3.7 Ariketak

1. Froga itzazu limitearen definizioa erabiliz:

   1) $\lim_{n\to\infty} \dfrac{n+1}{3n} = \dfrac{1}{3}$ 2) $\lim_{n\to\infty} \dfrac{1}{e^n} = 0$

   3) $\lim_{n\to\infty} \dfrac{2n^2+1}{n} = \infty$ 4) ez da existitzen $\lim_{n\to\infty}(-1)^n$

2. Esan ezazu, arrazoituz, ondoko baieztapenak egiazkoak ala faltsuak diren:

   (a) segida monotono oro konbergentea da;

   (b) segida bornatu oro konbergentea da;

   (c) segida konbergente guztiak ez dira monotonoak;

   (d) segida konbergente oro bornatua da;

   (e) infinitesimal guztiak ez dira baliokideak.

3. Froga ezazu $\{a_n\}$ segida hertsiki monotono gorakorra bada,

   (a) bornatua bada, konbergentea dela;

   (b) bornegabea bada, dibergentea dela.

4. Froga itzazu ondoko inplikazioak:

   (a) $\lim_{n\to\infty} |a_n| = 0 \Rightarrow \lim_{n\to\infty} a_n = 0$;

   (b) $\lim_{n\to\infty} a_n = 0 \Rightarrow \lim_{n\to\infty} |a_n| = 0$;

   (c) $\lim_{n\to\infty} a_n = 1 \Rightarrow \lim_{n\to\infty} |a_n| = 1$;

   (d) $\lim_{n\to\infty} |a_n| = 1$ betetzeak ez du esan nahi $\lim_{n\to\infty} a_n = 1$ beteko dela;

   (e) $\{a_n\}$ bornatua bada eta $\lim_{n\to\infty} b_n = \infty$ bada, $\lim_{n\to\infty} \dfrac{a_n}{b_n} = 0$ beteko da;

   (f) $\{a_n\}$ bornatua bada eta $\lim_{n\to\infty} b_n = 0$ bada, $\lim_{n\to\infty}(a_n b_n) = 0$ beteko da;

   (g) $\lim_{n\to\infty}(a_n b_n)$ existitzeak ez du esan nahi $\lim_{n\to\infty} a_n$ existituko denik;

   (h) $\lim_{n\to\infty} a_n$ ez existitzeak ez du esan nahi $\lim_{n\to\infty}(a_n b_n)$ existituko ez denik;

   (i) $\lim_{n\to\infty} a_n = 0$ eta $\lim_{n\to\infty} b_n = \infty$ badira, $\{a_n b_n\}$ segida konbergente, dibergente edo oszilatzailea izan daiteke;

   (j) $\lim_{n\to\infty} a_n = 0 \Rightarrow \lim_{n\to\infty} \dfrac{1}{a_n} = +\infty$, edo $\lim_{n\to\infty} \dfrac{1}{a_n} = -\infty$, edo ez da existitzen;

   (k) $\{a_n\}$ eta $\{b_n\}$ bornatuak izateak ez du esan nahi $\{\frac{a_n}{b_n}\}$ segida bornatua denik.

5. Kalkula itzazu gai orokor hauek dituzten segiden limiteak:

   1) $\frac{1/n}{\log_a(1+7/n)}$, $a>0$ 2) $\left(\cos\sqrt{\frac{2\ln 5}{n}}\right)^n$ 3) $\left(\frac{n}{3n^2+2}\right)^{\ln n}$

   4) $\frac{1}{\ln n}\sum_{k=1}^{n} \sin\frac{\pi}{k}$ 5) $n^2 e^{-\sqrt{n}}$ 6) $\frac{1}{\ln n}\sum_{k=1}^{n} \frac{\pi}{k}\cos\frac{\pi}{k}$

   7) $n^{-1/n}\ln\sqrt{n}$ 8) $\left(\frac{n-1}{n+3}\right)^{n+2}$ 9) $\left(\frac{3+2n^2}{3n+n^2}\right)^{\frac{n^3+2n}{n^2-5}}$

   10) $n(1-\sqrt[n]{a})$, $a>0$ 11) $\left(1-\frac{n+1}{n+2}\right)^{\frac{kn-1}{3}}$ 12) $\left(\frac{1}{n}\right)^{n^2+1}$

   13) $\frac{2^{2n}(n!)^2}{\sqrt{n}(2n)!}$ 14) $\frac{n+1}{\ln n}$ 15) $n-\ln n$

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   16) $\frac{1^2+2^2+\cdots+n^2}{n^3+\ln n}$ 17) $\frac{n^3+4n+2}{3n^2+7}$ 18) $\left(\frac{\ln n}{n}\right)^{\frac{n^3}{3n^2+2}}$

   19) $\sqrt[8]{n^2+1}-\sqrt[4]{n+1}$ 20) $n(\sqrt{n^2+1}-n)$ 21) $\frac{\ln n}{\cot(1/n)}$

   22) $n\left(\sqrt{\frac{n+1}{n}}-1\right)$ 23) $\left(\frac{2}{n+1}\right)^{\frac{2}{2+\ln n}}$ 24) $\sqrt[3]{n+\sqrt{n}}-\sqrt[3]{n}$

   25) $\left(\sqrt{\frac{1-n}{1-2n}}\right)^{\frac{2n-1}{3n+1}}$ 26) $\sin\frac{3n^2+1}{\ln n}$ 27) $\frac{n^{\frac{n+1}{n}}-n}{\ln n}$

   28) $\left(1+\ln\frac{n^2-5n+3}{n^2+5n-5}\right)^{2n-5}$ 29) $\sqrt[p]{(n+b_1)(n+b_2)\cdots(n+b_p)}-n$

   30) $\left(\frac{\sqrt[n]{a}+\sqrt[n]{b}}{2}\right)^n$, $a,b>0$ 31) $\sqrt[n]{\frac{1}{n^p}}$, $p<0, p=0$ eta $p>0$ kasuetan

   32) $\frac{1^2+2^2\sin\frac{1}{2}+\cdots+n^2\sin\frac{1}{n}}{n^2}$ 33) $\frac{\ln\left(\sqrt{a+\sqrt{b}}\,\sqrt[3]{a+\sqrt[3]{b}}\cdots\sqrt[n]{a+\sqrt[n]{b}}\right)}{\sin 1+\sin\frac{1}{2}+\cdots+\sin\frac{1}{n}}$, $a,b>0$

   34) $(\ln(n^2+3n-1))^{-\frac{n^2-1}{3n^2+4n-1}}$ 35) $\frac{1}{n^2}\left(\frac{2}{1}+\frac{3^2}{2}+\frac{4^3}{3^2}+\cdots+\frac{(n+1)^n}{n^{n-1}}\right)$

   36) $\frac{1^2+2^2+\cdots+n^2}{n^3}$ 37) $\frac{1^2+2^2+\cdots+n^2}{1+2+\cdots+n}\tan\frac{1}{n}$

6. Kalkula itzazu hurrengo segida errepikarien limiteak:

   (a) $a_1 = 1$, $a_{n+1} = \sqrt{2a_n}$;

   (b) $a_1 = 1$, $a_{n+1} = \dfrac{4-a_n}{3-a_n}$;

   (c) $a_1 = a > 0$, $a_{n+1} = \dfrac{n}{4n+1}a_n$.

7. $\{a_n\}$ segida ondoko errekurrentziak definitzen du: $a_1 = 5/2$, $5a_{n+1} = a_n^2 + 6$.

   (a) Froga ezazu $\forall n$, $2 < a_n < 3$;

   (b) froga ezazu beherakorra dela;

   (c) froga ezazu konbergentea dela eta kalkula ezazu bere limitea.

$\{a_n\} \to a$ eta $\{b_n\} \to b$ direla pentsatuko dugu.

Batuketa/Kenketa: $\{a_n \pm b_n\}$,

$+$	$-\infty$	b	$+\infty$
$-\infty$	$-\infty$	$-\infty$	?
a	$-\infty$	$a+b$	$+\infty$
$+\infty$	?	$+\infty$	$+\infty$

$-$	$-\infty$	b	$+\infty$
$-\infty$	?	$-\infty$	$-\infty$
a	$+\infty$	$a-b$	$-\infty$
$+\infty$	$+\infty$	$+\infty$	?

Biderketa: $\{a_n \cdot b_n\}$,

$\times$	$-\infty$	$b<0$	$b=0$	$0<b$	$+\infty$
$-\infty$	$+\infty$	$+\infty$	?	$-\infty$	$-\infty$
$a<0$	$+\infty$	$0<ab$	0	$ab<0$	$-\infty$
$a=0$	?	0	0	0	?
$0<a$	$-\infty$	$ab<0$	0	$0<ab$	$+\infty$
$+\infty$	$-\infty$	$-\infty$	?	$+\infty$	$+\infty$

Zatiketa: $\{a_n \div b_n\}$, $b_n \neq 0$ izanik,

$\div$	$-\infty$	$b<0$	$b=0$	$0<b$	$+\infty$
$-\infty$	?	$+\infty$	$\pm\infty$	$-\infty$	?
$a<0$	0	$0<a/b$	$\pm\infty$	$a/b<0$	0
$a=0$	0	0	?	0	0
$0<a$	0	$a/b<0$	$\pm\infty$	$0<a/b$	0
$+\infty$	?	$-\infty$	$\pm\infty$	$+\infty$	?

Logaritmoa: $\{\log_K a_n\}$, $K>0$ eta $a_n>0$ izanik,

$\log_K a_n$	$a=0$	$0<a$	$+\infty$
$0<K<1$	$+\infty$	$\log_K a$	$-\infty$
$1<K$	$-\infty$	$\log_K a$	$+\infty$

Esponentziala: $\{K^{a_n}\}$, $K>0$ izanik,

K^{a_n}	$-\infty$	a	$+\infty$
$0<K<1$	$+\infty$	K^a	0
$K=1$	1	1	1
$1<K$	0	K^a	$+\infty$

Berreketa: $\{a_n^{b_n}\}$, $a_n>0$ izanik,

$a_n^{b_n}$	$-\infty$	$b<0$	$b=0$	$0<b$	$+\infty$
$a=0$	$+\infty$	$+\infty$	?	0	0
$0<a<1$	$+\infty$	a^b	1	a^b	0
$a=1$	?	1	1	1	?
$1<a$	0	a^b	1	a^b	$+\infty$
$+\infty$	0	0	?	$+\infty$	$+\infty$

Baliokidetzak

Infinitesimalak: $\lim_{n\to\infty} a_n = 0$ betetzen da.

- $\{a_n\} \sim \{\sin a_n\} \sim \{\tan a_n\} \sim \{\arcsin a_n\} \sim \{\arctan a_n\}$
- $\{1-\cos a_n\} \sim \left\{\frac{a_n^2}{2}\right\}$
- $\{a_n\} \sim \{\ln(1+a_n)\}$
- Aurreko baliokidetzan, $b_n = 1 + a_n$ eginez, hau idatz dezakegu: $\{b_n - 1\} \sim \{\ln b_n\}$

Infinituak: $\lim_{n\to\infty} a_n = \infty$ betetzen da.

- $\{a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0\} \sim \{a_k n^k\}$
- Stirling-en baliokidetza: $\{n!\} \sim \{n^n e^{-n} \sqrt{2\pi n}\}$

Infinituen ordenak

Lau infinitu-mota bereiz ditzakegu, limitera hurbiltzeko "abiaduraren" arabera:

$\{(\ln a_n)^q\} <<< \{(a_n)^p\} <<< \{k^{a_n}\} <<< \{(a_n)^{ra_n}\}$, $q,p,r>0$ eta $k>1$ izanik.

Polinomioen arteko zatiduraren limitea

$$\lim_{n\to\infty} \frac{a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0}{b_l n^l + b_{l-1} n^{l-1} + \cdots + b_1 n + b_0} = \begin{cases} \infty & k>l \\ \frac{a_k}{b_l} & k=l \\ 0 & k<l \end{cases}$$

Stolz-en irizpideak

$\{a_n\}$ eta $\{b_n\}$ segidek baldintza hauek betetzen badituzte:

1) $\{b_n\}$ hertsiki monotonoa bada, 2) $\lim_{n\to\infty} \frac{a_{n+1}-a_n}{b_{n+1}-b_n}$ existitzen bada eta

3) hauetako bat ere betetzen bada:

3.1) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$ 3.2) $\lim_{n\to\infty} b_n = +\infty$ 3.3) $\lim_{n\to\infty} b_n = -\infty$

Stolz-en irizpideak dio berdintza hau beteko dela:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{a_{n+1}-a_n}{b_{n+1}-b_n}$$

e zenbakiaren erabilpena

$$e = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n \qquad e^k = \lim_{n\to\infty} \left(1+\frac{k}{n+l}\right)^{n+p}$$

$$\lim_{n\to\infty} a_n = 0 \Rightarrow e = \lim_{n\to\infty} (1+a_n)^{1/a_n} \qquad \lim_{n\to\infty} a_n = \pm\infty \Rightarrow e = \lim_{n\to\infty} \left(1+\frac{1}{a_n}\right)^{a_n}$$

(54)

D'Alembert $\to \lim_{n\to\infty} \frac{(\sqrt{2}-\sqrt[3]{2})(\sqrt{2}-\sqrt[5]{2})\cdots(\sqrt{2}-\sqrt[2n+1]{2})(\sqrt{2}-\sqrt[2(n+1)+1]{2})}{(\sqrt{2}-\sqrt[3]{2})(\sqrt{2}-\sqrt[5]{2})\cdots(\sqrt{2}-\sqrt[2n+1]{2})} =$

$= \lim_{n\to\infty}\left(\sqrt{2}-\sqrt[2(n+1)+1]{2}\right) = \lim_{n\to\infty}\left(\sqrt{2}-2^{\frac{1}{2n+3}}\right) = \sqrt{2}-2^{\frac{1}{\infty}} = \sqrt{2}-1 = \ell$

$\ell < 1$ denez, $\sum_n a_n$ seriea konbergentea da.

3
4 (2)

$\frac{(n!)^2}{(2n)!}\cdot a^n \quad a>0 \Rightarrow \frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!\,a^{n+1}}{2(n+1)!}}{\frac{(n!)^2\cdot a^n}{(2n)!}} = \frac{[(n+1)!]^2\cdot a^{n+1}\cdot(2n)!}{(2n+2)!\cdot(n!)^2 a^n} =$

$= \frac{(n^2+1+2n)\,a}{4n^2+6n+2} \Rightarrow \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2+6n+2}\cdot a = \frac{a}{4}$

$\ell = \frac{a}{4} \begin{cases} a>4 \\ a<4 \\ a=4 \text{ zalantzazkoa} \end{cases}$

D'Alembert-en irizpidearen arabera berez, Raabe-ren irizpidea erabiliko dugu.

$n\left(1-\frac{a_{n+1}}{a_n}\right) = n\left(1-\frac{(n^2+2n+1)\cdot 4}{4n^2+6n+2}\right) = n\left(\frac{-2n-2}{4n^2+6n+2}\right) = \frac{2n^2-2n}{4n^2-6n+2}$

$\lim_{n\to\infty} \frac{-2n^2-2n}{4n^2-6n+2} = -\frac{2}{4} = -\frac{1}{2}$

Raabe-ren irizpidea

$\lim n\left(1-\frac{a_{n+1}}{a_n}\right) = \ell \begin{cases} \ell>1 \text{ konbergentea} \\ \ell<1 \text{ dibergentea} \end{cases}$

$\ell = \frac{-1}{2} < 1$ berez, dibergentea da.

