrac{(m-1)!}{(n-1)!(m-n)!}\}$

$(m-1-n)! + (m-1-n)! n!$

$\dfrac{\cancel{(m-1)!}((m-1)!(m-n)! + (m-1-n)! n!)}{n!(m-1-n)!\cancel{(m-1)!}(m-n)!}$

8. $\{r1 \wedge r2\}$

$r := r1 + r2$

$\{r = \left(\dfrac{(m-1)!}{n!(m-1-n)!} + \dfrac{(m-1)!}{(n-1)!(m-n)!}\right)\}$

$\left(\dfrac{(m-1)!(m-1)!(m-n)! + (m-1)!(m-1-n)! n!}{n!(m-1-n)!(m-1)!(m-n)!}\right)$

9. $\left(r = \dfrac{(m-1)!}{n!(m-1-n)!} + \dfrac{(m-1)!}{(n-1)!(m-n)!}\right) \rightarrow \left(r = \dfrac{(m-n)!\,(m-1)! + (m-1)! \cdot n!}{n!(m-n)!}\right) \rightarrow \left(r = \dfrac{m!}{n!(m-n)!}\right)$

3.2

```
function Konb(m, n: Integer) return r: Integer is
{m ≥ n ≥ 0}
    r1, r2 : Integer;
    if m = n or n = 0 then
        r := 1;
    else
        r1 := Konb(m-1, n);
        r2 := Konb(m-1, n-1);
        r := r1 + r2;
    end if;
{r = m! / (n! * (m-n)!)}
```

$\left\{ r1 + r2 = \frac{m!}{n!\,(m-n)!} \right\}$

Kasu neberiaren frogapenaren eskema

1

$m = n \vee n = 0 \rightarrow (\{m \geq n \geq 0\}\ r := 1;\ \{r = \frac{m!}{n! * (m-n)!}\})$

Kasu induktiboa

$m \neq n \wedge n \neq 0 \rightarrow (\{m \geq n \geq 0\}\ r1 := Konb(m-1, n);\ r2 := Konb(m-1, n-1);\ r := r1 + r2;\ \{r = \frac{m!}{n! * (m-n)!}\})$

Frogapena

1. $(m \geq n \geq 0 \wedge (m = 0 \vee n = 0) \rightarrow$

$\rightarrow ((m \geq n \geq 0 \wedge m = 0) \vee (m \geq n \geq 0 \wedge n = 0) \rightarrow$

$\rightarrow (m = 0 \wedge n = 0) \vee (m \geq n = 0) \rightarrow (m \geq 0 \wedge n = 0) \rightarrow$

$\rightarrow (1 = \frac{m!}{n! * (m-n)!})$ (AA)

2. $\{1 = \frac{m!}{n! * (m-n)!}\}$

$r := 1;$

$\{r = \frac{m!}{n! * (m-n)!}\}$

Kasu induktiboa $x \neq 1 \land x \geq b$

(I.H) $\{\frac{x}{b} > 0 \land b > 1\} \; \log(b, \frac{x}{b}) \; \{\frac{x}{b} \geq b^{y} \land \frac{x}{b} < b^{y+1}\}$

4. $(x \neq 1 \land x \geq b \land \cancel{(\frac{x}{b} > 0 \land b > 1)} \; x > 0 \land b > 1) \rightarrow (\frac{x}{b} > 0 \land b > 1)$

5. $\{\frac{x}{b} > 0 \land b > 1\}$

$y := \log(b, \frac{x}{b});$

$\{\frac{x}{b} \geq b^{y} \land \frac{x}{b} < b^{y+1}\}$ (AA)

6. $\{x \neq 1 \land x \geq b \land x > 0 \land b > 1\}$

$y := \log(b, \frac{x}{b});$

$\{\frac{x}{b} \geq b^{y} \land \frac{x}{b} < b^{y+1}\}$ 4, 5 eta ODE

7. $\{\frac{x}{b} \geq b^{y} \land \frac{x}{b} < b^{y+1}\}$

$y := y+1;$

$\{\frac{x}{b} \geq b^{y-1} \land \frac{x}{b} < b^{y}\}$ (AA)

8. $(\frac{x}{b} \geq b^{y-1} \land \frac{x}{b} < b^{y}) \rightarrow (x \geq b^{y-1} \cdot b \land x < b^{y} \cdot b) \rightarrow$
$\rightarrow (x \geq b^{y} \land x < b^{y+1})$

