# Dualtasuna. Ariketak

1. Eredu lineal hauen dualak idatz itzazu:

1.1. $\min\ z = 2x_1 + 3x_2 - 4x_3$

hauen mende

$$\begin{aligned} x_1 + 2x_2 + 5x_3 &\geq 1 \\ 2x_1 - 2x_2 + 4x_3 &= 7 \\ x_1 + 2x_2 + x_3 &\geq 10 \\ x_1 \leq 0, x_2 \geq 0, x_3 &: \text{ez-mur.} \end{aligned}$$

1.2. $\min\ z = x_1 + 3x_2 + x_3$

hauen mende

$$\begin{aligned} 4x_1 - x_2 + 2x_3 &\leq -7 \\ 2x_1 - 4x_2 \qquad &\geq 12 \\ 2x_1 + 8x_2 + 4x_3 &\geq 5 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

1.3. $\max\ z = 2x_1 + 2x_2 + 5x_3$

hauen mende

$$\begin{aligned} 2x_1 + x_2 + 2x_3 &= 12 \\ -x_1 + 5x_2 - 2x_3 &\geq -8 \\ 3x_1 + 4x_2 - 6x_3 &\leq 10 \\ x_1 \leq 0, x_2, x_3 &\geq 0 \end{aligned}$$

1.4. $\max\ z = x_1 + x_2 + 5x_3$

hauen mende

$$\begin{aligned} x_1 + x_2 + 2x_3 &\leq -4 \\ -x_1 + 6x_2 + 2x_3 &\geq 2 \\ 4x_1 - x_2 + x_3 &= 6 \\ x_1, x_2 \geq 0, x_3 &: \text{ez-mur.} \end{aligned}$$

1.5. $\min\ z = 4x_1 + x_2 - x_3 + 2x_4$

hauen mende

$$\begin{aligned} 4x_1 - 2x_2 + 3x_3 + x_4 &\leq -6 \\ x_1 + x_2 + x_3 + x_4 &= 6 \\ 5x_1 + 2x_2 - x_3 - x_4 &\geq 10 \\ x_1, x_2 \leq 0, x_3, x_4 &\geq 0 \end{aligned}$$

1.6. $\max\ z = x_1 + 4x_2$

hauen mende

$$\begin{aligned} 2x_1 - 4x_2 &\leq 14 \\ -x_1 + 8x_2 &\geq -6 \\ 4x_1 + 6x_2 &\leq 10 \\ x_1 + 9x_2 &= 3 \\ x_1 \geq 0, x_2 &\leq 0 \end{aligned}$$



2. Eredu lineal hauen dualak kalkula itzazu, eta bai emandako ereduaren eta baita dualaren ebazpen grafikoak egin itzazu. Esan problema bakoitzerako zein motako soluzioa lortzen den (soluziorik ez, bakarra, anizkoitza, bornegabea).

2.1. $\min\ z = 4x_1 + 6x_2$

hauen mende

$$\begin{aligned} 2x_1 + x_2 &\geq 4 \\ x_1 + 4x_2 &\geq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2.2. $\max\ z = 4x_1 + 6x_2$

hauen mende

$$\begin{aligned} 10x_1 + 12x_2 &\leq 22 \\ 2x_1 + 6x_2 &\leq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2.3. $\max\ z = -2x_1 + 6x_2$

hauen mende

$$\begin{aligned} -x_1 + 3x_2 &\leq 9 \\ x_1 + x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2.4. $\max\ z = -3x_1 + 2x_2$

hauen mende

$$\begin{aligned} -4x_1 + 2x_2 &\geq 2 \\ x_1 - 2x_2 &\leq -4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

3. Eredu lineal hauen soluzio optimoa kalkula ezazu simplex dual algoritmoa erabiliz:

3.1. $\max\ z = -2x_1 - 4x_2 - 3x_3$

hauen mende

$$\begin{aligned} 2x_1 + x_2 + 2x_3 &\geq 8 \\ 4x_1 + 2x_2 + 2x_3 &\geq 10 \\ 6x_1 + x_2 + 4x_3 &\geq 12 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

3.2. $\min\ z = 2x_1 + x_2 + 3x_3 + 2x_4$

hauen mende

$$\begin{aligned} 2x_1 + 2x_2 + 2x_3 + 2x_4 &\geq 22 \\ 4x_1 + 4x_2 + x_3 + 4x_4 &\leq 20 \\ 2x_1 + 8x_2 + 2x_3 + x_4 &\geq 15 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

3.3. $\max\ z = -2x_1 - 3x_2 - x_3 - x_4$

hauen mende

$$\begin{aligned} x_1 + x_2 + 3x_3 + x_4 &\leq 40 \\ 2x_1 + 3x_2 + x_3 + x_4 &\geq 30 \\ 2x_1 \qquad + x_3 \qquad &\leq 25 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

3.4. $\max\ z = -6x_1 - 4x_2 - 5x_3 - 4x_4$

hauen mende

$$\begin{aligned} 2x_1 + 4x_2 + 2x_3 + 5x_4 &\leq 10 \\ x_1 + 2x_2 \qquad + x_4 &\geq 25 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

3.5. $\max\ z = -2x_1 - x_2 - 2x_3 - x_4$

hauen mende

$$\begin{aligned} 6x_1 + 2x_2 + 6x_3 + 3x_4 &\le 12 \\ 2x_1 + x_2 + 2x_3 + 2x_4 &\ge 12 \\ x_1 + 2x_2 + 6x_3 + 4x_4 &\ge 14 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

3.6. $\max\ z = -3x_1 + 4x_2 + 2x_3 + 5x_4$

hauen mende

$$\begin{aligned} 4x_1 + 2x_2 + 4x_3 + 3x_4 &\le 48 \\ -x_1 + 2x_2 - x_3 + 2x_4 &\ge 8 \\ 2x_1 - x_2 + x_3 + x_4 &\ge 6 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

3.7. $\max\ z = 3x_1 - 2x_2 + 2x_3 + x_4$

hauen mende

$$\begin{aligned} 3x_1 + 6x_2 + 3x_3 + 2x_4 &\le 36 \\ x_1 + 2x_2 + 3x_3 + x_4 &\ge 14 \\ x_1 + x_2 + x_3 + 2x_4 &\ge 10 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

3.8. $\max\ z = 6x_1 + 5x_2 + 5x_3$

hauen mende

$$\begin{aligned} -x_1 + x_2 + 2x_3 &\ge 40 \\ 2x_1 - 2x_2 - x_3 &\ge 30 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

3.9. $\max\ z = 4x_1 - 2x_2 + 3x_3 - 3x_4$

hauen mende

$$\begin{aligned} 6x_1 - 6x_2 + 9x_3 + 3x_4 &\ge 28 \\ 3x_1 + x_2 + x_3 - 3x_4 &\ge 22 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

4. Izan bedi eredu lineal hau:

$$\begin{aligned} &\max\ z = 10x_1 + 6x_2 \\ &\text{hauen mende} \\ &\quad x_1 + 2x_2 \le 2 \\ &\quad 2x_1 + x_2 \le 3 \\ &\quad 2x_1 + 2x_2 \le 3 \\ &\quad 4x_1 + x_2 \le 2 \\ &\quad x_1, x_2 \ge 0 \end{aligned}$$

4.1 Idatz ezazu eredu duala.

4.2 Ebatz ezazu eredu duala algoritmorik egokiena erabiliz: simplex primala edo simplex duala.

4.3 Dualaren taula optimotik eredu primalaren soluzio optimoa lor ezazu.



5. Izan bedi eredu lineal hau:

$$\begin{aligned} &\min\ z = 30x_1 + 28x_2 \\ &\text{hauen mende} \\ &\quad 4x_1 + 2x_2 \ge 20 \\ &\quad 6x_1 + 4x_2 \ge 16 \\ &\quad 4x_1 + 2x_2 \ge 18 \\ &\quad 4x_1 + 4x_2 \ge 21 \\ &\quad x_1, x_2 \ge 0 \end{aligned}$$

5.1 Idatz ezazu eredu duala.

5.2 Ebatz ezazu eredu duala algoritmorik egokiena erabiliz: simplex primala edo simplex duala.

5.3 Dualaren taula optimotik eredu primalaren soluzio optimoa lor ezazu.

6. Izan bitez eredu lineal hauek eta dagozkien taula optimoak:

6.1 Eredu lineal hau simplex primal algoritmoa erabiliz ebatzia izan da:

$\max\ z = 6x_1 + 5x_2 + 4x_3$

hauen mende

$$\begin{aligned} 15x_1 + 25x_2 + 30x_3 &\le 90 \\ 15x_1 + 5x_2 + 15x_3 &\le 60 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
|---|---|---|---|---|---|---|
| | 0 | 0 | $\frac{17}{4}$ | $\frac{3}{20}$ | $\frac{1}{4}$ | $\frac{57}{2}$ |
| $\mathbf{a}_2$ | 0 | 1 | $\frac{3}{4}$ | $\frac{1}{20}$ | $-\frac{1}{20}$ | $\frac{3}{2}$ |
| $\mathbf{a}_1$ | 1 | 0 | $\frac{3}{4}$ | $-\frac{1}{60}$ | $\frac{1}{12}$ | $\frac{7}{2}$ |

6.2 Eredu lineal hau simplex primal algoritmoa erabiliz ebatzia izan da, ereduari aldagai artifizial bat gehitu eta helburua zigortuz:

$\max\ z = 2x_1 + x_2 - x_3$

hauen mende

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &\le 12 \\ 4x_1 + 2x_2 \qquad &\ge 8 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $w_1$ | |
|---|---|---|---|---|---|---|---|
| | 0 | 3 | 9 | 2 | 0 | $M$ | 24 |
| $\mathbf{a}_5$ | 0 | 6 | 16 | 4 | 1 | $-1$ | 40 |
| $\mathbf{a}_1$ | 1 | 2 | 4 | 1 | 0 | 0 | 12 |

Horietako bakoitzerako erantzun itzazu honakoak:

(a) Taula optimoa aztertuz eman ereduaren soluzio optimoa.

(b) Idatz ezazu ereduaren duala, eta taula optimoa aztertuz eman dualaren soluzio optimoa.

(c) Itzal-prezioen interpretazioa egin ezazu.

# Simplex metodoa. Ariketak

1. Ondoko eredu linealak maximizatze-forma estandarra erabiliz idatz itzazu.

1.1
$$\begin{aligned} \max\ z &= 2x_1 + 4x_2 - 4x_3 \\ \text{hauen mende} & \\ 3x_1 + 2x_2 + 4x_3 &\geq 1 \\ 4x_1 - 3x_2 &= 2 \\ 2x_1 + x_2 + 6x_3 &\leq 3 \\ x_1, x_2 \geq 0, x_3 &: \text{ez-murriztua} \end{aligned}$$

1.2
$$\begin{aligned} \min\ z &= 2x_1 - 3x_2 + x_3 \\ \text{hauen mende} & \\ x_1 - 5x_2 + 6x_3 &\geq 8 \\ x_1 - 4x_2 &\leq -12 \\ 2x_1 - x_2 + 4x_3 &= 5 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

1.3
$$\begin{aligned} \min\ z &= 2x_1 + 2x_2 - 4x_3 \\ \text{hauen mende} & \\ 2x_1 + 2x_2 + 2x_3 &= 10 \\ -2x_1 + 6x_2 - x_3 &\leq -10 \\ -x_1 + 3x_2 &\geq 3 \\ x_1 \leq 0, x_2, x_3 &\geq 0 \end{aligned}$$

1.4
$$\begin{aligned} \max\ z &= 3x_1 - 7x_2 + 5x_3 \\ \text{hauen mende} & \\ x_2 - x_3 &\leq -9 \\ -x_1 - 2x_3 &\geq 5 \\ 4x_1 - x_2 &= 6 \\ x_1 \leq 0, x_2 \geq 0, x_3 &: \text{ez-murriztua} \end{aligned}$$

2. Izan bedi ondoko eredu lineala.

$$\begin{aligned} \max\ z &= x_1 + x_2 \\ \text{hauen mende} & \\ -x_1 + x_2 &\leq 4 \\ 2x_1 + 5x_2 &\leq 20 \\ 2x_1 - x_2 &\leq 2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

2.1 Ereduaren ebazpen grafikoa egin ezazu.

2.2 Kalkula itzazu oinarriko soluzio guztiak. Zein dira bideragarriak? Zein dira endekatuak?

2.3 Esan aurreko atalean kalkulatutako oinarriko soluzio bakoitza zein punturi dagokion grafikoan.



3. Izan bedi ondoko eredu lineala.

$$\begin{aligned} \max\ z &= 4x_1 + 3x_2 + 2x_3 \\ \text{hauen mende} & \\ x_1 + 2x_2 + 3x_3 &\leq 6 \\ 2x_1 + x_2 + x_3 &\leq 3 \\ x_1 + x_2 + x_3 &\leq 2 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Idatz ezazu eredua maximizatze forma estandarrean, eta $\mathbf{B} = \{\mathrm{a}_4, \mathrm{a}_1, \mathrm{a}_6\}$ oinarriari dagokion oinarriko soluzio bideragarritik hasita, hobekuntzaren teorema aplika ezazu soluzio optimora iritsi arte.