Bersten-en oinarria

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$\frac{\ln n}{e^n} = a_n$$

Erroaren irizpidea erabiliko dugu.

$$a_n = \frac{\ln n}{e^n} \Rightarrow \sqrt[n]{a_n} = \sqrt[n]{\frac{\ln n}{e^n}} = \frac{(\ln n)^{1/n}}{e^{n/n}} = \cancel{\frac{1}{n} \cdot \frac{\ln n}{e} = \frac{\ln n}{ne}}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{\ln n}{n e} = \lim_{n\to\infty} \frac{n-1}{ne} = \frac{1}{e} \qquad \frac{1}{e} = l$$

$$\boxed{\{\ln n\} \sim \{n-1\}}$$

$\frac{1}{e} < 1$ ~~bortze~~ serie konbergentea izango da.

$$\lim_{n\to\infty} \frac{\ln(n+1)}{\ln n}$$

$$\lim_{n\to\infty} \frac{(\ln n)^{1/n}}{e} = \text{indeterminazio}$$

$$\ln l = \lim_{n\to\infty} \ln \frac{(\ln n)^{1/n}}{e} = \lim_{n\to\infty} \frac{1}{n} \ln\left(\frac{\ln n}{e}\right) =$$

$$\cancel{\ln(n)} = \frac{1}{n}$$

$$\Rightarrow \lim_{n\to\infty} \frac{(\ln n)^{1/n}}{e} = \lim_{n\to\infty} \frac{(\ln n)^{1/n}}{e} = \text{indeterminazio.}$$

$$= \lim_{n\to\infty} \frac{n+1-1}{n-1} \cdot \frac{1}{e} = \lim_{n\to\infty} \frac{n}{en-e} = \frac{1}{e}$$

$$\lim_{n\to\infty} \frac{\ln(n+1)}{\ln(n)\cdot e} = \lim_{n\to\infty} \frac{\ln(n+1)}{\ln(n)}$$

ordena baliokideak dira

$$\ln l = \lim_{n\to\infty} \frac{1}{n} \ln\left(\cancel{\frac{(\ln n)^{1/n}}{e^{1/n}}}\right) = \cancel{\lim_{n\to\infty} \frac{1}{n}} \ln\left(\frac{\ln n}{e^n}\right) =$$

D'Alembert $= \lim_{n\to\infty}$

$$= \frac{1}{e}$$

$$a_n = \frac{\ln n}{e^n} \qquad \frac{a_{n+1}}{a_n} = \frac{\frac{\ln n+1}{e^{n+1}}}{\frac{\ln n}{e^n}} = \frac{\ln(n+1)\cdot e^n}{\ln n \cdot e^{n+1}} = \frac{\cancel{\ln n}\cdot e^n}{\ln n \cdot e^n \cdot e} =$$

$$= \frac{1}{e}$$

$l < 1$ bada, $\sum_n a_n$ seriea konbergentea izango da.

$$\frac{1}{e} < 1 \Rightarrow 1 < e$$

4) 33 Ariketak $\boxed{4}$ Aitzol Elu Etxano

$$\frac{n!}{n^n}$$

Ez zen atzorako, gaurko baizik

$\sum \frac{n!}{n^n}$ Korbengentea da, D'Alembert-en printzipioaren arabera:

$$\lim_{\cancel{n\to\infty}} \frac{(n+1)!\,n^n}{(n+1)^{n+1}\,n!} = \lim \frac{(n+1)n^n}{(n+1)^{n+1}} = \lim \frac{n^n}{(n+1)^n} = \frac{1}{e}$$

$\frac{1}{e} < 1$, beraz konbergentea da.

2)

b) $\sum_n a_n$ konbergentea bada $\sum_n \frac{a_n}{n}$-ren maiorantea denez

$\forall n \in \mathbb{N}$ $\quad a_n \geq \frac{a_n}{n}$ izango da.

Beraz, 14T arabera konb $\sum a_n >> \sum \frac{a_n}{n} \Rightarrow \sum_n \frac{a_n}{n}$ konb

c)

$\lim_{n\to\infty} n^3 a_n = \infty$ bada, $\sum_n a_n$ dibergentea da.

Baieztapena faltsua dela frogatzeko nahikoa da, $\lim n^3 a_n = \infty$ betetzen ~~dela~~ duen a_n bat bilatzea.

IZA agertzen den bezala $\lim_{n\to\infty} n^3 \frac{1}{n^2} = n = \infty$ izango da.
eta ~~beraz~~ badakigu $\sum \frac{1}{n^2}$ dibergentea dela.

5a)

$\sum_n \frac{1}{1+2^n}, k=3$

D'Alembert-en irizpidea.

$$\lim_{n\to\infty} \frac{\frac{1}{1+2^{n+1}}}{\frac{1}{1+2^n}} = \frac{1+2^n}{1+2^{n+1}} = \frac{1+2^n}{1+(2^n/2} = \frac{1}{2} < 1$$

$\rho < 1$ denez konbergentea da

Errorearen bornapena eskuinetik hurbiltzen da.

$$2 \cdot 2^{n+1} \geq 1 \cdot 2^{n+1} \Rightarrow \frac{1+2^{n+1}}{2} \leq 1+2^{n+1} \Rightarrow \frac{1+2^n}{1+2^{n+1}} \geq \frac{1}{2}$$

$R_k \leq \text{———}$

$0'125$

6

a)

$$\sum_n \frac{1}{(2n+1)!} \qquad R_k < 10^{-3}$$

D'Alembert

$$\frac{a_{n+1}}{a_n} = \frac{\frac{1}{(2(n+1)+1)!}}{\frac{1}{(2(n+1))!}} = \frac{1}{(2n+3)(2n+2)}$$

$$\lim_{n\to\infty} \frac{1}{(2n+3)(2n+2)} = 0 < 1 \text{ konbergentea izango da.}$$

2. kasua Demagun $\frac{a_{k+1}}{a_k} < 1$ dela

$$R_k \leq \frac{a_k \cdot a_{k+1}}{a_k - a_{k+1}} = A \Rightarrow R_k \leq A \quad R_k < 0'001 \Rightarrow A < 0'001$$

$$0'001 > \frac{\frac{1}{(2k+1)!} \cdot \frac{1}{(2k+3)!}}{\frac{1}{(2k+1)!} - \frac{1}{(2k+3)!}} = \frac{1}{(2k+3)! - (2k+1)!} < \frac{1}{1000} \Rightarrow 1000 < (2k+3)! - (2k+1)!$$

$k = 2$

$$\frac{a_{k+1}}{a_k} = \frac{a_3}{a_2} = \frac{\frac{1}{(2\cdot 3+1)!}}{\frac{1}{(2\cdot 2+1)!}} = 0'023 < 1 \qquad S_2 = 0'125$$

f)

$\lim_{n\to\infty} n^3 a_n = 0$ bada. $\sum a_n$ dibergentea da.

Gezurra da

Kontra Adibidea

$\sum_n a_n = \sum_n \frac{1}{n^4}$ hartuz gero, $\lim_{n\to\infty} \frac{1}{n^4} \cdot n^3 = \lim_{n\to\infty} \frac{1}{n} = 0$

baina S.h.o denez (eta $\alpha = 4 > 1$) $\sum_n \frac{1}{n^4}$ serie konbergentea da.

(2) *

b)

$\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ eta $\sum_{k\geq 1} a_n$ konbergentea bada, $\sum_n b_n$ konbergentea

Faltsoa $\sum_n a_n$ konbergentea bada $\Rightarrow \lim_{n\to\infty} a_n = 0$

$\left.\begin{array}{l}\lim_{n\to\infty} a_n = 0 \\ \lim_{n\to\infty} b_n = l\end{array}\right\} \frac{0}{l} = 0$

$\lim_{n\to\infty} a_n = 0 \Rightarrow \lim_{n\to\infty} \frac{1}{n} = 0$

7.4

$\sum_n \frac{n}{\alpha^n}$

$\alpha > 0$ Gai positibodo seriea da.

Cauchy-ren irizpidea

$\sqrt[n]{a_n} = \sqrt[n]{\frac{n}{\alpha^n}} = \frac{\sqrt[n]{n}}{\alpha} \Rightarrow \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{\alpha} = \frac{1}{\alpha}$

$\frac{1}{\alpha} < 1$ edo $1 < \alpha$ bada $\sum_n \frac{n}{\alpha^n}$ konbergentea

$1 < \frac{1}{\alpha}$ edo $1 > \alpha$ bada $\sum_n \frac{n}{a^n}$ dibergentea

$\frac{1}{\alpha} = 1$ edo $1 = \alpha$ bada, ? ezkutua, baina $\sum_n \frac{n}{\alpha^n} = \sum \frac{n}{1^n}$ edo $\sum n$ dib.

$\sum_n (-1)^{n-1} a_n$

$\alpha < 0$ Serie Alternatua $\alpha = -\beta,\ \beta \geq 0$ $\frac{1}{\alpha^n} = \frac{n}{(-\beta)^n} = \frac{n}{(-1)^n \beta^n} = (-1)^n \frac{n}{\beta^n}$;

$\sum (-1)^n \frac{n}{\beta^n}$

~~$0<\beta<1 \Rightarrow \infty$~~

Leibniz-en irizpidea. $\lim_{n\to\infty} \frac{n}{\beta^n} = \begin{cases} \infty & 0<\beta<1 \\ \infty & \beta = 1 \\ 0 & \beta > 1 \end{cases}$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ Leibniz ?? ez ⊙ ; $\beta > 1 \Rightarrow \sum_n (-1)^n \frac{n}{\beta^n}$ konberg

$\lim_{n\to\infty} \beta^n = \begin{cases} 0 & 0<\beta<1 \\ 1 & \beta = 1 \\ \infty & 1 < \beta \end{cases}$

⊙ esaten du $\nexists \lim (-1)^n \frac{n}{\beta^n} \Rightarrow \sum (-1)^n \frac{n}{\beta^n}$ ez da konbergentea

(23)

$(-1)^n \sin\frac{1}{n}$

$\sum \underbrace{(-1)^n \sin\frac{1}{n}}_{a_n}$ $\quad a_n = \sin\frac{1}{n} \Rightarrow \lim \sin\frac{1}{n} = 0 \quad \sin\frac{1}{n} > 0$

Leibniz-en irizpidearen arabera a_n segida beherakorra bada eta $\lim_{n\to\infty} a_n = 0$ bada, $\sum(-1)^n \sin\frac{1}{n}$ serie alternatiboa konbergentea da.

Beraz, $\sum(-1)^n \sin\frac{1}{n}$ serie alternatibo konbergentea da.

(9)

$(-1)^n \frac{\ln n}{n}$

$\sum (-1)^n \frac{\ln n}{n} \Rightarrow \frac{\ln n}{n} = a_n \Rightarrow$ Leibniz-en irizpidea

a_n segida beherakorra, $\frac{\ln n}{n} > 0$ eta $\lim_{n\to\infty} \frac{\ln n}{n} = 0$, $\sum(-1)^n \frac{\ln n}{n}$ serie alternatiboa konbergentea ~~da~~. izango da.

(36)

$0{,}001 + \sqrt{0'001} + \sqrt[3]{0'001} + \dots + \sqrt[n]{0'001} = \sum_{n\to\infty} \sqrt[n]{0'001}$

$\lim_{n\to\infty} 0'001^{1/n} = 0'001^{1/\infty} = 1$

7. K → $\lim_{n\to\infty} a_n \neq 0$ beraz seriea ez da konbergentea

Seriea beti positiboa denez, ez da oszilatzailea, beraz dibergentea da.

28)

$\frac{1}{n}\left(1-\cos\frac{\pi}{n}\right)=a_n \quad \left\{1-\cos\frac{\pi}{n}\right\} \sim \left\{\frac{\left(\frac{\pi}{n}\right)^2}{2}\right\} = \left\{\frac{\pi^2}{2n^2}\right\}$

~~$\lim_{n\to\infty} \frac{1}{n}\left\{\frac{\pi^2}{2n^2}\right\} \sim \left\{\lim \frac{\pi^2}{2n^3}\right\} =$~~ ~~$\alpha=3 \Rightarrow \frac{1}{n^3} \Rightarrow$ konbergentea ($\alpha>1$)~~

$\lim_{n\to\infty} ~~a_n~~ \quad \left\{\frac{1}{n}\left(1-\cos\frac{\pi}{n}\right)\right\} \sim \left\{\frac{1}{n}\frac{\pi^2}{2n^2}\right\} = \frac{\pi^2}{2n^3}$ Serie harmoniko korolarra

$\alpha=3 \Rightarrow \sum\frac{1}{n^3} \Rightarrow$ konbergentea

$\alpha>1$

5

$\sum\frac{1}{\sqrt[n]{\ln n}} = a_n \Rightarrow$ D'Alembert-en irizpidea $\frac{a_{n+1}}{a_n} = \frac{\frac{1}{\sqrt[n+1]{\ln n+1}}}{\frac{1}{\sqrt[n]{\ln n}}} = \frac{\sqrt[n]{\ln n}}{\sqrt[n+1]{\ln n+1}} =$

35

$a_n = \begin{cases} \frac{1}{n} & n=m^2 \\ \frac{1}{n^2} & n\neq m^2 \end{cases}$ $m\in\mathbb{N}$ kontsideratuz

$\sqrt{n}=m$ / $m\in\mathbb{N}$ betetzen denean $\frac{1}{n}$

$\sqrt{n}\neq m$ / $m\in\mathbb{N}$ betetzen denean $\frac{1}{n^2}$

$a_n=\left\{1, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4}, \frac{1}{5^2}, \frac{1}{6^2}, \frac{1}{7^2}, \frac{1}{8^2}, \frac{1}{9}\right.$

Seriearen izaera kalkulatzeko Seriea 2 zatitan, Horretarako trukatze-legea erabiliko dugu.

$\sum\frac{1}{n} \Rightarrow$ Serie harmonikoa, $\alpha\leq 1 \Rightarrow$ Dibergentea

$\sum\frac{1}{n^2} \Rightarrow$ serie harmonikoa, $\alpha>1 \Rightarrow$ Konbergentea

~~Batura~~ Serie. dibergente eta konbergentearen batura, izaera Dibergentea izango

(2)

g) $\sum a_n$ konbergentea $\Rightarrow \sum \ln(1+a_n)$ konbergentea ?

~~a_n~~ $\sum a_n$ konbergentea $\Rightarrow \lim_{n\to\infty} a_n = 0 \Rightarrow$ ~~ber~~ $\lim_{n\to\infty} \ln(1+a_n)$ konbergentea

$\boxed{\{a_n\} \sim \{\ln 1+a_n\}}$

izango da.

2.1

b)

$Errorea = 10^{-3}$

$q = 10$

$\cancel{\sum \frac{n^n}{\cancel{a_n} 10^n n!}} \quad a_n = \frac{n^n}{10^n n!}$

$\sum (-1)^n \frac{1}{10^n n} \Rightarrow a_n = \frac{1}{10^n n} \Rightarrow \lim_{n\to\infty} a_n = 0$ Gai positiboak direnez, eskuinetik hurbilduko da, beraz, 2. kasuan gaude.

$a_n < 1$ Pentsatuko dugu $a_k < 1$ dela.