9. $\{\frac{x}{b} \geq b^{y} \land \frac{x}{b} < b^{y+1}\}$

$y := y+1;$

$\{x \geq b^{y} \land x < b^{y+1}\}$ 7 eta 8 eta ODE

10. $\{x \neq 1 \land x \geq b \land x > 0 \land b > 1\}$

$y := \log(b, \frac{x}{b})$

$y := y+1;$

$\{x \geq b^{y} \land x < b^{y+1}\}$ 6, 9 eta KPE

4.2 PM

$\{\emptyset\} \equiv \{ x \geq 1 \wedge b > 1 \}$

```
function log(b, x : Integer) return y : integer is
    if x = 1 or x < b then
        y := 0;
    else
        y := log(b, x/b);
        y := y+1;
    end if
```

$\{\psi\} \equiv \{ x \geq b^y \wedge x < b^{y+1} \}$

Kasu Nebarie: $x = 1 \vee x < b$

1. $(x > 0 \wedge b > 1) \wedge (x = 1 \vee x < b)) \rightarrow$

$\rightarrow (x > 0 \wedge b > 1 \wedge x = 0) \vee (x > 0 \wedge b > 1 \wedge x < b) \rightarrow$

$\rightarrow (b > 1 \wedge x = 0) \vee (x > 0 \wedge b > 1 \wedge x < b) \rightarrow$

$\rightarrow (x = b^0 \wedge b > 1) \vee (x \geq b^0 \wedge x < b^1) \rightarrow$

$\rightarrow (x \geq b^0 \wedge x < b^{0+1})$

2. $\{ x \geq b^0 \wedge x < b^{0+1} \}$

$y := 0;$

$\{ x \geq b^y \wedge x < b^{y+1} \}$ (AA)

3. $\{ x > 0 \wedge b > 1 \wedge (x = 1 \vee x < b) \}$

$y := 0;$

$\{ x \geq b^y \wedge x < b^{y+1} \}$ 1, 2 eta (ODE)

4.3

```
function exp(x,y: Integer) return r: Integer is
Aurre ≡ {x>0 ∧ y≥0}
   m: Integer;
  if y=0 then
     r:=1;
 elsif y mod 2=0 then
     m:= exp(x, y/2);
     r:= m*m;
 else
     m:= exp(x, y/2);
     r:= m*m*x
 end if
Post ≡ {r = x^y}
```

Frogapenaren eskema

Kasu nabaria:

$y=0 \rightarrow \{x>0 \wedge y\geq 0\}\ r:=1\ \{r=x^y\}$

Kasu induktiboak:

$y\neq 0 \wedge y \bmod 2 = 0 \rightarrow \{x>0 \wedge y\geq 0\}\ m:=exp(x, y/2);\ r:=m*m\ \{\psi\}$

$y\neq 0 \wedge y \bmod 2 \neq 0 \rightarrow \{x>0 \wedge y\geq 0\}\ m:=exp(x, y/2);\ r:=m*m*x\ \{\psi\}$

Frogapena

1. $(x>0 \wedge y\geq 0 \wedge y=0) \rightarrow (x>0 \wedge y=0) \rightarrow (1=x^0 \wedge y=0) \rightarrow$

$\rightarrow (1=x^y)$ (AA)

2. $\{1=x^y\}$ $r:=1;$ $\{r=x^y\}$

3. $\{x>0 \wedge y\geq 0\}$ $r:=1;$ $\{r=x^y\}$ 1, 2 eta (ODE)

Kasu induktiboa $y\neq 0 \wedge y \bmod 2=0$

(I.H) $\{x>0 \wedge y>0 \wedge y \bmod 2=0\}\; exp(x,y/2)\; \{m=x^{y/2}\}$

4. $(x>0 \wedge y\geq 0 \wedge y\neq 0 \wedge y \bmod 2=0) \rightarrow (x>0 \wedge y>0 \wedge y \bmod 2=0) \rightarrow$

5. $\{x>0 \wedge y>0 \wedge y \bmod 2=0\}$

$m:=exp(x,y/2);$

$\{m=x^{y/2}\}$

6. $\{x>0 \wedge y\geq 0 \wedge y\neq 0 \wedge y \bmod 2=0\}$

$m:=exp(x,y/2);$

$\{m=x^{y/2}\}$ 4, 5 eta ODE

7. $(m=x^{y/2}) \rightarrow (m^2=x^y)$ (AA)