4. Izan bedi eredu lineal hau.

$$\begin{aligned} \max\ z &= 2x_1 + 2x_2 + 5x_3 \\ \text{hauen mende} & \\ x_1 \qquad + x_3 &\leq 2 \\ x_2 + x_3 &\leq 4 \\ x_1 \qquad + 2x_3 &\leq 3 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Ondoko matrizearen alderantzizkoa ezagutzen badugu,

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix}$$

Froga ezazu $\mathbf{B} = \{\mathrm{a}_1, \mathrm{a}_2, \mathrm{a}_3\}$ oinarriari dagokion oinarriko soluzioa optimoa dela. Kalkula itzazu soluzio optimoa eta helburu-funtzioaren balio optimoa.

5. Izan bedi eredu lineal hau.

$$\begin{aligned} \max\ z &= x_1 + 4x_2 + 3x_3 \\ \text{hauen mende} & \\ 3x_1 + 4x_2 + 6x_3 &\leq 10 \\ x_1 + 2x_2 + x_3 &\leq 4 \\ 2x_1 + 2x_2 + 3x_3 &\leq 8 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Demagun simplex algoritmoaren iterazio batean honako taulara iritsi garela.

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
|---|---|---|---|---|---|---|---|
| | 1 | 0 | | 0 | 2 | 0 | |
| $a_4$ | 2 | 0 | | 1 | $-2$ | 0 | |
| $a_2$ | 2 | 1 | | 0 | $\frac{1}{2}$ | 0 | |
| $a_6$ | 1 | 0 | | 0 | $-1$ | 1 | |

5.1 Froga ezazu $\mathbf{y}_1$ zutabea kalkulatzerakoan kalkulu-erroreak egin direla.

5.2 Froga ezazu $\mathbf{y}_5$ zutabea kalkulatzerakoan ez dela kalkulu-erroreik egin.

5.3 Taulako informazioa erabiliz, osa ezazu taula falta diren balio guztiak kalkulatuz.

6. Kalkula ezazu ondoko eredu linealen soluzio optimoa simplex algoritmoa erabiliz. Soluzioa baduten ereduetarako adieraz ezazu grafikan zein diren simplex algoritmoa erabiliz kalkulatu diren oinarriko soluzio bideragarriak (mutur-puntuak) optimora iritsi arte.

6.1 max $z = x_1 - x_2$

hauen mende

$$x_1 - 2x_2 \leq 2$$
$$4x_1 - 3x_2 \leq 12$$
$$x_1, x_2 \geq 0$$

6.2 max $z = x_1 + x_2$

hauen mende

$$x_1 + 6x_2 \geq 6$$
$$2x_1 - 3x_2 \geq -6$$
$$x_1 + 2x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

6.3 max $z = 4x_1 - 4x_2$

hauen mende

$$-2x_1 + 2x_2 \leq 4$$
$$2x_1 - 2x_2 \leq 6$$
$$-x_1 + 4x_2 \geq -2$$
$$x_1, x_2 \geq 0$$

6.4 max $z = x_1 + 2x_2$

hauen mende

$$x_1 + 2x_2 \leq 5$$
$$x_1 + x_2 \geq 2$$
$$x_1 - x_2 \leq 4$$
$$x_1, x_2 \geq 0$$



6.5 max $z = 2x_1 + 2x_2$

hauen mende

$$x_1 - x_2 \leq 2$$
$$2x_1 + 2x_2 \leq 6$$
$$x_1 + 2x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

6.6 max $z = 2x_1 - 2x_2$

hauen mende

$$x_1 + 3x_2 \geq 3$$
$$x_1 - 3x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

6.7 max $z = x_1 + 4x_2$

hauen mende

$$x_1 - x_2 \geq -4$$
$$3x_1 - x_2 \geq -3$$
$$x_1 - 2x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

6.8 max $z = 3x_1 + 4x_2$

hauen mende

$$x_1 - 2x_2 \leq 4$$
$$x_1 + x_2 \geq 6$$
$$2x_1 + 3x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

7. Kalkula ezazu ondoko eredu linealen soluzio optimoa simplex algoritmoa erabiliz.

7.1 max $z = 3x_1 + 2x_2 + x_3$

hauen mende

$$x_1 - x_2 + x_3 \leq 4$$
$$2x_1 + x_2 + 4x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0$$

7.2 min $z = 5x_1 - x_2 - 2x_3$

hauen mende

$$x_1 + x_2 - x_3 = 6$$
$$x_1 + 2x_2 - x_3 \geq 4$$
$$-x_1 + x_2 + 2x_3 \leq 8$$
$$x_1 \leq 0, x_2, x_3 \geq 0$$

7.3 min $z = x_1 - 3x_2 + 2x_3$

hauen mende

$$3x_1 - x_2 + 2x_3 \leq 9$$
$$-2x_1 + 4x_2 + x_3 \leq 14$$
$$-4x_1 + 4x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

7.4 min $z = 10x_1 + 8x_2 + 6x_3 + 4x_4$

hauen mende

$$2x_1 + 4x_2 + 2x_3 + x_4 \geq 10$$
$$-4x_1 + 4x_2 - x_3 + 2x_4 \geq 12$$
$$x_1, x_2, x_3, x_4 \geq 0$$

7.5 $\min\ z = -16x_1 + 2x_2 + x_3 - 2x_4 - 3x_5$

hauen mende

$$\begin{aligned} 3x_1 + x_2 + 3x_3 - 3x_4 + 9x_5 &\leq 12 \\ 2x_1 + 8x_2 + 4x_3 + 2x_4 - 4x_5 &\leq 10 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

7.6 $\max\ z = 9x_1 + 5x_2 + 4x_3 + x_4$

hauen mende

$$\begin{aligned} 2x_1 + x_2 + x_3 + 2x_4 &\leq 2 \\ 8x_1 + 4x_2 - 2x_3 - x_4 &\geq 10 \\ 4x_1 + 7x_2 + 2x_3 + x_4 &\leq 4 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

7.7 $\min\ z = -4x_1 + 2x_2 - x_3$

hauen mende

$$\begin{aligned} x_1 - x_2 + 2x_3 &\geq -8 \\ 2x_1 - 3x_2 + 4x_3 &\leq 5 \end{aligned}$$

$x_1, x_2 \geq 0, x_3$ : ez-murriztua

7.8 $\min\ z = 3x_1 + x_2 - 2x_3 - 2x_4 + x_5$

hauen mende

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + x_4 + x_5 &\leq 2 \\ 2x_1 + x_2 + 3x_3 + 2x_4 + 2x_5 &\geq 12 \\ x_1, x_2, x_3, x_4, x_5 &\geq 0 \end{aligned}$$

8. Galdera teorikoak.

8.1 Zer gertatuko litzateke $z_k - c_k > 0$ duen $\mathbf{a}_k$ bektore bat aukeratuko bagenu oinarrian sartzeko?

8.2 Zer gertatuko litzateke $z_k - c_k < 0$ guztien artetik minimoa ez duen $\mathbf{a}_k$ bektore bat aukeratuko bagenu oinarrian sartzeko?

8.3 Bektore baten irteera irizpidean, zergatik bete behar da $y_{rk} > 0$ izatea? Eta, $y_{rk} < 0$ bada? Eta, $y_{rk} = 0$ bada?

8.4 Bektore baten irteera irizpidean, zer gertatzen da $y_{rk} > 0$ izanik, irtetzeko aukeratutako $\mathbf{a}_r$ bektorearen $\frac{x_{Br}}{y_{rk}}$ ez bada $\frac{x_{Bi}}{y_{ik}}$ guztien artetik minimoa?

8.5 Behin oinarritik aldagai artifizial bat kendu eta gero, posible da, simplex algoritmoa jarraituz, berriro oinarrira bueltatzea.

## Sentikortasunaren analisia. Ariketak

1. Izan bitez honako eredu lineala eta dagokion taula optimoa:

$$\max\ z = 4x_1 + x_2 + 5x_3$$

hauen mende

$$x_1 + x_2 + x_3 \leq 4$$
$$2x_1 + x_2 + 3x_3 \leq 10$$
$$3x_1 + x_2 + 4x_3 \leq 16$$
$$x_1, x_2, x_3 \geq 0$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
|---|---|---|---|---|---|---|---|
| | 0 | 2 | 0 | 2 | 1 | 0 | 18 |
| $\mathbf{a}_1$ | 1 | 2 | 0 | 3 | −1 | 0 | 2 |
| $\mathbf{a}_3$ | 0 | −1 | 1 | −2 | 1 | 0 | 2 |
| $\mathbf{a}_6$ | 0 | −1 | 0 | −1 | −1 | 1 | 2 |

1.1 Ondoko aldaketa diskretuek taula optimoan sortzen duten eragina azter ezazu. Aldaketa bakoitzerako kalkula ezazu eredu berriaren soluzio optimoa.

1.1.1 $\mathbf{b} = \begin{pmatrix} 4 \\ 10 \\ 16 \end{pmatrix} \quad \rightarrow \quad \hat{\mathbf{b}} = \begin{pmatrix} 2 \\ 10 \\ 16 \end{pmatrix}$

1.1.2 $\mathbf{b} = \begin{pmatrix} 4 \\ 10 \\ 16 \end{pmatrix} \quad \rightarrow \quad \hat{\mathbf{b}} = \begin{pmatrix} 4 \\ 10 \\ 18 \end{pmatrix}$

1.1.3 $\mathbf{c}^T = (4\ \ 1\ \ 5) \quad \rightarrow \quad \hat{\mathbf{c}}^T = (3\ \ 3\ \ 5)$

1.1.4 $\mathbf{c}^T = (4\ \ 1\ \ 5) \quad \rightarrow \quad \hat{\mathbf{c}}^T = (5\ \ 1\ \ 7)$

1.1.5 $\mathbf{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \rightarrow \quad \hat{\mathbf{a}}_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

1.1.6 $\mathbf{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \rightarrow \quad \hat{\mathbf{a}}_2 = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$

1.1.7 Aldagai berria: $x_4$ $\quad c_4 = 6 \quad \mathbf{a}_4 = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}$



1.1.8 Aldagai berria: $x_4$ $\quad c_4 = 3 \quad \mathbf{a}_4 = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$

1.1.9 Murrizketa berria: $2x_1 + 4x_2 + x_3 \leq 8$

1.1.10 Murrizketa berria: $4x_1 + x_2 + 2x_3 \leq 8$

1.2 Kalkula itzazu itzal-prezioak.

1.3 Esan **c** eta **b** bektoreetako osagai bakoitza zein balio tartetan mugi daitekeen, beti ere emandako taula optimoko oinarriak optimo izaten jarraituko duelarik.

2. Izan bitez honako eredu lineala eta dagokion taula optimoa:

$$\max\ z = 4x_1 + 6x_2 + 5x_3$$

hauen mende

$$x_1 + 2x_2 + 2x_3 \leq 12$$
$$2x_1 + 4x_2 + 2x_3 \leq 14$$
$$x_1 + x_2 + 2x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 1 | 0 | 1 | 2 | 26 |
| $\mathbf{a}_4$ | 0 | 0 | 1 | 1 | $-\frac{1}{2}$ | 0 | 5 |
| $\mathbf{a}_2$ | 0 | 1 | −1 | 0 | $\frac{1}{2}$ | −1 | 1 |
| $\mathbf{a}_1$ | 1 | 0 | 3 | 0 | $-\frac{1}{2}$ | 2 | 5 |

2.1 Ondoko aldaketa diskretuek taula optimoan sortzen duten eragina azter ezazu. Aldaketa bakoitzerako kalkula ezazu eredu berriaren soluzio optimoa.

2.1.1 $\mathbf{b} = \begin{pmatrix} 12 \\ 14 \\ 6 \end{pmatrix} \quad \rightarrow \quad \hat{\mathbf{b}} = \begin{pmatrix} 7 \\ 14 \\ 6 \end{pmatrix}$

2.1.2 $\mathbf{b} = \begin{pmatrix} 12 \\ 14 \\ 6 \end{pmatrix} \quad \rightarrow \quad \hat{\mathbf{b}} = \begin{pmatrix} 12 \\ 18 \\ 10 \end{pmatrix}$

2.1.3 $\mathbf{c}^T = (4\ \ 6\ \ 5) \quad \rightarrow \quad \hat{\mathbf{c}}^T = (6\ \ 8\ \ 2)$

2.1.4 $\mathbf{c}^T = (4\ \ 6\ \ 5) \quad \rightarrow \quad \hat{\mathbf{c}}^T = (4\ \ 6\ \ 9)$

2.1.5 $\mathbf{a}_3 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \quad \rightarrow \quad \hat{\mathbf{a}}_3 = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$

2.1.6 $\mathbf{a}_3 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \longrightarrow \hat{\mathbf{a}}_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

2.1.7 Aldagai berria: $x_4$ $\quad c_4 = 2 \quad \mathbf{a}_4 = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$

2.1.8 Aldagai berria: $x_4$ $\quad c_4 = 5 \quad \mathbf{a}_4 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

2.1.9 Murrizketa berria: $x_1 + 2x_2 \leq 6$

2.1.10 Murrizketa berria: $x_1 + 3x_2 + 2x_3 \leq 10$

2.2 Kalkula itzazu itzal-prezioak.

2.3 Esan **c** eta **b** bektoreetako osagai bakoitza zein balio tartetan mugi daitekeen, beti ere emandako taula optimoko oinarriak optimo izaten jarraituko duelarik.