2.2

$$\sum_n \frac{n^n}{\alpha^n n!} \quad \alpha \neq 0,$$

~~$\alpha = 10$~~

$$\sum_n \frac{n^n}{\alpha^n n!} \Rightarrow \{n!\} \sim \{n^n e^{-n}\sqrt{2\pi n}\} \Rightarrow \frac{\cancel{n^n}}{\alpha^n(\cancel{n^n} e^{-n}\sqrt{2\pi n})} = \frac{1}{\alpha^n e^{-n}\sqrt{2\pi n}}$$

$$\lim_{n\to\infty} \frac{1}{\alpha^n e^{-n}\sqrt{2\pi n}} = 0 < 1$$ Beraz, ~~Seriea konbergentea izango da.~~

$$\left\{\sqrt[n]{a_n}\right\} = \left\{\frac{1}{\alpha^n e^{-n}\sqrt{2\pi n}}\right\}$$

2.1

$$\sum (-1)^n \frac{1}{\alpha^n n}$$

a)

$\alpha < 0$ bada,

Gai positiboko seriea izango da,

$$\sum (-1)^n \frac{1}{(-1)^n n} = \sum \frac{1}{n}$$ $\Rightarrow$ Serie harmoniko orokorra, $\alpha \leq 1$ bada seriea dibergentea izango da,

$\alpha \geq 0$ bada

Serie alternatua izango da.

$$\sum (-1)^n \frac{1}{\alpha^n n} \Rightarrow a_n = \frac{1}{\alpha^n n} \Rightarrow \lim_{n\to\infty} a_n = 0$$ eta a_n beherakorra izango da, Leibniz-en arabera, $\sum (-1)^n \frac{1}{\alpha^n n}$ $\alpha > 0$ serie konbergentea izango da.

5

c)

$$\frac{n^3}{(n+2)!} \xrightarrow{D'Alembert} \lim_{n\to\infty} \frac{\frac{(n+1)^3}{(n+1+2)!}}{\frac{n^3}{(n+2)!}} = \lim_{n\to\infty} \frac{(n+1)^3 (n+2)!}{(n+3)!\, n^3} =$$

$$\lim_{n\to\infty} \frac{(n+1)^3 \cancel{(n+2)!}}{\cancel{(n+2)!}\,(n+2+1)\cdot n^3} = \lim_{n\to\infty} \frac{(n+1)^3}{(n+3)\cdot n^3} \lim_{n\to\infty} \frac{n^3}{n^4} = 0 = 1$$

Seriea konbergentea da $0<1$ delako

$$\forall n > \overset{K=7}{K} \quad \frac{a_{n+1}}{a_n} \geq 1 \quad \frac{\frac{(n+1)^3}{(n+1+2)!}}{\frac{n^3}{(n+2)!}} \cancel{\geq} \leq \frac{1}{n} \geq 0$$

beraz 7. Ueswan gavde $\frac{a_{K+1}}{a_K} = \frac{\frac{8^3}{10!}}{\frac{7^3}{4!}} =$

$= 0'7$

$0'7<1$ denez, A ateleon gavde $R_K \leq \frac{a_K \cdot a_{K+1}}{a_K - a_{K+1}} = \text{\_\_\_\_\_\_} = 7'68 \cdot 10^{-3}$

$R_K \leq 8'08 \cdot 10^{3}$

Ariketak Serieak

9)

$$(-1)^n \cdot \frac{\ln n}{n}$$

$$\sum_n (-1)^n \cdot \frac{\ln n}{n} \; ; \; a = \frac{\ln n}{n} \quad r = -1 \Rightarrow \sum_n \frac{\ln n}{n} (-1)^n \text{ oszilatzailea da.}$$

$$\sum_n^{\infty} a_n = \lim_{n \to \infty} S_n \Rightarrow \sum_n^{\infty} (-1)^n \cdot \frac{\ln n}{n} = \lim_{n \to \infty} (-1)^n \cdot \frac{\ln n}{n} = \frac{\infty}{\infty}$$

$$\Rightarrow \cancel{\lim_{n \to \infty}} \ln l = \lim_{n \to \infty} \ln (-1)^n \cdot \ln \left(\frac{\ln n}{n} \right) = n \ln_{-1}$$

23)

$$(-1)^n \sin \frac{1}{n}$$

$$\sum_n^{\infty} (-1)^n \sin \frac{1}{n} = \lim_{n \to \infty} (-1)^n \sin \frac{1}{n} = \infty \cdot 0$$

$$\ln l = \lim_{n \to \infty} \ln (-1)^n \sin \frac{1}{n} = \lim_{n \to \infty} \ln (-1)^n + \ln \left(\sin \frac{1}{n} \right) =$$

$$= \lim_{n \to \infty} \ln \left(-1 \cdot \sin \frac{1}{n} \right) = \lim_{n \to \infty} n \ln \left(-\sin \frac{1}{n} \right) = \lim_{n \to \infty} n \ln \left(\frac{1}{n} \right) =$$

$$= \lim_{n \to \infty} n \ln$$

29)

$\ln\left(1+\frac{1}{n^2}\right)$ $\quad \sum_n \cancel{a_n} =$

$$\cancel{\sum_n^{\infty} a_n \quad \lim_{n\to\infty} a_n \Rightarrow \lim_{n\to\infty} \ln\left(1+\frac{1}{n^2}\right) = \lim_{n\to\infty} \frac{1}{n^2} = 0}$$

$\sum_n \ln\left(1+\frac{1}{n^2}\right)$ gai positibodun seriea da.

$$\left\{\ln\left(1+\frac{1}{n^2}\right)\right\} \sim \left\{\frac{1}{n^2}\right\} \Rightarrow \lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n^2}\right)}{1/n^2} = 1 \neq 0 \Rightarrow \sum_n \ln\left(1+\frac{1}{n^2}\right)$$

eta $\sum_n 1/n^2$ seriedk baliokideak dira,

$\sum_n \frac{1}{n^2}$ seriea harmonikoa ~~da~~ orokorra da. ~~Eta Cauchy-ren irizpidea betetzen~~

ez ~~denez ez da konbergentea beraz, dibergentea izango da~~

$\alpha = 2 > 1$, beraz, konbergentea izango d.

30)

$\frac{\ln n}{e^n}$

Erroaren irizpidea erabiliko dugu. $(\ln n)^{1/n}$

$$a_n = \frac{\ln n}{e^n} \Rightarrow \sqrt[n]{a_n} = \sqrt[n]{\frac{\ln n}{e^n}} = \frac{\cancel{\ln n}}{e^{n/n}} = \frac{\frac{1}{n}\ln n}{e} = \frac{\ln n}{ne}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{\ln n}{ne} = \lim_{n\to\infty} \frac{n-1}{ne} = \lim_{n\to\infty} \frac{1}{e} \Rightarrow \frac{1}{e} < 1 \Rightarrow 1 < e \Rightarrow \ln 1 < 1$$

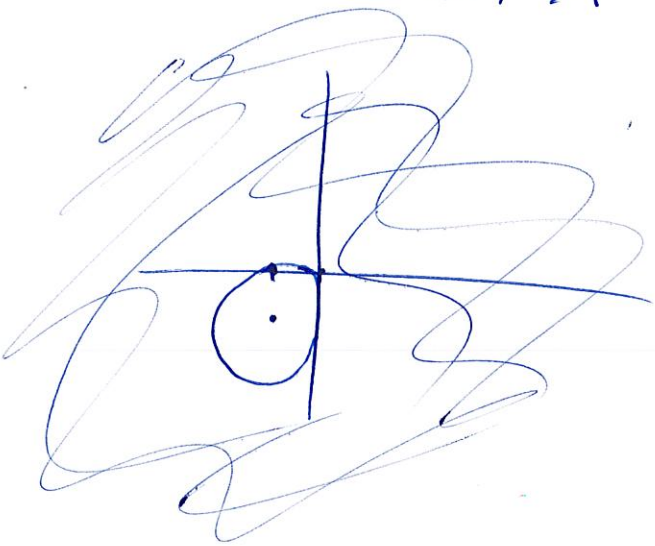
$\{\ln n\} \sim \{n-1\}$

$\sum_n \frac{\ln n}{e^n}$ serie konbergentea da.

$$\operatorname{Im}\left(\frac{-b}{(a-1)^2+b^2}\right) i$$

$$\frac{-b}{(a-1)^2+b^2} < \frac{1}{2} \Rightarrow -2b < (a-1)^2+b^2 \Rightarrow (a-1)^2+b^2+2b > 0 \Rightarrow$$

$$\Rightarrow (a-1)^2 + (b+1)^2 > 1$$

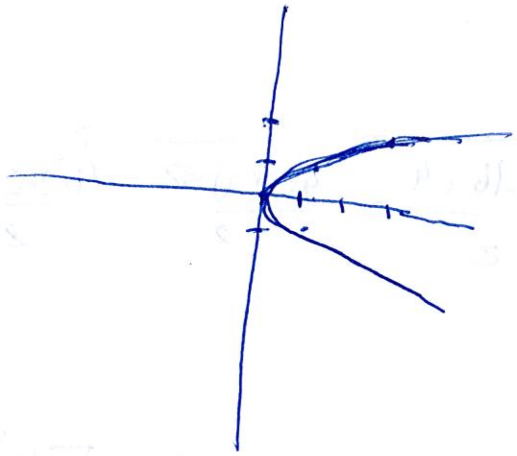


19)

$|z| < 1 - \operatorname{Re} z$

$|a+bi| < 1-a \Rightarrow \sqrt{a^2+b^2} < 1-a \Rightarrow a^2+b^2 < (1-a)^2 \Rightarrow$

$\Rightarrow a^2+b^2 < a^2-2a+1 \Rightarrow b^2+2a < 1 \Rightarrow a < \frac{b^2}{2}+1$



20

$\operatorname{Im}\left(\frac{1}{z-i}\right) < \frac{1}{2}$

$\frac{1}{z-1} = \frac{1}{z-1}\cdot\frac{z+1}{z+1} = \cancel{\frac{(a+1)+bi}{((a-1)+bi)(a+bi)}} = \cancel{\frac{(a+1)+bi}{a(a-1)(a+1)}}$

$= \frac{1}{(a-1)+bi}\cdot\frac{(a+1)+bi}{(a+1)+bi} = \cancel{\frac{(a+1)+bi}{(a-1)(a+1)-b^2+(a-1)bi+(a+1)bi}} =$

$= \cancel{\frac{(a+1)+bi}{a^2-1-b^2+abi+abi}} =$

$\frac{(a-1)-bi}{(a-1)^2+b^2}$

$\sin z = 2 \Rightarrow \dfrac{e^{iz} - e^{-iz}}{2i} = 2 \Rightarrow e^{iz} - e^{-iz} = 4i \Rightarrow e^{iz} - \dfrac{1}{e^{iz}} = 4i \Rightarrow$

$\Rightarrow \dfrac{e^{2iz} - 1}{\cancel{e^{iz}}} = 4i \Rightarrow e^{2iz} - 1 = 4ie^{iz} \Rightarrow e^{2iz} - 4ie^{iz} - 1 = 0$

$\boxed{e^{iz} = t}$

$t^2 - 4it - 1 = 0$

$t = \dfrac{4i \pm \sqrt{(4i)^2 - 4 \cdot -1 \cdot 1}}{2} = \dfrac{4i \pm \sqrt{-16 + 4}}{2} = \dfrac{4i \pm \sqrt{-12}}{2} = \dfrac{\overset{2}{4i} \pm 2\sqrt{3}i}{2} =$

$= \cancel{4i} \; 2i \pm \sqrt{3}i$

$e^{iz} = 2i \pm \sqrt{3}i \Rightarrow \cancel{\ln e}^{iz} = \ln(2i \pm \sqrt{3}i) = iz = \ln(|2 \pm \sqrt{3}|i)^{\frac{\pi}{2} + 2k\pi}$

$zi = \ln(2 \pm \sqrt{3}) + e^{\pi/2 + 2k\pi}$

$z = \ln(2 \pm \sqrt{3})i + \pi/2 + 2k\pi$

$\boxed{z = i^i}$

~~$t^2 k$~~

$z^1 = i \Rightarrow \begin{cases} |z| = \sqrt{i^2} = 1 \\ \text{Arctang}\left(\frac{b}{a}\right) = \frac{\pi}{2} \end{cases}$

~~z~~ i^i $z = i^i \Rightarrow \ln z = \ln i^i \Rightarrow \ln z = i \ln i \Rightarrow$

$\Rightarrow \ln z = i \; \cancel{\ln}\left(\frac{\pi}{2} + 2k\pi\right)i \Rightarrow \ln z = i \cdot i \cdot \left(\frac{\pi}{2} + 2k\pi\right) \Rightarrow \ln z = -\left(\frac{\pi}{2} + 2k\pi\right)$

$\Rightarrow z = e^{-\left(\frac{\pi}{2} + 2k\pi\right)}$

7)

$$1+\frac{1}{3}=\frac{3}{2}-\frac{1}{2\cdot 3}$$

$$\left.\begin{array}{l} 1+\frac{1}{3}+\frac{1}{9}=\frac{3}{2}-\frac{1}{2\cdot 9} \\ 1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}=\frac{3}{2}-\frac{1}{2\cdot 27} \end{array}\right\} 1+\frac{1}{3}+\dots+\frac{1}{3^n}=\frac{3}{2}-\frac{1}{2\cdot 3^n}$$

1) $P(k)$ egiazko.

2) $P(n)$ egiazkoa bada, $P(n+1)$ ere $\Big\}$ $\forall n \geq k$ $P(n)$ egiazkoa

1)

$P(k) = 1+\frac{1}{3}+\dots+\frac{1}{3^n}=\frac{3}{2}-\frac{1}{2\cdot 3^n}$

$\underline{k=2}$

Ezkerreko atala:

$$1+\frac{1}{3}+\frac{1}{9}=\frac{13}{9}$$

Eskuineko atala:

$$\frac{3}{2}-\frac{1}{2\cdot 9}=\frac{13}{9}$$

Beraz, frogatuta gelditzen da $P(k)$ egiazkoa dela.

2)

Suposatuz $P(n)$ egiazkoa dela frogatuko dugu, $P(n+1)$ egiazkoa dela.

$$1+\frac{1}{3}+\dots+\frac{1}{3^n}+\frac{1}{3^{n+1}} \underset{I.H}{=} \frac{3}{2}-\frac{1}{2\cdot 3^n}+\frac{1}{3^{n+1}}=\frac{3}{2}-\frac{3}{2\cdot 3^{n+1}}+\frac{2}{2\cdot 3^{n+1}}=$$

$$=\boxed{\frac{3}{2}-\frac{1}{2\cdot 3^{n+1}}}$$

Frogatuta gelditzen da.

Indukzioa

6)

$3^n - 1$ 2-ren multiploa da.

$\frac{3^n-1}{2} = m$ non $m \in \mathbb{Z}$

Indukzio-Hipotesiaren bidez frogatuko dugu $\forall n \geq k$ $P(n)$ egiazkoa da.

1) $P(k)$ egiazkoa da.

2) $P(n)$ egiazkoa izanik $P(n+1)$ egiazkoa da $\Big\}$ $\forall n \geq k$ $P(n)$ egiazkoa da.

1)

$\underline{k = 2}$

$$\frac{3^n-1}{2} = \frac{3^2-1}{2} = \frac{9-1}{2} = \boxed{4} \text{ beraz, } m = 4 \in \mathbb{Z}$$

Betetzen da, beraz, $P(k)$ egiazkoa da.