8. $\{m^2=x^y\}$

$r:=m*m;$

$\{r=x^y\}$

9. $\{x>0 \wedge y\geq 0 \wedge y\neq 0 \wedge y \bmod 2=0\}$

$m:=exp(x,y/2)$

$r:=m*m;$

$\{r=x^y\}$ 6, 7, 8 (ODE) eta KPE

(3.1)

dig_hand funtzioak zenbaki atzuntz baten digitu handiena kalkulatzen du.

function dig_hand (x : Integer) return y : Integer is

$\{x > 0\}$

if x<=9 then

y:=x;

else

w:= dig_hand(x/10);

if w > x mod 10 then

y:= w;

else

y:= x mod 10

end if;

end if

$\{y = max\{\frac{x}{10^{i-1}} \bmod 10 \mid i \geq 10\}\}$

Kasu nabaria:

$\{x > 0 \land x <= 9\} \rightarrow (x > 0 \land x <= 9 \land i = 0) \rightarrow$

$\rightarrow x =$

$@$: Sekuentzia(t) × Sekuentzia(t) → Sekuentzia(t)

~~@(s1, s2) = s~~ $s_1 @ s_2$ (t) = S(t)

$@(\langle\rangle, \langle\rangle) = \langle\rangle$ $@(e.s, \langle\rangle) = e.s$

Alderantzizkoa : Sekuentzia(t) → Sekuentzia(t)

Alderantzizkoa$(\langle\rangle) = \langle\rangle$

Alderantzizkoa$(e.s)$ = Alderantzizkoa(s) ⊢ 'e' ⇒ Alderantzizkoa(s) @$(e.\langle\rangle)$

+ : ~~Integer Sekuentzia(t)~~ → Sekuentzia(t)
Sekuentzia(t) × ~~Integer~~ t

$@$: Sekuentzia(t) × Sekuentzia(t) → Sekuentzia(t)

$@(\langle\rangle, \langle\rangle) = \langle\rangle$ $@(e.s, \langle\rangle) = \langle e.s\rangle$

$@(\langle\rangle, e.s) = \langle e.s\rangle$ $@(e_1.s_1, e_2.s_2) = \langle e_1.(s_1 @ e_2.s)\rangle$

Nahastu : Sekuentzia(t) × Sekuentzia(t) → Sekuentzia(t)

Nahastu$(\langle\rangle, \langle\rangle) = \langle\rangle$ Nahastu$(\langle\rangle, e.s) = e.s$

Nahastu$(e.s, \langle\rangle) = e.s$ Nahastu$(e.s_1, e_2 s_2) = e_1 . (e_2 @$ Nahastu$(s_1, s_2))$

Eragileak Errekurtsiboak

$$zip : \Sigma^* \times \Sigma^* \longrightarrow \Sigma^*$$

zip ("etxea", "argi") = "~~etxe~~ eatrxgei"

zip ("etxearen", "argi" = "etxearregni"

$$w \in \Sigma^+ \begin{cases} zip(\varepsilon, \varepsilon) = \varepsilon \\ zip(\varepsilon, w) = w \\ zip(w, \varepsilon) = w \end{cases}$$

$$w_1, w_2 \in \Sigma^+ \Big\{ zip(v_1 x, v_2 y) = zip(v_1, v_2) = \ell x \ell y \Rightarrow zip = (\text{"etxeare"}, \text{"arg"}) = \ell \text{ "i"}$$

```
function zip(S1, S2 : string) return String
begin
   if S1'length = 0 and S2'length = 0 then
      return S1;
   elsif S1'length = 0 and S2'length /= 0 then
      return S2;
   elsif S1'length /= 0 and S2'length = 0 then
      return S2;
   else
      return ( ~~zip(S1, S2) ℓx ℓy~~
         zip(S1(S1'first, ..., S1'Last-1), S2(S2'first ... S2'Last-1)) ℓ(S1'Last) ℓ(S2'Last)
```

