1. Gaia. Memoria gabeko konputazioa

Automata finitu deterministak

AFD-ren definizioa:

$M = (Q, \Sigma, \delta, q_0, F)$

non $\delta : Q \times \Sigma \dashrightarrow Q$ (δ trantsizio funtzio partziala)

Q egoera-multzo finitua.

Σ sarrerako alfabetoa.

$q_0 \in Q$ hasierako egoera

$F \subseteq Q$ bukaerako egoerak.

AFD-ren semantika:

Konfigurazioa: $(q, w) \in Q \times \Sigma^*$

q: oraingo egoera w: irakurtzeko dagoena.

Mugimendua: $(p, sw) \vdash (q, w)$ baldin $\delta(p, s) = q$

Konputazioa: Mugimendu segida $(p, w) \vdash^* (q, x)$

Onartutako hitza: M-k x onartzen du.

baldin $\exists p \in F \; (q_0, x) \vdash^* (p, \varepsilon)$

Onartutako lengoaia:

$L(M) = \{ w \in \Sigma^* : \exists p \in F \; (q, w) \vdash^* (p, \varepsilon) \}$

Automata finitu Ez-determinista

AFE-ren definizio formala: $M = \{Q, \Sigma, \delta, q_0, F\}$

$\delta : Q \times \Sigma \longrightarrow P(Q)$

Konfigurazioa: $(q, \omega) \in Q \times \Sigma^*$

Mugimendua: $(p, s\omega) \vdash (q, \omega)$ baldin $q \in \delta(p, s)$

M-k onartzen duen lengoaia:

$$L(M) = \{\omega \in \Sigma^* : \exists p \in F \; (q_0, \omega) \vdash^* (p, \varepsilon)\}$$

Trantsizio hutsun AFEak

ε-AFE-aren definizio formala:

$M = \{Q, \Sigma, \delta, q_0, F\}$ non $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \longrightarrow P(Q)$

Konfigurazioa:

$(q, \omega) \in Q \times \Sigma^*$

Mugimendua: $(p, u) \vdash (q, v)$ baldin

$u = v$ eta $q \in \delta(p, \varepsilon)$ edo

$u = sv$ eta $q \in \delta(p, s)$

M-k onartzen duen lengoaia:

$$L(M) = \{\omega \in \Sigma^* : \exists p \in F \; (q_0, \omega) \vdash^* (p, \varepsilon)\}$$

Froggapemo ε-AFE = AFD

$\varepsilon\text{-AFE} \overset{?}{=} \text{AFD}$

n AFD $\quad n = (Q, \varepsilon, \delta, q_0, q_f)$

$$\delta : Q \times \Sigma^* \dashrightarrow \mathcal{P}(Q)$$

$N : (P, \varepsilon, \gamma, q, F) \qquad (Q \subseteq P)$

$\delta(p, \varepsilon) = A \Rightarrow \gamma(p, \varepsilon) = A$

$\delta(p, s) = A \Rightarrow \gamma(p, s) = A$

$$\underset{\{n \geq 2\}}{\delta(p, s_1 \ldots s_n) = A} \Longrightarrow \{p_1, \ldots, p_n\} \subseteq P \begin{cases} p_1 \in \delta(p, s_1) \\ \gamma(\cancel{p_1}, s_2) = \{p_2\} \\ \gamma(p_{n-2}, s_{n-1}) = \{p_{n-1}\} \\ A \subseteq \gamma(p_{n-1}, s_n) \end{cases}$$

LKSA

Adierazpen erregularrak: RegEx

2) ε Adierazpen erregularra da.

3) $s \in \Sigma \Rightarrow s$ AdEr

1) $\emptyset$ AdEr

4) α, β AdEr dira $\Rightarrow \alpha \cup \beta$ AdEr

5) α, β AdEr dira $\Rightarrow \alpha \cdot \beta$ AdEr

6) α, AdEr da $\Rightarrow \alpha^*$ AdEr

α AdEr

α-a denotatzen duen lengoaia sortzen duen

$L(\alpha)$ a) $\alpha = \emptyset \Rightarrow L(\alpha) = L(\emptyset) = \emptyset$

b) $\alpha = \varepsilon \Rightarrow L(\alpha) = L(\varepsilon) = \{\varepsilon\}$

c) $\alpha = s \in \Sigma \Rightarrow L(\alpha) = L(s) = \{s\}$

d) $\alpha = \beta \cup \gamma \Rightarrow L(\alpha) = L(\beta \cup \gamma) = L(\beta) \cup L(\gamma)$

e) $\alpha = \beta \cdot \gamma \Rightarrow L(\alpha) = L(\beta \cdot \gamma) = L(\beta) \cdot L(\gamma)$

f) $\alpha = \beta^* \Rightarrow L(\alpha) = L(\beta^*) = L(\beta)^*$

Laboratorio LUSA

$\{0,1\} = \Sigma$

$\{x \in \Sigma^* : |x|_0 \geq 3\} \equiv \{0 \cup 1\}^* \cdot 0 \cdot \{0 \cup 1\}^* \cdot 0 \cdot \{0 \cup 1\}^* \cdot 0 \cdot \{0 \cup 1\}^*$

$\{x \in \Sigma^* : 000 \sqsubseteq_p x\} \equiv 000 \cdot \{0 \cup 1\}^*$

$\{x \in \Sigma^* : 000 \sqsubseteq_s x\} \equiv \{0 \cup 1\}^* \cdot 000$

$\{x \in \Sigma^* : 00 \sqsubseteq x \wedge 11 \sqsubseteq x\}$ $(0 \cup 1)^* \cdot 00 \cdot (0 \cup 1)^* \cdot 11 \cdot (0 \cup 1)^* \cup$

$(0 \cup 1)^* \cdot 11 \cdot (0 \cup 1)^* \cdot 00 \cdot (0 \cup 1)^*$

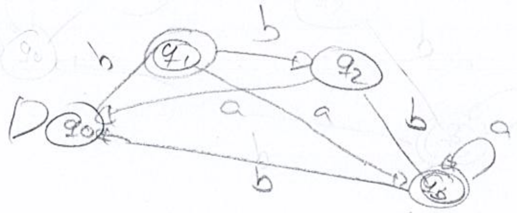
$\{x \in \Sigma^* : 000 \not\sqsubseteq x\}$ $(01 \cup 001 \cup 1)^* (00 \cup 0 \cup \varepsilon)$

$\{x \in \Sigma^* : |x|_{000} = 1\}$ $(01 \cup 001 \cup 1)^* \cdot 000 \cdot (10 \cup 100 \cup 1)^*$

$\{x \in \Sigma^* : |x|_{00} \geq 2\}$ $(0 \cup 1)^* \cdot 00 \cdot (0 \cdot (0 \cup 1)^* \cup 1 (0 \cup 1)^* \cdot 00 \cdot (0 \cup 1)^*)$

$\{x \in \Sigma^* : |x|_{00} = 2\}$ $(01 \cup 1)^* \cdot 00 \cdot (0 \quad \cup 1 (01 \cup 1)^* \cdot 00 (10 \cup 1)^*)$

$\{x \in \Sigma^* : |x|_{00} \leq 1\}$



$L_0 = aL_0 \cup bL_1$

$L_1 = aL_3 \cup bL_2 \cup \varepsilon$

$L_2 = aL_0 \cup bL_3$

$L_3 = aL_3 \cup bL_0 \cup \varepsilon$

Lengoaien ebazpenak ebazten

β-ko hitzak X-n daude

$w \in X \Rightarrow \alpha w$ motzako hitzak ere X-n badaude ere

$\beta \subset X$
$w \in X \Rightarrow \alpha \cdot w \subset X$

$\{a, b, c\}$

$X = a^* X \cup b$

$b \in X$
$ab \in X$ $\quad X = a^* b$ baina $X = a^* b \cup \ldots$
$aab \in X$ $\quad$ frogapena
$\vdots$ $\quad$ absurdura eramanez.

$X = a^* b \cup H$

$H \neq \emptyset$ $z \in H$ motzena

$\forall w \in H \ |w| \geq |z|$

z lengoaiakoa da.

$X = aX \cup b$

$\boxed{z \in aX}$ $z = az'_{\in X} \Rightarrow |z'| < |z| \Big| \Rightarrow \begin{cases} z' \in X \\ z' \notin H \end{cases}$

$\vee$

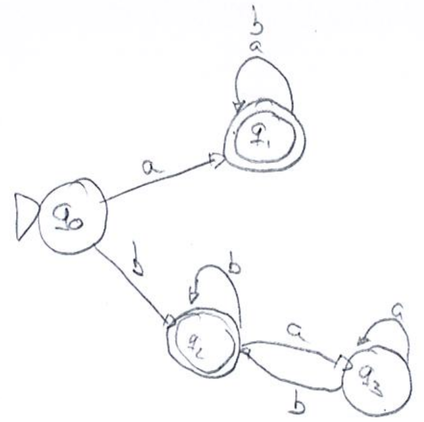
$z \in b \Rightarrow$ ez

$$\left.\begin{array}{l} z = a \cdot z' \\ z' \in X \end{array}\right| \Rightarrow z' \notin H$$

$$\boxed{z' \in a^*b} \Longrightarrow z = a \cdot z' \Rightarrow z \in a^+b \subset a^*b$$

Ondoriaz, a^*b dogo sotile X-on.

Adi:



$$\Rightarrow a(a \cup b)^* \cup b(a \cup b)^* b \cup b$$

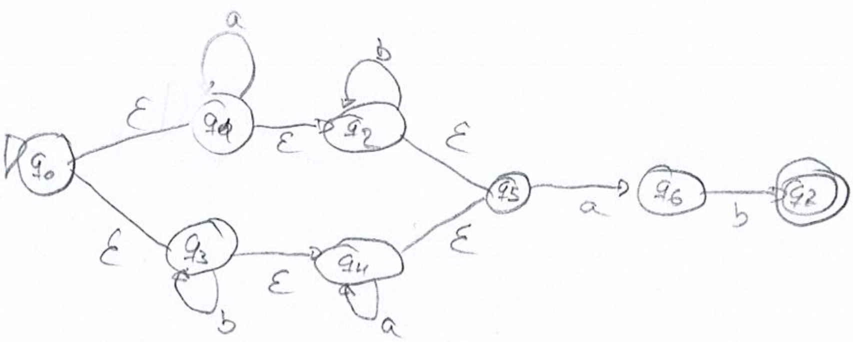
$$\left.\begin{array}{l} X_0 = aX_1 \cup bX_2 \\ X_1 = aX_1 \cup bX_1 \cup \varepsilon \\ X_2 = aX_3 \cup bX_2 \cup \varepsilon \\ X_3 = aX_3 \cup bX_2 \end{array}\right| \left.\begin{array}{l} X_0 = a(a \cup b)^* \cup bX_2 \\ X_1 = (a \cup b)X_1 \cup \varepsilon \Rightarrow X_1 = (a \cup b)^* \\ X_2 = aX_3 \cup bX_2 \cup \varepsilon \\ X_3 = aX_3 \cup bX_2 \end{array}\right| \left.\begin{array}{l} X_0 = a(a \cup b)^* \cup bX_2 \\ X_2 = a(a^*bX_2) \cup bX_2 \cup \varepsilon \\ X_3 = a\, bX_2 \Rightarrow X_3 = a^*bX_2 \end{array}\right|$$

$$X_2 = aa^*bX_2 \cup bX_2 \cup \varepsilon \Rightarrow X_2 = (aa^*b \cup b)X_2 \cup \varepsilon = (aa^*b \cup b)^* X_2 = a^*bX_2$$

$$X_2 = (a^*b)^* X_2$$

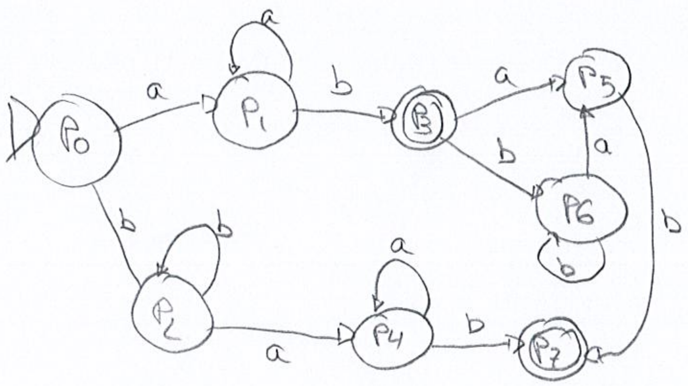
$$X_0 = a(a \cup b)^* \cup bX_2 \Rightarrow \boxed{X_0 = a(a \cup b)^* \cup b(a^*b)^*}$$

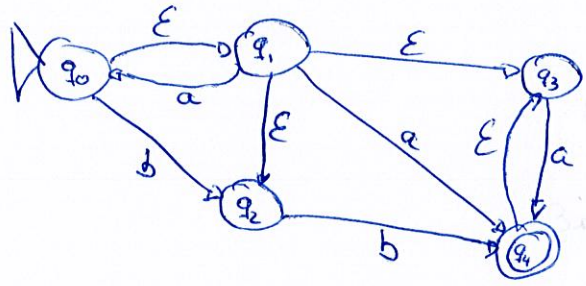
$(a^*b^* \cup b^*a^*)ab$



	a	b	ε	C(q)
q_0	-	-	$q_1 q_3$	$q_0 q_1 q_2 q_3 q_4 q_5$
q_1	q_1	-	q_2	$q_1 q_2 q_5$
q_2	-	q_2	q_5	$q_2 q_5$
q_3	-	q_3	q_4	$q_3 q_4 q_5$
q_4	q_4	-	q_5	$q_4 q_5$
q_5	q_6	-	-	q_5
q_6	-	q_7	-	q_6
q_7	-	-	-	q_7

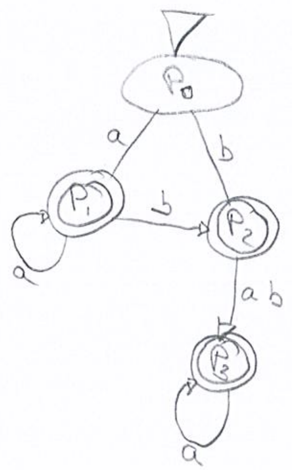
δ	a	b
$P_0=\{q_0,q_1,q_2,q_3,q_4,q_5\}$	P_1	P_2
$P_1=\{q_1,q_2,q_4,q_6\}$	P_1	P_3
$P_2=\{q_2,q_3,q_4,q_5\}$	P_4	P_2
$P_3=\{q_2,q_5,q_7\}$	P_5	P_6
$P_4=\{q_4,q_5,q_6\}$	P_4	P_7
$P_5=\{q_6\}$	-	P_7
$P_6=\{q_2,q_5\}$	P_5	P_6
$P_7=\{q_7\}$	-	-

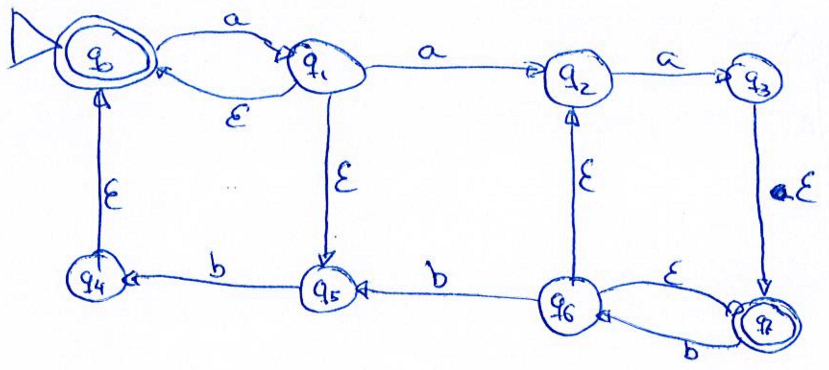




	a	b	ε	$C_{(q)}$
q_0	–	q_2	q_1	$q_0 q_1 q_2$
q_1	$q_0 q_4$	–	$q_2 q_3$	$q_2 q_3$
q_2	–	q_4	–	q_2
q_3	q_4	–	–	q_3
q_4	–	–	q_3	$q_3 q_4$

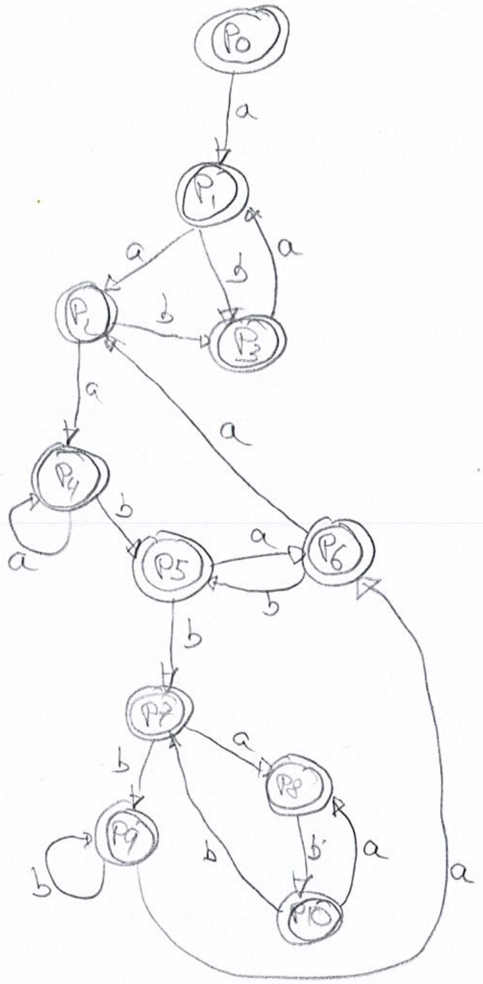
	a	b
$P_0 = \{q_0 q_1 q_2\}$	P_1	P_2
$P_1 = \{q_0 q_1 q_2 q_3 q_4\}$	P_1	P_2
$P_2 = \{q_2 q_3 q_4\}$	P_3	P_3
$P_3 = \{q_3 q_4\}$	P_3	–





	a	b	ε	$C(q)$
q_0	q_1	–	–	q_0
q_1	q_2	–	$q_0 q_5$	$q_0 q_1 q_5$
q_2	q_3	–	–	q_2
q_3	–	–	q_7	$q_3 q_7$
q_4	–	–	q_0	$q_0 q_4$
q_5	–	q_4	–	q_5
q_6	–	q_5	$q_2 q_7$	$q_2 q_6 q_7$
q_7	–	q_6	–	q_7

	a	b
$C(q_0) = P_0$	P_1	–
$P_1 = \{q_0, q_1, q_5\}$	P_2	P_3
$P_2 = \{q_0, q_1, q_2, q_5\}$	P_4	P_3
$P_3 = \{q_0 q_4\}$	P_1	–
$P_4 = \{q_0 q_1 q_2 q_3 q_5 q_7\}$	P_4	P_5
$P_5 = \{q_0 q_2 q_4 q_6 q_7\}$	P_6	P_7
$P_6 = \{q_0 q_1 q_3 q_5 q_7\}$	P_2	P_5
$P_7 = \{q_2 q_5 q_6 q_7\}$	P_8	P_9
$P_8 = \{q_3 q_7\}$	–	P_{10}
$P_9 = \{q_0 q_2 q_4 q_5 q_6 q_7\}$	P_6	P_9
$P_{10} = \{q_2 q_6 q_7\}$	P_8	P_7



Adierazpen Erregularrak. Ariketak

1

a) $(10)^* \cup (01)^*$

6 luzera edo gutxiago duten lengoaiako hitzak.