2)

Suposatuz $\frac{3^n-1}{2} = m$ egiazkoa dela frogatuko dugu, egiazkoa dela

$$\frac{3^{n+1}-1}{2} = p \Rightarrow \frac{3^{n+1}-1}{2} \Rightarrow \frac{3 \cdot 3^n - 1}{2} = p \Rightarrow 3 \cdot 3^n - 1 = 2p \Rightarrow 3 \cdot 3^n = 2p+1$$

$$\underset{3^n = 2m+1}{\Rightarrow} 3 \cdot (2m+1) = 2p+1 \Rightarrow 6m+3 = 2p+1 \Rightarrow 6m+2 = 2p \Rightarrow p = 3m+1$$

$\frac{3^n-1}{2} = m \Rightarrow m \in \mathbb{Z}$ dela suposatu dugunez eta $\frac{3^{n+1}-1}{2} = p$

$p = 3m+1$ denez frogatuta gelditzen da $p \in \mathbb{Z}$ dela beraz, $P(n+1)$ egiazkoa da. eta $\forall n \geq k$ $P(n)$ egiazkoa izango da.

$$\frac{5}{3}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\dots+\frac{1}{n}+\frac{1}{n+1}\right)$$

$$-\frac{3}{2}\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\dots+\frac{1}{n}+\frac{1}{n+1}\right.$$

$$-\frac{1}{6}\left(\frac{1}{4}+\frac{1}{5}+\dots+\frac{1}{n}+\frac{1}{n+1}\right)$$

$$\frac{5}{3}\left(1+\frac{1}{2}+\frac{1}{3}\right)-\frac{3}{2}\left(\frac{1}{2}+\frac{1}{3}\right)=\frac{5}{3}\cdot\frac{11}{6}-\frac{3}{2}\cdot\frac{5}{6}=\frac{55}{18}-\frac{15}{12}=\boxed{\frac{65}{36}}$$

4)

$$f(x)=\frac{1}{x^4-2x^2-1}$$

Definizio eremua, jarraitua eta deribagarria.
motor erlatiboak, minimo eta maximoak, gorakorra, beherakorra.
inflexio puntua. Grafika alurr.

$$x^4-2x^2-1=0 \underset{x^2=z}{\Rightarrow} z^2-2z-1=0$$

$$z=\frac{2\pm\sqrt{(-2)^2-4\cdot-1\cdot1}}{2}=\frac{2\pm\sqrt{8}}{2}=\begin{cases} z_1=1+\sqrt{2} \\ z_2=1-\sqrt{2}\end{cases}$$

$x_1=5'82$

$x_2=0'17$

-Definizio eremua ~~$(-\infty, 0'17)\cup(0'5$~~ $\cup(\mathbb{R}$, $\mathbb{R}-\{0'17, 5'82\}$

beraz, 2. kasuan gaude.

$a_n \to \sqrt[n]{a_n} =$

$R_k = a_{k+1} + a_{k+2} + a_{k+3} + < a_{k+1} + \frac{1}{5} a_{k+1} + \left(\frac{1}{5}\right)^2 a_{k+1} + =$

$= a_{k+1}\left(1 + \frac{1}{5} + \frac{1}{25} + \dots +\right) = a_{k+1}\left(\frac{1}{1-\frac{1}{5}}\right) = \frac{5}{4} a_{k+1}$

$R_5 = \frac{5}{4} a_6 = \frac{5}{4} \cdot \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12} \left(\frac{1}{5}\right)^6 = 1'8 \cdot 10^{-5}$ $= 0'00000$

$$\sum_{n=2}^{\infty} \frac{2n+3}{(n-1)n(n+2)} =$$

$$\frac{2n+3}{(n-1)n(n+2)} = \frac{A}{(n-1)} + \frac{B}{n} + \frac{C}{n+2}$$

$2n+3 = A(n \cdot (n+2)) + B(n-1)(n+2) + C(n-1)n$

$n=0 \quad 3 = \quad -2B \Rightarrow B = -\frac{3}{2}$

$n=1 \quad 5 = 2A \Rightarrow A = \frac{5}{2}$

$n=-2 \quad -1 = 6C \Rightarrow \boxed{C = -\frac{1}{6}}$

$\frac{5}{2} \sum \frac{1}{n-1} - \frac{3}{2} \sum \frac{1}{n} - \frac{1}{6} \sum \frac{1}{n+2} =$

3.1

$$\sum \frac{1\cdot 3\cdot 5\cdot 7\cdots(2n-1)}{2\cdot 4\cdot 6\cdot 8\cdots 2n} a^n = a_n$$

D'Alembert $\rightarrow \frac{a_{n+1}}{a_n} = \dfrac{\dfrac{1\cdot 3\cdot 5\cdots(2n-1)\cdot(2n+1-1)}{2\cdot 4\cdot 6\cdots 2(n+1)} a^{(n+1)}}{\dfrac{1\cdot 3\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n} a^n} = \frac{\cancel{1\cdot 3\cdot 5\cdots(2n-1)}\cdot 2n \cdot a^{n+1}\cdot \cancel{2\cdot 4\cdots 2n}}{\cancel{1\cdot 3\cdots(2n-1)}\cdot \cancel{2\cdot 4\cdot 6}\, 2n\cdot(2(n+1))\, a^n}$

$$= \frac{2n\cdot \cancel{a^n}\cdot a}{(2n+2)\cdot \cancel{a^n}} = \frac{2n\cdot a}{2n+2}$$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2n\cdot a+1}{2n+2} = \boxed{a}$ $\begin{cases} a<1 & \text{konbergentea} \\ a>1 & \text{dibergentea} \\ a=1 & \text{zalantzazko kasua.} \end{cases}$

Raaberen irizpidea!

$$n\left(1-\frac{a_{n+1}}{a_n}\right) = n\left(1-\frac{2n+1}{2n+2}\right) = n\left(\frac{2n+2-2n-1}{2n+2}\right) = n\left(\frac{2}{2n+2}\right) =$$

$$= \frac{n}{2n+2} = \boxed{\frac{1}{2}}$$ $a=1 \Rightarrow \frac{1}{2}<1$ dibergentea

b)

$$\sum_{n=1}^{\infty} \frac{1\cdot 3\cdots(2n-1)}{2\cdot 4\cdots 2n}\left(\frac{1}{5}\right)^n$$

~~a= 1/5 < 1 bera da~~ $a<1 \Rightarrow \frac{1}{5}<1$ beraz konbergentea da

$\dfrac{\frac{2n}{5}+1}{2n+1} \rightarrow$ eskuinetik hurbilduko da $\frac{1}{5}$-era.

$$\sum_{n=1}^{\infty} \frac{n+5}{n\cos\frac{1}{n}} = a_n$$

D'Alembert $\rightarrow \frac{a_{n+1}}{a_n} = \frac{\frac{n+1+5}{(n+1)\cos\frac{1}{n+1}}}{\frac{n+5}{n\cos\frac{1}{n+1}}} = \frac{(n+6)\cos\frac{1}{n}}{(n+5)\cos\frac{1}{n+1}}$

$$\lim_{n\to\infty} \frac{n+5}{n\cos\frac{1}{n}} = \frac{1}{1} = 1 \neq 0 \quad \text{Divergente}$$

$$\lim_{n\to\infty} \sqrt{n^3+2} - \sqrt{n^3-1} = \infty - \infty \Rightarrow \lim_{n\to\infty} \left(\sqrt{n^3+2} - \sqrt{n^3-1}\right) \cdot \frac{\left(\sqrt{n^3+2} + \sqrt{n^3-1}\right)}{\sqrt{n^3+2} + \sqrt{n^3-1}} =$$

$$= \frac{n^3+2-(n^3-1)}{\sqrt{n^3+2} + \sqrt{n^3-1}} = \frac{3}{\sqrt{n^3+2} + \sqrt{n^3-1}} = \boxed{0}$$

$$\lim_{n\to\infty} \frac{\ln\frac{1}{n}}{2(\ln n + \ln(n+1))} = \lim_{n\to\infty} \frac{\ln\frac{1}{n}}{2\ln(n(n+1))} = \lim_{n\to\infty} \frac{1}{2} \cdot \frac{\cancel{\ln 1}^{0} - \ln n}{\ln(n(n+1))} =$$

$\{\ln a + \ln b\} \sim \{\ln ab\}$

$$= \lim_{n\to\infty} -\frac{1}{2} \cdot \frac{\ln n}{\ln(n^2+n)} = \boxed{-\frac{1}{4}}$$

3)

a) $\sum_{n}^{\infty} \frac{7n+3}{n(n+1)(n+3)} = a_n$

D'Alembert

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{7(n+1)+3}{(n+1)(n+2)(n+4)}}{\dfrac{7n+3}{n(n+1)(n+3)}} = \frac{(7(n+1)+3)\cdot n(n+1)(n+3)}{(n+1)(n+2)(n+4)(7n+3)} =$$

$$= \frac{(7n+10)\cdot n\cancel{(n+1)}(n+3)}{\cancel{(n+1)}(n+4)(7n+3)} = \frac{7n^3+31n^2+30n}{7n^2+31n+12}$$

1) $\sum_{n=1}^{\infty} \frac{2}{n^2+3n+2}$

D'Alembert $\rightarrow \frac{a_{n+1}}{a_n} = \frac{\dfrac{\cancel{2}}{(n+1)^2+3(n+1)+2}}{\dfrac{\cancel{2}}{n^2+3n+2}} = \frac{n^2+3n+2}{(n+1)^2+3(n+1)+2} =$

$$= \frac{n^2+3n+2}{n^2+2n+1+3n+3+2} = \frac{n^2+3n+2}{n^2+5n+6}$$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n^2+3n+2}{n^2+5n+6} = \boxed{1}$ $1=1$ bada, zalantzazko kasuan gaude

Raaberen irizpidea erabiliko dugu:

$$n\left(1-\frac{a_{n+1}}{a_n}\right) = n\left(1-\frac{n^2+3n+2}{n^2+5n+6}\right) = n\left(\frac{\cancel{n^2}+\overset{2n}{\cancel{5n}}+\overset{4}{\cancel{6}}-(\cancel{n^2}+\cancel{3n}+\cancel{2})}{n^2+5n+6}\right) =$$

$$= n\left(\frac{2n+4}{n^2+5n+6}\right) = \frac{2n^2+4n}{n^2+5n+6}$$

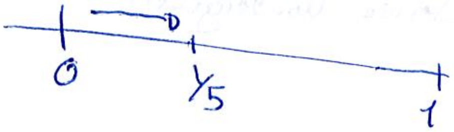
$\lim_{n\to\infty} n\left(1-\frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty} \frac{2n^2+4n}{n^2+5n+6} = \boxed{2}$ $2>1 \rightarrow$ Seriea ~~dibergentea d.~~ Konbergentea d.

2)

a) $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n \cdot \frac{1}{n} = a_n$

Cauchy-ren irizpidea $\Rightarrow \sqrt[n]{a_n} = \sqrt[n]{\left(\frac{1}{5}\right)^n \cdot \frac{1}{n}} = \frac{1}{5} \cdot \frac{1}{\sqrt[n]{n}} =$

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{1}{5} \cdot \frac{1}{\cancelto{1}{\sqrt[n]{n}}} = \frac{1}{5} \Rightarrow \frac{1}{5} < 1$ Konbergente!



$R_k \le \frac{l^{k+1}}{1-l} \Rightarrow R_5 \le \frac{\left(\frac{1}{5}\right)^6}{1-\frac{1}{5}} = 8 \cdot 10^{-5} = 0'00008$

b)

~~Badakigu~~

$\frac{5}{4}\left(\frac{1}{5}\right)^{k+1} \cdot \frac{1}{k+1}$

$R_1 = \frac{5}{4}\left(\frac{1}{5^2}\right) \cdot \frac{1}{2} = \frac{1}{40} > \frac{1}{10^2}$

$R_2 = \frac{5}{4}\left(\frac{1}{5^3}\right) \cdot \frac{1}{3} = \frac{1}{300} < \frac{1}{10^2}$

$S_2 = a_1 + a_2 = \frac{1}{5^2} \cdot \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{250} =$

b) $\sum_{n=1}^{\infty} \frac{(n+5)}{(n-8)} \left(\frac{e}{3}\right)^n = a_n$

- Cauchy-ren irizpidea $\to$ $\cancel{\frac{a_{n+1}}{a_n}}$ $\cdot \sqrt[n]{a_n} = \sqrt[n]{\frac{(n+5)}{(n-2)} \cdot \left(\frac{e}{3}\right)^n} =$

$= \sqrt[n]{\frac{n+5}{n-2}} \cdot \frac{e}{3}$

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \underbrace{\sqrt[n]{\frac{n+5}{n-2}}}_{\to 1} \cdot \frac{e}{3} = \frac{e}{3} < 1 \Rightarrow e < 3$

Seriea konbergentea da

c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+10)}} = a_n$

D'Alembert $\to$ $\frac{a_{n+1}}{a_n} = \frac{\frac{1}{\sqrt{(n+1)(n+11)}}}{\frac{1}{\sqrt{n(n+10)}}} = \sqrt{\frac{n(n+10)}{(n+1)(n+11)}} =$

$= \sqrt{\frac{n^2+10n}{n^2+12n+11}}$

$\lim_{n\to\infty} \sqrt{\frac{n^2+10n}{n^2+12n+11}} = \sqrt{1} = \boxed{1}$ zalantzazko kasua

Raaberen irizpidea $\to$ $n\left(1 - \sqrt{\frac{n^2+10n}{n^2+12n+11}}\right) = n\left(\cancel{\sqrt{\frac{n^2+12n+11}{n^2+12n+11}}} - \sqrt{\frac{n^2+10}{n^2+12}}\right.$

$= n$

Raaberen irizpidea

$$\lim_{n\to\infty} n\left(1-\frac{a_{n+1}}{a_n}\right) = n\left(1-\frac{(n^2+2n+1)\cdot 4}{4n^2+6n+2}\right) = n\left(1-\frac{4n^2+8n+4}{4n^2+6n+2}\right) =$$

$$= n\left(\frac{\cancel{4n^2}+6n+2-\cancel{4n^2}-8n-4}{4n^2+6n+2}\right) = -\frac{2n^2-2n}{4n^2+6n+2} = \boxed{-\frac{1}{2}}$$

$-2n-2$

$\boxed{-\frac{1}{2} < 1 \text{ dibergentea da}}$

1)

$$a_n = \frac{(n!)^2}{(2n)!}$$

D'Alembert $\to$ $$\frac{a_{n+1}}{a_n} = \frac{\frac{((n+1)!)^2}{(2n+2)!}}{\frac{(n!)^2}{(2n)!}} = \frac{((n+1)!)^2\cdot(2n)!}{(2n+2)!\cdot(n!)^2} = \frac{(n+1)^2}{4n^2+8n+4} =$$

$$= \frac{n^2+2n+1}{4n^2+8n+4}.$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{n^2+2n+1}{4n^2+8n+4} = \frac{1}{4}$$ $\frac{1}{4} < 1 \Rightarrow 1 < 4$ Konbergentea da

30)

$$\frac{\ln n}{e^n}$$

D'Alembert $\Rightarrow \dfrac{a_{n+1}}{a_n} = \dfrac{\frac{\ln n+1}{e^{n+1}}}{\frac{\ln n}{e^n}} = \dfrac{\ln(n+1)\cdot e^n}{\ln n \cdot e^{n+1}} =$

$$= \frac{\ln(n+1)\cdot \cancel{e^n}}{\ln n \cdot \cancel{e^n}\cdot e} = \frac{\ln(n+1)}{\ln n \cdot e}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\ln(n+1)}{\ln n \cdot e} = \frac{n}{\cancel{\ln}\, n\cdot e} = \frac{n}{n} = 1$$

Raaberen irizpidea

$$n\left(1-\frac{n}{n}\right) = n\left(\cancel{n}\,1-\frac{n}{n}\right) \Rightarrow n - \frac{n^2}{\cancel{n}} = n - \frac{\cancel{n}}{\cancel{n}}n = 0$$

$l<1$ beraz, dibergentea da.