3. Sukaldaritzako hiru liburu moten diseinuak egin nahi ditu argitaletxe batek: $L_1, L_2$ eta $L_3$. Horretarako sukaldaritzako hiru alorretan adituak diren sukaldariak kontratatu ditu: eguneko menuan adituak 40, postre berezietan adituak 20 eta pintxoetan adituak 10. $L_1$ motako liburuen diseinuetan denetariko errezetak agertuko dira. $L_2$ motako liburuetan, aldiz, ez da pintxoen errezetarik agertuko, eta $L_3$ motakoetan ez da postrerik egongo. Mota bakoitzeko liburuen diseinuak egiteko bost sukaldariz osatutako lan-taldeak behar dira, sukaldari-adituen boskoteak taulan adierazten den moduan multzokatuko direlarik.

| Liburu mota | Sukaldari-adituak | | |
|---|---|---|---|
| | Eguneko menuan | Postreetan | Pintxoetan |
| $L_1$ | 2 | 2 | 1 |
| $L_2$ | 4 | 1 | 0 |
| $L_3$ | 4 | 0 | 1 |
| Adituak guztira | 40 | 20 | 10 |

Boskote bakoitzak $L_i$ motako liburu baten diseinua egingo du. Guztira dagoen sukaldari-aditu kopurua kontuan hartuz erabaki beharko da zenbat boskote osa daitezkeen, eta ondorioz, $L_i$ motako zenbat liburu diseinatuko diren.



Honako erabaki aldagaiak definitu dira:

$x_i$ : diseinatuko den $L_i$ motako liburu kopurua, $(i = 1, 2, 3)$.

Merkatuan hiru liburu motek irabazi berbera sortzen dute. Argitaletxearen helburua diseinatutako liburuek sortutako irabazia maximizatzea da. Ondoko eredu lineala idatzi da problema adierazteko, eta murrizketa bakoitzean nasaitze-aldagai bat gehituz eta eredua ebatziz taula optimoa lortu da:

$$\begin{aligned} \max\ z = {} & x_1 + x_2 + x_3 \\ \text{hauen mende} & \\ & 2x_1 + 4x_2 + 4x_3 \leq 40 \\ & 2x_1 + x_2 \leq 20 \\ & x_1 + x_3 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | 14 |
| $\mathbf{a}_2$ | 0 | 1 | 0 | $\frac{1}{5}$ | $\frac{1}{5}$ | $-\frac{4}{5}$ | 4 |
| $\mathbf{a}_1$ | 1 | 0 | 0 | $-\frac{1}{10}$ | $\frac{2}{5}$ | $\frac{2}{5}$ | 8 |
| $\mathbf{a}_3$ | 0 | 0 | 1 | $\frac{1}{10}$ | $-\frac{2}{5}$ | $\frac{3}{5}$ | 2 |

3.1 Eredu lineala eta dagokion taula optimoa aztertuz esan $L_i$ motako liburuen diseinuak egiteko zenbat boskote osa daitezkeen, eta ondorioz, zenbat libururen diseinua egingo den. Zein izango da lortuko den $z$ irabazia? 14 bostkote

3.2 Sukaldariren bat gelditu al da inongo boskotetan parte hartu gabe? Ez, denak dau

3.3 $L_1$ motako liburuek merkatuan sortutako irabazia $L_2$ edo $L_3$ motakoek sortutakoaren bikoitza bada, $c_1 = 2$, aldatuko al da diseinatuko den $L_i$ motako liburu kopurua? Eta, hirukoitza bada, $c_1 = 3$? Zenbaterainoko handia izan daiteke $c_1$, diseinatuko den $L_i$ motako liburu kopurua ez aldatzeko?

3.4 Postreetan adituak 20 izan beharrean 30 badira, liburu mota bakoitzaren diseinuak egiteko osatuko diren boskote kopuruak aldatu egingo dira. $L_i$ motako zenbat liburu diseinatuko da kasu horretan? Zein izango da lortuko den $z$ irabazia?

$\hat{x}_B = \begin{pmatrix} 6 \\ 12 \\ -2 \end{pmatrix}$

4. Enpresa batean oinarrizko hiru pintura-kolore erabiliz (gorria, urdina eta horia) azken urteotan modan jarri diren beste lau kolore sortzen dira: $K_1, K_2, K_3$ eta $K_4$. Enpresan 26 kg pintura gorri, 14 kg urdin eta 32 kg hori dago erabilgarri. Nahasketak honela egiten dira:

Kilo bat $K_1$ sortzeko $\longrightarrow$ $\frac{1}{2}$ kg gorri + $\frac{1}{4}$ kg urdin + $\frac{1}{4}$ kg hori

Kilo bat $K_2$ sortzeko $\longrightarrow$ $\frac{3}{8}$ kg gorri + $\frac{1}{4}$ kg urdin + $\frac{3}{8}$ kg hori

Kilo bat $K_3$ sortzeko $\longrightarrow$ $\frac{1}{3}$ kg gorri + $\frac{1}{3}$ kg urdin + $\frac{1}{3}$ kg hori

Kilo bat $K_4$ sortzeko $\longrightarrow$ $\frac{3}{10}$ kg gorri + $\frac{2}{5}$ kg urdin + $\frac{3}{10}$ kg hori

Enpresako arduradunak badaki $K_1, K_2, K_3$ eta $K_4$ pintura kilo bakoitzak 3, 4, 1 eta 6 monetar unitateko irabazia emango diola, hurrenez hurren. Ez dakiena hau da: $K_1, K_2, K_3$ eta $K_4$ koloreak zein kantitatetan sortzea komeni zaion irabazia maximoa izan dadin. Horretarako problema eredu lineal baten bidez adierazi eta ebatzi du. Hona hemen eredua eta dagokion taula optimoa:

$$\begin{aligned}
\max\ z &= 3x_1 + 4x_2 + x_3 + 6x_4 \\
\text{hauen mende} & \\
\tfrac{1}{2}x_1 + \tfrac{3}{8}x_2 + \tfrac{1}{3}x_3 + \tfrac{3}{10}x_4 &\leq 26 \\
\tfrac{1}{4}x_1 + \tfrac{1}{4}x_2 + \tfrac{1}{3}x_3 + \tfrac{2}{5}x_4 &\leq 14 \\
\tfrac{1}{4}x_1 + \tfrac{3}{8}x_2 + \tfrac{1}{3}x_3 + \tfrac{3}{10}x_4 &\leq 32 \\
x_1, x_2, x_3, x_4 &\geq 0
\end{aligned}$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 0 | $\frac{13}{3}$ | $\frac{2}{5}$ | 0 | 16 | 0 | 224 |
| $\mathbf{a}_5$ | $\frac{1}{8}$ | 0 | $-\frac{1}{6}$ | $-\frac{3}{10}$ | 1 | $-\frac{3}{2}$ | 0 | 5 |
| $\mathbf{a}_2$ | 1 | 1 | $\frac{4}{3}$ | $\frac{8}{5}$ | 0 | 4 | 0 | 56 |
| $\mathbf{a}_7$ | $-\frac{1}{8}$ | 0 | $-\frac{1}{6}$ | $-\frac{3}{10}$ | 0 | $-\frac{3}{2}$ | 1 | 11 |

4.1 Taula optimoa aztertuz, esan al daiteke oinarrizko koloreak (gorria, urdina, horia) kantitate egokian erosten dituela? Nola alda daitezke kantitate horiek irabazia handitzeko? Itzal-prezioak kalkula itzazu eta arrazona ezazu zure erantzuna.

4.2 Pintura urdin gutxi erosten duela susmatzen du arduradunak. Gehienez zenbat eros dezake taula osoa birkalkulatzeko beharrik izan gabe?

4.3 Demagun pintura urdin gehiago eta pintura gorri eta hori gutxiago erostea erabaki duela. Honela aldatu nahi ditu erositako pintura kilo kantitateak.

$$\mathbf{b} = \begin{pmatrix} 26 \\ 14 \\ 32 \end{pmatrix} \quad \longrightarrow \quad \hat{\mathbf{b}} = \begin{pmatrix} 24 \\ 20 \\ 31 \end{pmatrix}$$

Oinarrizko pinturak kantitate horietan izanik, $K_1, K_2, K_3$ eta $K_4$ koloreak zein kantitatetan ekoiztea komeni zaio irabazia maximizatzeko?

# Dualtasuna. Ariketak

## 1.1. Ariketa

$\min\ z = 2x_1 + 3x_2 - 4x_3$

haren mende

$x_1 + 2x_2 + 5x_3 \geq 1$

$2x_1 - 2x_2 + 4x_3 = 2$

$x_1 + 2x_2 + x_3 \geq 10$

$x_1 \leq 0,\ x_2 \geq 0,\ x_3:$ ez-mur

$\Longrightarrow$

$\max\ G = 1y_1 + 7y_2 + 10y_3$

haren mende

$y_1 + 2y_2 + y_3 \geq 2$

$2y_1 - 2y_2 + 2y_3 \leq 3$

$5y_1 + 4y_2 + y_3 = -4$

$y_1 \geq 0$ $y_2:$ ez-mur $y_3 \geq 0$

## 1.2. Ariketa

$\min\ z = x_1 + 3x_2 + x_3$

haren mende

$4x_1 - x_2 + 2x_3 \leq -7$

$2x_1 - 4x_2 \quad \geq 12$

$2x_1 + 8x_2 + 4x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$

$\Longrightarrow$

$\max\ G = -7y_1 + 12y_2 + 5y_3$

haren mende

$4y_1 + 2y_2 + 2y_3 \leq 1$

$-y_1 - 4y_2 + 8y_3 \leq 3$

$2y_1 + \quad 4y_3 \leq 1$

$y_1 \leq 0,\ y_2, y_3 \geq 0$

## 1.3. Ariketa

$\max\ z = 2x_1 + 2x_2 + 5x_3$

haren mende

$2x_1 + x_2 + 2x_3 = 12$

$-x_1 + 5x_2 - 2x_3 \geq -8$

$3x_1 + 4x_2 - 6x_3 \leq 10$

$x_1 \leq 0,\ x_2, x_3 \geq 0$

$\Longrightarrow$

$\min\ G = 12y_1 - 8y_2 + 10y_3$

haren mende

$2y_1 - y_2 + 3y_3 \geq 2$

$y_1 + 5y_2 + 4y_3 \geq 2$

$2y_1 - 2y_2 - 6y_3 \geq 5$

$y_1:$ ez-mur $y_2 \leq 0$ $y_3 \geq 0$

## 1.4. Ariketa

$\max \ z = x_1 + x_2 + 5x_3$

hauen mende

$x_1 + x_2 + 2x_3 \leq 4$

$-x_1 + 6x_2 + 2x_3 \geq 2$

$4x_1 - x_2 + x_3 = 6$

$x_1, x_2 \geq 0, \ x_3$: ez-mur

$\Rightarrow$

$\min \ G = -4y_1 + 2y_2 + 6y_3$

hauen mende

$y_1 - y_2 + 4y_3 \geq 1$

$y_1 + 6y_2 - y_3 \geq 1$

$2y_1 + 2y_2 + y_3 = 5$

$y_1 \geq 0 \quad y_2 \leq 0 \quad y_3 = 0$

## 1.5. Ariketa

$\min \ z = 4x_1 + x_2 - x_3 + 2x_4$

hauen mende

$4x_1 - 2x_2 + 3x_3 + x_4 \leq -6$

$x_1 + x_2 + x_3 + x_4 \leq 6$

$5x_1 + 2x_2 - x_3 - x_4 \geq 10$

$x_1, x_2 \leq 0, \ x_3, x_4 \geq 0$

$\Rightarrow$

$\max \ G = -6y_1 + 6y_2 + 10y_3$

hauen mende

$4y_1 + y_2 + 5y_3 \geq 4$

$2y_1 + y_2 + 2y_3 \geq 1$

$3y_1 + y_2 - y_3 \leq -1$

$y_1 + y_2 - y_3 \leq 2$

$y_1 \leq 0, \ y_2$: ez-mur $\ y_3 \geq 0$

## 1.6. Ariketa

$\max \ z = x_1 + 4x_2$

hauen mende

$2x_1 - 4x_2 \leq 14$

$-x_1 + 8x_2 \geq -6$

$4x_1 + 6x_2 \leq 10$

$x_1 + 9x_2 = 3$

$x_1 \geq 0, \ x_2 \leq 0$

$\Rightarrow$

$\min \ G = 14y_1 - 6y_2 + 10y_3 + 3y_4$

hauen mende

$2y_1 - y_2 + 4y_3 + y_4 \geq 1$

$-4y_1 + 8y_2 + 6y_3 + 9y_4 \leq 4$

$y_1 \geq 0, \ y_2 \leq 0 \ \ y_3 \geq 0 \ \ y_4$ ez-mur

# 3. Ariketa

## 3.1. Ariketa

$$\max \; z = -2x_1 - 4x_2 - 3x_3$$

hauen mende

$$2x_1 + x_2 + 2x_3 \geq 8$$
$$4x_1 + 2x_2 + 2x_3 \geq 10$$
$$6x_1 + x_2 + 4x_3 \geq 12$$
$$x_1, x_2, x_3 \geq 0$$