- ε
- 01, 10
- 0101, 1010
- 010101, 101010

b) $(10 \cup 01)^*$

6 luzera edo gutxiago duten lengoaiako hitzak

- ε
- 01, 10
- 0101, 0110, 1001, 1010
- 010101, 010110, 011001, 011010, 100101, 100110, 101001, 101010

c) $[00 \cup 11 \cup (01 \cup 10)(00 \cup 11)^*(01 \cup 10)]^*$

- ε
- 00, 11
- 0000, 0101, 0110, 1001, 1010, 1111
- 000101, 000110, 001001, 001010, 010001, 010010, 011101, 011110, 100001, 101110, 110101, 110110, 111001, 111010

d) $(11 \cup 00)^*(00 \cup 1)^*$

6 luz. edo gutxiagoko lengoaiako hitzak:

- ε
- 0, 1
- 00, 01, 11
- 000, 001, 011, 110, 111
- 0000, 0001, 0011, 0100, 0111, 1001, 1100, 1101, 1111
- 00000, 00001, 00011, 00100, 00110, 00111, 01100, 01001, 01101, 01111, 10000, 10011, 11000, 11001, 11100, 11110, 11111
- 000000, 000001, 000011, 000100, 000110, 000111, 001001, 001100, 001101, 001111, 010000, 010011, 010100, 011000, 011001, 011011, 011100, 011110, 011111, 100001, 100100, 100111, 101001, 101100, 101111, 110000, 110001, 110011, 110100, 110111, 111001, 111101, 111111

e) $(1 \cup 01 \cup 001)^*(\varepsilon \cup 0 \cup 00)$

- ε
- 0, 1
- 00, 01, 10, 11
- 001, 010, 100, 101, 110, 111
- 0010, 0011, 0100, 0101, 0110, 0111, 1001, 1010, 1101, 1110, 1111,
- 00100, 00101, 00110, 00111, 01001, 01010, 01011, 01100, 01101, 01110, 01111, 10010, 10011, 10100, 10101, 10110, 10111, 11001, 11010, 11011, 11100, 11101, 11110, 11111

Adierazpen Erregularra

Aribketak

$\{x \in \Sigma^* : |x|_0 \geq 3\} \equiv (0 \cup 1)^* \cdot 0 \cdot (0 \cup 1)^* 0 \cdot (0 \cup 1)^* \cdot 0 \cdot (0 \cup 1)^*$

$\{x \in \Sigma^* : 000 \sqsubseteq_p x\} \equiv 000(0 \cup 1)^*$

$\{x \in \Sigma^* : 000 \sqsubseteq_s x\} \equiv (0 \cup 1)^* 000$

$\{x \in \Sigma^* : 00 \sqsubseteq x \wedge 11 \sqsubseteq x\} \equiv [(0 \cup 1)^* (00(0 \cup 1)^* 11 (0 \cup 1)^* \cup 11 (0 \cup 1)^* 00 (0 \cup 1)^*)]^+$

$\{x \in \Sigma^* : 000 \not\sqsubseteq x\} \equiv [1^* 0 (0 \cup \varepsilon) 1^+]^* (1^* \cup 0 \cup 00)$

$\{x \in \Sigma^* : |x|_{000} = 1\} \equiv (01 \cup 001 \cup 1)^* 000 (10 \cup 100 \cup 1)^*$

$\{x \in \Sigma^* : |x|_{00} \geq 2\} \equiv (0 \cup 1)^* 00 (0 \cup 1)^* 00 (0 \cup 1)^*$

$\{x \in \Sigma^* : |x|_{00} = 2\} \equiv [(0 \cup \varepsilon) 1^+]^* \cdot 00 (0 \cup 1 [(0 \cup \varepsilon) 1^+]^* 00) [1^+ (0 \cup \varepsilon)]^*$

$\Sigma = \{a, b, c, d\}$

$\{x \in \Sigma^* : |x|_a \% 3 = 0\} \equiv (b \cup c \cup d)^* a (b \cup c \cup d)^* a (b \cup c \cup d)^* a (b \cup c \cup d)^+ \cup (b \cup c \cup d)^*$

$\{x \in \Sigma^* : aaa \not\sqsubseteq x\} \equiv (b \cup c \cup d)^* \cdot [a (a \cup \varepsilon)(b \cup c \cup d)^+]^* (aa \cup a \cup (b \cup c \cup d)^*)$

$\{x \in \Sigma^* : |x|_{aaa} \leq 1\} \equiv [(b \cup c \cup d)^* \cdot a(a \cup \varepsilon) (b \cup c \cup d)^+]^* (aaa \cup aa \cup a \cup \varepsilon) [(b \cup c \cup d)^+ a (a \cup \varepsilon) (b \cup c \cup d)^*]^*$

$\{x \in \Sigma^* : |x|_a \not\sqsubseteq x \wedge |x|_{aa} \not\sqsubseteq x\} \approx (b \cup c \cup d)^* [aaa^+ (b \cup c \cup d)^*]^*$

$\{x \in \Sigma^* : |x| \% 2 \neq 0 \wedge |x|_a \geq 2 \wedge |x|_c \geq 2\} \equiv$

$\equiv (a \cup b \cup c \cup \varepsilon) [(a \cup b \cup c)(a \cup b \cup c)]^* aa [(a \cup b \cup c)(a \cup b \cup c)]^* cc [(a \cup b \cup c)(a \cup b \cup c)]^*$
$\cup [(a \cup b \cup c)(a \cup b \cup c)]^* aa [(a \cup b \cup c)(a \cup b \cup c)]^* cc [(a \cup b \cup c)(a \cup b \cup c)]^* (a \cup b \cup c)$

$\{x \in \Sigma^* : 101 \not\sqsubseteq x\} \equiv (1^* 00^+ 1^*)^* \cup 1^* \cup 0^* \cup 1^+ 0^+ \cup 0^+ 1^+$

$(1^* \cup 0^*) [(001 \cup 100)(1^* \cup 0^*)]^+ \cup 10 \cup 01$

②

a) 3-az zatigarria den a kopurua duten hitzak.

0, 3, 6, 9, 12

$$(((b \cup c \cup d)^* a (b \cup c \cup d)^* a (b \cup c \cup d)^* a (b \cup c \cup d)^*)^* \cup (b \cup c \cup d)^*)$$

$$L(M) = \{ x \in \Sigma^* : |x|_a \bmod 3 = 0 \}$$

b)

aaa azpikatearen agerpenik ez duten hitzak.

$$L(M) \{ x \in \Sigma^* ; aaa \not\subseteq x \}$$

$$[(b \cup c \cup d)^* a (a \cup \varepsilon)(b \cup c \cup d)^+]^* (aa \cup a \cup (b \cup c \cup d)^*$$

c) gehienez aaa azpikatearen agerpen bat duten hitzak

$$L(M) = \{ x \in \Sigma^* : |x|_{aaa} \leq 1 \}$$

$$((a \cup aa \cup (((b \cup c \cup d)^* (aaa \cup \varepsilon)(b \cup c \cup d)(a(a(b \cup c \cup d) \cup b \cup c \cup d))^*)$$

$$(\quad [[(aa \cup a \cup \varepsilon)(b \cup c \cup d)]^* (aaa \cup aa \cup a \cup \varepsilon)][(b \cup c \cup d)(aa \cup a \cup \varepsilon)]^*$$

d) a ikurrak gutxienez hirunaka bilduta dauden Σ^*-eko hitzak.

$$((b \cup c \cup d)^* [aa^+ (b \cup c \cup d)^*]^*$$

3

a) $\{w \in \{a,b,c\}^*$: w-n ez dago a-ren agerpenik c-ren aurretik eta, b-ren agerpenik a-ren aurretik eta c-ren agerpenik b-ren aurretik

~~ac~~ ~~bc~~ ~~cb~~

$(c^+a^* \cup a^+b^* \cup b^+a^*)$

b)

$(aa)^*(bb(a \cup b))^*$

c) Luzera bakoitia, gutxienez bi a eta horien ostean gutxienez bi c dituzten $\{a,b,c\text{-ren}\}$ gaineko hitzak

$((b(bb)^*(aa)^+(cc)^+(bb)^*) \cup ((bb)^*(aa)^+(cc)^+b(bb)^*))$

e) $\{w \in \{0,1\}^*$: w-k ez dauka 101 azpikatea

$(1^*00^+1^*)^*$

f) $\{w \in \{1,0\}^*$: w-eko hitz guztiak 101-ekin hasi eta bukatzen dira

$101((110)^*101)$

g) $\{w \in \{a,b,c\}^*$: b-aren artean dagoen c kopurua beti 3-ren multiploa.

$((b \cup c)^* a\, b^*(c(b^* c\, b^* c\, b^*))^* a\, (b \cup c)^*)^*$

h) gehienez jota bi b eta c-z hiltzen ez diren hitzak

$(a \cup c)^*((b(a \cup c)^* b(a \cup c)^* a) \cup (b(a \cup c)^*(a \cup b) \cup (a))$

5

b) 0 bera kenduta 0 -korik ez dituzten zenbakiak

$0^* \cup [1-9]^*$

c) at notablo onargarriak diren zenbaki osoak

$(-1)[\backslash d\{9\}]$

(6)

a) $(aab \cup b^*)(a^* \cup bb^+)$ eta $(ab \cup bbb \cup baa^*bb)^*$

$$\underset{M1}{\uparrow} \cap \underset{M2}{\uparrow}$$

$L(M) = (bbb)^*$

b)

$ab^*(a \cup ba)^*$ eta $(a \cup b)^* bbab^*b$

$L(M) = ab^*bbab^*b$

c)

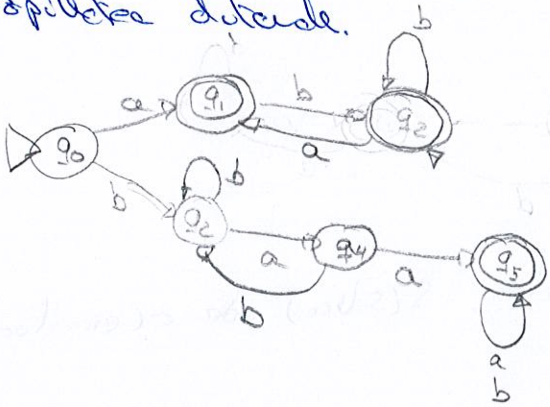
$(a \cup b)b(aa \cup bb)^*)$ eta $b(ab \cup ba)^* a(abbb \cup ba)^* b$

$L(M) = \emptyset$

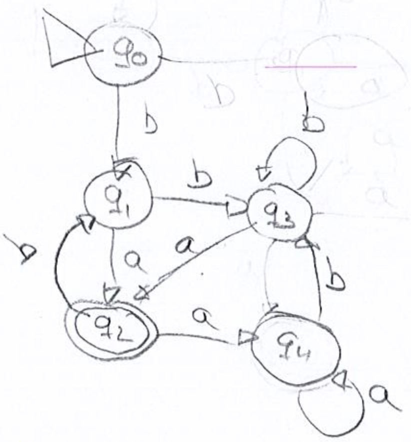
Automata finitu Deterministak, Ariketak

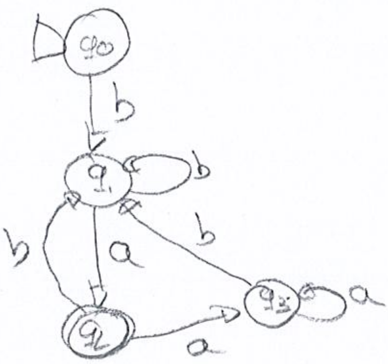
1. ariketa

a) a-z hasi, ondoren aa azpikaterik ez, eta b-z hasi gero b azpikatea dutenak.

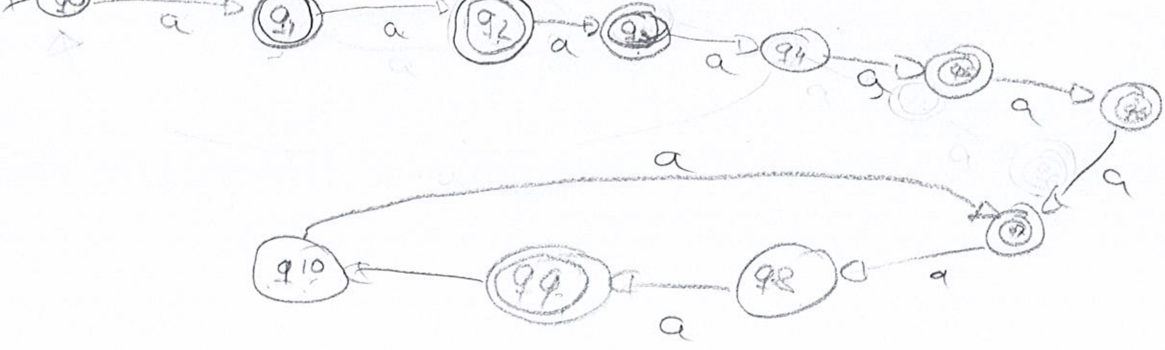


b) $\{x \in \Sigma^* : b \sqsubseteq_p x \wedge ba \sqsubseteq_s x\}$

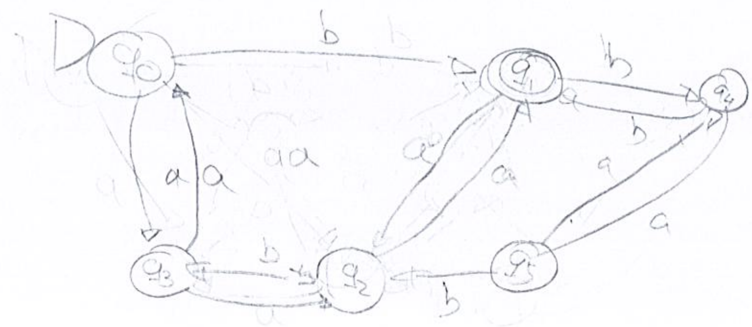




c) Bi a, sei a edo a kopuru bikoitia



g) $\{x \in \Sigma^* : |x|_a \% 2 = 0 \wedge |x|_b \% 2 = 1\}$

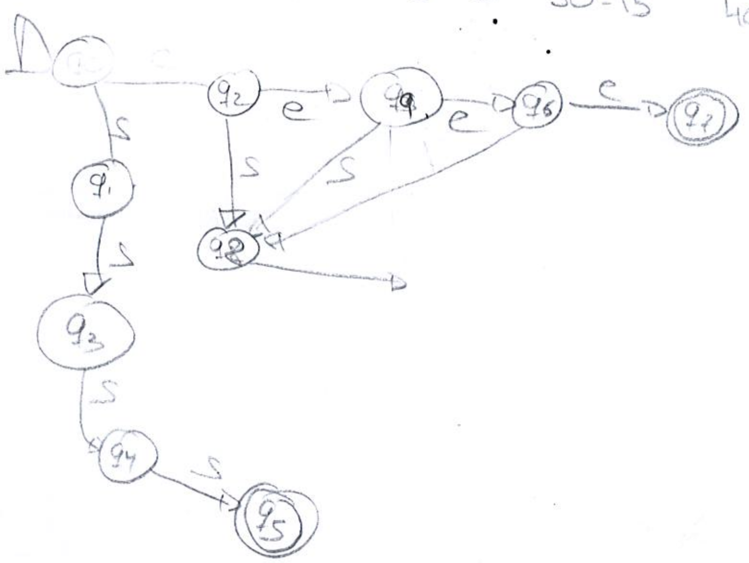


(3)

teniseko partidua, joko bakoitza, S(Sakea) edo e(errestoa) definitzeko.

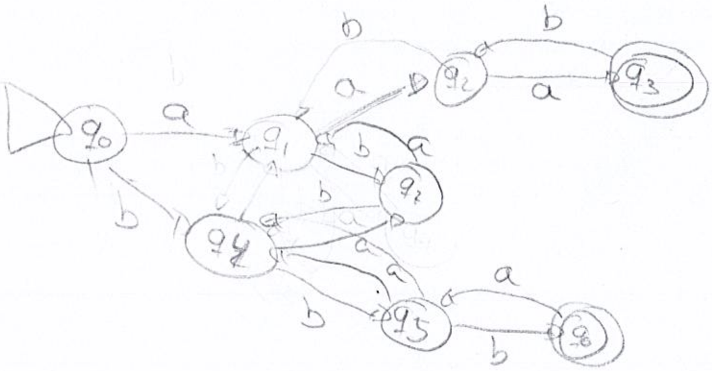
0-0 0-15 15-15 30-15 30-30 40-30 40-40 AD-40

0-0 15-0 15-15 30-15 40-15



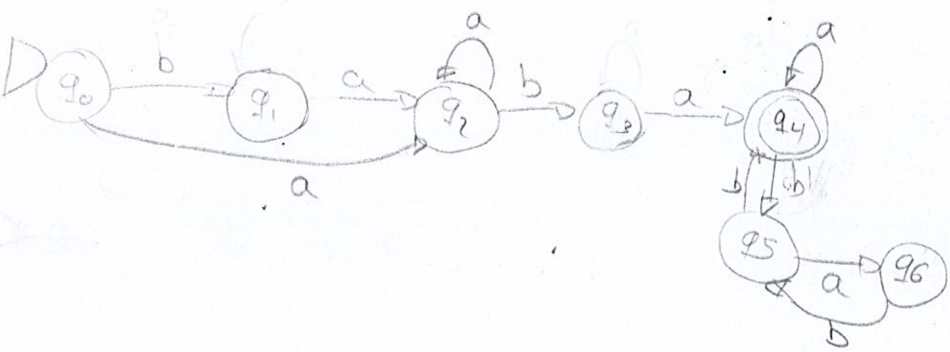
d)

$x \in \Sigma^*$: x-ek ez duen inongo aurrizkirik eta $|x|_a$ eta $|x|_b$-ren arteko aldea 3 baino handiagoa den.

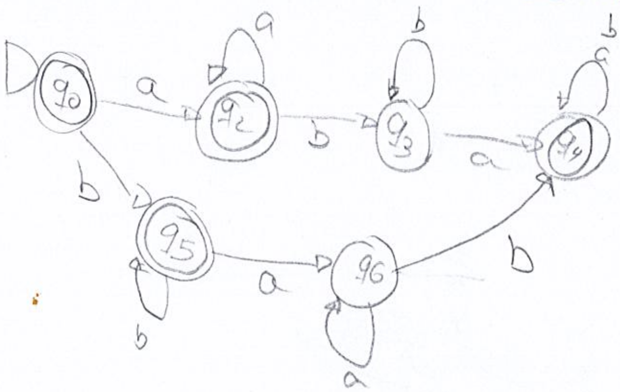


e)

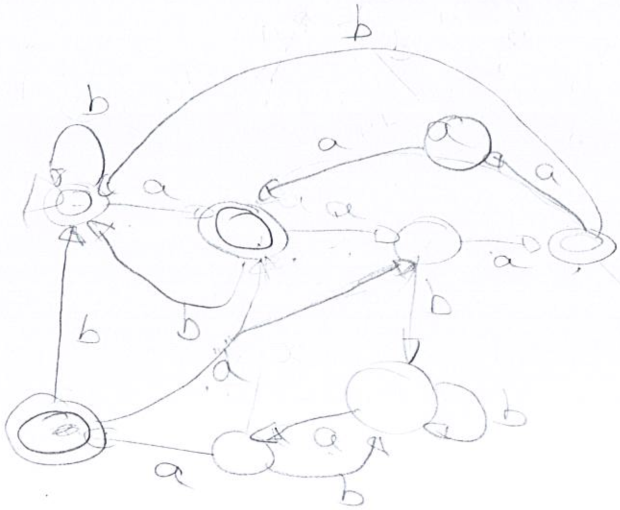
aba-ren edozein agerpen bb-ren aurretik edo hitzaren bukaeran.