2) $a_n = \dfrac{(n!)^2}{(2n)!}a^n$

D'Alembert $\Rightarrow \dfrac{a_{n+1}}{a_n} = \dfrac{\frac{((n+1)!)^2}{(2(n+1)!)}a^{n+1}}{\frac{(n!)^2}{(2n)!}\cdot a^n} = \dfrac{(n+1)\cancel{!}^2\cdot \cancel{a^{n+1}}^{\,a}\cdot \cancel{(n!)^2}}{\cancel{(n!)^2}\cdot \cancel{a^n}\cdot (2n+2)!} =$

$$= \frac{(n+1)^2\cdot a}{\cancel{2n+2}\; 4n^2+6n+2} = \frac{(n^2+2n+1)\,a}{4n^2+6n+2}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{an^2+2an+1}{4n^2+6n+2} = \boxed{\frac{a}{4}}$$

$\frac{a}{4}<1 \Rightarrow a<4 \;\; \Sigma \frac{a_{n+1}}{a_n}$ dibergentea

$\frac{a}{4}>1 \Rightarrow a>4 \cdot \Sigma \frac{a_{n+1}}{a_n}$ konbergentea

$a=4$ Zalantzazko.

(33)

$$a_n = \frac{n!}{n^n}$$

D'Alembert

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} = \frac{1}{n+1} \cdot \frac{\cancel{(n+1)}\,(n+1)! \cdot n^n}{(n+1)^n \cdot \cancel{n+1}} = \frac{n^n}{(n+1)^n} = \left(\frac{n}{n+1}\right)^n = \boxed{\frac{1}{e}}$$

(29)

$\sum_n^{\infty} \ln\left(1+\frac{1}{n^2}\right)$ gai positiboko seriea denez

$\left\{\sum_n^{\infty} \ln\left(1+\frac{1}{n^2}\right)\right\} \sim \left\{\sum_n^{\infty} \frac{1}{n^2}\right\}$ Serie harmoniko orokorra da,

$\epsilon = 2 > 1$ denez, konbergentea da.

(34)

$$a_n = (\sqrt{2}-\sqrt[3]{2})(\sqrt{2}-\sqrt[5]{2})\cdots(\sqrt{2}-\sqrt[2n+1]{2})$$

D'Alembert $\rightarrow \frac{a_{n+1}}{a_n} = \frac{\cancel{(\sqrt{2}-\sqrt[3]{2})\cdots(\sqrt{2}-\sqrt[2n+1]{2})}(\sqrt{2}-\sqrt[2n+2]{2})}{\cancel{(\sqrt{2}-\sqrt[3]{2})\cdots(\sqrt{2}-\sqrt[2n+1]{2})}} =$

$= \sqrt{2} - \sqrt[2n+2]{2}$

$$\lim_{n\to\infty} \sqrt{2} - \sqrt[2n+2]{2} = \sqrt{2}-1$$

$\sqrt{2}-1 < 1$ denez, Serie konbergentea da.

$\sum_n \frac{300}{n^n}$ emanik, $R_k < 0'0001$ izateko kalkulatu S_k.

D'alembert,

$$\frac{a_{n+1}}{a_n} = \frac{\frac{300}{(n+1)^{(n+1)}}}{\frac{300}{n^n}} = \frac{n^n}{(n+1)^{(n+1)}} \Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n^n}{(n+1)^{(n+1)}} =$$

$$= \lim_{n\to\infty} \frac{1}{n+1} \cdot \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \frac{1}{n+1} \cdot \left(\frac{n}{n+1}\right)^n = 0 < 1$$

0 ra doa eskuinetik beraz, 2. klasean gaude.

$$R_k \leq \frac{\frac{300}{k^k} \cdot \frac{300}{(k+1)^{(k+1)}}}{\frac{300}{k^k} - \frac{300}{(k+1)^{(k+1)}}} = A$$

$$\cancel{\frac{\frac{300}{k^k} \cdot \frac{300}{(k+1)^{k+1}}}{\frac{300}{k^k} \cdot \frac{300}{(k+1)^{(k+1)}}} < 0'001} \Rightarrow \frac{\frac{300}{k^k} \cdot \frac{300}{(k+1)^{k+1}}}{\frac{300}{k^k} - \frac{300}{(k+1)^{(k+1)}}} < 0'001 \Rightarrow$$

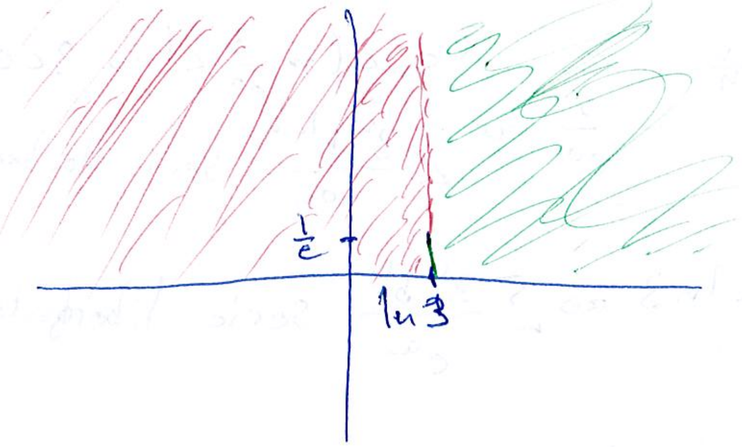
$$\Rightarrow \frac{\frac{\cancel{300} \cdot 300}{\cancel{k^k \cdot (k+1)^{(k+1)}}}}{\cancel{300}\left(\frac{k^k - (k+1)^{(k+1)}}{\cancel{k^k (k+1)^{k+1}}}\right)} < 0'001 \Rightarrow 300 < 0'001\left(k^k - (k+1)^{(k+1)}\right) \Rightarrow$$

$$\Rightarrow 300\,000 < -k^k + (k+1)^{(k+1)}$$

$k=6$ denean ~~k~~ ~~...~~ 776887 da beraz $k=6$, $\frac{a_7}{a_6} = 0'056 <$

denez,

$\ln \frac{1}{b} = 1 \Rightarrow \frac{1}{b} = e \Rightarrow b = \frac{1}{e}$, beraz, $\sum_n \left(\frac{1}{e}\right)^{\ln n} = \sum \frac{1}{n}$ harmonikoa



$\sum_n \frac{300}{n^n}$ emanik, R_3 bornatuko dugu,

a) $k=3$, $S_3 = \frac{300}{1^1} + \frac{300}{2^2} + \frac{300}{3^3}$

Cauchy-ren irizpidea;

$\sqrt[n]{a_n} = \frac{\sqrt[n]{300}}{n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{\sqrt[n]{300}}{n} = 0 < 1$ konbergentea da.

Eta estimazioa bornatuko da.

beraz, $2 \leq ...$

b)

$R_k \leq a_{k+1} + a_{k+2} + \dots + a_j + \frac{(\sqrt[j]{a_j})^{j+1}}{1 - \sqrt[j]{a_j}}$

$R_3 \leq a_4 + a_5 + \frac{(\sqrt[5]{a_5})^6}{1 - \sqrt[5]{a_5}} = \frac{300}{4^4} + \frac{300}{5^5} + \frac{\left(\sqrt[5]{\frac{300}{5^5}}\right)^6}{1 - \sqrt[5]{\frac{300}{5^5}}} = 1$

$\boxed{1'428}$

$$a_n = \frac{3^n b^{\ln n}}{e^{an}} \Rightarrow \sqrt[n]{a_n} = \sqrt[n]{\frac{3^n b^{\ln n}}{e^{an}}} = \frac{3 \cdot b^{\frac{\ln n}{n}}}{e^a}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{3 \cdot b^{\frac{\ln n}{n}}}{e^a} = \frac{3}{e^a}$$

$a_n < l \Rightarrow \frac{3}{e^a} < l \Rightarrow 3 < e^a \Rightarrow \ln 3 < a$ $\Rightarrow \sum \frac{3^n b^{\ln n}}{e^{an}}$ serie konbergentea da.

$1 < \frac{3}{e^a} \Rightarrow e^a < 3 \Rightarrow a < \ln 3 \Rightarrow \sum \frac{3^n b^{\ln n}}{e^{an}}$ serie dibergentea d.

$1 = \frac{3}{e^a} \Rightarrow a = \ln 3$, zalantzazko kasua.

$\sum_n \frac{3^n b^{\ln n}}{e^{n \ln 3}} = \sum_n \frac{\cancel{3^n} b^{\ln n}}{\cancel{3^n}} = \sum_n b^{\ln n}$ seria dugu.

D'Alembert-en irizpidea,

$$a_n = b^{\ln n} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{b^{\ln n+1}}{b^{\ln n}} = b^{\ln(n+1) - \ln(n)} = b^{\ln\left(\frac{n+1}{n}\right)} =$$

$\Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} b^0 = 1$ zalantzazko kasua.

Raaberen irizpidea,

$$n\left(1 - \frac{a_{n+1}}{a_n}\right) = n\left(1 - b^{\ln\left(\frac{n+1}{n}\right)}\right) \sim -n\left(\ln b^{\ln\left(\frac{n+1}{n}\right)}\right) =$$

$$= -n \ln\left(\frac{n+1}{n}\right) \ln b = \boxed{-\ln b}$$

$$\lim_{n\to\infty} n\left(1 - \frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty} -\ln b = \boxed{\ln \frac{1}{b}}$$

$\ln \frac{1}{b} < 1 \Rightarrow \frac{1}{b} < e^1 \Rightarrow \frac{1}{e} < b \Rightarrow \sum b^{\ln n}$ konbergentea

$1 < \ln \frac{1}{b} \Rightarrow \frac{1}{e} > b \Rightarrow \sum b^{\ln n}$ dibergentea

1)

a) $\ln(\sqrt{2}+\sqrt{2}i)^5 = 5\ln(\underbrace{\sqrt{2}+\sqrt{2}i}_{z'}) =$

$z' = \sqrt{2}+\sqrt{2}i \Rightarrow |z'| = \sqrt{\sqrt{2}^2+\sqrt{2}^2} = \sqrt{4} = \boxed{2}$

$\mathrm{Arg}(z') = \arctan\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \arctan\left(\sqrt{\frac{2}{2}}\right) = \arctan(1) =$

$= \boxed{\frac{\pi}{4}}$

$= \cancel{5 \cdot 2 \ln e^{\pi/4}}$ $= 5\ln 2 + i\left(\frac{\pi}{4}+2k\pi\right)$

$k=0$ denean, $A = 5\ln 2 + i\left(\frac{\pi}{4}\right) =$

b)

$A = \sqrt{(2+2i)} = \sqrt{z}$

$z = 2+2i \Rightarrow |z| = \sqrt{2^2+2^2} = \sqrt{8}$

$\mathrm{Arg}(z) = \arctan\left(\frac{2}{2}\right) = \frac{\pi}{4}$

$l = \sqrt[5]{8}$ $\left(\frac{\frac{\pi}{4}+2k\pi}{5}\right)$

$$\lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n} = \lim_{n\to\infty} \frac{2^n\cdot 2+3^n\cdot 3}{2^n+3^n} \underset{\uparrow}{=} \boxed{3}$$

4

30)

$$\sqrt{\frac{(a+1)(a+2)\cdots(a+n)}{n!}} \Rightarrow \lim_{n\to\infty}\sqrt{\frac{(a+1)(a+2)\cdots(a+n)}{n!}} \underset{\uparrow}{=} \lim_{n\to\infty}\sqrt{\frac{(a+n+1)}{n^n e^{-n}\sqrt{2\pi n}}} =$$

$\boxed{\text{Stolz}}$

$\boxed{\text{Stirling}}$

$$= \lim_{n\to\infty}\sqrt{\frac{a+n+1}{n^n e^{-n}\sqrt{2\pi n}}} = \sqrt{\frac{(a+n+1)e^n}{n^n\sqrt{2\pi n}}}$$

34)

$$\lim_{n\to\infty} n\left[\sqrt{n^2+1}-n\right] = \lim_{n\to\infty} n^2\left[\sqrt{\frac{n^2+1}{n}}-1\right] = 1$$

$$x^2 - 9x^4 + 8x = 0$$

$$x(x^6-9x^3+8) = 0 \Rightarrow x_1 = 0$$

$$x^3 = \frac{9\pm\sqrt{(-9)^2-4\cdot 1\cdot 8}}{2} = \frac{9\pm 7}{2} = \begin{cases} x_2^3 = 8 \\ x_3^3 = 1 \end{cases}$$

$$x_2 = \sqrt[3]{8} = \boxed{2}$$

$$x_3 = \boxed{1}$$

$$x_2 = 2 \Rightarrow 2_{\frac{0+2k\pi}{3}} = 2_{\frac{2k\pi}{3}}$$

$k=0$ deneca, 2_0

$k=1$ deneci $2_{\frac{2\pi}{3}}$

$k=2$ denecn $2_{\frac{4\pi}{3}}$

4)

c) $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n^3} = \lim_{n\to\infty} 1^{\infty}$ = indeterminazioa

$\ln l = \ln\left(1-\frac{1}{n}\right)^{n^3} \Rightarrow \lim_{n\to\infty} n^3 \ln\left(1-\frac{1}{n}\right) = \lim_{n\to\infty} -\frac{n^3}{n} =$

$= -n^2 = \boxed{-\infty}$

$\ln l = -n^2$

$e^{l} = e^{-n^2} \Rightarrow l = e^{-n^2} = \frac{1}{e^{n^2}} \Rightarrow \lim_{n\to\infty} e^{\frac{1}{n^2}} = \boxed{0}$

3)

b) $\{a_n\} \sim \{\ln(a_n+1)\} \to \{b_n - 1\} \sim \{\ln b_n\}$

$\lim_{n\to\infty} n \ln\sqrt{\frac{n+a}{n-a}} = \lim_{n\to\infty} \frac{n}{2}\ln\left(\frac{n+a}{n-a}\right) = \lim_{n\to\infty} \frac{n}{2}\ln\left(\frac{n}{n-a}+\frac{a}{n-a}\right) =$

$= \frac{n}{2}\cdot\left(\frac{n+a}{n-a}-1\right) = \frac{n}{2}\cdot\left(\frac{\cancel{n}+a-\cancel{n}+a}{n-a}\right) = \frac{n}{2}\cdot\frac{2a}{n-a} = \frac{2an}{2n-2a} = \boxed{a}$

3)

a) $\lim_{n\to\infty} \frac{\sqrt{1\cdot2\cdot3}+\sqrt{2\cdot3\cdot4}+\dots+\sqrt{n(n+1)(n+2)}}{n^2\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{1\cdot2\cdot3}+\dots+\sqrt{n(n+1)(n+2)}+\sqrt{(n+1)(n+2)(n+3)}-(a_n)}{(n+1)^2\sqrt{n+1}-n^2\sqrt{n}} =$

$\boxed{\text{Stolz}}$

$\{b_n\}$

1) b_n hertsiki monotonoa da

2) Existitzen da $\lim_{n\to\infty}\frac{a_{n+1}-a_n}{b_{n+1}-b_n}$

3) $\lim_{n\to\infty} b_n = \infty$

$\lim_{n\to\infty} \frac{\sqrt{(n+1)(n+2)(n+3)}}{(n+1)^2\sqrt{n+1}-n^2\sqrt{n}} =$