$\Rightarrow$

$$\max \; z = -2x_1 - 4x_2 - 3x_3$$
$$-2x_1 - x_2 - 2x_3 - x_4 = -8$$
$$-4x_1 - 2x_2 - 2x_3 - x_5 = -10$$
$$-6x_1 - x_2 - 4x_3 - x_6 = -12$$
$$x_1, x_2, x_3, x_4, x_5, x_6 = 0$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 2 | 4 | 3 | 0 | 0 | 0 | 0 | $m_0 = -\frac{1}{3}$ |
| $a_4$ | -2 | -1 | -2 | 1 | 0 | 0 | -8 | $m_1 = \frac{1}{3}$ |
| $a_5$ | -4 | -2 | -2 | 0 | 1 | 0 | -10 | $m_2 = \frac{2}{3}$ |
| $a_6$ | $\boxed{-6}$ | -1 | -4 | 0 | 0 | 1 | -12 | |
| | 0 | 11/3 | 5/3 | 0 | 0 | $\frac{1}{3}$ | -4 | -1 |
| $a_4$ | 0 | -2/3 | -2/3 | 1 | 0 | $\boxed{-1/3}$ | -4 | |
| $a_5$ | 0 | -4/3 | $\frac{2}{3}$ | 0 | 1 | $-\frac{2}{3}$ | -2 | 2 |
| $a_1$ | 1 | $\frac{1}{6}$ | $\frac{2}{3}$ | 0 | 0 | $-\frac{1}{6}$ | 2 | 1/2 |
| | 0 | 3 | 1 | 1 | 0 | 0 | -8 | |
| $a_6$ | 0 | 2 | 2 | -1/3 | 0 | 1 | 12 | |
| $a_5$ | 0 | 0 | 2 | -2 | 1 | 0 | 6 | |
| $a_1$ | 1 | 1/2 | 1 | -1/2 | 0 | 0 | 4 | |

$x_1 = 4$, $x_2 = 0$ $x_3 = 0$ $z = -8$

# 3.2. Ariilete

min $z = 2x_1 + x_2 + 3x_3 + 2x_4$

haven meude

$$2x_1 + 2x_2 + 2x_3 + 2x_4 \geq 22$$

$$4x_1 + 4x_2 + x_3 + 4x_4 \leq 20$$

$$2x_1 + 8x_2 + 2x_3 + x_4 \geq 15$$

$$x_1, x_2, x_3, x_4 \geq 0$$

max $z = -2x_1 - x_2 - 3x_3 - 2x_4$

haven meude

$$-2x_1 - 2x_2 - 2x_3 - 2x_4 + x_5 = -22$$

$$4x_1 + 4x_2 + x_3 + 4x_4 + x_6 = 20$$

$$-2x_1 - 8x_2 - 2x_3 - x_4 + x_7 = -15$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $z$ | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 1 | 3 | 2 | 0 | 0 | 0 | 0 | $m_0 = -\frac{1}{2}$ |
| $a_5$ | -2 | $\boxed{-2}$ | -2 | -2 | 1 | 0 | 0 | -22 | |
| $a_6$ | 4 | 4 | 1 | 4 | 0 | 1 | 0 | 20 | $m_1 = -2$ |
| $a_7$ | -2 | -8 | -2 | -1 | 0 | 0 | 1 | -15 | $m_2 = 4$ |
| | 1 | 0 | 2 | 1 | $\frac{1}{2}$ | 0 | 0 | -11 | $m_0 = -\frac{2}{3}$ |
| $a_2$ | 1 | 1 | 1 | 1 | $-\frac{1}{2}$ | 0 | 0 | 11 | $m_1 = -\frac{1}{3}$ |
| $a_6$ | 0 | 0 | $\boxed{-3}$ | 0 | 2 | 1 | 0 | -24 | |
| $a_7$ | 6 | 0 | 6 | 7 | -4 | 0 | 1 | 73 | $m_3 = -2$ |
| | 1 | 0 | 1 | 1 | $\frac{5}{3}$ | $\frac{1}{3}$ | 0 | -27 | |
| $a_2$ | 1 | 1 | 1 | 1 | $\frac{1}{6}$ | $\frac{1}{3}$ | 0 | 3 | |
| $a_3$ | 0 | 0 | 1 | 0 | $-\frac{2}{3}$ | $-\frac{1}{3}$ | 0 | 8 | |
| $a_7$ | 2 | 0 | 1 | 7 | 0 | 2 | 1 | 25 | |

$x_1^* = x_4^* = 0$

$x_3^* = 8 \quad x_2^* = 3 \quad z^* = 27$

## 3.3. Ariketa

$\max\ z = -2x_1 - 3x_2 - x_3 - x_4$
hauen mende

$x_1 + x_2 + 3x_3 + x_4 \leq 40$

$2x_1 + 3x_2 + x_3 + x_4 \geq 30$

$2x_1 + x_3 \leq 25$

$\max\ z = -2x_1 - 3x_2 - x_3 - x_4$
hauen mende

$-x_1 + x_2 + 3x_3 + x_4 + x_5 = 40$

$-2x_1 - 3x_2 - x_3 - x_4 + x_6 = -30$

$2x_1 + x_3 + x_7 = 25$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 1 | 1 | 0 | 0 | 0 | 0 |
| $a_5$ | -1 | 1 | 3 | 1 | 1 | 0 | 0 | 40 |
| $a_6$ | $\boxed{-2}$ | -3 | -1 | -1 | 0 | 1 | 0 | -30 |
| $a_7$ | 2 | 0 | 1 | 0 | 0 | 0 | 1 | 25 |
| | 0 | 0 | 0 | 0 | 0 | 1 | 0 | -30 |
| $a_5$ | 0 | $-\frac{1}{2}$ | $\frac{5}{2}$ | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | 0 | 25 |
| $a_1$ | 1 | $\frac{3}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | 15 |
| $a_7$ | 0 | -3 | 0 | -1 | 0 | 1 | 1 | -5 |

$m_0 = -1$

$m_1 = -\frac{1}{2}$

$m_3 = -1$

$z^* = -30$ Anizkoitz.

$x_1^* = 15 \quad x_5^* = 25$

$x_7^* = -5$

Soluzio optimo anizkoitza daude ondorioz, emaitza $z^* = -30$ izango da.

## 3.4. Ariketa

$\max\ z = -6x_1 - 4x_2 - 5x_3 - 4x_4$
hauen mende

$2x_1 + 4x_2 + 2x_3 + 5x_4 \leq 10$

$x_1 + 2x_2 + x_4 \geq 25$

$x_1, x_2, x_3, x_4 \geq 0$

$\max\ z = -6x_1 - 4x_2 - 5x_3 - 4x_4$
hauen mende

$2x_1 + 4x_2 + 2x_3 + 5x_4 + x_5 = 10$

$-x_1 - 2x_2 - x_4 + x_6 = -25$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | -6 | -4 | -5 | -4 | 0 | 0 | 0 |
| $a_5$ | 2 | 4 | 2 | 5 | 1 | 0 | 10 |
| $a_6$ | -1 | -2 | 0 | -1 | 0 | 1 | -25 |

Ez dago $y_{rj}$ negatiborik ondorioz, bornagabea izango da.

## 3.6

max $z = -3x_1 + 4x_2 + 2x_3 + 5x_4$

hauen mende

$4x_1 + 2x_2 + 4x_3 + 3x_4 \leq 48$

$-x_1 + 2x_2 - x_3 + 2x_4 \geq 8$

$2x_1 - x_2 + x_3 + x_4 \geq 6$

$x_1, x_2, x_3, x_4 \geq 0$

max $z = -3x_1 + 4x_2 + 2x_3 + 5x_4$

hauen mende

$4x_1 + 2x_2 + 4x_3 + 3x_4 + x_5 = 48$

$x_1 - 2x_2 + x_3 - 2x_4 + x_6 = -8$

$-2x_1 + x_2 - x_3 - x_4 + x_7 = -6$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 3 | -4 | -2 | -5 | 0 | 0 | 0 | 0 | $m_0 = 2$ |
| $a_5$ | 4 | 2 | 4 | 3 | 1 | 0 | 0 | 48 | $m_1 = -1$ |
| $a_6$ | 1 | $\boxed{-2}$ | $\boxed{1}$ | -2 | 0 | 1 | 0 | -8 | |
| $a_7$ | -2 | 1 | -1 | -1 | 0 | 0 | 1 | -6 | $m_3 = -\frac{1}{2}$ |
| | 1 | 0 | -4 | -1 | 0 | -2 | 0 | 16 | $m_0 = \frac{1}{2}$ |
| $a_5$ | 5 | 0 | 5 | 1 | 1 | 1 | 0 | 40 | $m_1 = -\frac{1}{2}$ |
| $a_2$ | $-\frac{1}{2}$ | 1 | $-\frac{1}{2}$ | 1 | 0 | $-\frac{1}{2}$ | 0 | 4 | $m_2 = -\frac{1}{2}$ |
| $a_7$ | $-\frac{3}{2}$ | 0 | $-\frac{1}{2}$ | $\boxed{-2}$ | 0 | $\frac{1}{2}$ | 1 | -2 | |
| | 3 | 0 | $-\frac{7}{2}$ | 0 | 0 | $-\frac{5}{2}$ | $-\frac{1}{2}$ | -17 | |
| $a_5$ | $\frac{17}{4}$ | 0 | $\frac{19}{4}$ | 0 | 1 | $\frac{3}{4}$ | $-\frac{1}{2}$ | 41 | |
| $a_2$ | -1 | 1 | $-\frac{1}{2}$ | 0 | 0 | $-\frac{3}{4}$ | $-\frac{1}{2}$ | 5 | |
| $a_4$ | $\frac{3}{4}$ | 0 | $\frac{1}{4}$ | 1 | 0 | $-\frac{1}{4}$ | $-\frac{1}{2}$ | 1 | |

## 3.7.

max $z = 3x_1 - 2x_2 + 2x_3 + x_4$

hoven mende

$$3x_1 + 6x_2 + 3x_3 + 2x_4 + x_5 = 36$$

$$-x_1 - 2x_2 - 3x_3 - x_4 + x_6 = -14$$

$$-x_1 - x_2 - x_3 - 2x_4 + x_7 = -10$$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | -3 | 2 | -2 | -1 | 0 | 0 | 0 | 0 | 0 | $m_0 = -3$ |
| $a_5$ | 3 | 6 | 3 | 2 | 1 | 0 | 0 | 0 | 36 | $m_1 = 3$ |
| $a_6$ | -1 | -2 | -3 | -1 | 0 | 1 | 0 | 0 | -14 | $m_2 = -1$ |
| $a_7$ | -1 | -1 | -1 | -2 | 0 | 0 | 1 | 0 | -10 | $m_3 = 1$ |
| $a_8$ | $\boxed{1}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | M | |
| | 0 | 2 | 1 | 2 | 0 | 0 | 0 | 3 | 3M | $m_0 = -1$ |
| $a_5$ | 0 | 6 | 0 | -1 | 1 | 0 | 0 | $\boxed{-3}$ | 36-3M | |
| $a_6$ | 0 | -2 | -2 | 0 | 0 | 1 | 0 | 1 | M-14 | $m_2 = -\frac{1}{3}$ |
| $a_7$ | 0 | -1 | 0 | -1 | 0 | 0 | 1 | 1 | M-10 | $m_3 = -\frac{1}{3}$ |
| $a_1$ | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | M | $m_4 = -\frac{1}{3}$ |
| | 0 | 8 | 1 | 1 | 1 | 0 | 0 | 0 | 36 | $m_0 = -\frac{1}{2}$ |
| $a_8$ | 0 | -2 | 0 | $\frac{1}{3}$ | $-\frac{1}{3}$ | 0 | 0 | 1 | M-12 | $m_1 = 0$ |
| $a_6$ | 0 | 0 | $\boxed{-2}$ | -1/3 | 1/3 | 1 | 0 | 0 | -2 | |
| $a_7$ | 0 | 1 | 0 | -4/3 | 1/3 | 0 | 1 | 0 | +2 | $m_3 = 0$ |
| $a_1$ | 1 | 2 | 1 | $\frac{2}{3}$ | 1/3 | 0 | 0 | 0 | 12 | $m_4 = -\frac{1}{2}$ |
| | 0 | 8 | 0 | 5/6 | 7/6 | 1/2 | 0 | 0 | 35 | |
| $a_8$ | 0 | -2 | 0 | 1/3 | -1/3 | 0 | 0 | 1 | M-12 | |
| $a_3$ | 0 | 0 | 1 | $\frac{1}{6}$ | $-\frac{1}{6}$ | $-\frac{1}{2}$ | 0 | 0 | 1 | |
| $a_7$ | 0 | 1 | 0 | -4/3 | 1/3 | 0 | 1 | 0 | 2 | |
| $a_1$ | 1 | 2 | 0 | 1/2 | 1/2 | 1/2 | 0 | 0 | 11 | |