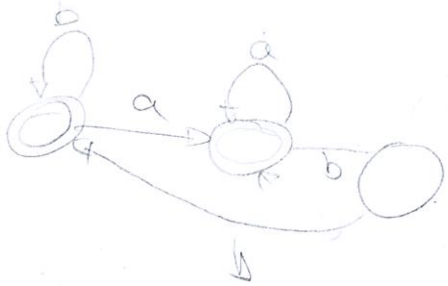
f) $\{x \in \Sigma^* : ab \sqsubseteq x \to ba \sqsubseteq x\}$



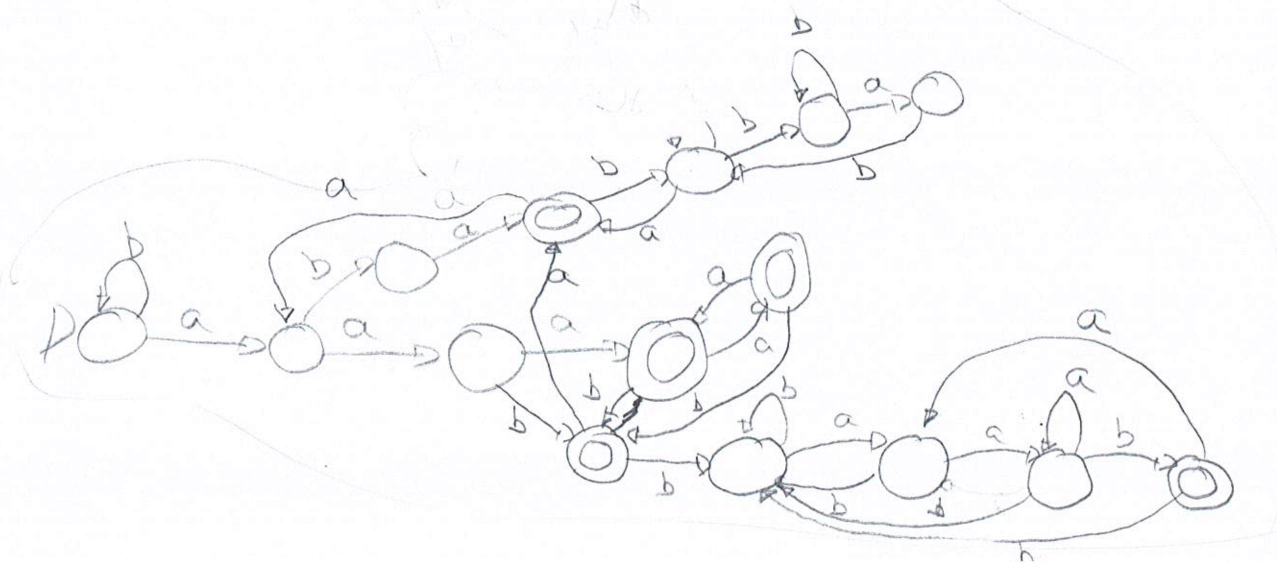
h) $\{x \in \Sigma^* : |x|_{aa} \% 2 = 0\}$



i) $\{x \in \Sigma^* : aba \not\sqsubseteq x \wedge bab \not\sqsubseteq_s x\}$



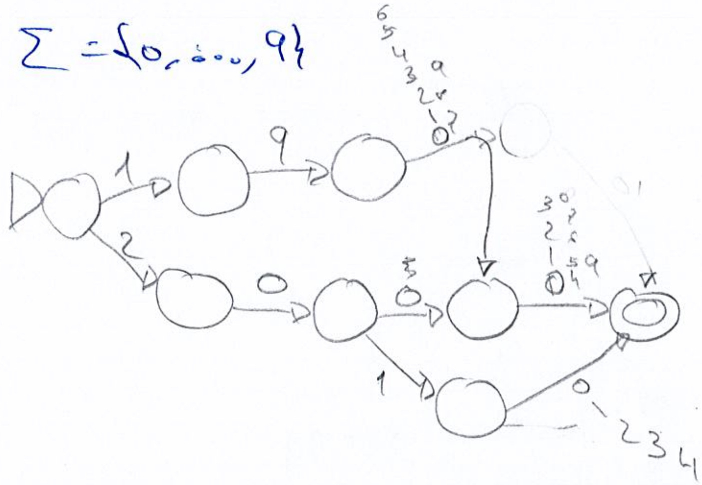
j) $\{x \in \Sigma^* : aaa \sqsubseteq_s x \vee aab \sqsubseteq_s \vee aba \sqsubseteq_s x\}$



l)

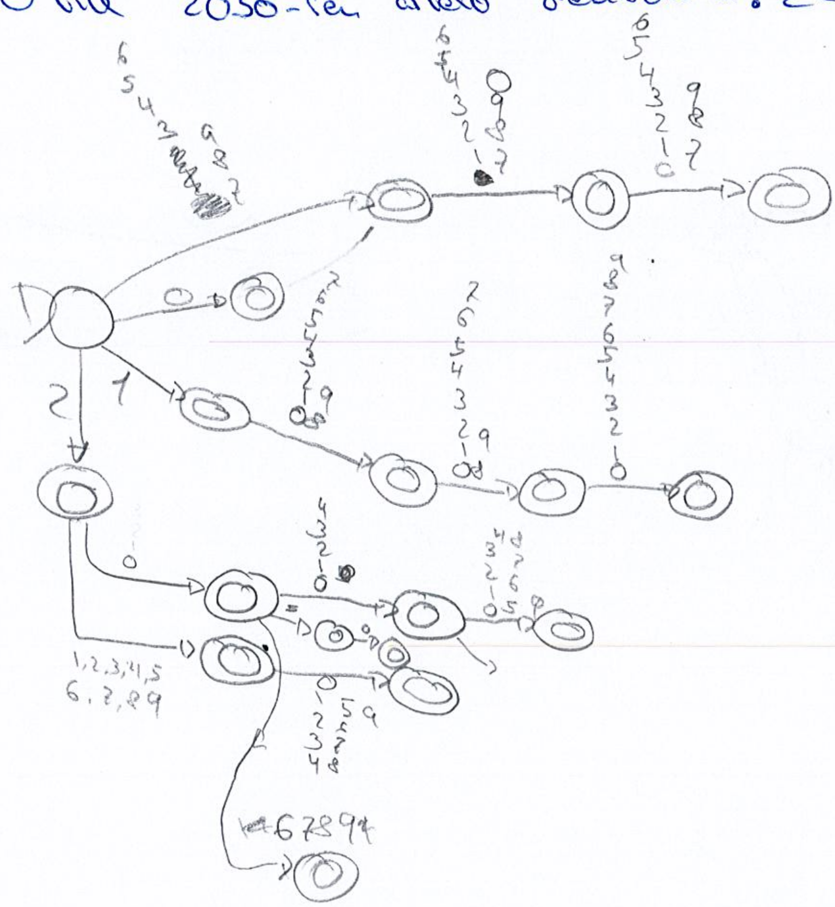
1900-etik 2014-rac arteko lau digituko zenbakiak

$\Sigma = \{0, \ldots, 9\}$



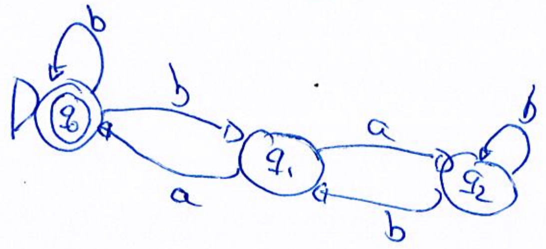
m)

0 tik 2050-ean arteko zenbakiak. $\Sigma = \{0, \ldots, 9\}$



209

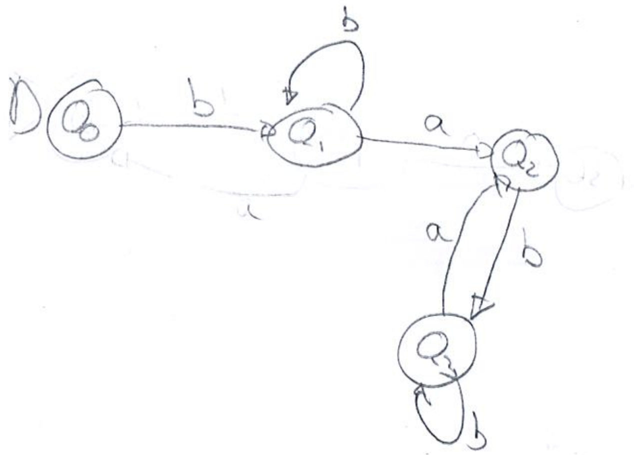
AFN → AFD



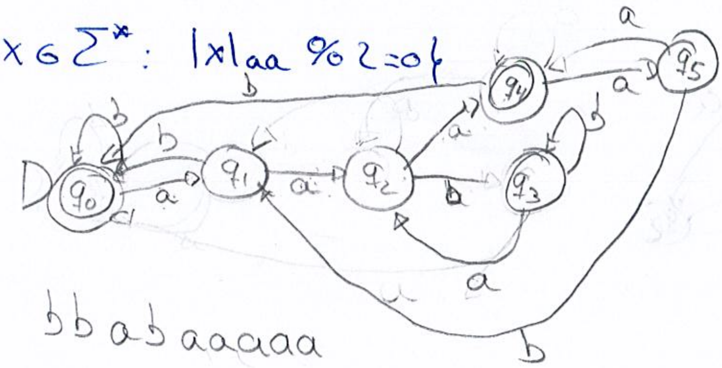
	a	b
q_0	–	$\{q_0, q_1\}$
q_1	$\{q_0, q_2\}$	–
q_2	–	$\{q_1, q_2\}$

⟹

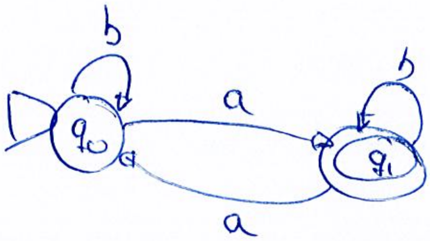
	a	b
$Q_0 \Leftarrow q_0$	–	$\{q_0, q_1\}$
$Q_1 \Leftarrow q_0, q_1$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
$Q_2 \Leftarrow q_0, q_2$	–	$\{q_0, q_1, q_2\}$
$Q_3 \Leftarrow q_0, q_1, q_2$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$



$\{x \in \Sigma^* : |x|_{aa} \% 2 = 0\}$



bbabaaaaa



$X_0 = aX_1 \cup bX_0$

$X_1 = aX_0 \cup bX_1 \cup \mathcal{E}$ | $X_1 = bX_1 \cup aX_0 \cup \mathcal{E} = b^*(aX_0 \cup \mathcal{E})$

$X_0 = a(b^*(aX_0 \cup \mathcal{E})) \cup bX_0 \Rightarrow$

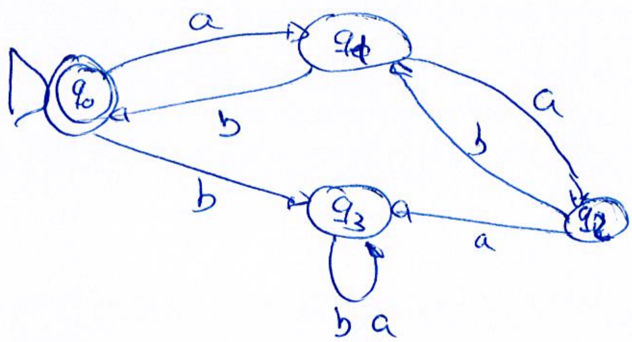
$\Rightarrow X_0 = ab^*(aX_0 \cup \mathcal{E}) \cup bX_0 \Rightarrow$

$\Rightarrow X_0 = ab^*aX_0 \cup ab^*a \cup bX_0 \Rightarrow$

$\Rightarrow X_0 = ab^*aX_0 \cup bX_0 \cup ab^*a \Rightarrow$

$\Rightarrow X_0 = (ab^*a \cup b)X_0 \cup ab^*a \Rightarrow$

$\Rightarrow X_0 = (ab^*a \cup b)^* \cup ab^*a$



$$X_0 = aX_1 \cup bX_3 \cup \varepsilon \quad \Big| \quad x_0 = ax_1 \cup bx_3 \cup \varepsilon \quad \Big| \quad x_0 = ax_1 \cup \varepsilon$$

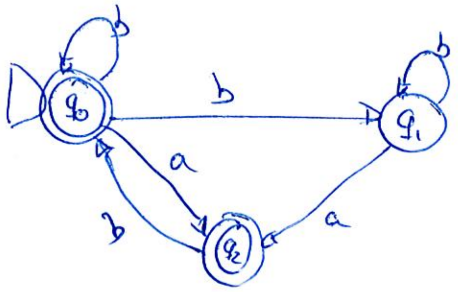
$$X_1 = aX_2 \cup bX_0 \quad \Big| \quad x_1 = ax_2 \cup bx_0 \quad \Big| \quad x_1 = abx_1 \cup bx_0$$

$$X_2 = aX_3 \cup bX_1 \quad \Big| \quad x_2 = \varnothing \cup bx_1 \quad \Big| \quad x_2 = bx_1$$

$$X_3 = aX_3 \cup bX_3 \Rightarrow X_3 = (a \cup b)X_3 \Rightarrow X_3 (a \cup b)^* \cdot \varnothing = \varnothing$$

$$x_0 = ax_1 \cup \varepsilon \quad \Big| \quad x_0 = a(ab)^* bx_0 \cup \varepsilon = (a(ab)^* b)^*$$

$$x_1 = (ab)^* bx_0$$



$$X_0 = bX_0 \cup aX_2 \cup bX_1 \cup \varepsilon$$

$$X_1 = bX_1 \cup aX_2$$

$$X_2 = bX_0 \cup \varepsilon =$$

$$x_0 = bX_0 \cup a(bx_0 \cup \varepsilon) \cup bx_1 \cup \varepsilon \quad \Big| \quad x_0 = bx_0 \cup abx_0 \cup bx_1 \cup a \cup \varepsilon$$

$$X_1 = bx_1 \cup a(bx_0 \cup \varepsilon) \quad \Big| \quad x_1 = b^*(a(bx_0 \cup \varepsilon))$$

$$x_0 = bx_0 \cup abx_0 \cup bb^*(a(bx_0 \cup \varepsilon)) \cup a \cup \varepsilon \Rightarrow$$

$$\Rightarrow x_0 = bx_0 \cup abx_0 \cup bb^*abx_0 \cup bb^*a \cup a \cup \varepsilon \Rightarrow$$

$$\Rightarrow x_0 = (b \cup ab \cup bb^*ab)^* (bb^*a \cup a \cup \varepsilon)$$

LKSA

Hitzaren definizioa

Σ alfabetoa emanda.

ε Σ-ren gaineko hitza da.

baldin w Σ-ren gaineko edozein hitza ~~izanda~~ bada eta $s \in \Sigma$, orduan w eta ε-ren edozein batura hitza izango da $w \cdot s \in \Sigma$-ren gaineko hitza

Alfabetoaren gaineko hitz guztien multzoari lengoaia unibertsala deitzen diogu:

$\Sigma = \{a, b, c\}$ alfabetoa bada, $\Sigma^* = \{a, b, c, aa, ab, ac, \dots\}$

{Lengoaia Unibertsala}

Ordena lexikografikoak:

adb: $\overset{\text{lehena}}{a}\ \overset{\text{isatsa}}{\overbrace{b\,b\,c\,a\,b}}$ — $\underbrace{a\,b\,b\,c\,a}_{\text{burua}}\ \underbrace{b}_{\text{azkena}}$

$Lehena(\varepsilon) = \perp$ (bottom) $\rightarrow$ ikurrik ez dago

Definizio formala:

$$azken(x) = \begin{cases} \perp & x = \varepsilon \\ s & x = ws \end{cases}$$

$$Buru(x) = \begin{cases} \perp & x = \varepsilon \\ w & x = ws \end{cases} \Rightarrow burua(a) = \varepsilon$$

Hitzaren luzera $\underline{|x| \rightarrow luzera(x)}$

Duen karaktere kopurua da.

$$\underbrace{Luzera(x)}_{|x|} = \begin{cases} 0 & x = \varepsilon \\ \underbrace{luzera(w)}_{|x|}+1 & x = ws \end{cases}$$

Hitzen Kateamendua $x \cdot y$

Σ alfabetoa izanik, $x, y \in \Sigma^*$

$y = \varepsilon \Rightarrow x \cdot y = x$

$y = ws \Rightarrow x \cdot y = (x \cdot w)s$

Ikurren ilarra $x[i]$

Definizio aljebraikoa:

$$x[i] = s \Leftrightarrow \exists y, z \in \Sigma^* \ (x = ys \cdot z \wedge |y| = i)$$

adb.

$x = abcba$

$x[3] = ? \Rightarrow \left| \begin{array}{l} y = abc \Rightarrow |y| = 3 \\ z = a \end{array} \right. \Rightarrow x[3] = s = b$

Definizioa induktiboa:

$$x[i] = \begin{cases} \perp & x = \varepsilon \\ s & x = ws \wedge w = \varepsilon \wedge i = 0 \\ \perp & x = ws \wedge w = \varepsilon \wedge i > 0 \\ w[i] & x = ws \wedge w \neq \varepsilon \end{cases}$$

Ez dugo lortzen induktiboa izatea

Ikurraren agerpen kopurua $|x|_t$

Σ alfabetoa, $s, t \in \Sigma$, $w \in \Sigma^*$

$$|x|_t = \begin{cases} 0 & x = \varepsilon \\ |w|_t & x = ws \wedge s \neq t \\ |w|_t + 1 & x = ws \wedge s = t \end{cases}$$

Hitzaren berredura x^n

Σ alfabetoa, $x \in \Sigma^*$, $n \geq 0$

$$x^n = \begin{cases} \varepsilon & n = 0 \\ x^k \cdot x & n = k+1 \end{cases}$$

$\rightarrow x^0 = \varepsilon$ izango da elemento neutro delako

Hitzaren Kateamendua x^R

Definizio induktiboa

$x = \varepsilon \qquad x^R = \varepsilon$

$x = ws \qquad x^R = s \cdot w^R$

Atzizkia atzizki$[x,y]$ edo $x \sqsubseteq_s y$

Azpikatearen agerpen kopurua $|x|_y$

$x = b\underbrace{aa}\underbrace{aa}a\ bc\ \underbrace{aa}$

Definizio induktiboa: Σ alfabetoa, $x, y \in \Sigma^*$

$$|x|_y = \begin{cases} \perp & y = \varepsilon \\ 0 & x = \varepsilon \wedge y = ws \\ |r|_y + 1 & x = rt \wedge y = ws \wedge t = s \wedge r \sqsubseteq_s w \\ |r|_y & x = rt \wedge y = ws \wedge (t \neq s \vee r \not\sqsubseteq_s w) \end{cases}$$

Lengoaia

$L \subseteq \Sigma^*$

$\Sigma = \{a, b, c\}$

$L_1 = \{x \in \Sigma^* : |x|_a \% 3 = 0\}$

$L_2 = \emptyset$

$L_3 = \{\varepsilon\}$ } Ez dira berdinak, L_2 lengoaia hutsa da, L_3 ez.