Sekuentzia

Datu-mota

Mota

Sekuentzia

Eragiketak

$\langle\rangle : \_ \rightarrow Sekuentzia(T)$

$\cdot : Integer \times Sekuentzia(T) \rightarrow Sekuentzia(T)$

Txertatuketan sekuentzia batean balioak txertatu

Izanik: ~~<4>~~ $4.\langle\rangle \rightarrow \langle 4 \rangle$

$4.\langle 5,8,3,2\rangle \rightarrow \langle 4,5,8,3,2\rangle$

Adibidea

Luzera: Sekuentzia ~~[illegible]~~ $\rightarrow$ Nat

Luzera(<>) = 0

Luzera(e.s)

e ∈ Integer — e.s ∈ Sekuentzia motakoa

s ∈ Sekuentzia

Hutsa_da: Sekuentzia $\rightarrow$ Boolean

Hutsa_da (<>) = true

Hutsa_da (e.s) = False

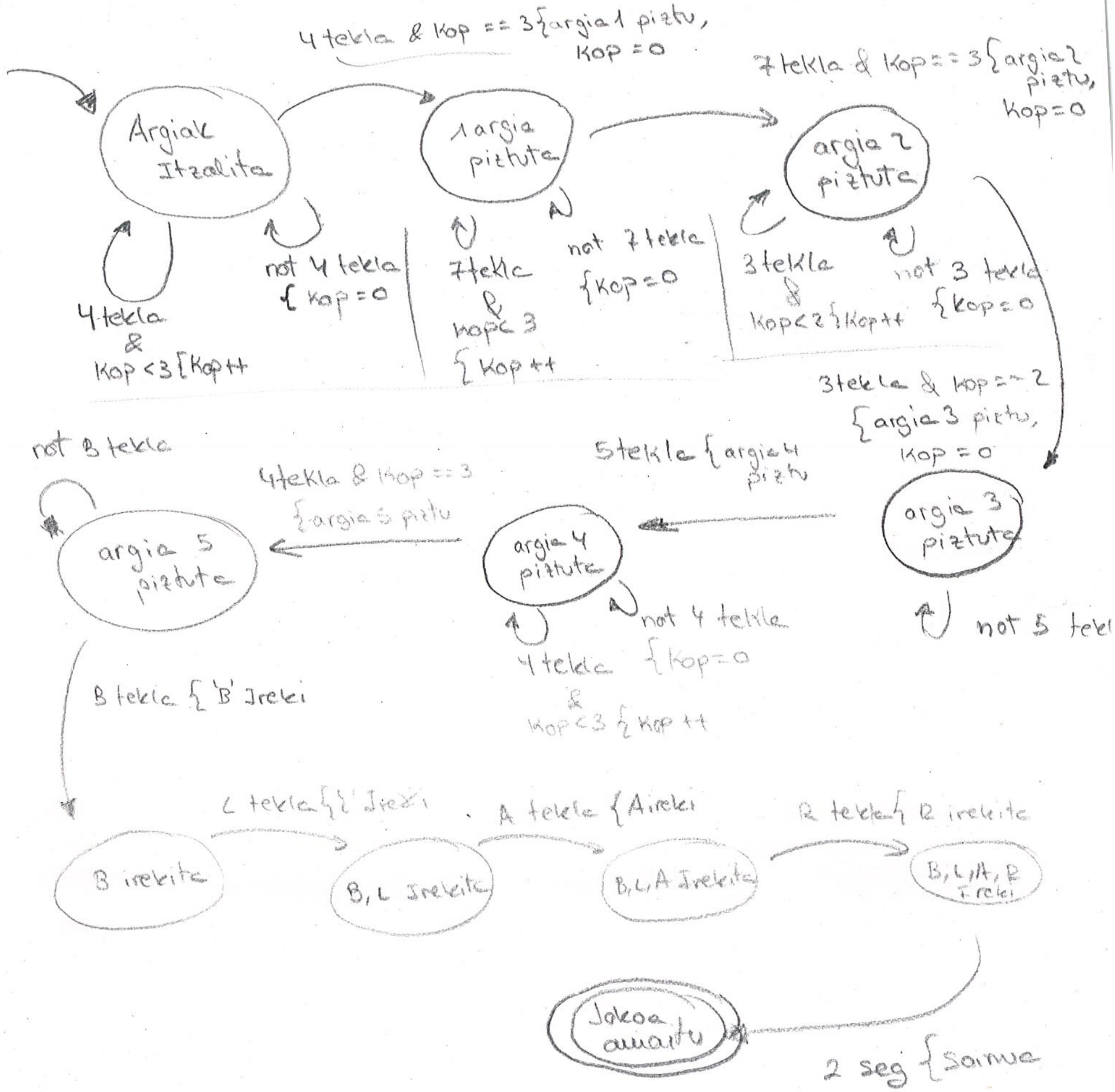
$\in : T \times Sekuentzia(T) \rightarrow Boolean$