$$x_1^* = 11 \quad x_2^* = 0 \quad x_3^* = 1 \quad x_4^* = 0 \quad z^* = 35$$

# 4. Ariketa

$\max \ z = 10x_1 + 6x_2$

hauen mende

$x_1 + 2x_2 \leq 2$

$2x_1 + x_2 \leq 3$

$2x_1 + 2x_2 \leq 3$

$4x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

$\Longrightarrow$ Eredu Duala

**4.1**

$\min \ G = 2y_1 + 3y_2 + 3y_3 + 2y_4$

hauen mende

$y_1 + 2y_2 + 2y_3 + 4y_4 \geq 10$

$2y_1 + y_2 + 2y_3 + y_4 \geq 6$

$y_1, y_2, y_3, y_4 \geq 0$

**4.2**

Simplex dual algoritmoa erabiliko dugu

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $-2$ | $-3$ | $-3$ | $-2$ | $0$ | $0$ | $0$ | $m_0 = 1$ |
| $a_5$ | $-1$ | $-2$ | $-2$ | $-4$ | $1$ | $0$ | $-10$ | $m_1 = \frac{1}{2}$ |
| $a_6$ | $\boxed{-2}$ | $-1$ | $-2$ | $-1$ | $0$ | $1$ | $-6$ | |
| | $0$ | $-2$ | $-1$ | $-1$ | $0$ | $-1$ | $+6$ | $m_0 = \frac{2}{7}$ |
| $a_5$ | $0$ | $-\frac{3}{2}$ | $-1$ | $\boxed{-\frac{7}{2}}$ | $1$ | $-\frac{1}{2}$ | $-7$ | $m_1 = -\frac{1}{7}$ |
| $a_1$ | $1$ | $\frac{1}{2}$ | $1$ | $\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $3$ | |
| | $0$ | $-\frac{11}{7}$ | $-\frac{5}{7}$ | $0$ | $-\frac{2}{7}$ | $-\frac{6}{7}$ | $8$ | |
| $a_4$ | $0$ | $\frac{3}{7}$ | $\frac{2}{7}$ | $1$ | $-\frac{2}{7}$ | $\frac{1}{7}$ | $2$ | |
| $a_1$ | $1$ | $\frac{2}{7}$ | $\frac{6}{7}$ | $0$ | $\frac{1}{7}$ | $-\frac{4}{7}$ | $2$ | |

Solusio optimoa aurkitu da, $y_1^* = 2 \quad y_4^* = 2 \quad z^* = 8$

**4.3**

$x_1^* = \frac{2}{7} \qquad x_2^* = \frac{6}{7} \qquad z^* = 8$

Dira eredu primalaren soluzioak

# 5. Ariketa

$\min z = 30x_1 + 28x_2$

haren mende

$4x_1 + 2x_2 \geq 20$

$6x_1 + 4x_2 \geq 16$

$4x_1 + 2x_2 \geq 18$

$4x_1 + 4x_2 \geq 21$

$x_1, x_2 \geq 0$

## 5.1

$\max G = 20y_1 + 16y_2 + 18y_3 + 21y_4$

haren mende

$4y_1 + 6y_2 + 4y_3 + 4y_4 \leq 30$

$2y_1 + 4y_2 + 2y_3 + 4y_4 \leq 28$

$y_1, y_2, y_3, y_4 \geq 0$

## 5.2

Simplex primal algoritmoa erabiliko da.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $-20$ | $-16$ | $-18$ | $-21$ | $0$ | $0$ | $0$ | $m_0 = -\frac{21}{4}$ |
| $a_5$ | $4$ | $6$ | $4$ | $4$ | $1$ | $0$ | $30$ | $m_1 = 1$ |
| $a_6$ | $2$ | $4$ | $2$ | $\boxed{4}$ | $0$ | $1$ | $28$ | $m$ |
| | $-\frac{19}{2}$ | $5$ | $-\frac{15}{2}$ | $0$ | $0$ | $\frac{21}{4}$ | $147$ | $m_0 = -\frac{19}{4}$ |
| $a_4$ | $\boxed{2}$ | $2$ | $2$ | $0$ | $1$ | $-1$ | $12$ | $m_1 = \frac{1}{4}$ |
| $a_6$ | $\boxed{\frac{1}{2}}$ | $1$ | $\frac{1}{2}$ | $1$ | $0$ | $\frac{1}{4}$ | $7$ | |
| | $0$ | $\frac{29}{2}$ | $2$ | $0$ | $\frac{19}{4}$ | $\frac{1}{2}$ | $313/2$ | |
| $a_1$ | $1$ | $1$ | $1$ | $0$ | $1/2$ | $-1/2$ | $1$ | |
| $a_6$ | $0$ | $1/2$ | $0$ | $1$ | $-\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{13}{2}$ | |

$y_1^* = 1 \quad y_2^* = y_3^* = y_4^* = y_5^* = 0 \quad y_6^* = \frac{13}{2} \qquad z^* = \frac{313}{2}$

## 5.3

Ereduu primalaren balioa optimoa: $x_1^* = \frac{19}{4} \quad x_2^* = \frac{1}{2} \quad z^* = \frac{313}{2}$

# 6. Ariketa

## 6.1

a) Taula optimotik eredu dualaren soluzio optimoa, Ondorengoa da.

$$x_1^* = \frac{7}{2} \quad x_2^* = \frac{3}{2} \quad x_3^* = 0 \quad z^* = \frac{57}{2}$$

b)

Eredu Duala

$$\min z = 90y_1 + 60y_2$$

hauen mende

$$15y_1 + 15y_2 \geq 6$$

$$25y_1 + 5y_2 \geq 5$$

$$30y_1 + 15y_2 \geq 4$$

Eredu Dualaren soluzioa

$$y_1^* = \frac{3}{20} \quad y_2^* = \frac{1}{4} \quad z^* = \frac{57}{2}$$

c)

Itzal-prezioen interpretazioa egin

$$\hat{b} = \begin{pmatrix} 91 \\ 60 \end{pmatrix} \quad x_B = B^{-1}\hat{b} = \begin{pmatrix} \frac{1}{20} & -\frac{1}{20} \\ -\frac{1}{60} & \frac{1}{12} \end{pmatrix} \begin{pmatrix} 91 \\ 60 \end{pmatrix} = \begin{pmatrix} \frac{31}{20} \\ \frac{209}{60} \end{pmatrix} \geq 0$$

91-ra pasatzen bada $b_1$, Soluzio optimo berria $\hat{z} = z + y_1^* = \frac{57}{2} + \frac{3}{20} = \frac{573}{20}$

$$x_1^* = \frac{209}{60} \quad x_2^* = \frac{31}{20} \quad \hat{z} = \frac{573}{20}$$

$$\hat{b} = \begin{pmatrix} 90 \\ 61 \end{pmatrix} \quad x_B = B^{-1}\hat{b} = \begin{pmatrix} \frac{1}{20} & -\frac{1}{20} \\ -\frac{1}{60} & \frac{1}{12} \end{pmatrix} \begin{pmatrix} 90 \\ 61 \end{pmatrix} = \begin{pmatrix} \frac{29}{20} \\ \frac{215}{60} \end{pmatrix} \geq 0$$

$$\hat{z} = z^* + y_2^* = \frac{57}{2} + \frac{1}{4} = \frac{115}{4}$$

$$x_1^* = \frac{215}{60} \quad x_2^* = \frac{29}{20} \quad z^* = \frac{115}{4}$$

6.2

$\max\ z = x_1 + x_2$

havan mende

$x_1 + 6x_2 \geq 6$

$2x_1 - 3x_2 \geq -6$

$x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$

$\max\ z = x_1 + x_2 - Mw_1$

havan mende

$x_1 + 6x_2 - x_3 + w_1 = 6$

$-2x_1 + 3x_2 + x_4 = 6$

$x_1 + 2x_2 + x_5 = 6$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $-M-1$ | $-6M-1$ | $M$ | 0 | 0 | 0 | $-6M$ | $m_0 = -6M - 1/6$ |
| $-M$ | $a_{w_1}$ | 1 | $\boxed{6}$ | $-1$ | 0 | 0 | 1 | 6 | |
| 0 | $a_4$ | $-2$ | 3 | 0 | 1 | 0 | 0 | 6 | $m_2 = \frac{1}{2}$ |
| 0 | $a_5$ | 1 | 2 | 0 | 0 | 1 | 0 | 6 | $m_3 = \frac{1}{3}$ |
| | | $-5/6$ | 0 | $1/6$ | 0 | 0 | $M+1/6$ | 1 | $m_0 = -5$ |
| 1 | $a_2$ | $\boxed{1/6}$ | 1 | $-1/6$ | 0 | 0 | $1/6$ | 1 | |
| 0 | $a_4$ | $-5/2$ | 0 | $1/2$ | 1 | 0 | $-1/2$ | 3 | $m_2 = -15$ |
| 0 | $a_5$ | $+2/3$ | 0 | $1/3$ | 0 | 1 | $-1/3$ | 4 | $m_3 = 4$ |
| | | 0 | 5 | $-1$ | 0 | 0 | $M+1$ | 6 | $m_0 = -1$ |
| 1 | $a_1$ | 1 | 6 | $-1$ | 0 | 0 | 1 | 6 | $m_1 = -1$ |
| 0 | $a_4$ | 0 | 15 | $-2$ | 1 | 0 | 2 | 18 | $m_2 = -2$ |
| 0 | $a_5$ | 0 | $-4$ | $\boxed{1}$ | 0 | 1 | $-1$ | 0 | |
| | | 0 | 1 | 0 | 0 | 1 | $M$ | 6 | |
| 1 | $a_1$ | 1 | 2 | 0 | 0 | $-1$ | 0 | 6 | |
| 0 | $a_4$ | 0 | 7 | 0 | 1 | $-2$ | 0 | 18 | |
| 0 | $a_3$ | 0 | $-4$ | 1 | 0 | 1 | $-1$ | 0 | |

$x_1^* = 6 \quad x_4^* = 18 \quad z^* = 6$

7.4

$\min z = 10x_1 + 8x_2 + 6x_3 + 4x_4$

[illegible]

$2x_1 + 4x_2 + 2x_3 + x_4 \geq 10$

$-4x + 4x_2 - x_3 + 2x_4 \geq 12$

$x_1, x_2, x_3, x_4 \geq 0$

$\max -z = -10x_1 - 8x_2 - 6x_3 - 4x_4$

[illegible]

$2x_1 + 4x_2 + 2x_3 + x_4 - x_5 + Mw_1 = 10$

$-4x_1 + 4x_2 - x_3 + 2x_4 - x_6 + Mw_2 = 12$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 2M+10 | -8M+8 | M+6 | -3M+4 | -M | -M | 0 | 0 | -22M |
| -M | $aw_1$ | 2 | $\boxed{4}$ | 2 | 4 | -1 | 0 | 1 | 0 | 10 |
| -M | $aw_2$ | -4 | 4 | -1 | 2 | 0 | -1 | 0 | 1 | 12 |
| | | 6M+10 | 0 | 3M+2 | -M+2 | -M+2 | M | -2 | 0 | -20-2M |
| -8 | $x_2$ | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $-\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 0 | $\frac{10}{4}$ |
| -M | $aw_2$ | -6 | 0 | -3 | $\boxed{1}$ | 1 | -1 | -1 | 1 | 2 |
| | | 18 | 0 | 8 | 0 | 0 | 2 | M | M-2 | -24 |
| -8 | $x_2$ | 2 | 1 | $\frac{5}{4}$ | 0 | $-\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $-\frac{1}{4}$ | 2 |
| -4 | $x_4$ | -6 | 0 | -3 | 1 | 1 | -1 | -1 | 1 | 2 |

$m_2 = 1$

$m_1 = \frac{1}{4}$

$x_2^* = 2$

$x_4^* = 4$ $\qquad z^* = 24$

## 6.3

Max $z = 4x_1 - 4x_2$

haven wende

$-2x_1 + 2x_2 \leq 4$

$2x_1 - 2x_2 \leq 6$

$-x_1 + 4x_2 \geq -2$

$x_1, x_2 \geq 0$

max $z = 4x_1 - 4x_2 + Mw_1$

haven wende

$-2x_1 + 2x_2 + x_3 = 4$

$2x_1 - 2x_2 + x_4 = 6$

~~$6x_1 + 4x_2 - x_5 + Mw_1 = 2M$~~

$x_1 - 4x_2 + x_5 = 2$

| | | 4M | -4-4M | 0 | 0 | M | 0 | 2M |
|---|---|---|---|---|---|---|---|---|
| 0 | $a_3$ | -2 | 2 | 1 | 0 | 0 | 0 | 4 |
| 0 | $a_4$ | 2 | -2 | 0 | 1 | 0 | 0 | 6 |
| -M | $aw_1$ | -1 | 4 | 0 | 0 | -1 | 1 | -2 |

Ez da korrek egin behar.