$L_4 = \{\varepsilon, a, b, c, aa\}$ $L_8 = \{x \in \Sigma^* : |x|_b < 3\}$

$L_5 = \{x \in \Sigma^* : |x| \leq 100\}$

$L_6 = \{x \in \Sigma^* : |x| \geq 100\}$

$L_7 = \{x \in \Sigma^* : bbb \not\trianglelefteq x\}$

Aurreko Lengoaien definizioen aplikazioak, ordena lexikografikoa:

- $L_1 = \{\varepsilon, b, c, bb, bc, cc, aaa, bbb, bbc \ldots\}$
- $L_5 = \{\varepsilon, a, b, c, aa, ab, ac, ba, bb, bc, \ldots\}$
- $L_6 = \{a^{101}, a^{100}b, a^{100}c, a^{99}ba, \}$

Adb.

$\{x \in \{a,b\}^* : \exists y, z \in \Sigma^* (x = y \cdot z \wedge |y| = |z|)\} \equiv \{x \in \{a,b\}^* : |x| \% 2 = 0\}$

$\{\varepsilon, aa, ab, ba, bb\}$

$\{x \in \{a,b\}^* : \exists y, z \in \Sigma^* (aa \not\trianglelefteq y \wedge bb \not\trianglelefteq z \wedge x = y \cdot z)\}$

$\{\varepsilon, a, b, aa, ab, ba, bb, \underset{3\text{ luz}}{\ldots\ldots}, aaaa, aaab, aaba, abaa, abab, abba, abbb$

Beste definitzeko modu bat:

$\{x \in \{a,b\}^* : aa \not\trianglelefteq x \vee \forall y, z \in \Sigma^* (x = yaaz \rightarrow bb \not\trianglelefteq z)\}$

Adibideak

ab gutxienez bi aldiz daukaten hitzak

$\{ x \cdot ab \cdot y \cdot ab \cdot z : x, y, z \in \Sigma^* \}$

ab Z.m bi aldiz daukaten hitzak

$\{ x \in \cancel{\Sigma} \{a,b\}^* : |x|_{ab} = 2 \}$

$\{ b^n a^m \cdot ab \cdot b^p a^q \cdot ab \cdot b^r \cdot a^s : n, m, p, q, r, s \geq 0 \} = \{ b^n a^{\hat{m}} b^{\hat{p}} a^{\hat{q}} b^{\hat{r}} a^s : n, s \geq 0 \wedge m, p, q, r \geq 1 \}$

$L_{12} = \{ 0^n 1^n : n \geq 0 \}$

- $\varepsilon \in L_{12}$
- $w \in L_{12} \Rightarrow 0w1 \in L_{12}$

$L_{13} = \{ a^n : \exists i \geq 0 \ \ n = 2^i \}$

- $a \in L_{13}$
- $x \in L_{13} \Rightarrow xx \in L_{13}$

$\boxed{|x|_{ab} \geq 2}$

$x \in \Sigma^* \Rightarrow ab\,x\,ab \in L_{14}$

$y \in L_{14} \Rightarrow ay \in L_{14}$

$by \in L_{14}$

$ya \in L_{14}$

$yb \in L_{14}$

$\{ x \in \Sigma^* : |x|_a = |x|_b \}$

Induktiboa;

$\varepsilon \in L$

$x \in L \Rightarrow xx \in L$

$\Rightarrow axb \in L$

$\Rightarrow bxa \in L$

LKSA Lengoaiak

1. Definizioa. L1 eta L2 lengoaien Bildura

Informalki: L1 eta L2-n dauden hitzak bilduz lortzen den lengoaia.

Formalki: ~~$\forall x \in$~~ $L_1 \cup L_2 = \{ x \in \Sigma^* : x \in L_1 \vee x \in L_2 \}$

2. Definizioa: L_1 eta L_2 lengoaien Ebakidura.

Informalki: L_1 eta L_2 lengoaietako hitz komunez osaturiko lengoaia.

Formalki: $L_1 \cap L_2 = \{ x \in \Sigma^* : x \in L_1 \wedge x \in L_2 \}$

3. Definizioa: L_1 eta L_2 lengoaien arteko Diferentzia.

Informalki: L_1 ekoak izanik L_2-n agertzen ez diren hitzen multzoa.

Formalki: $L_1 - L_2 = \{ x \in \Sigma^* : x \in L_1 \wedge x \notin L_2 \}$

4. Definizioa: L_1 lengoaiaren Osagarria

Informalki: L_1 lengoaian agertzen ez diren hitzen multzoa.

Formalki: $\overline{L} = \{ x \in \Sigma^* : x \notin L)$

5. Definizioa: L lengoaiaren Alderantzizkoa

Informalki: L ko hitz guztiak alderantziz lortutako lengoaia

Formalki: $L^R = \{ x \in \Sigma^* : \exists y \in L \;\; x = y^R \} = \{ y^R : y \in L \}$

6. Definizioa: L_1 eta L_2 lengoaien kateamendua

Informalki: Binaka L_1 eko eta L_2 ko hitzak kateatuz lor daitezkeen hitz guztien hitzak

Formalki: $L_1 \cdot L_2 = \{ x \in \Sigma^* : \exists y \in L_1, \exists z \in L_2 \; x = y \cdot z \}$

7. Definizioa: Lengoaiaren n-garren berredura

Informalki: Lkoak diren zehatz-mehatz n zati kateatuz osa daitezkeen hitzen lengoaia.

Formalki: $L^0 = \{\varepsilon\}$ $L^{k+1} = L^k \cdot L$

8. Definizioa: L lengoaiaren Itxidura Positiboa

Informalki: Lko hitzak kateatuz eraiki daitezkeen hitzak.

Formalki: $w \in L \Rightarrow w \in L^+$

$x \in L^+ \wedge w \in L \Rightarrow x \cdot w \in L^+$

9. Itxidura

Informalki: Lko hitz guztiak kateatuz lor daitezkeen hitz guztiak ε barne

Formalki:

$\varepsilon \in L^*$

$x \in L^* \wedge w \in L \Rightarrow x \cdot w \in L^*$

informatika fakultatea — facultad de informática

Lengoaia eta Sistema Informatikoak Saila

LKSA – Lengoaien eragiketak

Elkarrirakurketa

1. FASEA: A ataleko aditu bihurtu (30 minutu)

Izen-abizenak: Aitzol Elu

Ekintza ebaluagarri honek hiru fase dauka, eta gaur eta bihar artean burutuko dugu. Lehendabiziko fasean emandako dokumentazioa aztertu eta landu beharko duzu. Ondo ulertu ondoren hemen zehazten diren ariketak ebatz itzazu. Orduan **A ataleko aditua** izango zara. Azken helburua, **atala irakurri ez duten ikaskideei azaltzea** izango da. Orrialde hau irakasleari entregatzekoa da.

EBAZTEKO ARIKETAK: Emandako definizioak esplikatzeko bezain ondo, ulertu dituzula egiaztatzeko ondoko eragiketa hauek buru itzazu, eta emaitzez sortutako lengoaiak defini itzazu. Baten batean zalantzak izanez gero, edo ez bazaizu ateratzen, dokumenta itzazu zure dudak. Hurrengo fasean A ataleko beste aditu batzuekin argitzeko aukera izango duzu.

G ∪ B = $\{0,1\} \cup \{0^n1^n : n \geq 0\}$

F ∪ G = $\{x \in \{0,1\}^* : |x|_{10} = 0 \vee x = 0^n1^n : n \geq 0\}$

A ∩ G = $\{\emptyset\}$

$F \cap G = \{0^n 1^n : n \geq 0\} = G \quad (F \sqsubseteq G)$

$D - E = \{0 \cdot y \cdot 0 : y \in \{0,1\}^*\}$

$G - B = \{0^n 1^n : n > 0\}$

$\overline{F} = \{x \in \{0,1\}^* : |x|_{10} \neq 0\}$

$\overline{D} = \{x \in \{0,1\}^* : 0 \not\sqsubseteq_r x\}$

Enunciato Antileta

$L = \{x \in \Sigma^* : abb \sqsubseteq_s x\}$

$M = \{x \in \Sigma^* : bba \sqsubseteq_p x\}$

$L \cdot \{a,b\}^* \cap \{a,b\}^* \cdot M = \cancel{\{bba \cdot y \cdot abb : y \in \{a,b\}^*\}}$

$= \{x \in \{a,b\}^*$

$L = \{x \in \Sigma^* : aba \sqsubseteq x\}$

$(\overline{L})^+ = (\{x \in \Sigma^* : aba \not\sqsubseteq x\})^+ = \Sigma^*$

$L = \{x \in \Sigma^* : a8 \sqsubseteq x\}$

$(\overline{L})^3 = \cancel{(\{x \in \Sigma^* : a8 \not\sqsubseteq x\})^3 = \Sigma^*}$

$= \{x \in \Sigma^* : a$

$L = \{abx : x \in \Sigma^*\}$

$L \cdot L^R - \{x : x = x^R\} = abx \cdot xba - \{x : x = x^R\} = abxba - \{x : x = x^R\} =$

$= \cancel{abxba}\ \{x \in \Sigma^* : abxba \wedge x \neq x^R$

LKSA – Lengoaien eragiketak

Elkarrirakurketa

A atala

Gogora ezazu **lengoaia** emandako Σ alfabetoaren gaineko hitzen edozein multzo dela. Lengoaiak konbinatuz beste lengoaia batzuk lortzeko balio duten hainbat **eragiketa** definituko dugu, hitzak eta haien arteko eragiketekin egin dugun tankeran.

LENGOAIA ADIGARRIEN ZERRENDA: (alfabetoa beti $\Sigma = \{\mathbf{0},\mathbf{1}\}$ izango da)

$\mathbf{A} = \{ \mathbf{0}, \mathbf{10}, \mathbf{100}, \mathbf{001} \}$ $\quad$ $\mathbf{B} = \{ \varepsilon, \mathbf{0}, \mathbf{1} \}$ $\quad$ $\mathbf{C} = \varnothing$

$\mathbf{D} = \{ \mathbf{x} \in \{0,1\}^*: \mathbf{0} \sqsubseteq_P \mathbf{x} \}$ (**0**z hasten diren hitzak)

$\mathbf{E} = \{ \mathbf{x} \in \{0,1\}^*: \mathbf{0} \not\sqsubseteq_S \mathbf{x} \}$ (**0**z bukatzen ez diren hitzak)

$\mathbf{F} = \{ \mathbf{x} \in \{0,1\}^*: |\mathbf{x}|_{10} = 0 \}$ (**10** azpihitza ez duten hitzak)

$\mathbf{G} = \{ \mathbf{0}^n\mathbf{1}^n: n \geq 0 \}$ (lehenengo erdia **0**z osatuta eta bigarren erdia **1**z osatuta dauzkaten hitzak)

1. DEFINIZIOA: **L1** eta **L2** lengoaien **BILDURA** $[\mathbf{L1} \cup \mathbf{L2}]$

Informalki **L1** eta **L2**ko hitzak bilduz lortzen den lengoaia da (bietan daudenak errepikatu gabe, noski).

Formalki: $\mathbf{L1} \cup \mathbf{L2} = \{ \mathbf{x} \in \Sigma^*: \mathbf{x} \in \mathbf{L1} \vee \mathbf{x} \in \mathbf{L2} \}$

BILKETAREN ADIBIDEAK:

$\mathbf{A} \cup \mathbf{B} = \{ \varepsilon, \mathbf{0}, \mathbf{1}, \mathbf{10}, \mathbf{100}, \mathbf{001} \}$ $\quad$ $\mathbf{B} \cup \mathbf{C} = \{ \varepsilon, \mathbf{0}, \mathbf{1} \} = \mathbf{B}$

$\mathbf{A} \cup \mathbf{D} = \{ \mathbf{10}, \mathbf{100} \} \cup \{ \mathbf{0y} : \mathbf{y} \in \{0,1\}^* \}$

$\mathbf{B} \cup \mathbf{F} = \{ \mathbf{x} \in \{0,1\}^*: |\mathbf{x}|_{10} = 0 \} = \mathbf{F}$ (**B** $\subseteq$ **F** betetzen delako)

$\mathbf{D} \cup \mathbf{E} = \{ \mathbf{x} \in \{0,1\}^*: \neg\exists \mathbf{y}\ (\mathbf{x} = \mathbf{1}\cdot\mathbf{y}\cdot\mathbf{0}) \}$ (aldi berean **1**z hasten eta **0**z bukatzen direnak kenduta, beste hitz guztiak)

2. DEFINIZIOA: L1 eta **L2** lengoaien **EBAKIDURA** [**L1 ∩ L2**]

Informalki **L1** eta **L2**ko hitz komunez osaturiko lengoaia da

Formalki: $L1 \cap L2 = \{ x \in \Sigma^{*}: x \in L1 \wedge x \in L2 \}$

EBAKETAREN ADIBIDEAK:

$A \cap B = \{ 0 \}$

$B \cap F = \{ \varepsilon, 0, 1 \} = B$

$A \cap F = \{ 0, 001 \}$

$B \cap C = \varnothing = C$

(**B ⊆ F** betetzen delako)

$D \cap E = \{ 0 \cdot y \cdot 1 : y \in \{0,1\}^{*} \}$

3. DEFINIZIOA: L1 eta **L2** lengoaien arteko **DIFERENTZIA** [**L1 – L2**]

Informalki **L1**ekoak izanik **L2**koak ez diren hitzen multzoa da

Formalki: $L1 - L2 = \{ x \in \Sigma^{*}: x \in L1 \wedge x \notin L2 \}$

DIFERENTZIAREN ADIBIDEAK:

$A - B = \{ 10, 100, 001 \}$

$D - F = \{ 0 \cdot y \cdot 10 \cdot z : y,z \in \{0,1\}^{*} \}$

$G - F = \varnothing$

$D - B = D - \{ 0 \} = \{ 0 \cdot y : \varepsilon \neq y \in \{0,1\}^{*} \}$

(**G ⊆ F** betetzen delako)

4. DEFINIZIOA: L lengoaiaren **OSAGARRIA** [$\overline{L}$]

Informalki **L**koak ez diren hitzen multzoa da (alfabeto beraz osa daitezkeen artean)

Formalki: $\overline{L} = \{ x \in \Sigma^{*}: x \notin L \}$

OSAGARRIAREN ADIBIDEAK:

$\overline{B} = \{ x \in \Sigma^{*}: |x| > 1 \}$

$\overline{C} = \{0,1\}^{*}$

$\overline{E} = \{ x \in \{0,1\}^{*}: 0 \sqsubseteq_S x \}$

$\overline{G} = \{ x \in \{0,1\}^{*}: |x| \ \% \ 2 = 1 \} \cup \{ y \cdot z : y,z \in \{0,1\}^{*} \wedge |y|=|z| \wedge (1 \sqsubseteq y \vee 0 \sqsubseteq z) \}$ (luzera bakoitia duten, edo beren lehenengo erdian **1**ren bat duten, edo beren bigarren erdian **0**ren bat duten hitzak)

LKSA – Lengoaien eragiketak

Elkarrirakurketa

B atala

Gogora ezazu **lengoaia** emandako Σ alfabetoaren gaineko hitzen edozein multzo dela. Lengoaiak konbinatuz beste lengoaia batzuk lortzeko balio duten hainbat **eragiketa** definituko dugu, hitzak eta haien arteko eragiketekin egin dugun tankeran.

LENGOAIA ADIGARRIEN ZERRENDA: (alfabetoa beti $\Sigma = \{0,1\}$ izango da)

$A = \{ 0, 1, 00, 10, 001 \}$ $B = \{ \varepsilon, 0 \}$ $C = \{ \varepsilon \}$ $D = \varnothing$

$E = \{ x \in \{0,1\}^*: |x|_{01} = 0 \}$ (**01** azpihitza ez duten hitzak)

$F = \{ x \in \{0,1\}^*: 01 \sqsubseteq_P x \} = \{ 01 \cdot y: y \in \{0,1\}^* \}$ (**01**ez hasten direnak)

$G = \{ 0^n 1^n: n \geq 0 \}$ (lehenengo erdia **0**z eta bigarren erdia **1**z osatuta dauzkaten hitzak)

$H = \{ x \in \{0,1\}^*: |x|_0 \ \% \ 2 = 0 \}$ (**0** kopuru bikoitza duten hitzak)

5. DEFINIZIOA: L lengoaiaren ALDERANTZIZKOA [L^R]

Informalki: Lko hitz guztiak alderantziz lortutako lengoaia da

Formalki: $L^R = \{ x \in \Sigma^*: \exists y \in L \ \ x = y^R \} = \{ y^R: y \in L \}$

ALDERANTZIKETAREN ADIBIDEAK:

$A^R = \{ 0, 1, 00, 01, 100 \}$ $C^R = \{ \varepsilon \}$ $D^R = \varnothing$

$E^R = \{ x \in \{0,1\}^*: |x|_{10} = 0 \}$ (**10** azpikatea ez daukaten hitzak)

$F^R = \{ x \in \{0,1\}^*: 10 \sqsubseteq_S x \} = \{ y \cdot 10: y \in \{0,1\}^* \}$ (**10**z bukatutakoak)

$H^R = \{ x \in \{0,1\}^*: |x|_0 \ \% \ 2 = 0 \} = H$

6. DEFINIZIOA: **L1** eta **L2** lengoaien **KATEAMENDUA** [$L1 \bullet L2$]

Informalki binaka hartuz **L1**eko hitz guztiak **L2**ko hitz guztiekin kateatuz gero lor daitezkeen hitzen lengoaia da

Formalki: $L1 \bullet L2 = \{ x \in \Sigma^*: \exists y \in L1 \; \exists z \in L2 \;\; x = y \bullet z \} = \{ y \bullet z: y \in L1 \wedge z \in L2 \}$

KATEAMENDUAREN ADIBIDEAK:

$A \bullet B = \{ 0, 1, 00, 10, 001 \} \cup \{ 00, 10, 000, 100, 0010 \} = \{ 0, 1, 00, 10, 000, 001, 100, 0010 \}$

$A \bullet C = \{ 0, 1, 00, 10, 001 \} = A$ $\quad A \bullet D = \varnothing$ $\quad F \bullet B = F$

$B \bullet G = \{ 0^n 1^m: n,m \geq 0 \wedge (n=m \vee n=m+1) \}$

$E \bullet G = \{ 1^n 0^m 1^p: n,m,p \geq 0 \wedge m \geq p \}$ $\quad H \bullet H = H$

7. DEFINIZIOA: **L** lengoaiaren **n-GARREN BERREDURA** [L^n]

Informalki **L**koak diren zehatz-mehatz **n** zati kateatuz osa daitezkeen hitzen lengoaia da. Hau da, **L** bere buruarekin **n** aldiz kateatzeko emaitza da.