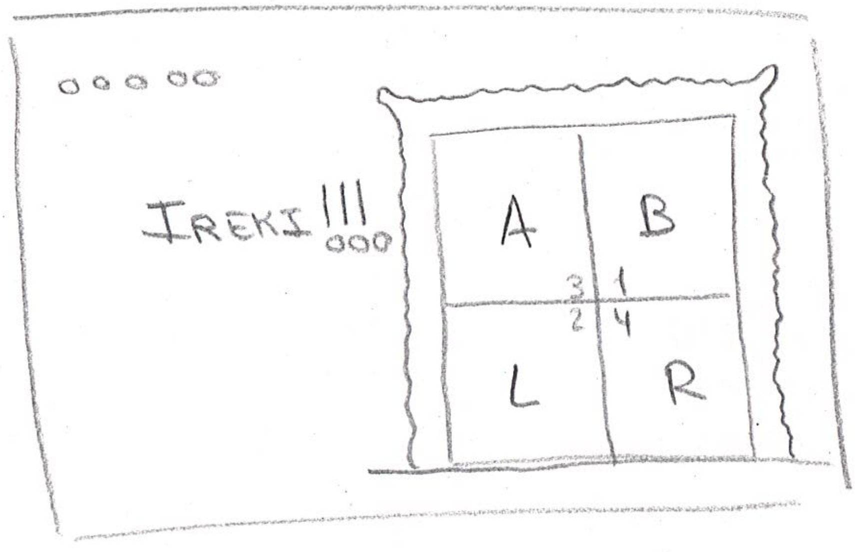
$y \in (\langle\rangle) = False$

~~$y \in (e, e.s) = true$~~

$y \in (e.s) = (y = e \wedge y \in s)$

$$Barne(e1, e2.s) = \begin{cases} true & \text{baldin } e1 = e2 \\ Barne(e1, s) & \text{bestela} \end{cases}$$


Argiak Itzalita
1 argia piztuta
argia 2 piztuta
argia 3 piztuta
argia 4 piztuta
argia 5 piztuta
4 tekla & Kop == 3 {argia 1 piztu, Kop = 0
7 tekla & Kop == 3 {argia 2 piztu, Kop = 0
4 tekla & Kop < 3 {Kop ++
not 4 tekla {Kop = 0
7 tekla & kop < 3 {Kop ++
not 7 tekla {Kop = 0
3 tekla & Kop < 2 {Kop ++
not 3 tekla {Kop = 0
3 tekla & Kop == 2 {argia 3 piztu, Kop = 0
not 5 tekl
5 tekla {argia 4 piztu
4 tekla & Kop == 3 {argia 5 piztu
not 4 tekla {Kop = 0
4 tekla & Kop < 3 {Kop ++
not B tekla
B tekla {'B' Ireki
B irekita
L tekla {'L' Ireki
B, L Irekita
A tekla {A ireki
B, L, A Irekita
R tekla {R irekita
B, L, A, R Ireki
2 seg {Soinua
Jokoa amaitu


IREKI !!!
A
B
L
R
3
1
2
4

8. Ondoko baieztapenetan, froga ezazu zuzena dena eta bila ezazu kontradibidea okerra denarentzat.

(A) $\{ z = x + y \}$

```
y := y/2;
if y mod 2 /= 0 then
   x := x+1;
end if;
x := x+y;
```

$\{ z = x + y \}$

(B) $\{ z = x + y \}$

```
if y mod 2 /= 0 then
   x := x+1;
end if;
y := y/2;
x := x+y;
```

$\{ z = x + y \}$

(A) zuzena da eta (B) faltsua da [ ]

(B) zuzena da eta (A) faltsua da [ ]

9. Asmatu inferentzi erregela egokia honako aginduentzat:

9.1. Izan bitez B_i adierazpen boolearrak eta P_i programak:

```
if B1 then I1;
elsif B2 then I2;
   ...
elsif Bn then In;
else In+1;
end if;
```

9.2. Kasu-hautaketa aginduen aldaera desberdina da honako hau. E datu-mota diskretu bateko espresioa da, eta b_i balioak berekoak dira (bere artean desberdinak).

```
case E is
   when b1 => I1;
      ...
   when bn => In;
   when others => In+1;
end case;
```

Aginduaren funtzionamendua honako hau da:

P_i exekutatzen da E b_i denean. E ebaluatzean $b_1, \ldots, b_n$ ez den balioa lortzen bada, P_{n+1} exekutatzen da.