## 6.4

max $z = x_1 + 2x_2$

haven wende

$x_1 + 2x_2 \leq 5$

$x_1 + x_2 \geq 2$

$x_1 - x_2 \leq 4$

max $z = x_1 + 2x_2 - Mw_2$

haven wende

$x_1 + 2x_2 + x_3 = 5$

$x_1 + x_2 - x_4 + w_2 = 2$

$x_1 - x_2 + x_5 = 4$

| | | -1-M | -2-M | 0 | M | 0 | 0 | -2M | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | $a_3$ | 1 | 2 | 1 | 0 | 0 | 0 | 5 | $m_1 = 1$ |
| -M | $aw_2$ | $\boxed{1}$ | 1 | 0 | -1 | 0 | 1 | 2 | |
| 0 | $a_5$ | 1 | -1 | 0 | 0 | 1 | 0 | 4 | $m_3 = 1$ |
| | | 0 | -1 | 0 | -1 | 0 | M | 2 | |
| 0 | $a_3$ | 0 | 1 | 1 | 1 | 0 | -1 | 3 | $m_1 = 1$ |
| -1 | $a_1$ | 1 | $\boxed{1}$ | 0 | -1 | 0 | 1 | 2 | $m_2 = 1$ |
| 0 | $a_5$ | 0 | -2 | 0 | $\boxed{1}$ | 1 | -1 | 2 | $m_3 = -2$ |

| | | $\varnothing$ | 0 | 0 | +2 | 0 | M+2 | 4 |
|---|---|---|---|---|---|---|---|---|
| 0 | $a_3$ | -1 | 0 | 1 | 2 | 0 | -2 | 1 |
| 2 | $a_2$ | 1 | 1 | 0 | -1 | 0 | 1 | 2 |
| 0 | $a_5$ | 2 | 0 | 0 | -1 | 1 | 1 | 6 |

6.8

max $z = 3x_1 + 4x_2$

kısıtlar

$x_1 - 2x_2 \leq 4$

$x_1 + x_2 \geq 6$

$2x_1 + 3x_2 \leq 6$

$x_1, x_2 \geq 0$

max $z = 3x_1 + 4x_2 - Mw_1$

kısıtlar

$x_1 - 2x_2 + x_3 = 4$

$x_1 + x_2 - x_4 + w_1 = 6$

$2x_1 + 3x_2 + x_5 = 6$

$x_1, x_2, x$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | -3-M | -4-M | 0 | M | 0 | 0 | -6M |
| 0 | $a_3$ | 1 | -2 | 1 | 0 | 0 | 0 | 4 |
| -M | $a_{w_1}$ | 1 | 1 | 0 | -1 | 0 | 1 | 6 |
| 0 | $a_5$ | 2 | $\boxed{3}$ | 0 | 0 | 1 | 0 | 6 |
| | | $-\frac{1}{3}-\frac{1}{3}M$ | 0 | 0 | M | $\frac{4}{3}+M$ | 0 | 8-4M |
| 0 | $a_3$ | 7/3 | 0 | 1 | 0 | 2/3 | 0 | 8 |
| -M | $a_{w_1}$ | $\boxed{1/3}$ | 0 | 0 | -1 | -1/3 | 1 | 4 |
| 4 | $a_2$ | $\frac{2}{3}$ | 1 | 0 | 0 | $\frac{1}{3}$ | 0 | 2 |
| | | 0 | 0 | 0 | 7 | 1 | 1 | 12 |
| 0 | $a_3$ | 0 | 0 | 1 | 7 | 3 | -7 | -20 |
| 3 | $a_1$ | 1 | 0 | 0 | -1/3 | -1 | 3 | 12 |
| 4 | $a_2$ | 0 | 1 | 0 | 2 | 1 | -2 | -6 |

$m_1 = -\frac{2}{3}$

$m_2 = \frac{1}{3}$

$m_1 = 7$

$m_3 = 2$

Problem bidercedir.

7.2

$\min\ z = 5x_1 - x_2 - 2x_3$

hoven mende

$x_1 + x_2 - x_3 = 6$

$x_1 + 2x_2 - x_3 \geq 4$

$-x_1 + x_2 + 2x_3 \leq 8$

$x_1 \leq 0,\ x_2, x_3 \geq 0$

$\max\ -z = +5x_1 + x_2 + 2x_3$

hoven mende

$-x_1' + x_2 - x_3 + w_1 = 6$

$-x_1' + 2x_2 - x_3 - x_4 + w_2 = 4$

$x_1' + x_2 + 2x_3 + x_5 = 8$

$x_1', x_2, x_3 \geq 0$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 2M-5 | -1-3M | 2M-2 | M | 0 | 0 | 0 | -10M | |
| -M | $a_{w_1}$ | -1 | 1 | -1 | 0 | 0 | 1 | 0 | 6 | $m_1 = \frac{1}{2}$ |
| -M | $a_{w_2}$ | -1 | $\boxed{2}$ | -1 | -1 | 0 | 0 | 1 | 4 | |
| 0 | $a_5$ | 1 | 1 | 2 | 0 | 1 | 0 | 0 | 8 | $m_2 = \frac{1}{2}$ |
| | | $\frac{M}{2} - \frac{9}{2}$ | 0 | $\frac{M}{2} - 2$ | $-M + \frac{1}{2}$ | 0 | 0 | $\frac{3}{2}M - \frac{1}{2}$ | -2-4M | |
| -M | $a_{w_1}$ | -1/2 | 0 | -1/2 | $\boxed{+1/2}$ | 0 | 1 | -1/2 | 4 | |
| -1 | $a_2$ | -1/2 | 1 | -1/2 | -1/2 | 0 | 0 | $\frac{1}{2}$ | 2 | $m_2 = -1$ |
| 0 | $a_5$ | 3/2 | 0 | 5/2 | 1/2 | 1 | 0 | -1/2 | 6 | $m_3 = 1$ |
| | | 6 | 0 | 3 | 0 | 0 | M+1M | | -6 | |
| 0 | $a_4$ | -1 | 0 | -1 | 1 | 0 | 2 | -1 | 8 | |
| -1 | $a_2$ | -1 | 1 | -1 | 0 | 0 | 1 | 0 | 6 | |
| 0 | $a_5$ | $\boxed{2}$ | 0 | 3 | 0 | 1 | -1 | 0 | 2 | |
| | | | | | | | | | | |
| 0 | $a_4$ | | | | | | | | | |
| -1 | $a_2$ | | | | | | | | | |
| -5 | $a_1$ | | | | | | | | | |

4. Gaia

# Sentikortasunaren analisia

## Planteamendu orokorra

max $z = c^t x$

haren mende

$Ax < b$

$x \geq 0$

max $z = c^t x + 0^t x_h$

haren mende

$Ax + I x_h = b$

$x, x_h \geq 0$

### Hasierako taula

| | $x_1 \dots x_n$ | $x_{n+1} \dots x_{n+m}$ | |
|---|---|---|---|
| | $-c^T$ | $0$ | $0$ |
| | $A$ | $I$ | $b$ |

### taula optimoa

| | | | |
|---|---|---|---|
| | $c_B^T B^{-1} A - c^T$ | $c_B^T B^{-1}$ | $z = c_B^T x_B$ |
| $B$ | $B^{-1} A$ | $B^{-1}$ | $x_B = B^{-1} b$ |

$z_1 - c_1 = c_B^T B^{-1} a_1 - c_1$

Taula optimoa $\rightarrow$ bideragarritasun primala eta duala

$x_B \geq 0$ $\qquad z_j - c_j \geq 0$

## Adibidea

| Baliabideak | Produktuak A | B | C | Baliabide erabilgarritasuna |
|---|---|---|---|---|
| 1 | 4 | 2 | 3 | 40 |
| 2 | 2 | 2 | 1 | 30 |
| Irabazia | 3 | 2 | 1 | |

$\max \ z = 3x_1 + 2x_2 + x_3$

hauen mende

$4x_1 + 2x_2 + 3x_3 \leq 40$

$2x_1 + 2x_2 + x_3 \leq 30$

$x_1, x_2, x_3 \geq 0$

1. Kasuan. $\hat{X}_B \geq 0$ bada, taula optimoa da, 2. Ereduarako: $\hat{X}_B$ eta $\hat{z}$.

2. Kasuan. $\hat{X}_B \not\geq 0$ bada, ez dago bideragarritasun primalik $\rightarrow$ Simplex dual

$b^t = (40, 30) \longrightarrow \hat{b}^t = (38, 36)$

$\hat{X}_B = B^{-1}\hat{b} = \begin{pmatrix} 1 \\ 17 \end{pmatrix} \geq 0$

$\hat{z} = C_B^t \hat{X}_B = 37$

2. Ereduari dagokion lehenengo taula optimoa da.

Soluzio optimoa: $x_1^* = 1$, $x_2^* = 17$, eta $z^* = 37$

## Aldaketak kostu-bektorean: $c \rightarrow \hat{c}$

1. Eredua

$\max \ z = c^t x$

hauen mende

$Ax \leq b$

$x \geq 0$

2. Eredua

$\max \ z = \hat{c}^t x$

hauen mende

$Ax \leq b$

$x \geq 0$

$\hat{z}_j - \hat{c}_j = \hat{c}_B^t B^{-1} a_j - c_j = \hat{c}_B^t y_j - \hat{c}_j$

$\hat{z} = \hat{c}_B^t x_B$

1. kasuan. $\hat{z}_j - \hat{c}_j \geq 0$ bada, $x_B$ eta $\hat{z} = \hat{c}_B^t x_B$ optimoak

2. kasuan: $\hat{z}_j - \hat{c}_j < 0$ ~~bada~~ existitzen bada, ez dago bideragarritasun duala → simplex primala.

## Aldagaietako oinarrizkoa ez den bektore batean: $a_j \to \hat{a}_j$

1. Eredua

$\max z = c^t x$

hauen mende

$a_1 x_1 + \dots + a_j x_j + \dots + a_n x_n < b$

$x_1, \dots, x_n \geq 0$

2. Eredua

$\max z = c^t x$

hauen mende

$a_1 x_1 + \dots + \hat{a}_j x_j + \dots + a_n x_n \leq b$

$x_1, \dots, x_n \geq 0$

1. kasua. $\hat{z}_i - c_j \geq 0$ bada, bideragarritasun duala dago, soluzio optimoa $x_B$ eta $z$.

2. kasuan $\hat{z}_j - \hat{c}_j < 0$ existitzen bada, ez dago bideragarritasun duala → Simplex primala.

## 11

### 1.1.1

$$b = \begin{pmatrix} 4 \\ 10 \\ 16 \end{pmatrix} \longrightarrow \hat{b} = \begin{pmatrix} 2 \\ 10 \\ 16 \end{pmatrix}$$

$$\hat{x}_B = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 10 \\ 16 \end{pmatrix} = \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix}$$

$$\cancel{x_B = B^{-1}\hat{b} = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}}$$

$$\hat{z} = c_B^T \hat{x}_B = (4 \; 5 \; 0) \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix} = 14$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | 2 | 0 | 2 | 1 | 0 | 14 |
| $a_1$ | 1 | 2 | 0 | 3 | $\boxed{-1}$ | 0 | -4 |
| $a_3$ | 0 | -1 | 1 | -2 | 1 | 0 | 6 |
| $a_6$ | 0 | -1 | 0 | -1 | -1 | 1 | 4 |
| | 1 | 4 | 0 | 5 | 0 | 0 | 10 |
| $a_5$ | -1 | -2 | 0 | -3 | 1 | 0 | 4 |
| $a_3$ | 1 | 1 | 1 | 1 | 0 | 0 | 2 |
| $a_6$ | -1 | -3 | 0 | -4 | 0 | 1 | 8 |

$\Rightarrow z^* = 10 \quad x_3^* = 2 \quad x_5^* = 4 \quad x_6^* = 8$

### 1.1.2

$$b = \begin{pmatrix} 4 \\ 10 \\ 16 \end{pmatrix} \longrightarrow \hat{b} = \begin{pmatrix} 4 \\ 10 \\ 18 \end{pmatrix}$$

$$\hat{x}_B = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 16 \\ 18 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

$$\hat{z} = c_B^T \hat{x}_B = (4 \; 5 \; 0) \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} = 18$$

$x_1^* = 2 \quad x_3^* = 2 \quad x_6^* = 4 \quad z^* = 18$

1.1.3

$c^t = (4\ 1\ 5) \longrightarrow \hat{c}^t = (3\ 3\ 5)$

$\hat{z}_j - \hat{c}_j = \hat{c}_B^{\,t} B^{-1} a_j - c_j = \hat{c}_B^{\,t} y_j - \hat{c}_j = \cancel{(3\ 5\ 0)}$