Formalki: $L^0 = \{ \varepsilon \}$ $\quad L^{k+1} = L^k \bullet L$

BERREKETAREN ADIBIDEAK:

$B^3 = \{\varepsilon \bullet \varepsilon \bullet \varepsilon, \varepsilon \bullet \varepsilon \bullet 0, \varepsilon \bullet 0 \bullet \varepsilon, \varepsilon \bullet 0 \bullet 0, 0 \bullet \varepsilon \bullet \varepsilon, 0 \bullet \varepsilon \bullet 0, 0 \bullet 0 \bullet \varepsilon, 0 \bullet 0 \bullet 0 \} = \{ \varepsilon, 0, 00, 000 \}$

$C^7 = \{ \varepsilon \} = C$

$E^2 = \{ x \in \{0,1\}^*: |x|_{01} \leq 1 \}$ (**01** azpihitza gehienez behin daukaten hitzak)

$F^2 = \{ 01 \bullet y \bullet 01 \bullet z : y,z \in \{0,1\}^* \}$ (erne, esandakoa eta $\{ x \in \{0,1\}^*: 01 \sqsubseteq_P x \wedge 01 \sqsubseteq x \}$ <u>ez dira baliokide</u>)

$F^4 = \{ 01 \bullet y \bullet 01 \bullet z \bullet 01 \bullet u \bullet 01 \bullet v : y,z,u,v \in \{0,1\}^* \} = \{ x \in \{0,1\}^*: 01 \sqsubseteq_P x) \wedge |x|_{01} \geq 4 \}$

$G^3 = \{ 0^n 1^n 0^m 1^m 0^p 1^p: n,m,p \geq 0 \}$

informatika fakultatea · facultad de informática

Lengoaia eta Sistema Informatikoak Saila

LKSA – Lengoaien eragiketak

Elkarrirakurketa

C atala

Gogora ezazu **lengoaia** emandako Σ alfabetoaren gaineko hitzen edozein multzo dela. Lengoaiak konbinatuz beste lengoaia batzuk lortzeko balio duten hainbat **eragiketa** definituko dugu, hitzak eta haien arteko eragiketekin egin dugun tankeran.

LENGOAIA ADIGARRIEN ZERRENDA: (alfabetoa beti $\Sigma = \{0,1\}$ izango da)

$A = \{0, 11\}$ $B = \{\varepsilon\}$ $C = \varnothing$ $D = \{1, 01\}$

$E = \{x \in \{0,1\}^*: 01 \sqsubseteq_P x\} = \{01 \cdot y: y \in \{0,1\}^*\}$ (**01**z hasten direnak)

$F = \{x \in \{0,1\}^*: |x|_{01} = 0\}$ (**01** azpikatea ez duten hitzak)

$G = \{x \in \{0,1\}^*: |x|_0 \% 2 = 0\}$ (**0** kopuru bikoitia duten hitzak)

$H = \{0^n 1^n: n \geq 0\}$ (lehenengo erdia **0**z osatuta eta bigarren erdia **1**z osatuta dauzkaten hitzak)

$I = \{x \in \{0,1\}^*: 111 \not\sqsubseteq x\}$ (**111** azpikatea ez duten hitzak)

8. DEFINIZIOA: L lengoaiaren **ITXIDURA POSITIBOA** $[L^+]$

Informalki nahi adin **L**ko hitzen segida kateatua eraikiz lor daitezkeen hitz guztien lengoaia da (bat, bi, hirurogei, ... **L**ko hitz kateatuen segidak)

Formalki:

- $w \in L \Rightarrow w \in L^+$ (**L**ko hitzak elementu bateko segidak dira)
- $x \in L^+ \wedge w \in L \Rightarrow x \cdot w \in L^+$ (**L⁺**eko segida izanik, beste **L**ko hitz batez luza dezakegu segida berria lortzeko)

ITXIDURA POSITIBOAREN ADIBIDEAK:

$A^+ = \{ 0, 11 \} \cup \{ 00, 011, 110, 1111 \} \cup \{ 000, 0011, 0110, 01111, 1100, 11011, 11110, 111111 \} \cup \{ 0000, \ldots \} \cup \ldots$ (hitz ez hutsak non **1**zko taldeek beti luzera bikoitia duten)

$B^+ = \{ \varepsilon \} = B$ (hitz hutsa bere buruarekin kateatzen tematu arren ...)

$C^+ = \varnothing = C$ (multzo hutsetik ezin da hitzik atera segidak osatzeko)

$E^+ = \{ x \in \{0,1\}^*: 01 \sqsubseteq_P x \} = E$

$F^+ = \{0,1\}^*$ (alfabetoaren gaineko hitz guztiak suertatzen dira)

9. DEFINIZIOA: **L** lengoaiaren **ITXIDURA** [L^*]

Informalki nahi adin **L**ko hitzen segida kateatua eraikiz lor daitezkeen hitz guztien lengoaia da (baina zero hitzetako segidak ere onarturik). Eta zer da "zero hitzetako segida"? ε hitz hutsa, alegia.

Formalki:

- $\varepsilon \in L^*$ (zero hitzek **L**ko hitzen segida osatzen dute)
- $x \in L^* \wedge w \in L \Rightarrow x \bullet w \in L^*$ (L^*ko segida baldin badaukagu, beste **L**ko hitz batez luza dezakegu segida berria lortuz)

ITXIDURAREN ADIBIDEAK:

$A^* = \{ \varepsilon \} \cup \{ 0, 11 \} \cup \{ 00, 011, 110, 1111 \} \cup \{ 000, 0011, 0110, 01111, 1100, 11011, 11110, 111111 \} \cup \{ 0000, \ldots \} \cup \ldots = A^+ \cup \{ \varepsilon \}$

(hitzak non **1**zko taldeek beti luzera bikoitia duten)

$B^* = \{ \varepsilon \} = B$ (hitz hutsa bere buruarekin kateatzen tematu arren, zero aldiz bada ere ...)

$C^* = \{ \varepsilon \}$ (hemen zerbait aldatzen da; multzo hutsetik posible baita "zero hitz ateratzea")

$F^* = \{0,1\}^*$ (alfabetoaren gaineko hitz guztiak suertatzen dira)

$G^* = \{ x \in \{0,1\}^*: |x|_0 \bmod 2 = 0 \} = G$

$H^* = \{ \varepsilon, 01, 0011, 000111, \ldots, 0101, 010011, 01000111, \ldots, 001101, \ldots, 010000111001101001101, \ldots \}$ (luzera bikoitzeko k≥0 zati ez hutsetan deskonposa daitezkeen hitzak, zeinetan zati bakoitzaren lehenengo erdia **0**z soilik osatuta dagoen, eta bigarren erdia **1**z soilik) d)

$$L=\{x\in\{0,1\}^*\ \exists y,z\in\{0,1\}^*: (x=y\cdot z \wedge y=z^R \wedge |x|\%2=0) \vee (|x|\%2\neq 0 \wedge (x=y\cdot 0\cdot z \vee x=y\cdot 1\cdot z))\}$$

e)

$$L=\{x\in\{0,1\}^*: |x|\%2\neq 0 \wedge 1\sqsubseteq_p x \wedge 1\sqsubseteq_s x \wedge x_{[\frac{|x|-1}{2}]}=1\}$$

f)

$$L=\{x\in\{0,1\}^*: |x|_0=|x|_1\}$$

g) $L=\{x\in\{0,1\}^*: |x|_0\geq 3\}$

h) $L=\{x\in\{0,1\}^*: |x|_0=3\}$

i) $L=\{x\in\{0,1\}^*: |x|_0\leq 3\}$

j) $L=\{x\in\{0,1\}^*: 000\sqsubseteq x \wedge 001\not\sqsubseteq x$

5) a)

$$L=\{x\in\Sigma^*: \exists u\in\Sigma^*: \wedge |x|_u\geq 3 \wedge |u|\geq 3\}$$

b)

$$L=\{x\in\Sigma^*: \exists u\in\Sigma^*: u\sqsubseteq_s x \wedge |u|\geq 3 \wedge u=u^R\}$$

c)

$$\{x\in\Sigma^*: true\}$$

6

a)

$aa, bb\in L1$

$x,y\in L1 \Rightarrow x\cdot y, y\cdot x\in L1 \Rightarrow \{aa,bb\}^*$

c) $a\in L_3$

$x\in L_3 \Rightarrow b\,x\,b\in L_3$

$\{b^n a b^n : n\geq 0\}$.

(1)

a)

$L_1 = \{ x \in \{0,1\}^* : |x|_{11} \geq 1 \wedge |x|_{111} = 1 \}$

Bai:

{111}

000111

111000

00110111 0

b) $L_2 = \{ x \in \{0,1\}^* : 1 \sqsubseteq_p x \wedge 1 \sqsubseteq_s x \}$

1

11

111

c) $L_3 = \{ 0^i 1^j 0^k : j \geq i \geq 0 \wedge k \geq 0 \}$

ε

1

01

11

000

111

000111

001111

11100

(2)

a) $\{ 1^{k+1} 0^k : k \geq 1 \wedge |k+1+k| \% 2 \neq 0$

$\{ x \in \{0,1\}^* : |x| \% 2 \neq 0 \wedge \forall u \leq (|x|/2 + 1) (x[u] = 1)$

b) $\{ x \in \{0,1\}^* : |x| \% 3 = 0 \}$

c) Induktiboa

$\varepsilon \in L$

$x \in L \Rightarrow xxx \in L$

6. Definizioa: Kateamendua

Informalki: L_1-eko hitzak eta L_2-ko hitzak banaka hartuz, eta beste lengoaiakoarekin kateatuz lortzen den lengoaia

Formalki: $L_1 \cdot L_2 = \{ x \in \Sigma^* \ \exists y \in L_1 \wedge \exists z \in L_2 : x = y \cdot z \}$

7. Definizioa: n-garren ~~berredura~~ Berredura

Informalki: L-eko hitzak hartu eta bere buruarekin n aldiz kateatuz lortzen den lengoaia.

Formalki: $L^n = \{ x \in \Sigma^* : x^n = x^{n-1} \cdot x \}$

8. Definizioa: Itxidura Positiboa

Informalki: L lengoaia hartuz, bertako hitz guztiak elkarrekin kateatuz lortu daitekeen lengoaia, $\not\varepsilon$

Formalki: $w \in L \Rightarrow w \in L^+$

$x \in L^+ \wedge w \in L \Rightarrow x \cdot w \in L^+$

9. Definizioa: Itxidura

Informalki: L-ko hitzak ~~beraien~~ kateatuz lortzen diren hitzen multzoa, ε barne.

$\varepsilon \in L^*$

$x \in L^* \wedge w \in L \Rightarrow x \cdot w \in L^*$

Hitzaren Definizioa:

Σ alfabetoa emanda:

ε - Σ-ren gaineko hitza da.

Baldin w Σ-ren edozein hitza bada eta $s \in \Sigma$, orduan, $w \cdot s \in \Sigma$-ren gaineko edozein hitza izango da.

LUSA lengoaiak

1. Definizioa: Bildura

Informala: L_1 ko eta L_2 ko hitz guztiak hartzen ditu.

$L = L_1 \cup L_2$ Bilduraren bidez.

Formala: ~~{xes txel~~ $L_1 \cup L_2 = \{x \in \Sigma^*: x \in L_1 \lor x \in L_2\}$

2. Definizioa: Ebakidura

Informala: L_1-eko eta L_2-ko hitz komunez osatutako lengoaia

Formala: $L_1 \cap L_2 = \{x \in \Sigma^*: x \in L_1 \land x \in L_2\}$

3. Definizioa: Diferentzia

Informala: L_1 ean dauden ~~hitzez~~ eta L_2 an ez dauden hitzez osatutako lengoaia.

Formala: $L_1 - L_2 = \{x \in \Sigma^*: x \in L_1 \land x \notin L_2\}$

4. Definizioa: Osagarria

Informala: L_1 eko hitzak ez diren hitzez osatutako multzoa

Formala: $\overline{L_1} = \{x \in \Sigma^*: x \notin L_1\}$

5. Definizioa: Alderantzizkoa

Informala: L_1 eko hitzeak alderantzia hartzen du lengoaia da.

Formala: $L_1^R = \{x \in \Sigma^*: \exists y \in L_1\ x = y^R\}$

h) $\varepsilon \in L$

$x, y \in L \Rightarrow 0x0y0 \in L$

4

a) $txertatu(x,y) = \begin{cases} \varepsilon & x=\varepsilon \wedge y=\varepsilon \\ u \circ t & x=\varepsilon \wedge y=u \circ t \\ w \circ s & x=ws \wedge y=\varepsilon \\ w \circ (u \circ t) \circ s & x=ws \wedge y=u \circ t \end{cases}$

b) $azkHasierara(x) = \begin{cases} \perp & x=\varepsilon \\ s & x=w \circ s \wedge w=\varepsilon \\ s \circ w & x=w \circ s \wedge w \neq \varepsilon \end{cases}$

c) $atzAborill(x) = \begin{cases} \varepsilon & x=\varepsilon \\ \varepsilon & x=w \circ s \wedge s=a \\ atzAborill(w) \circ s & x=w \circ s \wedge s \neq a \end{cases}$

d) $baEz?(x) = \begin{cases} true & x=\varepsilon \\ true & x=w \circ s \wedge w=\varepsilon \\ true & x=w \circ s \wedge s=a \wedge azkLen(w)=b \\ baEz?(w) & x=w \circ s \wedge (s \neq a \vee azkLen(w) \neq b) \end{cases}$

PO2. Orria

e) $\overbrace{\{x \in \{a,b\}^*: |x|_a \% 2 = 0\}}^{L_1} \circ \overbrace{\{x \in \{a,b\}^*: |x|_a \% 3 = 0\}}^{L_2}$ baba∘aaa

$L = \{x \in \{a,b\}^* : \exists y \in L_1 \wedge \exists z \in L_2 : x = y \circ z\}$

Pol. Orria

(1)

d) $L_4 = \{x \in \{0,1\}^*: \forall i < |x| (i \% 2 = 0 \rightarrow x[i] = 1)\}$

Betetzen duten hitzak:

- 1
- 11
- 111
- ε

e) $L_5 = \{x \in \{0,1\}^*: \exists u \in \{0,1\}^* (1 \sqsubseteq_5 x \wedge u = u^R \wedge |u| = 3)\}$

Betetzen duten hitzak:

- 111
- 001111

f) $01, 10 \in L_6$

$x, y \in L_6 \Rightarrow x \cdot y \in L_6$

Betetzen duten hitzak:

- 01

g) $\varepsilon \in L_7$

$x \in L_7 \Rightarrow 0 \cdot x \cdot 1,\ 1 \cdot x \cdot 0 \in L_7$

Betetzen duten hitzak:

- ε
- 01
- ~~0~~ 000111
-

(2)

g) $000 \in L$

$x \in L \Rightarrow 1x \in L$

$0x \in L$

$x1 \in L$

$x0 \in L$

c)

$\{x \in \{0,1\}^* : |x|_{01} = 0\} - \{x \in \{0,1\}^* : |x|_{10} = 0\}$

$\{x \in \{0,1\}^* : |x|_{01} = 0\}$

③

$L_1 = \{x \in \{0,1\}^* : |x| \% 2 = 0 \land 00 \sqsubseteq_p x\}$ $L_2 = \{x \in \{0,1\}^* : |x|_0 \geq 2\}$

a) $L_2 \cup L_1 = \{x \in \{0,1\}^* : |x| \% 2 = 0 \land 00 \sqsubseteq_p x \lor |x|_0 \geq 2\}$

b) $L_1 \cap L_2 = L_1$

c) $L_2 - L_1 = \{x \in \{0,1\}^* : |x|_0 \geq 2 \land |x| \% 2 \neq 0 \land 00 \not\sqsubseteq_p x\}$

d) $L_1 - L_2 = \emptyset$

e) $\overline{L_1} = \{x \in \{0,1\}^* : |x| \% 2 \neq 0 \land 00 \not\sqsubseteq_p x\}$

f) $\overline{L_2} = \{x \in \{0,1\}^* : |x|_0 < 2\}$

g) $L_1 \cdot L_2 = \{00y = x \land |y|_0 \geq 2 : y \in \{0,1\}^*\}$

h) $L_2 \cdot L_1 = \{y00z \land |y|_0 \geq 2 : y, z \in \{0,1\}^*\}$

i) $L_1^2 = \{x = 00z00y \land |x| \% = 2 : y, z \in \{0,1\}^*\}$

o) $L_1^R \cap (L_1 \cdot L_2) = \{x \in \{0,1\} : |x| \% 2 = 0 \land 00 \sqsubseteq_s x\} \cap L_1 \cdot L_2 = \{00y00 = x \land |x| \% = 2 : y \in \{0,1\}^*$

p) $(L_1 \cdot L_2^R)^* = (L_1 \cdot \underset{L_2^R}{\underset{\shortparallel}{L_2}})^* = \{x \in \{0,1\}^* \ \exists y, z \in \{0,1\}^* : 00y00z \land |y|_0 \geq 2 \land |z|_0 \geq 2\}$

④

$L_1 \cdot L_2 = \{x \in \{0,1\}^* \ \exists y \in \Sigma^* (00y = x \land |y|_0 \geq 2)\}$

a)

$= \{x \in \{0,1\}^* \ \exists y, z \in \{0,1\}^* : 00y00z \land |y|_0 \geq 2 \land |z|_0 \geq 2\}$

$\{x \in \{0,1\}^* : |x|_{01} = 0 \cup |x|_{10} = 0\}$

b) $\{x \in \{0,1\}^* : |x|_{01} = 0 \land |x|_{10} = 0\}$

LKSA

Problema orria 1.