$\lim_{n\to\infty} \frac{\sqrt{(n^2+3n+2)(n+3)}}{(n^2+2n+1)\sqrt{n+1}-n^2\sqrt{n}} =$

$\lim_{n\to\infty} \frac{\sqrt{n^3+3n^2+2n+3n^2+9n+6}}{}$

$= \boxed{0}$

a) $\lim_{n\to\infty} (n+\sqrt{n}) \cos\left(\frac{\pi}{n}\right) \log\left(1+\frac{1}{n}\right) =$

$= \lim_{n\to\infty} (n+\sqrt{n}) \left(1-\frac{\left(\frac{\pi}{n}\right)^2}{2}\right) \log\left(1+\frac{1}{n}\right) = \lim_{n\to\infty} (n+\sqrt{n}) \left(1-\frac{\pi^2}{2n^2}\right) \log\left(1+\frac{1}{n}\right) =$

$= \lim_{n\to\infty} \left(1-\frac{\pi^2}{2n^2}\right) \log\left(1+\frac{1}{n}\right)^{(n+\sqrt{n})} = \lim_{n\to\infty} \left(1-\frac{\pi^2}{2n^2}\right) \log\left(\left(1+\frac{1}{n}\right)^n\right)^{\frac{n+\sqrt{n}}{n}} =$

$= \lim_{n\to\infty} \left(1-\frac{\pi^2}{2n^2}\right) \cdot \log e^{\frac{n+\sqrt{n}}{n}} = \log e = \boxed{1}$

23)

$\lim_{n\to\infty} \frac{4}{n}\left[\left(\frac{4}{n}\right)^2 + \left(\frac{8}{n}\right)^2 + \left(\frac{12}{n}\right)^2 + \dots + \left(\frac{4n}{n}\right)^2\right] = \frac{4}{n}\left[\frac{4^2+8^2+\dots+(4n)^2}{n^2}\right] =$

$= 4 \lim_{n\to\infty} \frac{4^2+8^2+\dots+(4n)^2}{n^3} = 4 \lim_{n\to\infty} \frac{4^2+8^2+\dots+(4n)^2+(4(n+1))^2-(4^2+8^2+\dots+(4n)^2)}{(n+1)^3-n^3} =$

$= 4 \lim_{n\to\infty} \frac{(4(n+1))^2}{(n^2+2n+1)(n+1)-n^3} = 4 \lim_{n\to\infty} \frac{16n^2+32n+16}{\cancel{n^3}+2n^2+n+n^2+2n+1-\cancel{n^3}} =$

$= 4 \lim_{n\to\infty} \frac{16n^2+32n+16}{3n^2+3n+1} = \frac{4\cdot 16}{3} = \frac{64}{3}$

3)

$$|z-3|-|z+3|=4$$

$$|(a-3)+bi|-|(a+3)+bi|=4 \Rightarrow \cancel{\sqrt{(a-3)^2+b^2}-\sqrt{(a+3)^2+b^2}=4} \Rightarrow$$

$$\cancel{\Rightarrow (a-3)^2+b^2-(a+3)^2-b^2=4} \Rightarrow a^2$$

$$\Rightarrow \sqrt{(a-3)^2+b^2}=4+\sqrt{(a+3)^2+b^2} \Rightarrow$$

$$\Rightarrow (a-3)^2+b^2=\left[4+\sqrt{(a+3)^2+b^2}\right]^2 \Rightarrow$$

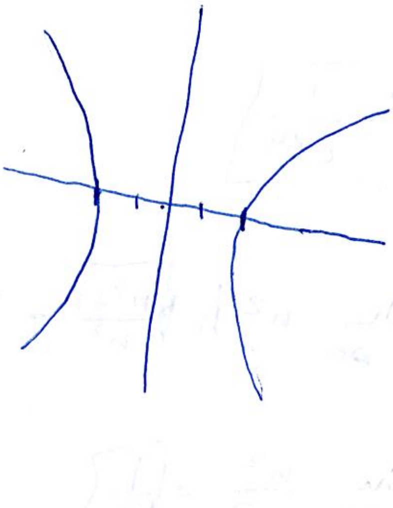
$$\Rightarrow (a-3)^2+b^2=16+(a+3)^2+b^2+8\sqrt{(a+3)^2+b^2} \Rightarrow$$

$$\Rightarrow \cancel{a^2}-6a+\cancel{9}+\cancel{b^2}=16+\cancel{a^2}+6a+\cancel{9}+\cancel{b^2}+8\sqrt{(a+3)^2+b^2} \Rightarrow$$

$$\Rightarrow 12a+16=-8\sqrt{(a+3)^2+b^2} \Rightarrow \Rightarrow -3a-4=2\sqrt{(a+3)^2+b^2} \Rightarrow$$

$$\Rightarrow (-3a-4)^2=4((a+3)^2+b^2)^2 \Rightarrow 9a^2+24a+16=4(a^2+6a+9+b^2) \Rightarrow$$

$$\Rightarrow 5a^2-4b^2=20 \Rightarrow \frac{5a^2}{20}-\frac{4b^2}{20}=1 \Rightarrow \frac{a^2}{4}-\frac{b^2}{5}=1 \Rightarrow$$



$$\Rightarrow \frac{a^2}{2^2}-\frac{b^2}{(\sqrt{5})^2}$$

2)

1) $\lim_{n\to\infty}\left(1+\ln\frac{n^2-3n+5}{n^2+9n}\right)^{\frac{2n^2+3}{n+1}} =$

$= \lim_{n\to\infty}\left(1+\ln 1\right)^{\infty} = 1^{\infty} =$ Indeterminación $= \ell$

$\ln \ell = \ln\left(1+\ln\frac{n^2-3n+5}{n^2+9n}\right)^{\frac{2n^2+3}{n+1}} = \frac{2n^2+3}{n+1}\ln\left(1+\ln\frac{n^2-3n+5}{n^2+9n}\right)$

$\Rightarrow \lim_{n\to\infty}\ln\ell = \lim_{n\to\infty}\frac{2n^2+3}{n+1}\ln\frac{n^2-3n+5}{n^2+9n} = \lim_{n\to\infty}\frac{2n^2+3}{n+1}\cdot\left(\frac{n^2-3n+5}{n^2+9n}-1\right)$

$= \lim_{n\to\infty}\frac{2n^2+3}{n+1}\cdot\frac{n^2-3n+5-n^2-9n}{n^2+9n} = \lim_{n\to\infty}\frac{2n^2+3}{n+1}\cdot\frac{-12n+5}{n^2+9n} =$

$= \lim_{n\to\infty}\frac{-24n^3+10n^2-36n+15}{n^3+9n^2+n^2+9n} = \boxed{-24}$

$\ln\ell=-24 \Rightarrow \cancel{\ln}\,\ell = e^{-24} \Rightarrow \ell = e^{-24} \Rightarrow \boxed{\ell=\frac{1}{e^{24}}}$

2) $\lim_{n\to\infty} n\left[\sqrt{n^2+1}-n\right] = \lim_{n\to\infty} n^2\left[\frac{\sqrt{n^2+1}}{n}-1\right] = \lim_{n\to\infty} n^2\ln\sqrt{\frac{n^2+1}{n^2}} =$

$= \lim_{n\to\infty}\frac{n^2}{2}\ln\frac{n^2+1}{n^2} = \lim_{n\to\infty}\frac{n^2}{2}\ln 1+\frac{1}{n^2} = \lim_{n\to\infty}\frac{n^2}{2n^2} = \boxed{\frac{1}{2}}$

10)

$$\cancel{\lim_{n\to\infty} \frac{3n \sin^2\left(\frac{1}{n}\right) \ln\left(1+\frac{1}{n}\right)}{n+5}} = \lim_{n\to\infty} \frac{3n \sin\left(\frac{1}{n}\right)\cdot \sin\left(\frac{1}{n}\right)\cdot \ln\left(1+\frac{1}{n}\right)}{n+5} =$$

$=$

14)

$$\lim_{n\to\infty} \left(\sqrt{\frac{1-n}{1-2n}}\right)^{\frac{2n-1}{3n+1}} = \left(\sqrt{\frac{1}{2}}\right)^{2/3} = \left(\frac{1}{2}\right)^{\frac{1}{2}\cdot\frac{2}{3}} = \frac{1}{2}^{1/3} = \frac{1}{\sqrt[3]{2}}$$

17)

$$\lim_{n\to\infty} \frac{\sin(n^{-2})\left(\sqrt[3]{1+n^2} - \sqrt[3]{(2+n)^2}\right)}{\sqrt[3]{1+n^2}\cdot \ln\left(\frac{n}{n+1}\right)} = \lim_{n\to\infty} \frac{\frac{1}{n^2}\cdot\left(\sqrt[3]{1+n^2} - \sqrt[3]{n^2+4n+4}\right)}{\sqrt[3]{1+n^2}\cdot \ln\left(\frac{n}{n+1}\right)} =$$

$$\boxed{\{\sin(n^{-2})\} \sim \left\{\frac{1}{n^2}\right\}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n^2}\cdot\left(\sqrt[3]{1+\cancel{n^2}-\cancel{n^2}-4n-4}\right)}{\sqrt[3]{1+n^2}\cdot \ln\left(\frac{n}{n+1}\right)} = \lim_{n\to\infty} \frac{\frac{1}{n^2}\cdot \sqrt[3]{-4n-3}}{\sqrt[3]{1+n^2}\cdot \ln\left(\frac{n}{n+1}\right)} =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n^2}\cdot \sqrt[3]{-4n-3}}{\sqrt[3]{1+n^2}\cdot \left(\frac{n}{n+1} - 1\right)} = \lim_{n\to\infty} \frac{(n+1)\cdot \sqrt[3]{-4n-3}}{n^3\cdot \sqrt[3]{1+n^2} - \sqrt[3]{1+n^2}} = \lim_{n\to\infty} \frac{n\sqrt[3]{-4n-3} + \sqrt[3]{-4n-3}}{n^3\cdot\sqrt[3]{1+n^2} - \sqrt[3]{1+n^2}} =$$

$$= \lim_{n\to\infty} \frac{n}{n^3} = 0$$

dallo polinomio allo stesso grado denom. infinitore emette o grado di finito ordine hand... dello

$$\lim_{n\to\infty} \frac{2^n - 5^n}{10^n} = \lim_{n\to\infty} \frac{2^n}{10^n} - \lim_{n\to\infty} \frac{5^n}{10^n} = \lim_{n\to\infty} \left(\frac{1}{5}\right)^2 - \lim_{n\to\infty} \left(\frac{1}{2}\right)^2 =$$

$= \boxed{0}$

5) $\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right) = \lim_{n\to\infty}\sqrt{n}\left(\frac{\sqrt{n+1}}{\sqrt{n}}-\frac{\sqrt{n}}{\sqrt{n}}\right) = \lim_{n\to\infty}\sqrt{n}\left(\sqrt{\frac{n+1}{n}}-1\right) =$

$= \lim_{n\to\infty}\sqrt{n}\cdot\ln\sqrt{\frac{n+1}{n}} = \lim_{n\to\infty}\frac{\sqrt{n}}{2}\cdot\ln\frac{n+1}{n} = \lim_{n\to\infty}\frac{\sqrt{n}}{2}\cdot\ln\left(1+\frac{1}{n}\right) =$

$= \lim_{n\to\infty}\frac{\sqrt{n}}{2n} = \cancel{\lim_{n\to\infty}}\ 0$

6) $\lim_{n\to\infty}\frac{2^{2n}(n!)\sqrt{n}}{(2n+1)!} \underset{\text{Stirling}}{=} \lim_{n\to\infty}\frac{2^{2n}\left(n^n e^{-n}\sqrt{2\pi n}\right)\sqrt{n}}{(2n+1)^{2n+1}e^{-(2n+1)}\sqrt{2\pi\cdot(2n+1)}} =$

$= \lim_{n\to\infty}\frac{2n\cdot 2n^{2n}\,e^{-n}\cdot n\sqrt{2\pi}}{(2n+1)^{2n}\cdot(2n+1)e^{-2n-1}\sqrt{4\pi\cdot n+2\pi}} =$

7) $\lim_{n\to\infty}\left(\frac{n^2-1}{n^2}\right)^{\sqrt{n}} = 1^\infty$ = indeterminación

$\ln l = \lim \ln\left(\frac{n^2-1}{n^2}\right)^{\sqrt{n}} \Rightarrow \lim_{n\to\infty}\ln l = \lim_{n\to\infty}\sqrt{n}\ln\left(1-\frac{1}{n^2}\right) = \lim_{n\to\infty}\frac{\sqrt{n}}{-n^2} = \boxed{0}$

8) $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n^3} = \lim_{n\to\infty} 1^\infty$ = indeterminación

$\ln l = \ln\left(1-\frac{1}{n}\right)^{n^3} \Rightarrow \lim_{n\to\infty} n^3\ln\left(1-\frac{1}{n}\right) = \lim_{n\to\infty}\frac{n^3}{-n} = n^2 = \boxed{\infty}$

9) $\lim_{n\to\infty}\frac{(2n+1)^3-(2n-1)^3}{3n^2+1} = \lim_{n\to\infty}\frac{(4n^2+4n+1)(2n+1)-(4n^2-4n+1)(2n-1)}{3n^2+1} = \lim$

$\lim_{n\to\infty}\frac{\cancel{8n^3}+8n^2+\cancel{2n}+4n^2+\cancel{4n}+1-\cancel{8n^3}+8n^2-\cancel{2n}+4n^2-\cancel{4n}+1}{3n^2+1} =$

$\lim_{n\to\infty}\frac{\cancel{4n^2}\ 16n^2+2}{3n^2+1} = \boxed{\frac{16}{3}}$

1)

$$\lim_{n\to\infty} \frac{2^n - 3^n}{\ln n} = -\infty$$

2)

$$\lim_{n\to\infty} \frac{2^{2n}(n!)^2}{\sqrt{n}\,(2n)!} \underset{\boxed{\text{Stirling}}}{=} \lim_{n\to\infty} \frac{2^{2n}\left(n^n \cdot e^{-n}\sqrt{2\pi n}\right)^2}{\sqrt{n}\left(2n^{2n}\cdot e^{-2n}\sqrt{2\pi \cdot 2n}\right)} = \lim_{n\to\infty} \frac{\cancel{2^{2n}}\cancel{n^{2n}}\cdot \cancel{e^{-2n}}\, 2\pi n}{\sqrt{n}\cdot \cancel{2^{2n}}\cancel{n^{2n}}\cdot \cancel{e^{-2n}}\cdot 2\sqrt{\pi n}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{2}\pi n}{\sqrt{n}\cdot \cancel{2}\sqrt{\pi n}} = \frac{\pi n}{\sqrt{\pi n n}} = \frac{\pi \cancel{n}}{\cancel{n}\sqrt{\pi}} = \frac{\pi}{\sqrt{\pi}} = \boxed{\sqrt{\pi}}$$

3)

$$\lim_{n\to\infty} \left(\frac{1}{n^2}\right) \frac{\frac{1}{n^3}\sin\frac{1}{n}}{\left(1-\cos\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{1}{n^5}\cdot\frac{\sin\frac{1}{n}}{1-\cos\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{n^6\cdot\left(1-\cos\frac{1}{n}\right)} =$$

$$= \lim_{n\to\infty} \frac{1}{n^6\cdot\left(\frac{\frac{1}{n}}{2}\right)} = \frac{2n}{n^6} = 0$$