$\hat{z} = \hat{c}_B^{\,T} x_B =$

$= (3\ 5\ 0)\begin{pmatrix}2\\2\\2\end{pmatrix} = 16$

$\hat{z}_1 - \hat{c}_1 = (3\ 5\ 0)\begin{pmatrix}1\\0\\0\end{pmatrix} - 3 = 0$

$\hat{z}_2 - \hat{c}_2 = (3\ 5\ 0)\begin{pmatrix}2\\-1\\-1\end{pmatrix} - 3 = 1 - 3 = -2$

$\hat{z}_3 - \hat{c}_3 = (3\ 5\ 0)\begin{pmatrix}0\\1\\0\end{pmatrix} - 5 = 0$

$\hat{z}_4 - \hat{c}_4 = (3\ 5\ 0)\begin{pmatrix}3\\-2\\-1\end{pmatrix} - 0 = -1$

$\hat{z}_5 - \hat{c}_5 = (3\ 5\ 0)\begin{pmatrix}-1\\1\\-1\end{pmatrix} - 0 = 2$

$\hat{z}_6 - \hat{c}_6 = (3\ 5\ 0)\begin{pmatrix}0\\0\\1\end{pmatrix} - 0 = 0$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0 | -2 | 0 | -1 | 2 | 0 | 16 |
| $a_1$ | 1 | $\boxed{2}$ | 0 | 3 | -1 | 0 | 2 |
| $a_3$ | 0 | -1 | 1 | -2 | 1 | 0 | 2 |
| $a_5$ | 0 | -1 | 0 | 1 | -1 | 1 | 2 |
| | 1 | 0 | 0 | 2 | 1 | 0 | 18 |
| $a_2$ | 1/2 | 1 | 0 | 3/2 | -1/2 | 0 | 1 |
| $a_3$ | 1/2 | 0 | 1 | -1/2 | 1/2 | 0 | 3 |
| $a_5$ | 1/2 | 0 | 0 | 1/2 | -3/2 | 1 | 3 |

$x_2^* = 1 \quad x_3^* = 3 \quad x_5^* = 3 \qquad z^* = 18$

1.2

$\hat{z}^* = \hat{z}^* + y_1^* =$

$$\bar{X}_B = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4+1 \\ 10 \\ 14 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} \geq \bar{0}$$

$y_1^*$ 1. baliabidearen itzal-prezioa, $\hat{z}^* = 18 + y_1^* = 20$

$$\hat{z}^* = (4\ 5\ 0) \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} = 20$$

Bigarrenean egingo da aldaketa

$$\bar{X}_B = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 11 \\ 16 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \geq \bar{0}$$

$y_2^*$ 2. baliabidearen itzal-prezioa $\hat{z}^* = 18 + y_2^* = 19$

$$\hat{z}^* = (4\ 5\ 0) \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = 19$$

$y_3^*$ 3. baliabidearen itzal-prezioa $\hat{z} = 18 + \cancel{y_3}\,0 = 18$

Bideragarritasuna galtzen ez bada

1.3

$$\begin{pmatrix} b_1 \\ 10 \\ 16 \end{pmatrix}; \quad \hat{X}_B = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ 10 \\ 16 \end{pmatrix} = \begin{pmatrix} 3b_1 - 10 \\ -2b_1 + 10 \\ -b_1 + 6 \end{pmatrix} \geq 0$$

$[10/3,\ 5]$

$c_1$-ek. ze balio hartu ditzake bideragarritasuna galdu gabe.

$$\hat{z}_2 - c_2 = (\cancel{4}c_1, 5, 0)\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} - 1 = 2c_1 - 6 \geq 0$$

$$\hat{z}_4 - c_4 = (c_1, 5, 0)\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} - 0 = 3c_1 - 10 \geq 0 \qquad \left[\frac{10}{3}, 5\right]$$

$$\hat{z}_5 - c_5 = (c_1, 5, 0)\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} - 0 = -c_1 + 5 \geq 0 \Rightarrow$$

$c_2$

$$\hat{z}_2 - c_2 = (4, 5\ 0)\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} - c_2 = 3 - c_2 \geq 0 \Rightarrow c_2 \leq 3$$

$$\hat{z}_4 - c_4 = (4, 5, 0)\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} - 0 = 2 \qquad [-\infty, 3]$$

$$\hat{z}_5 - c_5 = (4\ 5\ 0)\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} - 0 = 1$$

$a_2$ Voordendeel

$$\begin{pmatrix}2\\1\\1\end{pmatrix}=\begin{pmatrix}1&1&1\\0&2&0\\0&1&1\end{pmatrix}\begin{pmatrix}y_{12}\\y_{22}\\y_{32}\end{pmatrix}$$

$$Y_2=\begin{pmatrix}y_{12}\\y_{22}\\y_{32}\end{pmatrix}=\begin{pmatrix}1&-1/2&0\\0&1/2&0\\0&-1/2&1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\1\end{pmatrix}=\begin{pmatrix}3/2\\1/2\\1/2\end{pmatrix};\ (0,4,0)\cdot\begin{pmatrix}3/2\\1/2\\1/2\end{pmatrix}-3=-1$$

$a_3$ Voordenative

$$\begin{pmatrix}3\\1\\1\end{pmatrix}=\begin{pmatrix}1&1&1\\0&2&0\\0&1&1\end{pmatrix}\begin{pmatrix}y_{13}\\y_{23}\\y_{33}\end{pmatrix}$$

$$Y_3=\begin{pmatrix}y_{13}\\y_{23}\\y_{33}\end{pmatrix}=\begin{pmatrix}1&-1/2&0\\0&1/2&0\\0&-1/2&1\end{pmatrix}\cdot\begin{pmatrix}3\\1\\1\end{pmatrix}=\begin{pmatrix}5/2\\1/2\\1/2\end{pmatrix};\quad z_3-c_3=0$$

Ez de interesatzen informazio hau.

$a_5$ Voordenative

$$\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}1&1&1\\0&2&0\\0&1&1\end{pmatrix}\begin{pmatrix}y_{15}\\y_{25}\\y_{35}\end{pmatrix}$$

$$Y_5=\begin{pmatrix}y_{15}\\y_{25}\\y_{35}\end{pmatrix}=\begin{pmatrix}1&-1/2&0\\0&1/2&0\\0&-1/2&1\end{pmatrix}\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}-1/2\\1/2\\-1/2\end{pmatrix};\quad z_5-c_5=2$$

~~nata~~ $$\frac{x_{B_3}}{y_{32}}=\min\left\{\frac{x_{B_1}}{y_{12}}=\frac{9/2}{3/2};\ \frac{x_{B_2}}{y_{22}}=\frac{3/2}{1/2};\ \frac{x_{B_3}}{y_{32}}=\frac{1/2}{1/2}\right\}$$

3.

Morrizbetalk

$$x_1 + 2x_2 + 3x_3 + x_4 = 6$$

$$2x_1 + x_2 + x_3 + x_5 = 3$$

$$x_1 + x_2 + x_3 + x_6 = 2$$

$$\max z = 4x_1 + 3x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6$$

haven mende

$$x_1 + \cancel{x}\,2x_2 + 3x_3 + x_4 \qquad = 6$$

$$2x_1 + x_2 + x_3 + x_5 \qquad = 3$$

$$x_1 + x_2 + x_3 + x_6 \qquad = 2$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

$B = \{a_4, a_1, a_6\}$

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \frac{1}{\cancel{4}\,2} \cdot \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}^T =$$

$$= \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \cdot \frac{1}{2} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix}$$

$$X_B = B^{-1} \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 9/2 \\ 3/2 \\ 1/2 \end{pmatrix}; \quad z = \bar{c}_3^T X_B = (0\ 4\ 0) \begin{pmatrix} 9/2 \\ 3/2 \\ 1/2 \end{pmatrix} = 6$$

7.1

$\max \; z = 3x_1 + 2x_2 + x_3$

hoven werde

$x_1 - x_2 + x_3 \le 4$

$2x_1 + x_2 + 4x_3 \le 8$

$x_1, x_2, x_3 \ge 0$

$\Rightarrow$

$x_1 - x_2 + x_3 + x_4 = 4$

$2x_1 + x_2 + 4x_3 + x_5 = 8$

$x_1, x_2, x_3, x_4, x_5 \ge 0$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $z$ | |
|---|---|---|---|---|---|---|---|
| | $-3$ | $-2$ | $-1$ | $0$ | $0$ | $0$ | $m_0 = -3$ |
| $\bar{a}_4$ | $\boxed{1}$ | $-1$ | $1$ | $1$ | $0$ | $4$ | |
| $\bar{a}_5$ | $2$ | $1$ | $4$ | $0$ | $1$ | $8$ | $m_2 = 2$ |
| | $0$ | $-5$ | $2$ | $3$ | $0$ | $12$ | $m_0 = -5/3$ |
| $\bar{a}_1$ | $1$ | $-1$ | $1$ | $1$ | $0$ | $4$ | $m_1 = -1/3$ |
| $\bar{a}_5$ | $0$ | $\boxed{3}$ | $2$ | $-2$ | $1$ | $0$ | |
| | $0$ | $0$ | $16/3$ | $-1/3$ | $5/3$ | $12$ | $m_0 = -1$ |
| $\bar{a}_1$ | $1$ | $0$ | $5/3$ | $\boxed{1/3}$ | $1/3$ | $4$ | |
| $\bar{a}_2$ | $0$ | $1$ | $2/3$ | $-2/3$ | $1/3$ | $0$ | $m_2 = -2$ |
| | $1$ | $0$ | $7$ | $0$ | $2$ | $16$ | |
| $a_4$ | $3$ | $0$ | $5$ | $1$ | $1$ | $12$ | |
| $a_2$ | $2$ | $1$ | $4$ | $0$ | $1$ | $8$ | |

$x_1^* = 0 = x_3^*$

$x_2^* = 8$

$x_4^* = 12$

$\Bigg\} \; z = 16$

# 6.1 Ariketa

$$\max z = x_1 - x_2$$

hauen mende

$$\begin{aligned} x_1 - 2x_2 &\le 2 \\ 4x_1 - 3x_2 &\le 12 \\ x_1, x_2 &\ge 0 \end{aligned} \implies \begin{aligned} x_1 - 2x_2 + x_3 &= 2 \\ 4x_1 - 3x_2 + x_4 &= 12 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

$$B = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{x_{Br}}{y_{r1}} = \min \left\{ \boxed{\frac{x_{B1}}{y_{11}}} = \frac{2}{1},\ \frac{x_{B2}}{y_{21}} = \frac{12}{4} \right\}$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $z$ |
|---|---|---|---|---|---|
| | -1 | 1 | 0 | 0 | 0 |
| $a_3$ | $\boxed{1}$ | -2 | 1 | 0 | 2 |
| $a_4$ | 4 | -3 | 0 | 1 | 12 |
| | 0 | -1 | 1 | 0 | 2 |
| $a_1$ | 1 | -2 | 1 | 0 | 2 |
| $a_4$ | 0 | $\boxed{5}$ | -4 | 1 | 4 |
| | 0 | 0 | 1/5 | 1/5 | 14/5 |
| $a_1$ | 1 | 0 | -3/5 | 2/5 | 18/5 |
| $a_2$ | 0 | 1 | -4/5 | 1/5 | 4/5 |

$m_1 = \frac{1}{-1}$ ; $m_2 = \frac{4}{1}$

$m_1 = -1$ $\quad Y_{ij} = \begin{pmatrix} y_{11} \\ y_{21} \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$m_2 = 4$

$m_0 = \frac{-1}{5}$

$m_1 = \frac{-2}{5}$

$$\left.\begin{aligned} x_1^* &= 18/5 \\ x_2^* &= 4/5 \\ x_3^* &= x_4^* = 0 \end{aligned}\right\} \quad z^* = 14/5$$

Soluzio optimo bilatze

$$\max z = \underset{\substack{\parallel \\ c_1}}{6}x_1 + 4x_2 + 5x_3 + 5x_4$$

haren mende

$$x_1 + x_2 + x_3 + x_4 \leq 3$$

$$2x_1 + x_2 + 4x_3 + x_4 \leq 4$$

$$x_1 + 2x_2 - 2x_3 + 3x_4 \leq 10$$

$\Rightarrow$

$$\max z = 6x_1 + 4x_2 + 5x_3 + 5x_4 + 0x_5 + 0x_6 + 0x_7$$

haren mende

$$x_1 + x_2 + x_3 + x_4 + x_5 = 3$$

$$2x_1 + x_2 + 4x_3 + x_4 + x_6 = 4$$

$$x_1 + 2x_2 - 2x_3 + 3x_4 + x_7 = 10$$

$$B = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$z = c_B^t x_B = (0\ 0\ 0)\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix} = 0$$

$$z_1 - c_1 = c_B^t y_1 - c_1 = (0\ 0\ 0)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - 6 = -6$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | |
|---|---|---|---|---|---|---|---|---|
| | $-6$ | $-4$ | $-5$ | $-5$ | 0 | 0 | 0 | 0 |
| $a_5$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 3 |
| $a_6$ | $\boxed{2}$ | 1 | 4 | 1 | 0 | 1 | 0 | 4 |
| $a_7$ | 1 | 2 | $-2$ | 3 | 0 | 0 | 1 | 10 |
| $a_5$ | 0 | $1/2$ | $-1$ | $1/2$ | 1 | $-1/2$ | 0 | 1 |
| $a_1$ | 1 | $1/2$ | 2 | $1/2$ | 0 | $1/2$ | 0 | 2 |
| $a_7$ | 0 | $3/2$ | $-4$ | $5/2$ | 0 | $-1/2$ | 1 | 8 |

$$\bar{Y}_1 = \begin{pmatrix} y_{11} \\ y_{21} \\ y_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}; \quad \bar{X}_B = \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix} = \begin{pmatrix} x_{B_1} \\ x_{B_2} \\ x_{B_3} \end{pmatrix}$$