3. Ariketa

g) $\{w \in \Sigma^*: w \cdot w = w \cdot w \cdot w\} = \{\varepsilon\}$

i) $\{w \in \Sigma^*: \exists u \in \Sigma^* \; w \cdot w \cdot w = u \cdot u\} \equiv \{xx : x \in \Sigma^*\}$

ε

$w \cdot w \cdot w = a^6 \cdot a^6 \cdot a^6$

$u \cdot u = a^9 \cdot a^9 \cdot a^9$

Lenguaien Ariketak

1. $L_1 = \{x \in \{0,1\}^* : |x| \geq 2\}$, $L_2 = \{00x : x \in \{0,1\}^*\}$.

a) $L_1^2 = L_1 \rightarrow$ Faltsua

e) $L_2^2 = \{x \in \{0,1\}^* : |x|_{00} \geq 2\} \rightarrow$ Faltsua

2. $L = \{0^n 1^m : |n-m| = 1\}$

$L^2 = L \cdot L = \{0^n 1^m 0^p 1^q : |n-m| = 1 \wedge |p-q| = 1\}$

$L \cap L^R = \{0,1\}$

8

a) $\Sigma = \{A, B, \dots, z, \cancel{b}, -\}$

$I = \{x \in \Sigma : |x| \leq 40 \wedge x \neq \varepsilon \wedge -\cancel{\exists}_p x \wedge -\cancel{\exists}_s x \wedge -\cancel{\exists} x\}$

$\Sigma = \{A, B, C, \dots, Z, -, ,\}$

b)

$\{x \in \{1, \dots, 9, A, B, \dots, Z\} \; \exists y \in \{A, B, \dots, Z\} : x = y \wedge \cancel{\vee} \; (\exists z \in \{0, \dots, 9\} : x = y \cdot z \wedge |z| = 1$ $|y| = 8$

c) $\{x \in \{0, \dots, 9\}^* : |x| = 5\}$

1

a) $L_1^2 = L_1 \Rightarrow 00 \notin L_1^2 \Rightarrow L_1^2 \{x \in \{0,1\}^* : |x| \geq 2\}$

$\quad 00 \in L_1 \Rightarrow 0000 \in L_1^2$

$\quad 11 \in L_1 \Rightarrow 1111 \in L_1^2$

b)

$L_1 \cdot L_3 = L_3 \Rightarrow \varepsilon \in L_3$ ba... $\varepsilon \notin L_1 \cdot L_3$

c) $L_2^R = L_2 \Rightarrow 001 \in L_2$ bažive $001 \notin L_2^R \Rightarrow 100 \in L_2^R$

4

d)

$\{x \in \{a,b\}^* : ab \sqsubseteq_p x\} \cap \{x \in \{a,b\}^*\ ba \sqsubseteq_s x\} \cap \{x \in \{a,b\}^*\ |x| \leq 5\}$

$L = \{x \in \{a,b\}^* : ab \sqsubseteq_p x \wedge ba \sqsubseteq_s x \wedge |x| \leq 5\}$

e) $\{x \in \{a,b\}^* : |x|_a \% 2 = 0\} \cdot \{x \in \{a,b\}^* : |x|_a \% 3 = 0\}$

$aba \cdot abaa = abaabaa$

$L = \{x \in \{a,b\}\ \exists y, z \in \{a,b\} : x = y \cdot z \wedge |y|_a \% 2 = 0 \wedge |z|_a \% 3 = 0\}$

f) $\{01^m : m \geq 0\}^+$

$L = \{x \in \{a,b\}^+ : 0 \sqsubseteq_p$

9

A $L_1 = \{x \in \Lambda : x = AR \vee x = B1 \vee x = GI\}$

$L_2 = \{y \in \Delta^* : |y| = 3\}$

$L_3 = \{x \in \Lambda \wedge y \in \Sigma, \exists z \in \Sigma : (y \sqsubseteq z \wedge |y| \geq 1 \wedge |z| = 3\}$

$L_4 = \{-\}$

$L_5 = L_1 \cdot L_2 \cdot L_4 \cdot L_3$

Luzera bakoitia duten hitzak

$|x| \% 2 = 1$

$1 \ldots 1 \; 0 \ldots 0$

$\{1^{n+1} 0^n : n \geq 0\} \xrightarrow{\text{Induktiboan}}$ $1 \in L$

$x \in L \Rightarrow 1x0 \in L$

Luzera 3-ren multiploa den hitza

$|x| \% 3 = 1$

Induktiboki:

$\varepsilon \in L$

$\left.\begin{array}{l} x \in L \\ s_1, s_2, s_3 \in \{0,1\} \end{array}\right| \Rightarrow x s_1 s_2 s_3 \in L$

Luzera 3-ren multiploa ez den

$|x| \% 3 \neq 0$

Induktiboki:

$x \in \Sigma \cup \Sigma^2 \Rightarrow x \in L$

$\left.\begin{array}{l} x \in L \\ y \in \Sigma^3 \end{array}\right| \Rightarrow xy \in L$

Hitz Palindromoak

$\{x \in \Sigma^* : x = x^R\}$

$\varepsilon \in L, \; 0 \in L, \; 1 \in L$

$x \in L \Rightarrow 1x1, \; 0x0 \in L$

Kuzesa bakoitza dator!

$000 \in X$

$001 \notin X$

Induktiboki:

$000 \in L$

$x \in L \Rightarrow x0 \in L$

$x \in L \Rightarrow 01x \in L$

$x \in L \Rightarrow 1x \in L$

4. Ariketa

lehbukaerara(x) = isats(x) lehen(x)

$x = \varepsilon$

$x = ws$

$$lehbukaerara(x) = \begin{cases} \perp & x = \varepsilon \\ x & x = ws \wedge |w| = 0 \\ isats(w) \cdot s \cdot lehen(w) & x = ws \wedge |w| \neq 0 \end{cases}$$

Azkenhasierara(x) = azken(x) Buru(x)

$$kapikua(x) = \begin{cases} true & x = \varepsilon \\ true & x = ws \wedge w = \varepsilon \\ kapikua(isats(w)) & x = ws \wedge lehen(w) = s \\ False & x = ws \wedge lehen(w) \neq s \end{cases}$$

$$atzAborik(x) = \begin{cases} \varepsilon & x = \varepsilon \\ \varepsilon & x = ws \wedge s = a \\ atzAborik(w) \cdot s & x = ws \wedge s = b \end{cases}$$

$$atzikomun(x,y) = \begin{cases} \varepsilon & x = \varepsilon \wedge y = \varepsilon \\ \varepsilon & x = \varepsilon \wedge y = ws \\ \varepsilon & x = ws \wedge y = \varepsilon \\ \varepsilon & x = ws \wedge y = ut \wedge s \neq t \\ atzikomun(w,u) & x = ws \wedge y = ut \wedge s = t \end{cases}$$

5. edo 3-ren multiploa den luzera duten hitzak.

$L = \{x \in \Sigma^* : |x| \% 5 = 0\}$

$L_1 = \{x \in \Sigma^* : |x| \% 3 = 0\}$

$L \cup L_1$

$\{x \in \Sigma^* : |x| = 3\}^* \cup \{x \in \Sigma^* : |x| = 5\}^*$

$\{\Sigma^3\}^* \cup \{\Sigma^5\}^*$

$101 \sqsubseteq x$ - baina, $010 \not\sqsubseteq x$ eta $202 \not\sqsubseteq x$

$\{x \in \Sigma^* : 101 \sqsubseteq x\} - \{x \in \Sigma^* : 010 \sqsubseteq x\} - \{x \in \Sigma^* : 202 \sqsubseteq x\}$

$\Sigma^* \circ 101 \circ \Sigma^* - \Sigma^* \circ 010 \circ \Sigma^* - \Sigma^* \circ 202 \circ \Sigma^*$

Turingen Makina

$\varphi_M(x) = y$

$f(x) = x$

$f(x) = x^R$

abbaa
a X bbaa
b a X X baa
b b a X X X aa
a b b a X X X X a a
a a b b a X X X X X a
a a b b a X X X X X X □
q_0

⇒ □aabbba □□□

$\varphi_M(x, y, z)$

□-z bereizi

abba □ bbb

abba □ b bb
q_1

abbab □ bb
q_1

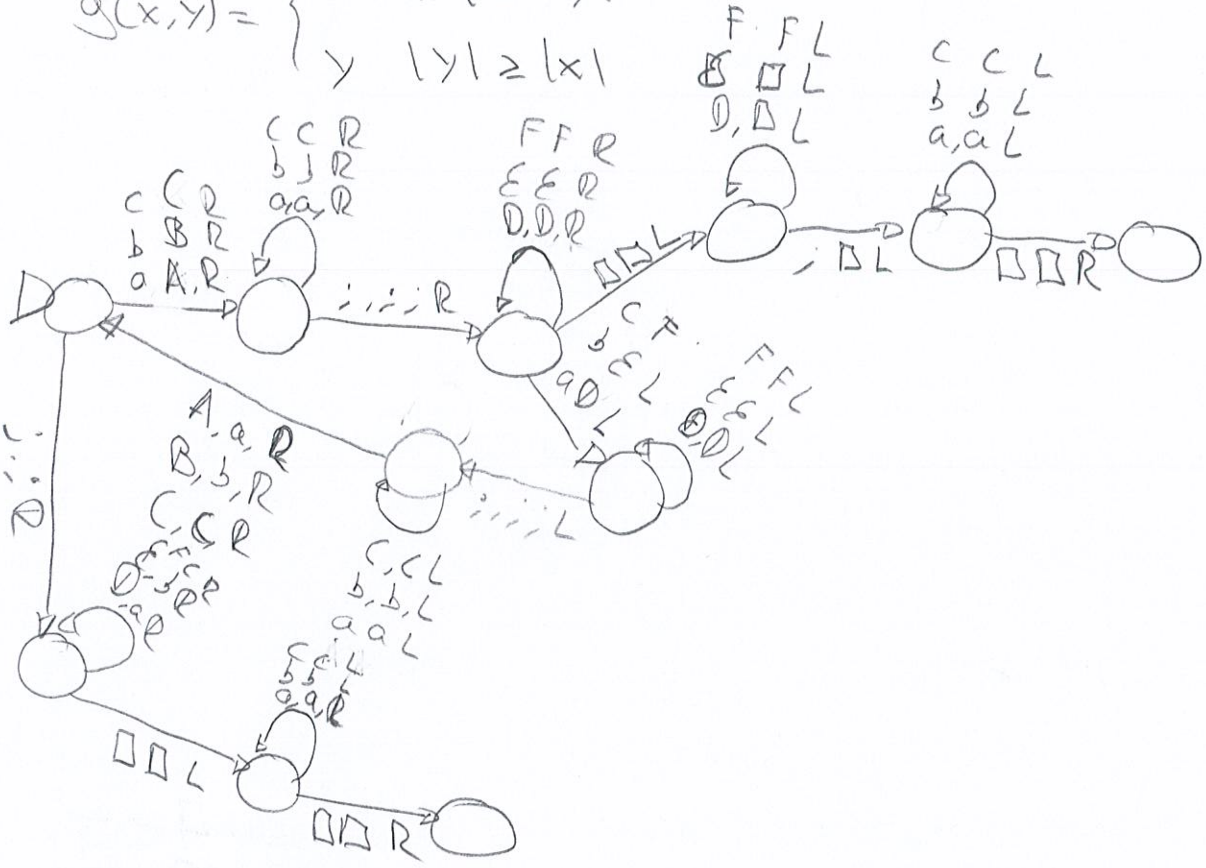
abba bb □ b
q_1

abba bbb □ □
q_1

⇒ abbabbb
q_F

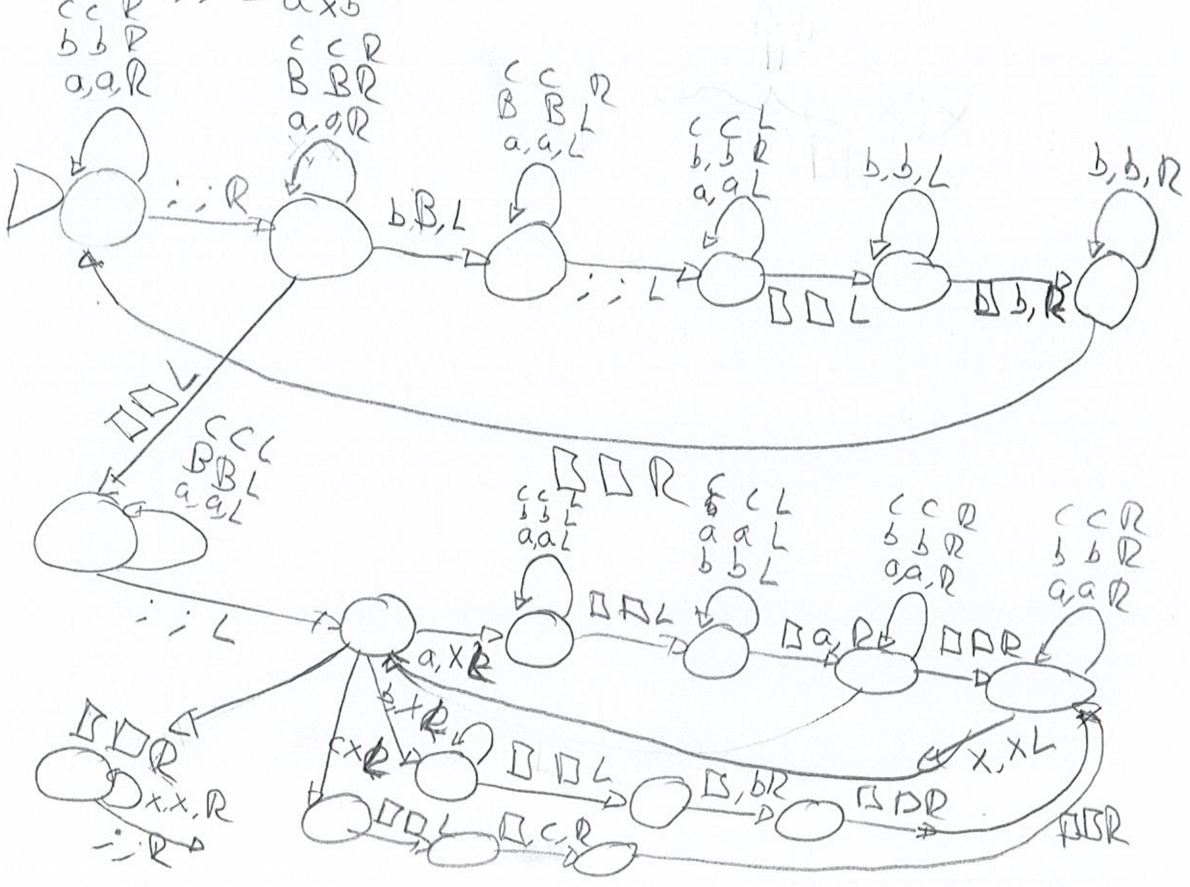
2.b

$$g(x,y) = \begin{cases} x & |x| \geq |y| \\ y & |y| \geq |x| \end{cases}$$

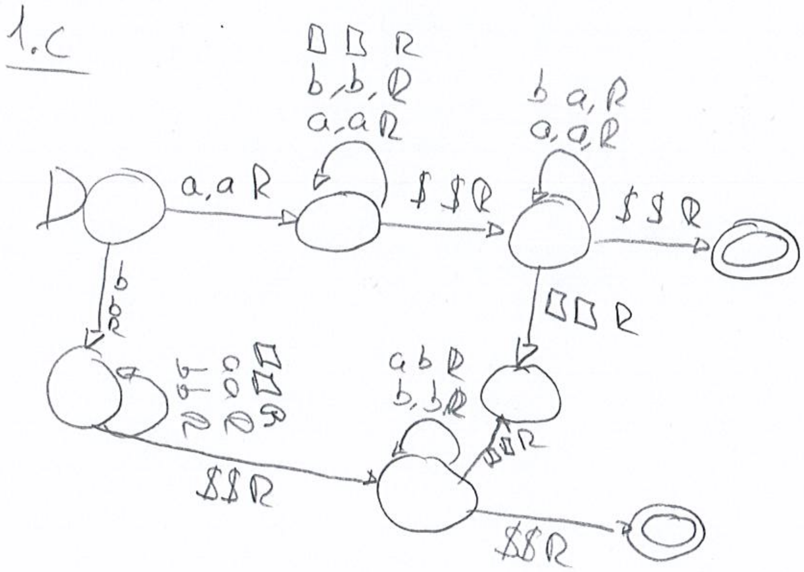


2.f

$$g(x,y) = a^n x b^m$$



1.c

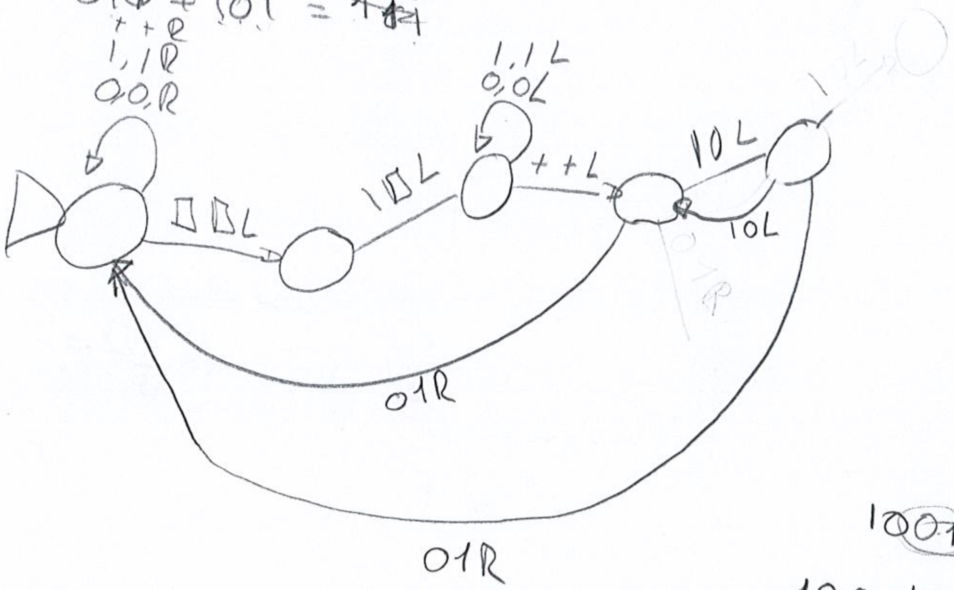


b)

00

1000

011 + 101 = 111



011 + 10□
100 + 10□
+ 1□□

□ 10

010

1001

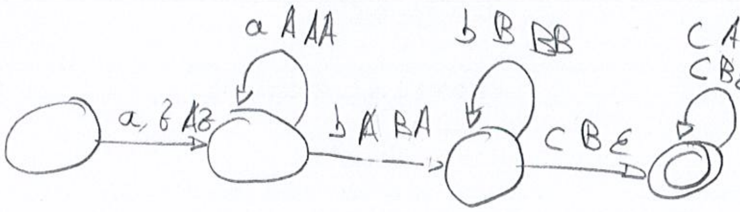
100 + 101 +

011 + 101

011
111
1010

011 + 101 ⇒ 10□01□+10□

5) $\{a^i b^j c^k : i,j,k>0 \wedge i+j=k \wedge k \% 2 \neq 0\}$



A
A
Z

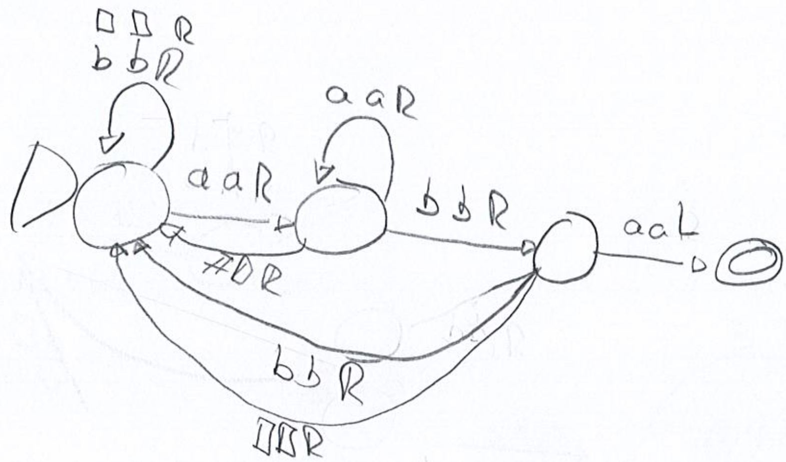
a ~~B~~ AAZ

a AAAA

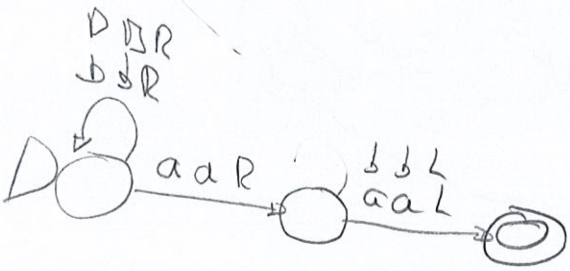
bA ε

aab ccc
abb ccc

1.a)



1b)



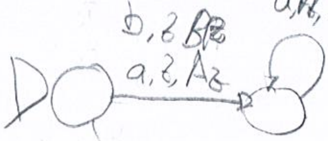
1)