4)

$$\lim_{n\to\infty} n\left(\sqrt{\frac{n+1}{n}} - 1\right) = \infty \cdot 0 = \text{indeterminado}$$

$$\cancel{\lim\left(\sqrt{\frac{n+1}{n}}-1\right) = \ln n + \ln\left(\sqrt{\frac{n+1}{n}}\right)}$$

$$\lim_{n\to\infty} n\cdot\ln\left(\sqrt{\frac{n+1}{n}}\right) = \frac{n}{2}\ln\frac{n+1}{n} = \lim_{n\to\infty}\frac{n}{2}\ln\frac{n}{n}+\frac{1}{n} =$$

$$= \lim_{n\to\infty}\frac{n}{2}\ln\left(1+\frac{1}{n}\right) = \lim_{n\to\infty}\frac{\cancel{n}}{2\cdot\cancel{n}} = \boxed{\frac{1}{2}}$$

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$$\lim_{n\to\infty} \frac{1^2+2^2+3^2+\dots+n^2}{n^3} \underset{\boxed{Stolz}}{=} \lim_{n\to\infty} \frac{1^2+2^2+3^2+\dots+n^2+(n+1)^2-(1^2+2^2+\dots+n^2)}{(n+1)^3-n^3} =$$

1) $\{b_n\}$ hertsiki monotonoa

2) Existitzen da $\lim_{n\to\infty} \frac{a_{n+1}-a_n}{b_{n+1}-b_n}$

3) $\lim_{n\to\infty} b_n = \infty$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{(n+1)^3-n^3} = \lim_{n\to\infty} \frac{(n+1)^2}{(n^2+2n+1)(n+1)-n^3} = \lim_{n\to\infty} \frac{(n+1)^2}{\cancel{n^3}+2n^2+n+2n^2\dots-\cancel{n^3}} =$$

$$\lim_{n\to\infty} \frac{n^2+2n+1}{3n^2+3n+2} = \boxed{\frac{1}{3}}$$

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$$\lim_{n\to\infty} \frac{1^2+2^2+\dots+n^2}{1+2+\dots+n} \tan\frac{1}{n} \underset{\boxed{Stolz}}{=} \lim_{n\to\infty} \frac{\cancel{1^2+2^2+\dots+n^2}+(n+1)^2-\cancel{(1^2+2^2+\dots+n^2)}}{\cancel{1+2+\dots+n+n+1}\ \cancel{(1+2+3+\dots+n)}(n)} \cancel{\tan}\frac{1}{4} =$$

1)

2)

3)

$$\cancel{= \lim_{n\to\infty} \frac{(n+1)^2}{(n+1)} \cdot \tan\frac{1}{n}} = \lim_{n\to\infty} (n+1)\cdot\tan\frac{1}{n} = 0\cdot\infty$$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{1+2+\dots+(n+1)\cdot(n+1)-(1+2+\dots+n)} = \lim_{n\to\infty} \frac{(n+1)^2}{\frac{(n+2)(n+1)(n+1)}{2} - \frac{(n+1)(n)}{2}} = \lim_{n\to\infty} \frac{(n+1)^2}{\frac{(n^2+3n+2)(n+1)}{2} - n\dots}$$

$$(n+1)\left(\frac{(n+1)(n+2)}{2} - \dots\right)$$

$$= \lim_{n\to\infty} \frac{(n+1)^2(n+1)}{n^3+3n^2+2n+n^2+3n+2-n^2-n} = \lim_{n\to\infty} \frac{2(n+1)^2}{\frac{n^2+3n+2}{2n}} = \boxed{\frac{2}{3}}$$

$$\frac{n^2+3n+2}{2} - n^2$$

32)

$$\frac{1^2+2^2\sin\frac{1}{2}+\dots+n^2\sin\frac{1}{n}}{n^2}$$

$$\lim_{n\to\infty}\frac{\overbrace{1^2+2^2\sin\frac{1}{2}+\dots+n^2\sin\frac{1}{n}}^{a_n}}{\underbrace{n^2}_{\{b_n\}}}=\lim_{n\to\infty}\frac{1^2+2^2\sin\frac{1}{2}+\dots+n^2\sin\frac{1}{n}+(n+1)^2\sin\frac{1}{n+1}-(a_n)}{(n+1)^2-n^2}=$$

$\boxed{\text{stolz}}$

1) $\{b_n\}$ hertzilli monotonon d.

2) $\lim\limits_{n\to\infty}\frac{a_{n+1}-a_n}{b_{n+1}-b_n}$ existitzen d.

3) $\lim\limits_{n\to\infty} b_n=\infty$

33) *

$=\lim\limits_{n\to\infty}\ln\left(a\cdot\sqrt[n+1]{b}\right)=$

$=\lim\limits_{n\to\infty}\ln a+\ln\left(\sqrt[n+1]{b}\right)=\ln a+\lim\limits_{n\to\infty}\left(\sqrt[n+1]{b}\right)=$

$=\ln a+0=\boxed{\ln a}$

$$=\lim_{n\to\infty}\frac{(n+1)^2\sin\frac{1}{n+1}}{(n+1)^2-n^2}=\frac{\frac{(n+1)^2}{(n+1)}}{(n+1)^2-n^2}=\lim\frac{n+1}{n^2+2n+1-n^2}=\frac{n+1}{2n+1}=$$

$\boxed{\frac{1}{2}}$

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$$\lim_{n\to\infty}\frac{\ln\left(\sqrt{a+\sqrt{b}}\cdot\sqrt[3]{a+\sqrt[3]{b}}\cdot\dots\cdot\sqrt[n]{a+\sqrt[n]{b}}\right)}{\sin 1+\sin\frac{1}{2}+\dots+\sin\frac{1}{n}}=\boxed{\text{stolz}}$$

1)

2)

3)

$$=\lim_{n\to\infty}\frac{\ln\left(\sqrt{a+\sqrt{b}}\cdot\sqrt[3]{a+\sqrt[3]{b}}\cdot\dots\cdot\sqrt[n]{a+\sqrt[n]{b}}\cdot\sqrt[n+1]{a+\sqrt[n+1]{b}}\right)-\ln\left(\sqrt{a+\sqrt{b}}\cdot\sqrt[3]{a+\sqrt[3]{b}}\cdot\dots\cdot\sqrt[n]{a+\sqrt[n]{b}}\right)}{\left(\sin 1+\sin\frac{1}{2}+\dots+\sin\frac{1}{n}+\sin\frac{1}{n+1}\right)-\left(\sin 1+\dots+\sin\frac{1}{n}\right)}=$$

$$=\lim_{n\to\infty}\frac{\cancel{\ln\left(\sqrt{a+\sqrt{b}}\right)}+\cancel{\ln\left(\sqrt[3]{a+\sqrt[3]{b}}\right)}+\dots+\cancel{\ln\sqrt[n]{a+\sqrt[n]{b}}}+\ln\left(\sqrt[n+1]{a+\sqrt[n+1]{b}}\right)-\cancel{\left(\ln\left(\sqrt{a+\sqrt{b}}\right)+\dots+\ln\left(\sqrt[n]{a+\sqrt[n]{b}}\right)\right)}}{\sin\frac{1}{n+1}}=$$

$\{\ln(a\cdot b)=\ln a+\ln b\}$

$$=\lim_{n\to\infty}\frac{\ln\left(\sqrt[n+1]{a}\cdot\sqrt[n+1]{b}\right)}{\sin\frac{1}{n+1}}=\lim_{n\to\infty}\frac{1}{n+1}\cdot\frac{\ln\left(a\cdot\sqrt[n+1]{b}\right)}{\sin\frac{1}{n+1}}=\lim_{n\to\infty}\frac{1}{\cancel{n+1}}\cdot\frac{\ln\left(a\sqrt[n+1]{b}\right)}{\frac{1}{\cancel{n+1}}}=*$$

$\left\{\sin\frac{1}{n+1}\right\}\sim\left\{\frac{1}{n+1}\right\}$

Analisi Matematica

Aritmetica . Segiodi

5.

20)

$$n\left(\sqrt{n^2+1}-n\right)$$

$$\lim_{n\to\infty} n\left(\sqrt{n^2+1}-n\right) = \lim_{n\to\infty} n^2\left(\frac{\sqrt{n^2+1}}{n}-\frac{n}{n}\right) = \lim_{n\to\infty} n^2\left(\sqrt{\frac{n^2+1}{n^2}}-1\right) =$$

$$= \lim_{n\to\infty} n^2\left(\ln\sqrt{\frac{n^2+1}{n^2}}\right) = \lim_{n\to\infty} n^2\left(\ln\left(\frac{n^2+1}{n^2}\right)^{1/2}\right) = \lim_{n\to\infty} \frac{n^2}{2}\ln\frac{n^2+1}{n^2} =$$

$$\left\{\sqrt{\frac{n^2+1}{n^2}}-1\right\} \sim \left\{\ln\sqrt{\frac{n^2+1}{n^2}}\right\}$$

$$= \lim_{n\to\infty} \frac{n^2}{2}\ln\left(1+\frac{1}{n^2}\right) = \lim_{n\to\infty} \frac{n^2}{2}\cdot\frac{1}{n^2} = \boxed{\frac{1}{2}}$$

$$\left\{\left\{\ln\left(\frac{1}{n^2}+1\right)\right\} \sim \left\{\frac{1}{n^2}\right\}\right\}$$

23)

$$\left(\frac{2}{n+1}\right)^{\frac{2}{2+\ln n}}$$

$$\lim_{n\to\infty}\left(\frac{2}{n+1}\right)^{\frac{2}{2+\ln n}} = 0^{\infty}$$

$$\ln l = \lim_{n\to\infty} \ln\left(\frac{2}{n+1}\right)^{\frac{2}{2+\ln n}} = \lim_{n\to\infty}\frac{2}{2+\ln n}\cdot\ln\left(\frac{2}{n+1}\right) =$$

$$= \lim_{n\to\infty}\frac{2}{2+\ln n}\cdot\left(\ln(2)-\ln(n+1)\right) = \lim_{n\to\infty}\frac{2\ln(2)-2\ln(n+1)}{2+\ln n} =$$

$$= \lim_{n\to\infty}\frac{2\ln(2)-2n}{2+n-1} = \cancel{\lim_{n\to\infty}\frac{2}{n+1}}\ \boxed{-2}$$

$$\ln l = -2$$

$$\cancel{\ln} e^{l} = e^{-2}$$

$$l = e^{-2}$$

24)

$$\sqrt[3]{n+\sqrt{n}} - \sqrt[3]{n}$$

$$\cancel{\lim_{n\to\infty}\sqrt[3]{n+\sqrt{n}}-n}$$

$$\lim_{n\to\infty}\sqrt[3]{n}\left(\frac{\sqrt[3]{n+\sqrt{n}}}{\sqrt{n}}-1\right) = \lim_{n\to\infty}\sqrt[3]{n}\ \ln\left(\sqrt[3]{\frac{n+\sqrt{n}}{\sqrt{n}}}\right) = \frac{1}{3}\sqrt[3]{n}\ \ln\left(\frac{n+\sqrt{n}}{\sqrt{n}}\right) =$$

$$= \lim_{n\to\infty}\frac{\sqrt[3]{n}}{3}\ \ln\left(\frac{\sqrt{n}}{\sqrt{n}}+1\right) = \lim_{n\to\infty}\frac{\sqrt[3]{n}}{3}\cdot\frac{n}{\sqrt{n}} = \cancel{\frac{n^{\frac{1}{3}+\frac{3}{3}}}{3n^{\frac{1}{2}}} = \frac{n^{1/3}}{3n^{1/2}}} =$$

$$= \lim_{n\to\infty}\frac{\sqrt[3]{n}\cdot\sqrt{n}}{3n} = \lim_{n\to\infty}\frac{n^{\frac{1}{3}+\frac{1}{2}}}{3n} = \frac{n^{\frac{2}{6}+\frac{3}{6}}}{3n} = \frac{n^{5/6}}{3n} = \boxed{0}$$

13)

$$\lim_{n\to\infty} \frac{2^{2n}(n!)^2}{\sqrt{n}\,(2n)!} = \lim_{n\to\infty} \frac{2^{2n}\left(n^n e^{-n}\sqrt{2\pi n}\right)^2}{\sqrt{n}\,(2n)^{2n}\, e^{-2n}\sqrt{2\pi\cdot 2n}} = \lim_{n\to\infty} \cancel{\frac{2^{2n}\, n^{2n}\, e^{-2n}\, 2\pi\cdot n}{\sqrt{n}\, 2^{2n} n^{2n}\, e^{-2n}\sqrt{2\pi\cdot 2n}}}$$

$\left\{ n! \sim \left(n^n e^{-n}\sqrt{2\pi n}\right) \right\}$ stirling

$$= \lim_{n\to\infty} \frac{2\cancel{n}^{2n}\cdot \cancel{e^{-2n}}\cdot 2\pi\cdot n}{\sqrt{n}\; 2\cancel{n}^{2n}\; \cancel{e^{-2n}}\sqrt{2\pi 2n}} = \lim_{n\to\infty} \frac{2\pi\cdot n}{2\sqrt{n}\cdot\sqrt{2\pi n}} = \lim_{n\to\infty} \frac{2\pi\, n}{\sqrt{2\pi n^2}} =$$

$$= \lim_{n\to\infty} \frac{2\pi\, \cancel{n}}{\cancel{n}\sqrt{2\pi}} = \frac{2\pi}{\sqrt{2\pi}}\cdot\frac{\sqrt{2\pi}}{\sqrt{2\pi}} = \frac{\cancel{2\pi}\sqrt{2\pi}}{\cancel{2\pi}} = \sqrt{2\pi}$$

20)

$n\left(\sqrt{n^2+1}-n\right)$

$$\lim_{n\to\infty} n\left(\sqrt{n^2+1}-n\right) = \lim_{n\to\infty} n^2\left(\frac{\sqrt{n^2+1}}{n}-\frac{n}{n}\right) = \lim_{n\to\infty} n^2\left(\frac{\sqrt{n^2+1}}{n}-1\right) =$$

$$= \lim_{n\to\infty} n^2\left(\sqrt{\frac{n^2+1}{n^2}}-1\right) = \lim_{n\to\infty} \frac{n^2}{2}\ln\left(\frac{n^2+1}{n^2}\right) = \lim_{n\to\infty} \frac{n^2}{2}\ln\left(\frac{n^2}{n^2}+\frac{1}{n^2}\right) =$$

$$= \lim_{n\to\infty} \frac{n^2}{2}\ln\left(1+\frac{1}{n^2}\right) = \lim_{n\to\infty} \frac{n^2}{2n^2} = \boxed{\frac{1}{2}}$$

22)

$n\left(\sqrt{\frac{n+1}{n}}-1\right)$

$$\lim_{n\to\infty} n\ln\sqrt{\frac{n+1}{n}} = \lim_{n\to\infty} \frac{n}{2}\ln\frac{n+1}{n} = \lim_{n\to\infty} \frac{n}{2}\ln\left(\frac{n}{n}+\frac{1}{n}\right) =$$

$$= \lim_{n\to\infty} \frac{n}{2}\ln\left(\frac{1}{n}+1\right) = \lim_{n\to\infty} \frac{\cancel{n}}{2}\cdot\frac{1}{\cancel{n}} = \boxed{\frac{1}{2}}$$

14)

$$\frac{n+1}{\ln n}$$

Infinitu ordena handiagokoa denez $n+1$ $\ln n$ baino limitea infinitora joango da.

$$\lim_{n\to\infty} \frac{n+1}{\ln n} = \infty$$

15)

$n - \ln n$

$$\lim_{n\to\infty} n - \ln n = \infty$$

16)

$$\lim_{n\to\infty} \frac{1^2+2^2+\dots+n^2}{n^3+\ln n} \underset{\boxed{\text{Stolz}}}{=} \lim_{n\to\infty} \frac{\cancel{1^2+2^2+\dots+n^2}+(n+1)^2-(\cancel{1^2+2^2+\dots+n^2})}{(n+1)^3+\ln(n+1)-(n^3+\ln n)} =$$

1) $\{b_n\}$ hertsiki monotona da

2) $\lim_{n\to\infty} \frac{a_{n+1}-a_n}{b_{n+1}-b_n}$ existitzen da.