Bi beltore horrien zartello zatikia

$$\left\{ \frac{x_{B_1}}{y_{11}} = \frac{3}{1}, \boxed{\frac{x_{B_2}}{y_{21}} = \frac{4}{2}}, \frac{x_{B_3}}{y_{31}} = \frac{10}{1} \right\} \min = \frac{x_{B_2}}{y_{21}}.$$

$$\bar{X}_B = \begin{pmatrix} x_{B_1} \\ x_{B_2} \\ x_{B_3} \end{pmatrix} \qquad \bar{Y}_1 = \begin{pmatrix} y_{11} \\ y_{21} \\ y_{31} \end{pmatrix}$$

$\hat{B} =$

→ errenkada berria

$$\hat{X}_B = \begin{cases} \hat{x}_{B_1} = x_{B_1} - x_{Br}\left(\frac{y_{11}}{y_{21}}\right) = 3 - 4\left(\frac{1}{2}\right) = 1 \\ \hat{x}_{B_2} = \frac{x_{B_2}}{y_{21}} \\ \hat{x}_{B_3} = x_{B_3} - x_{Br}\left(\frac{y_{31}}{y_{21}}\right)\frac{1}{2} \end{cases}$$

Simplex metodoaren taula:

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $z$ | |
|---|---|---|---|---|---|---|
| | -2 | -3 | 0 | 0 | 0 | |
| $\bar{a}_3$ | 3 | $\boxed{1}$ | 1 | 0 | $\boxed{2}$ | $x_{B_1}$ |
| $\bar{a}_4$ | 1 | $\boxed{-1}$ | 0 | 1 | $\boxed{3}$ | $x_{B_2}$ |

Ez hartu ⇒ negatiboa delako

Balio adierazle negatiboak daudelako ez da soluzio optimoa

$z_1 - c_1 = -c_1$

$z_2 - c_2 = c_2$

$\frac{x_{B_1}}{y_{12}}$ ⇒ Soluzio positiborik ez dago ⇒ bornegabea

$\cancel{\frac{x_{B_2}}{y_{22}}}$

# Simplex adibidea

$\max\ z = 2x_1 + 3x_2 + \underbrace{0x_3 + 0x_4}_{c_B^T = 0^T}$

baren mende

$$3x_1 + x_2 + x_3 = 2$$
$$x_1 - x_2 + x_4 = 3$$

nasoitze aldagaiak $\Rightarrow$ gehitu
Oinarri kanonikoa

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$B = I \rightarrow B^{-1} = I$

$B = (a_3\ a_4) = I \rightarrow$ Oinarri kanonikoa

1.

$x_B = B^{-1}b = I\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \rightarrow$ Soluzio bideragarri.
$b > 0$

2

Helburu funtzioaren balioa $\rightarrow z = c_B^T x_B = (0\ 0)\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 0$

3. $y_j$ eta $z_j - c_j$

$a_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow y_1 = B^{-1}\cancel{a_1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$z_1 - c_1 = c_B^T y_1 - c_1 = 0 - c_1 = -2$

$a_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \rightarrow y_2 = B^{-1}a_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$z_2 - c_2 = c_B^T y_2 - c_1 = (0\ 0)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(Apunteak)

$$\hat{B}^{-1} = \begin{pmatrix} 1 & 1 & -3 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

$$x_{\hat{B}} = \begin{pmatrix} 1 & 1 & -3 \\ 0 & 1 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$

$$\hat{z} = \bar{c}_B^T \, \bar{x}_B = (0 \; 4 \; 3) \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = 7$$

$$\bar{a}_3 \longrightarrow \bar{y}_3 = \hat{B}^{-1} \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

## 3.6

max $z = -3x_1 + 4x_2 + 2x_3 + 5x_4$

haven merde

$$4x_1 + 2x_2 + 4x_3 + 3x_4 + x_5 = 48$$

$$x_1 - 2x_2 + x_3 - 2x_4 + x_6 = -8$$

$$-2x_1 + x_2 - x_3 - x_4 + x_7 = -6$$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 3 | -4 | -2 | -5 | 0 | 0 | 0 | 0 |
| $a_5$ | 4 | 2 | 4 | 3 | 1 | 0 | 0 | 48 |
| $a_6$ | 1 | -2 | 1 | $\boxed{-2}$ | 0 | 1 | 0 | -8 |
| $a_7$ | -2 | 1 | -1 | -1 | 0 | 0 | 1 | -6 |
| | $1/2$ | 1 | $-\frac{9}{2}$ | 0 | 0 | $-\frac{5}{2}$ | 0 | 20 |
| $a_5$ | $11/2$ | -1 | $11/2$ | 0 | 1 | $\frac{3}{2}$ | 0 | 36 |
| $a_4$ | $-\frac{1}{2}$ | 1 | $-\frac{1}{2}$ | 1 | 0 | $-\frac{1}{2}$ | 0 | 4 |
| $a_7$ | $-\frac{5}{2}$ | 2 | $-\frac{3}{2}$ | 0 | 0 | $\boxed{-\frac{1}{2}}$ | 1 | -2 |
| | 13 | -9 | 3 | 0 | 0 | 0 | -5 | 30 |
| $a_5$ | | | | | | 0 | 3 | 30 |
| $a_4$ | | | | | | 0 | -1 | 6 |
| $a_6$ | 5 | -4 | 3 | 0 | 0 | 1 | -2 | 4 |

$m_0 = \frac{5}{2}$

$m_1 = -\frac{3}{2}$

$m_3 = \frac{1}{2}$

$m_0 = 5$

$m_1 = -3$

$m_2 = 1$

4.Arikete

# 3. Ariketa

## 3.1

Ezin dutenez izan diren hostoak kopurua:

Kontuan izanik, 70 sukaldari aditu kontratatu direla, eta ~~Soluzio optimo~~ 5 pertsonako taldeak egin direla, $70/5 = 14 = 8^*$

## 3.2

Ez da geldituko sukaldaririk parte hartu gabe.

## 3.3

$$c_1 = 1 \longrightarrow \hat{c}_1 = 2$$

$$c^t = (1 \;\; 1 \;\; 1) \longrightarrow \hat{c}^t = (2 \;\; 1 \;\; 1)$$

$$\hat{z}_j - \hat{c}_j = \hat{c}_B^t y_j - \hat{c}_j = (1 \;\; 2 \;\; 1) \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} - 2 = 7 \geq 0$$

$$\hat{z} = \hat{c}_B^t x_B = (1 \;\; 2 \;\; 1) \begin{pmatrix} 4 \\ 8 \\ 2 \end{pmatrix} = 22 \;\; \cancel{\text{talde}}$$

**IKERKETA OPERATIBOA**
**2016ko maiatzaren 25a**
**Ebaluazio jarraitua: 15 puntu**
**Nota minimoa: 7,5 puntu**

**1. PROBLEMA.** (3 puntu). Garraio-problemarako ondoko kostuen taula izanik, kalkula ezazu hasierako oinarriko soluzio bideragarri bat Vogel-en metodoa erabiliz.

| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | $H_5$ | $H_6$ | Eskaintzak |
|---|---|---|---|---|---|---|---|
| $I_1$ | 5 | 10 | 15 | 8 | 9 | 7 | 30 |
| $I_2$ | 14 | 13 | 10 | 9 | 20 | 21 | 40 |
| $I_3$ | 15 | 11 | 13 | 25 | 8 | 12 | 10 |
| $I_4$ | 9 | 19 | 12 | 8 | 6 | 13 | 100 |
| Eskariak | 50 | 20 | 10 | 35 | 15 | 50 | |

**2. PROBLEMA** (4 puntu). Demagun garraio-problema baten ondoko kostuen taula:

| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | Eskaintzak |
|---|---|---|---|---|---|
| $I_1$ | – | 8 | 2 | 10 | 30 |
| $I_2$ | 7 | 3 | 4 | 3 | 60 |
| $I_3$ | 3 | 1 | 7 | 5 | 40 |
| Eskariak | 20 | 50 | ~~40~~ 45 | 20 | |

Problema orekatu eta Vogel-en metodoa aplikatu eta gero ondoko soluzioa lortu da:

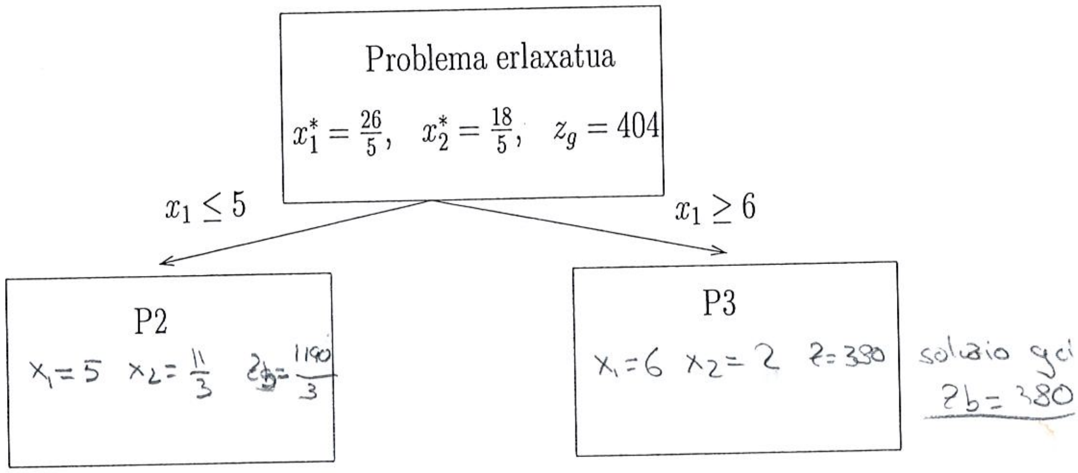
| | $H_1$ | $H_2$ | $H_3$ | $H_4$ | Eskaintzak |
|---|---|---|---|---|---|
| $I_1$ | | | 30 | | 30 |
| $I_2$ | | 30 | 10 | 20 | 60 |
| $I_3$ | 20 | 20 | | | 40 |
| $I_4$ | | | 5 | | 5 |
| Eskariak | 20 | 50 | 45 | 20 | |

Esan ezazu soluzioa optimoa al den. Hala ez bada, kalkula ezazu soluzio optimoa garraio-problemarako algoritmoa erabiliz.

**3. PROBLEMA** (8 puntu) Izan bedi ondoko eredu lineal osoa:

$$
\begin{aligned}
&\max z = 50x_1 + 40x_2\\
&\text{hauen mende}\\
&x_1 + 3x_2 \leq 16\\
&4x_1 + 2x_2 \leq 28\\
&x_1, x_2 \geq 0 \text{ eta osoak}
\end{aligned}
$$

Adarkatze- eta bornatze-algoritmoa aplikatzen hasi da (ikusi diagrama):




$x_1$ aldagaia aukeratu da P2 eta P3 problemak sortzeko. Problema horien azken taulak ondokoak dira:

| P2 | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
|---|---|---|---|---|---|---|
| | 0 | 0 | $\frac{40}{3}$ | 0 | $\frac{20}{3}$ | $\frac{1190}{3}$ |
| $\mathbf{a}_2$ | 0 | 1 | $\frac{1}{3}$ | 0 | $-\frac{1}{3}$ | $\frac{11}{3}$ |
| $\mathbf{a}_1$ | 1 | 0 | 0 | 0 | 1 | 5 |
| $\mathbf{a}_4$ | 0 | 0 | $-\frac{2}{3}$ | 1 | $-\frac{10}{3}$ | $\frac{2}{3}$ |

| P3 | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | |
|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 20 | 30 | 380 |
| $\mathbf{a}_2$ | 0 | 1 | 0 | $\frac{1}{2}$ | 2 | 2 |
| $\mathbf{a}_1$ | 1 | 0 | 0 | 0 | $-1$ | 6 |
| $\mathbf{a}_3$ | 0 | 0 | 1 | $-\frac{3}{2}$ | $-5$ | 4 |

Jarraitu ezazu algoritmoaren aplikazioarekin problema osoaren soluzio optimoa lortzeko eta diagrama osatzeko.