$\{w \in \{a,b\}^* : |w|_b \leq |w_a| \leq 2*|w_b|\}$

a B AB
b A BA
b B BB
a A, AA

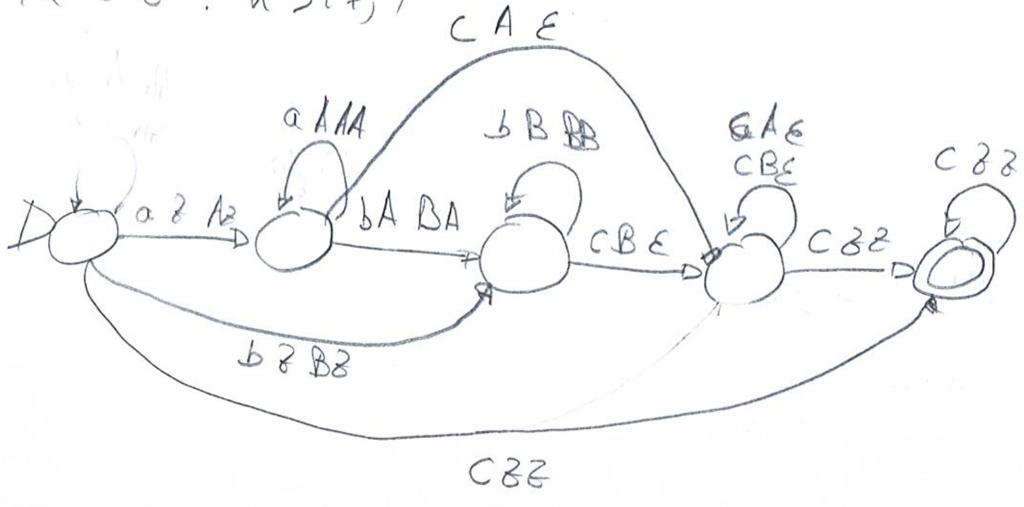
babaabab

bbbbaaaaaaaa



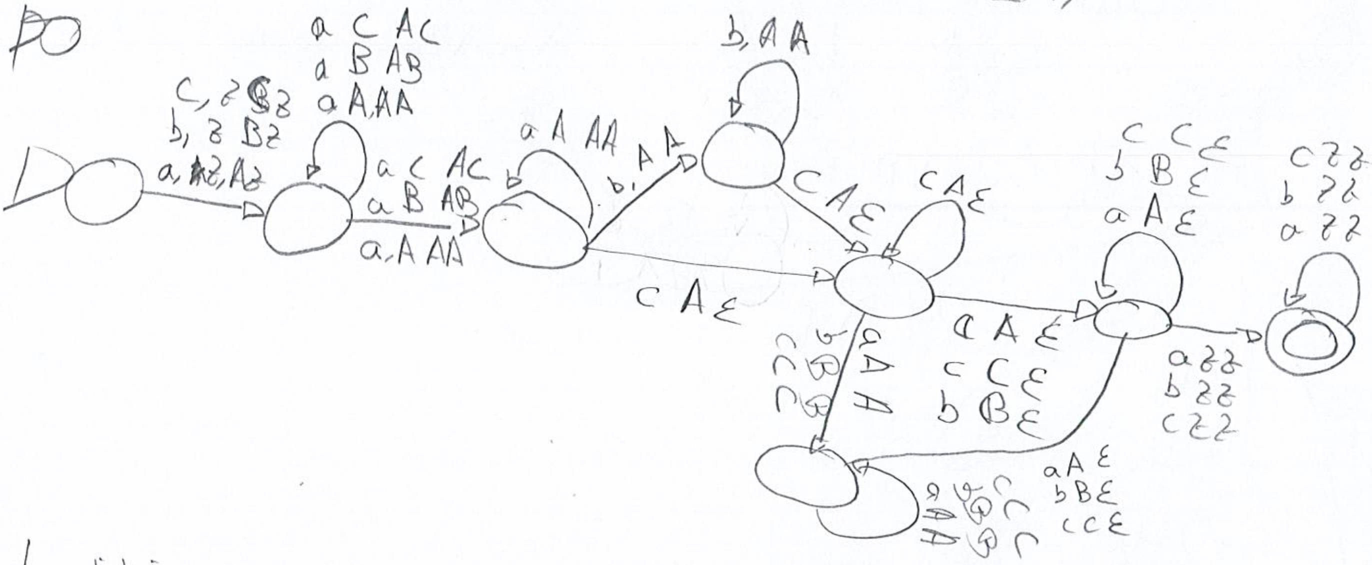
4)

$\{a^i b^j c^k : k > i+j\}$



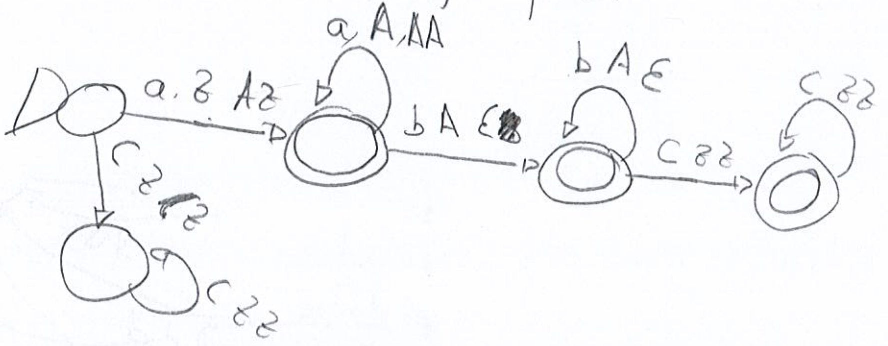
5) $\{x a^n b^m c^n y : x, y \in \{a,b\}^* \wedge n, m \geq 0 \wedge y^R \sqsubseteq x\}$

PO



l)

$\{a^i b^j c^k : i=j \vee j=k\}$



aaabaaaa

1. c) $\{a^n b^m c^m d^n : n, m \geq 0\}$

4. d) $\{a^n b^m c^{n+m} : n, m \geq 0\}$

c) $\{a^n b^m \# a^i : m, n \geq 1, \; i \geq n+m\}$

g)

$x a^{2n+1} c^{2n}$

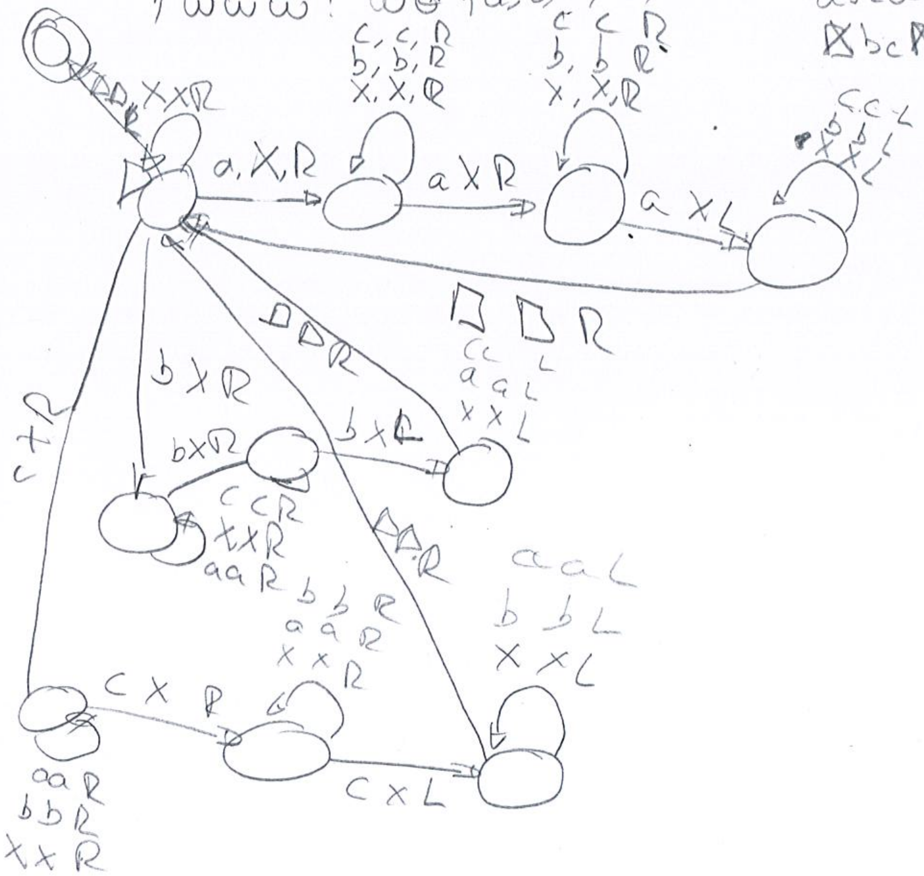
$x \in \{a,b\}^*$

w)

{www : w ∈ {a,b,c}*}

abcaabcaabca

Xbc X X bcaabca



u)

S → Eu

E → awaaE

E → bwbbE

E → cwccE

waab → abwaa

wbba → awbb

wcca → awcc

wbbc → cwbb

waac → cwaa

wccb → bwcc

waa → awa

wbb → bwb

wcc → cwc

wab → bwa

wba → awb

wac → cwa

wca → awc

wbc → cwb

wcb → bwc

waw → aw

wbw → bw

wcw → cw

wcu → c

wau → a

wbu → b

abcaabcaabccu

awaabwbbcawccu

abwaaawbbwccu

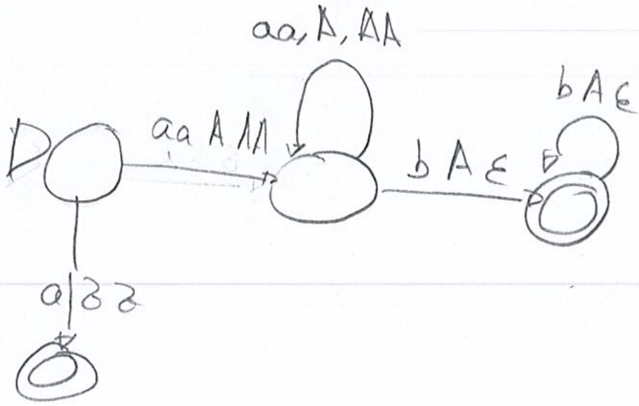
abcwaawbbwccu

abcawabwbwcwcu

abcabwacwbwcu

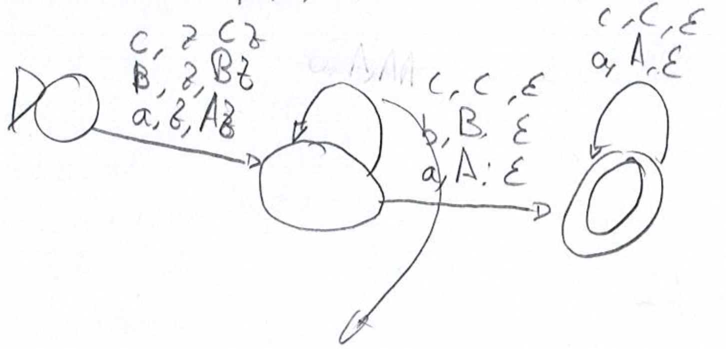
abcabcwawbwcu

h) $\{a^{2*i+1} b^i : i \geq 0\}$



e)

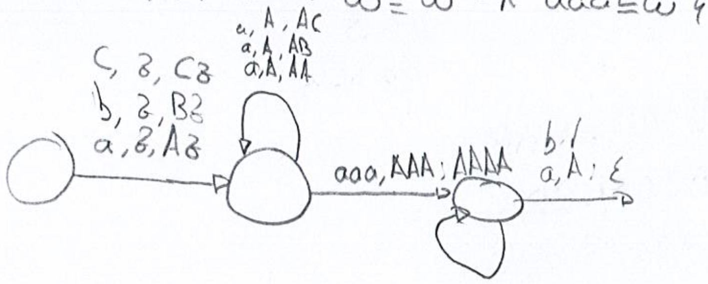
$\{ w \in \{a,b,c\}^* : w = w^R \}$



a, A, AA
a, A, AB
a, A, AC
b, B, BB
b, B, BA
b, B, BC
c, C, CC
c, C, CA
c, C, CB

f)

$\{ w \in \{a,b,c\}^* : w = w^R \wedge aaa \sqsubseteq w \}$



J) $\{a^n b^{n \cdot n} : n \geq 1\}$

$S \to aAb$

$A \to aBbb$

$B \to$

$S \to SAB \mid ab$

bA

$S \to KE$

$E \to aAEBE$

$AB \to ab$

$bA \to Ab$

$KA \to a$

$BA \to AB$

AB

$AB \to$

$aB \to ab$

$bA \to Abbb$

$bB \to bb$

$aA \to aa$

$S \to KE$

$E \to ABE$

$AB \to aB$

$bA \to Abb$

$bbB \to bbbb$

$aA \to aa$

abab

KABAB

aBAB

abAB

aAbbB

aabbbb

AB

KABABAB

aBABAB

abABAB

~~aAbbbBAB~~

~~aabbbbAB~~

abAABB

abAAbbbABB

aAbbAbbbBB

aAbAbbbbbbBB

aAAbbbbbbbbbBB

aaa

KABABABE →

aBABAB

aABBAB

aAABBB

aaaBBB → aaabbbBB

aaabbBB

aaabbbbbb

aaB

aabbbb

aaabbbbbb

aaabbB

aaabbbbbbB

KABABABABE

aAAAABBBB

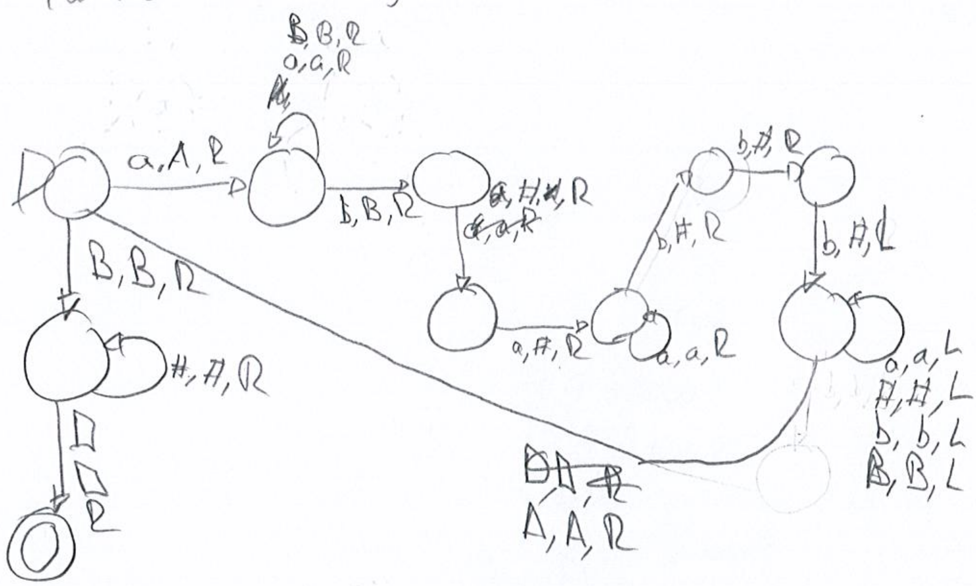
aaaaBBBB

aaaabbbBBB

aaaabbb

aa bbb

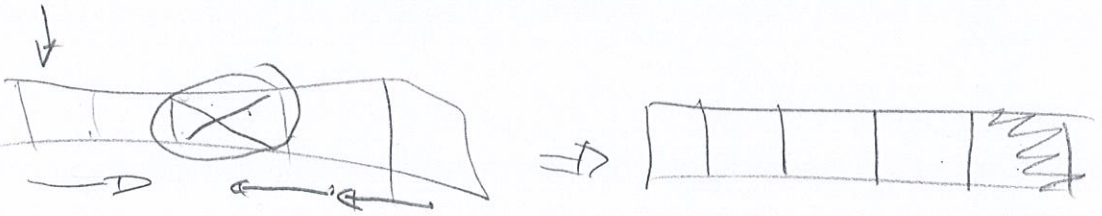
5) $\{a^i b^j a^{2*i} b^{3*j} : i, j \geq 0\}$



$\{a^n b^n c^n : n \geq 0\}$

	a	b	c	□	X	Y
q_0	q_b,□,R	-	-	q_F,□,R	q_M,□,R	~~q_0,Y,R~~
q_b	q_b,a,R	q_c,X,R	-	-	q_b,X,R	-
q_c	-	q_c,b,R	q_L,Y,L	-	-	q_c,Y,R
q_L	q_L,a,L	q_L,b,L	-	q_0,□,R	q_L,X,L	q_L,Y,L
q_M	-	-	-	q_F,□,R	q_M,□,R	q_M,□,R
q_F	-	-	-	-	-	-

1. Sor



Side-1

~~S → wctcv~~

~~w → aw, bw, ε~~

~~t → at, bt, ε~~

~~v → av, bv, ε~~

S → wc

w → w

$\{ a^i b^j a^{2i} b^{2j} \}$

S → wt

w → awaa

aat → taa

t → bbbbt

wb → bw

bw → b

t → ε

ba → ab

wt

awAbBt

awbABt

aabbb

awaa b

ab aa bbb

wbbbbt

bwbbbt

bawaa

abwaa bbb

abaabbb

ab bbb aaaabbbbbbbbbb

AB

ABb AABBB

AbBb

bA →

wt

awA bBt

aawAA bBbBt

aaAbA bbBBt

aaAbAA

h)

$\{ a^i b^j c^{i \cdot j} \}$

S → AB

aaab ccc

aaabbb ccccccccc

AFD

$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

$\delta : Q \times \Sigma \times \Gamma \to Q \times \Gamma^*$

$\delta(p, s, A) = (q, B)$

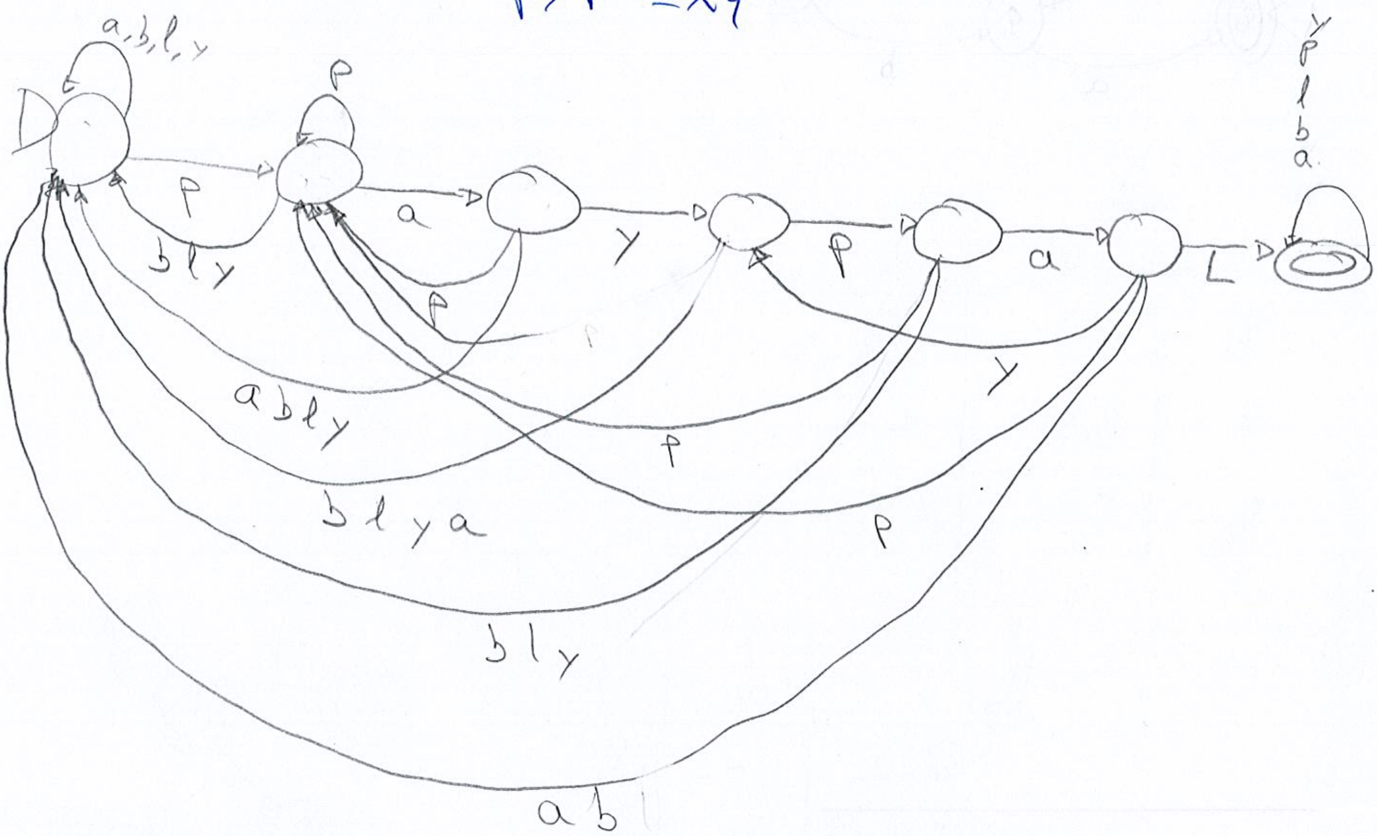
p egoeran, s irakurri eta pilan A badago, q egoerara pasatu eta pilan B idatzi