3) $\lim_{n\to\infty} b_n = \infty$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{(n+1)^3+\ln(n+1)-n^3-\ln n} = \lim_{n\to\infty} \frac{n^2+2n+1}{\cancel{n^3}+3n^2+3n+1+n-\cancel{n^3}-(n-1)} =$$

$(n^2+2n+1)(n+1) =$

$= n^3+2n^2+n+n^2+2n+1$

$\{\ln(n+1)\} \sim \{n\}$

$\{\ln(n)\} \sim \{n-1\}$

$$= \lim_{n\to\infty} \frac{n^2+2n+1}{3n^2+3n} = \boxed{\frac{1}{3}}$$

5)

$n^2 e^{-\sqrt{n}}$

$$\lim_{n\to\infty} \frac{n^2}{e^{\sqrt{n}}} = 0$$

8)

$$\lim_{n\to\infty} \left(\frac{m-1}{m+3}\right)^{m+2} = \lim_{n\to\infty} \left(\frac{m+3-4}{m+3}\right)^{n+2} = \lim_{n\to\infty} \left(1-\frac{4}{m+3}\right)^{m+2} =$$

$$= \lim_{n\to\infty} e^{-4} = \boxed{\frac{1}{e^4}}$$

10)

$$\lim_{n\to\infty} n\left(1-\sqrt[n]{a}\right) = \infty \cdot 0 = \text{Indeterminazioa}$$

$$\lim_{n\to\infty} -n\left(\sqrt[n]{a}-1\right) = -n\left(\ln(\sqrt[n]{a})\right) = -\cancel{n}\frac{1}{\cancel{n}} \ln a = \boxed{-\ln a}$$

$$\boxed{\{a_n - 1\} \sim \{\ln a_n\}}$$

12)

$$\left(\frac{1}{n}\right)^{n^2+1}$$

$$\lim_{n\to\infty} \left(\frac{1}{n}\right)^{n^2+1} = \cancel{\lim_{n\to\infty} \frac{1}{n^{n^2+1}}} = 0^{\infty} = \text{Indeterminazioa}$$

$$\cancel{\lim \ln \ell = \ln\left(\frac{1}{n}\right)^{n^2+1} = (n^2+1)\cdot \ln\left(\frac{1}{n}\right) = \frac{n^2+1}{n}\cdot\left(\frac{1}{n}-1\right) = \frac{n^2+1}{n} - (n^2+1)} =$$

$$\cancel{= \lim_{n\to\infty} \frac{n^2+1}{n} - \frac{n(n^2+1)}{n}} =$$

$$= \lim_{n\to\infty} \frac{1}{n^{n^2+1}} = \frac{1}{\infty} = \boxed{0}$$

(4)

$$\frac{1}{\ln n}\sum \sin\frac{\pi}{k}$$

$$\frac{1}{\ln n}\sum^{n}\sin\frac{\pi}{k} = \lim_{n\to\infty}\frac{\sin\frac{\pi}{1}+\sin\frac{\pi}{2}+\dots+\sin\frac{\pi}{n}}{\underbrace{\ln n}_{b_n}} =$$

$$= \lim_{n\to\infty}\frac{\sin\frac{\pi}{1}+\cancel{\sin\frac{\pi}{2}+\dots+\sin\frac{\pi}{n}}+\sin\frac{\pi}{n+1}-\cancel{\left(\sin\frac{\pi}{1}+\dots+\sin\frac{\pi}{n}\right)}}{\ln(n+1)-\ln n} =$$

{Stolz}

1) $\{b_n\}$ hertsilki monotonoo d..

2) $\lim_{n\to\infty}\frac{a_{n+1}-a_n}{b_{n+1}-b_n}$

3)

2) $\lim_{n\to\infty} b_n = \infty$

$$= \lim_{n\to\infty}\frac{\sin\frac{\pi}{n+1}}{\ln(n+1)-\ln n} = \lim_{n\to\infty}\frac{\sin\frac{\pi}{n+1}}{\ln\left(\frac{n+1}{n}\right)} = \lim_{n\to\infty}\frac{\sin\frac{\pi}{n+1}}{\frac{n+1}{n}-1} =$$

$\cancel{\{\ln(n+1)\}\sim\{n+1-1\}}$ $\left\{\ln\left(\frac{n+1}{n}\right)\right\}\sim\left\{\frac{n+1}{n}-1\right\}$

$$= \lim_{n\to\infty}\frac{\sin\frac{\pi}{n+1}}{\frac{\cancel{n}+1}{n}-\frac{\cancel{n}}{n}} = \lim_{n\to\infty}\frac{\sin\frac{\pi}{n+1}}{\frac{1}{n}} = \lim_{n\to\infty} n\sin\frac{\pi}{n+1} =$$

$$= \lim_{n\to\infty} n\cdot\frac{\pi}{n+1} = \boxed{\pi}$$

$\{\sin a_n\}\sim\{a_n\}$

$$\lim_{n\to\infty}\left(\cos\sqrt{\frac{2\ln 5}{n}}\right)^{n} = 1^{\infty} = \ell = \text{indeterminazione}$$

$$\ln \ell = \ln\left(\cos\sqrt{\frac{2\ln 5}{n}}\right)^{n} = n\ln\left(\cos\sqrt{\frac{2\ln 5}{n}}\right) \Rightarrow$$

$$\Rightarrow \lim_{n\to\infty} n\ln\left(\cos\sqrt{\frac{2\ln 5}{n}}\right) = \infty \cdot \ln 1 = \infty\cdot 0 = \text{Indeterminazione}$$

$$= \lim_{n\to\infty} n\left(\cos\sqrt{\frac{2\ln 5}{n}} - 1\right) = \lim_{n\to\infty} n\cdot\left(-\frac{\sqrt{\frac{2\ln 5}{n}}^{2}}{2}\right) = \lim_{n\to\infty} \frac{\cancel{n}2\cdot(-\ln 5)}{\cancel{n}2} = \ln 5$$

$\boxed{\{a_n \ln(a_n)\} \sim \{a_n - 1\}}$ $\boxed{\{1-\cos(a_n)\} \sim \left\{\frac{a_n^2}{2}\right\}}$

$$\ln \ell = -\ln 5$$

$$\ell = 5^{-1} \Rightarrow \boxed{\ell = \frac{1}{5}}$$

3)

$$\left(\frac{n}{3n^2+2}\right)^{\ln n}$$

$$\lim_{n\to\infty}\left(\frac{n}{3n^2+2}\right)^{\ln n} = 0^{\infty} = \text{indeterminazione} = \ell$$

$$\ln \ell = \ln\left(\frac{n}{3n^2+2}\right)^{\ln n} = \lim_{n\to\infty} \cancel{\ln} n \cdot \ln\left(\frac{n}{3n^2+2}\right) = \cancel{\infty\cdot 0 = \text{Indeterminazione}}$$

$$\cancel{\lim_{n\to\infty} \ln n \cdot \ln\left(\frac{n}{3n^2+2}\right) = (n-1)\cdot(\ln n - \ln\{3n^2+2\})} =$$

$\{\ln n\} \sim \{n-1\}$

$= \infty$

$z^3 \cdot \bar{z} = 1 \Rightarrow (a+bi)^3 \cdot (a-bi) = 1 \Rightarrow (a^2 + abi - b^2) \cdot (a+bi) \cdot (a-bi) = 1 \Rightarrow$

$\Rightarrow (a^3 + a^2bi - ab^2 + a^2bi - ab^2 - b^3i) \cdot (a-bi) = 1 \Rightarrow (a^3 + 2a^2bi - 2ab^2 - b^3i) \cdot (a-bi) = 1$

$\Rightarrow a^4 + 2a^3bi - \cancel{2a^2b^2} - \cancel{ab^3i} - \cancel{a^3bi} + \cancel{2a^2b^2} + 2ab^3i - b^4 = 1 \Rightarrow$

$\Rightarrow a^4 + a^3bi + ab^3i - b^4 = 1$

18)

$$\mathrm{Re}\left(\frac{1}{z}\right) < \frac{1}{4}$$

$$\boxed{\frac{1}{z} = \frac{1}{a+bi} = \frac{1}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{a-bi}{a^2+b^2}}$$

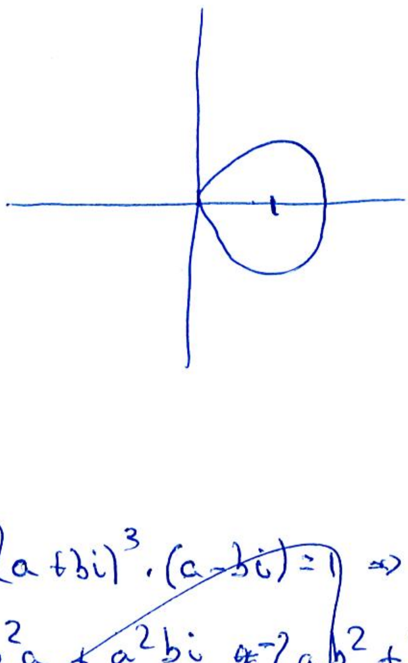
$$\mathrm{Re}\,\frac{1}{z} = \mathrm{Re}\left(\frac{a}{a^2+b^2}\right) - \mathrm{Im}\left(\frac{b}{a^2+b^2}\right) i$$

$$\mathrm{Re}\left(\frac{a}{a^2+b^2}\right) < \frac{1}{4}$$

$$\frac{a}{a^2+b^2} < \frac{1}{2} \Rightarrow 2a < a^2+b^2 \Rightarrow a^2-2a+b^2 > 0 \Rightarrow$$

$$\Rightarrow (a-1)^2+b^2 > 1$$

$\alpha = 1$
$\beta = 0$
$R = 1$



16)

$$z^3 \bar{z} = 1$$

$$z^3 \cdot \bar{z} = 1 \Rightarrow (a+bi)^3 \cdot (a-bi) = 1 \Rightarrow (a^2+2abi-b^2)(a+bi)(a-bi) = 1 \Rightarrow$$

$$\Rightarrow (a^3+2a^2bi-b^2a+a^2bi-2ab^2+b^3)(a-bi) = 1 \Rightarrow$$

$\cancel{\Rightarrow a^4 + 2a^3bi - a^2b^2 + a^3bi - 2a^2b^2 + ab^3 - a^3bi + 2a^2b^2 + b^3a + a^2b^2 + 2ab^2}$

$\Rightarrow (a^4$

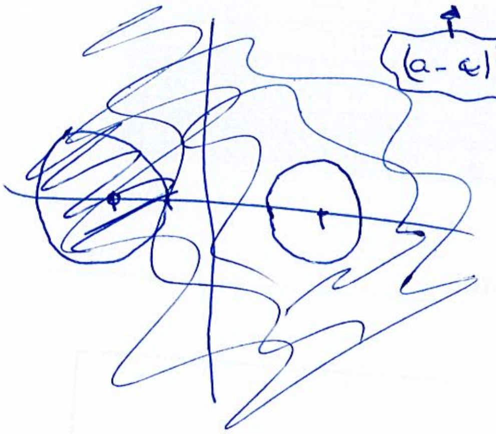
$$\mathrm{Re}\left(\frac{a-1}{(a-1)^2+b^2}\right) < \frac{1}{2}$$

$$\frac{a-1}{(a-1)^2+b^2} < \frac{1}{2} \Rightarrow 2a-2 < (a-1)^2+b^2 \Rightarrow 2a-2 < a^2-2a+1+b^2 \Rightarrow$$

$$\Rightarrow 0 < a^2-4a+3+b^2 \Rightarrow a^2-4a+3+b^2 > 0 \Rightarrow (a-2)^2-1+b^2 > 0 \Rightarrow$$

$$\Rightarrow (a-2)^2+b^2 > 1 \Rightarrow (a-2)^2+(b-0)^2 > 1^2$$

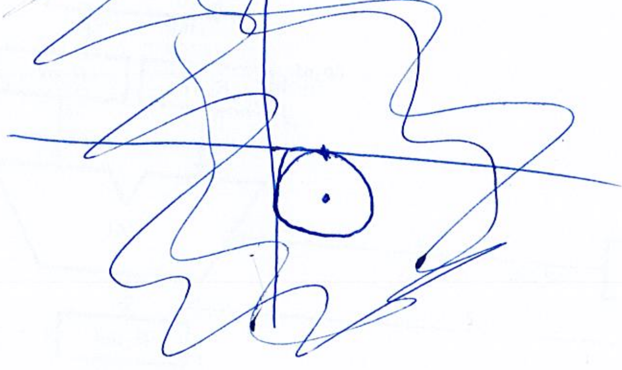
$$\boxed{(a-\alpha)^2+(b-\beta)^2 = R^2}$$



$$\mathrm{Im}\left(\frac{-b}{(a-1)^2+b^2}\right)i < \frac{1}{2}$$

$$\frac{-b}{(a-1)^2+b^2} < \frac{1}{2} \Rightarrow -2b < (a-1)^2+b^2 \Rightarrow (a-1)^2+b^2+2b > 0 \Rightarrow$$

$$\Rightarrow (a-1)^2+(b+1)^2-1 > 0 \Rightarrow (a-1)^2+(b+1)^2 > 1^2$$



Conjunto de Instrucciones del Procesador elemental

CO	Codificación		Ensamblador	Significado	Formato máquina
add	9	001001	`add rd,rf1,rf2`	rd := rf1 + rf2	op / rd / rf1 / ////// / rf2
and	19	010011	`and rd,rf1,rf2`	rd := rf1 and rf2	op / rd / rf1 / ////// / rf2
beq	26	011010	`beq rf1,etiq`	if (rf1=0) PC:=dir	26 / //// / rf1 / dir
div	15	001111	`div rd,rf1,rf2`	rd := rf1 / rf2	op / rd / rf1 / ////// / rf2
ld	0	000000	`ld rd,etiq`	rd := MEM[dir]	0 / rd / //// / dir
mov	7	000111	`mov rd,rf1`	rd := rf1	7 / rd / rf1 / //////////
mul	13	001101	`mul rd,rf1,rf2`	rd := rf1 * rf2	op / rd / rf1 / ////// / rf2
or	21	010101	`or rd,rf1,rf2`	rd := rf1 or rf2	op / rd / rf1 / ////// / rf2
st	3	000011	`st rf1,etiq`	MEM[dir] := rf1	3 / //// / rf1 / dir
sub	11	001011	`sub rd,rf1,rf2`	rd := rf1 - rf2	op / rd / rf1 / ////// / rf2
xor	23	010111	`xor rd,rf1,rf2`	rd := rf1 xor rf2	op / rd / rf1 / ////// / rf2

Estructura (Unidad de Proceso) del Procesador Elemental