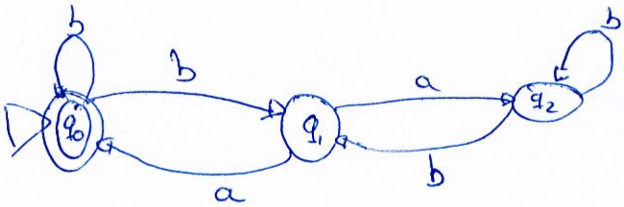
TA

$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

$\delta : Q \times (\Sigma \cup \Gamma) \to Q \times (\Sigma \cup \Gamma) \times \{R, L\}$

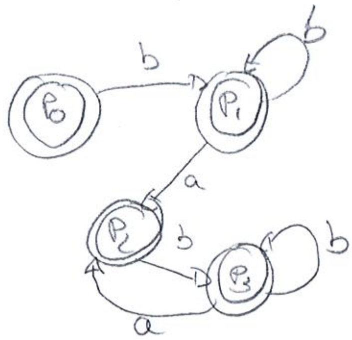
o) $\{x \in \{a,b,l,p,y\}^* : paypal \sqsubseteq x\}$





	a	b
q_0	–	$q_0 q_1$
q_1	$q_0 q_2$	–
q_2	–	$q_1 q_2$

	a	b
$P_0 = q_0$	–	P_1
$P_1 = \{q_0 q_1\}$	P_2	P_1
$P_2 = \{q_0 q_2\}$	–	P_3
$P_3 = \{q_0 q_1 q_2\}$	P_2	P_3

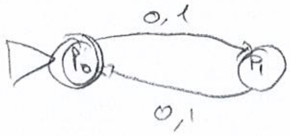


$L=\{x\in\{0,1\}^*: |x|_1 \% 2=0 \wedge |x|_0 \% 3=0\} = L_1 \cap L_2$

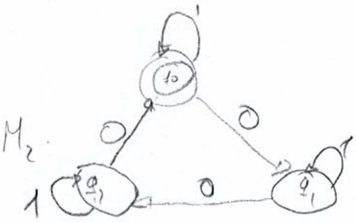
$L_1=\{x\in\{0,1\}^*: |x| \% 2=0\}$

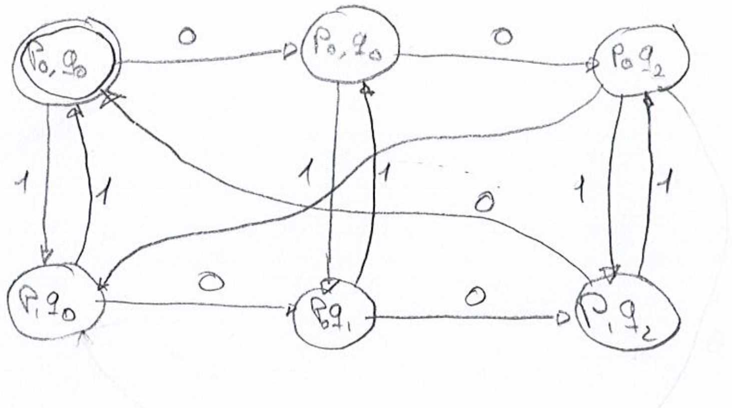
$L_2=\{x\in\{0,1\}^*: |x|_0 \% 3=0\}$

M_1:



M_2:





$M_1=(Q_1, \Sigma, \delta_1, q_1, F_1)$

$M_2=(Q_2, \Sigma, \delta_2, q_2, F_2)$

$M=(Q_1\times Q_2, \Sigma, \delta, (q_1,q_2), F_1\times F_2)$

$\delta((p,q),s)=(\delta_1(p,s), \delta(q,s))$

$L=\{0^n1^n : n\geq 0\}$

n emande

$w=0^n1^n$

$w=xyz$, $|xy|\leq n$, $|y|>0$ $\Rightarrow$ $x=0^i$, $y=0^j$, $j>0$, $z=0^{n-i-j}1^n$

$\boxed{xy^0z=0^{n-j}1^n}\notin L$

$k=0$

Ulsa Ariketa orria

1. ariketa

a)

$S \to aaSA \mid \varepsilon$

$A \to bA \mid cA \mid cc$

$\{a^{2n} x\, c^{2n} : x \in \{b,c\}^* \wedge n \geq 0\}$

b)

$S \to aSd \mid A \mid B$

$A \to aAc \mid \varepsilon$

$B \to bBd \mid C$

$C \to bCc \mid \varepsilon$

$a^{n} a^{m} b^{x} c^{m+x} d^{n}$

$a^{n} b^{m+x} c^{x} d^{n+m}$

$a^{n} a^{m} b^{i} b^{j} c^{j} d^{j} c^{i} d^{m} d^{n}$

$\{a^{n} a^{m} b^{i} b^{j} c^{j} d^{i} c^{m} d^{n} : n, m, i, j \geq 0 \wedge (m=0 \vee i=0)\}$

c)

$S \to aS \mid bS \mid A$

$A \to cA \mid c \mid S$

$\{x \in \{a,b,c\}^* : c \sqsubseteq_s x\}$

d)

$S \to ABC$

$A \to aAb \mid \varepsilon$

$B \to bBc \mid \varepsilon$

$C \to cC \mid \varepsilon$

$\{a^{n} b^{n+m} c^{m+i} : n, m, i \geq 0\}$

e)

$S \to aSbb \mid A$

$A \to bAa \mid \varepsilon$

$\{a^{n} b^{m} a^{m} b^{2n} : n, m \geq 0\}$

f)

$S \to abSdc \mid c$

$A \to cdAba \mid \varepsilon$

$\{(ab)^{n} c (dc)^{n} : n \geq 0\}$

g)

$S \to aSbb \mid dAd$

$A \to cAc \mid d$

$\{a^n d^m c^i d c^i d^m b^{2n} : n, i \geq 0 \wedge m \geq 1$

h)

$S \to aSbb \mid B$

$B \to bbB \mid bb$

$\{ a^n b^{2(n+m)} : n \geq 0 \wedge m \geq 1 \}$

i)

$S \to BC$

$C \to cC \mid c$

$B \to aBb \mid aAb$

$A \to aA \mid a$

$\{ a^{n+m} b^n c^i : m, n, i \geq 1 \}$.

k)

$S \to aaSd \mid B$

$B \to bbB \mid b$

$\{ (aa)^n (bb)^m b \, d^n : n, m \geq 0 \}$

l)

$S \to aSd \mid bSd \mid B$

$B \to bBc \mid \varepsilon$

abbcdd

$\{ x^n b^m c^m d^n : n, m \geq 0 \wedge x = \{a, b\} \}$

m)

$S \to abB$

$A \to aaBb \mid \varepsilon$

$B \to bbAb$

$\{ abb (aabb)^n b^{2n} b : n \geq 0 \}$

l)

$\{uawb : u, w \in \{a,b\}^* \wedge |u| = |w|\}$

$S \to Ub$

$U \to aUa \mid aUb \mid bUa \mid bUb \mid a$

m) $\{w \in \{a,b\}^* : \exists x (x \sqsubseteq_p w \wedge |x|_a < |x|_b)\}$

$S \to XA$

$A \to aA \mid bA \mid \varepsilon$

$X \to aXb \mid bXb \mid B \mid XBY$

$Y \to bYa \mid \varepsilon$

$B \to bB \mid b$

aabbbbbbaa

abaabbbb

n)

$\{x \in \{a,b\}^* : x = uv \wedge |u| = |v| \wedge u \neq v\}$

$S \to aSa \mid bSb \mid aXb \mid bXa$

$X \to S \mid \varepsilon \mid aXa \mid bXb$

aabaaa

abaaa

abaaaab

i)

$\{w \in L(a^* b^* a^* b^*) : |w|_a = |w|_b\}$

$S \to aSb \mid aS_1b \mid S_1S \mid \varepsilon$

$S_1 \to bS_1a \mid \varepsilon$

e)

$\{a^i b^j c^k : i,j \geq 1 \wedge (i=j \vee j=k)\}$

$S \to S_1 \mid S_2$

$S_1 \to aS_1 \mid aA$

$A \to bAc \mid bc$

$S_2 \to S_2c \mid B$

$B \to aBb \mid ab$

m)

$\{w \in \{a,b\}^* : \exists x (x \sqsubseteq_p w \wedge |x|_a < |x|_b)\}$

$S \to XS_1$

$S_1 \to aS_1 \mid bS_1 \mid \varepsilon$

$X \to aXb \mid bYb \mid bXa \mid b$

$Y \to a \mid bYb \mid b \mid \varepsilon \mid bXb$

a b b a b

a b b b

a b b

a b b b

b a b

b b a b b

b a b b

p) $\{w \in \{a,b,c\}^* :$ w-ko azken hirugarren ikurra c den$\}$

ab | b a b a b

$S \to S_1c \mid S_1b \mid S_1a$

$S_1 \to S_2c \mid S_2b \mid S_2a$

$S_2 \to S_3c$

$S_3 \to S_3a \mid S_3b \mid S_3c \mid \varepsilon$

g)

$\{w \in \{a,b,c\}^* : w$ .U bi illor berdin sagidan ?

$S \to aS \mid bS \mid cS$

$S \to aaA \mid bbA \mid ccA$

$A \to aA \mid bA \mid cA \mid \varepsilon$

s) $\{a^m b^n c^{m+n} : n, m \geq 0\}$

$S \to aSc \mid \varepsilon$

$S \to bS_1c$

$S_1 \to bS_1c \mid \varepsilon$

t)

$\{w \in \{a,b,c\}^* : w[2] = w[|w|-2]\}$

$S \to DAD$

$A \to aBa \mid bBb \mid cBc$

$B \to DCD$

$C \to D \mid \varepsilon$

$D \to a \mid b \mid c$

l) $\{w \in \{a,b,c\}^* ; ca \sqsubseteq w \wedge bb \sqsubseteq w\}$

$S \to CacaCbbC \mid CbbCcaC$

$C \to aC \mid bC \mid cC \mid \varepsilon$

e) $\{a^i b^j c^k : i, j \geq k \wedge (i=j \vee j=k$

$S \to S_1 \mid S_2$

$S_1 \to aAbBc$

$A \to aAb \mid \varepsilon$

$B \to Bc \mid \varepsilon$

$S_2 \to aCbDc$

$C \to aC \mid \varepsilon$

$D \to bDc \mid \varepsilon$

g) $\{a^i b^j c^k : j > i+k\}$

$S \to ABC$

$A \to aAb \mid \varepsilon$

$B \to bB \mid b$

$C \to bCc \mid \varepsilon$

h) $\{w \in \{a,b\}^* : |w|_a + 1 = |w|_b\}$

$S \to AbA$

$A \to aAb \mid BA \mid AB \mid \varepsilon$

$B \to bBa \mid \varepsilon$

baaaabbbabb

abbba

bbaaaabbb

aabbbba

j) $\{xcy : x, y \in \{a,b\}^* \wedge x^R E y\}$

$S \to XY$

$X \to aXa \mid bXb \mid cY$

$Y \to aY \mid bY \mid \varepsilon$

abacbbbababa

ab

ba

3. Ariketa

a)

$\{a^i b^j : i,j \geq 1 \wedge i \neq j\}$

$S \rightarrow aAb$

$A \rightarrow aAb \mid a \mid b$

b) $\{a^i b^i c^j d^j : i,j \geq 1\}$

$S \rightarrow AB$

$A \rightarrow aAb \mid ab$

$B \rightarrow cBd \mid cd$

c) $\{a^i b^j c^j d^i : i,j \geq 1\}$

$S \rightarrow aAd$

$A \rightarrow aAd \mid B$

$B \rightarrow bBc \mid bc$

d) $\{a^i b^j c^k : i,j \geq 1 \wedge (i \neq j \vee j \neq k)\}$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow aAbBc$

$A \rightarrow aA \mid a$

$B \rightarrow bBc \mid b \mid c$

$S_2 \rightarrow aCbDc$

$C \rightarrow aCb \mid b \mid a$

$D \rightarrow Dc \mid c$

$\{w \in \{a,b,c\}^* : w = w^R\}$

$\forall n$

$w = \overbrace{\underbrace{a^n}_{y}}^{xy} b^n a^n$

$xyz = w^2$

$|xy| \leq n$

$w = \underbrace{aa}_{x}\underbrace{aa}_{y}\underbrace{aaa\ b^n a^n}_{z}$

$xy^0z \notin L$

$\{a^n b^m : n > m \geq 0\}$

$\forall n$

$a^n b^m \quad m > n \geq 0$

$w = a^n b^{n+1}$

$w = \underbrace{aa \ldots a}_{x}\underbrace{a}_{y}\underbrace{ab \ldots b}_{z}$ ($n+1$)

$w = \underbrace{a^i}_{x} \underbrace{a^j}_{y} a^{n-i-j} b^{n+1}$

$y \Downarrow y^2$

$k = 2 \quad w = xy^2z = a^i a^{2 \cdot j} a^{n-i-j} b^{n+1} =$

$= a^{n+j} b^{n+1} \notin L$

$j > 0$

TGG adibidea

$L = \{a^n b^n : n \geq 0\}$

$G = (\{S\}, \{a,b\}, P, S)$

$P = \{S \to aSb \mid \varepsilon\}$

$L = \{w \in \{a,b\}^* : w = w^R\}$

$P = \{S \to aSa \mid bSb \mid \varepsilon \mid a \mid b\}$

$G = (\{S\}, \{a,b\}, P, S)$

$L = \{ w \in \{a,b\}^* : |w|_a = |w|_b \}$

$S \rightarrow \varepsilon \mid aSb \mid bSa \mid SS$

Beste modu bat

$S \rightarrow aB \mid bA \mid \varepsilon$

$A \rightarrow aS \mid bAA$

$B \rightarrow bS \mid aBB$

abbbaabbbaaa

$S \Rightarrow aB \Rightarrow abS \Rightarrow abbA \Rightarrow abbbAA \Rightarrow abbbaA \Rightarrow abbbaaS \Rightarrow$

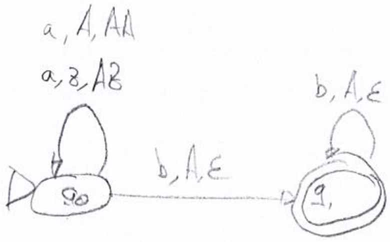
$\Rightarrow abbbaabA \Rightarrow abbbaabbAA \Rightarrow abbbaabbbAAA \Rightarrow abbbaabbbaAA \Rightarrow$

$\Rightarrow abbbaabbbaaA \Rightarrow abbbaabbbaaaS \Rightarrow abbbaabbbaaa$

Automata Piladone

$\{a^n b^n : n \geq 1\}$

	a	b
q_0, z	q_0, Az	–
q_0, A	q_0, AA	q_1, ε
q_1, A	q –	q_1, ε



Automata Piladon Deterministe

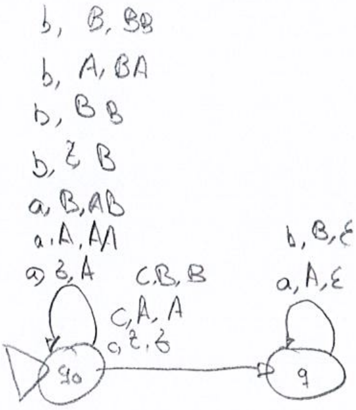
$$(Q, \Sigma, \Gamma, \delta, q_0, F)$$

$$\delta : Q \times \Sigma \times (\Gamma \cup \{z\}) \longrightarrow Q \times \Gamma^* \cup \{z, \varepsilon\}$$

Aribat

$\{wcw^R : w \in \{a,b\}^*\}$

	a	b	c
q_0, z	q_0, Az	$q_0 Bz$	q_1, z
q_0, A	$q_0 AA$	$q_0 BA$	q_1, A
q_0, B	q_0, AB	$q_0 BB$	q_1, B
$q_1 A$	$q_1 \varepsilon$	–	–
$q_1 B$	–	$q_1 \varepsilon$	–



Javier Marin

De:	Sherpa <info@sher.pa>
Enviado el:	lunes, 02 de noviembre de 2015 11:55
Para:	Javier Marin
Asunto:	Confirmación de inscripción en Sherpa Keynote 2015

SHERPA
keynote

INSCRIPCIÓN CONFIRMADA

Confirmamos que su inscripción se ha realizado correctamente. Imprima el documento adjunto y muéstrelo a la entrada del evento junto con su DNI.

9 de Noviembre, Bilbao

Teatro Campos Elíseos
De 12:00 a 13:30 aprox.

Nombre	Apellidos	DNI
Aitzol	Elu Etxano	72558782S
Empresa (entidad)	Cargo	Ciudad
invitado	invitado	Donosti

Identificador
SHP151102105453

Nota: Le recomendamos que llegue con suficiente antelación.

Buscamos talento

En Sherpa estamos creciendo y buscamos el mejor talento para unirse a nuestro equipo. Si estás interesado en trabajar con nosotros, o conoces a alguien que lo esté, contáctanos en jobs@sher.pa

$\Sigma = \{a, b, c\}$ $\Gamma = \{\square, \#\}$

δ	a	b	c	$\square$	#	*
q_0	$q_1, \square, R$	–	–	q_f, R	$q_\varepsilon, \#, R$	–
q_1	R	$q_2, \#, R$	–	–	$q_1, \#, R$	
q_2	–	$q_2, \#, R$	$q_L, \#, L$	–	–	R
q_L	q_L, a, L	q_L, b, L	–	$q_0, \square, R$	$q_2, \#, L$	L
q_ε	–	–	–	q_f, R	R	R
q_f						

Hitza onartzen da.