2.- Mathematical Induction

- Mathematical Induction: Two steps for proving that an infinite interval of (natural) numbers satisfies a property:

  Step 1 Show that the property is true for the **first one**.

  Step 2 Show that **if any one satisfies the property, then** the next one also satisfies it.

- The "Domino Effect"

  Step 1 The **first domino** falls.

  Step 2 **When any domino falls, then** the next domino also falls.

  Hence, **all dominos falls**.

Example: For all $n \geq 1$: $3^n - 1$ is even.

- Show it is true for $n = 1$: $3^1 - 1 = 3 - 1 = 2$ and 2 is even.
- Assume that the property is true for $n = k$, that is:

  $3^k - 1$ *is even*

  is the **induction hypothesis** (an assumption that we treat as a fact) for proving that (then) the property is true for $n = k + 1$, that is $3^{k+1} - 1$ is even.

$$\begin{aligned} & 3^{k+1} - 1 \\ = \; & 3.(3^k) - 1 \\ = \; & \underbrace{2.(3^k)}_{even\ 2.x} + \underbrace{3^k - 1}_{even\ by\ I.H.} \end{aligned}$$

Since the sum of two even numbers is also even, *then* $3^{k+1} - 1$ *is even.*

Matemathical Induction

Let n_0 be a natural number. To prove that

"Every n such that $n \geq n_0$ satisfies a property P"

Base Step Prove that "n_0 satisfies P".

Inductive Step ($k \mapsto k+1$) Prove that "$k+1$ satisfies P", for any natural number k such that $k \geq n_0$, UNDER THE HYPOTHESIS that "k satisfies P" (induction hypothesis).

or optionally

Inductive Step ($n-1 \mapsto n$) Prove that "n satisfies P", for any natural number n such that $n > n_0$, supposing that "$n-1$ satisfies P" (induction hypothesis).

EXERCISES

Using the version "$n - 1 \mapsto n$" of the inductive step, you should prove that

1. Every $n \geq 0$ satisfies that $3^n - 1$ is divisible by 2. (Then, in Dafny)
2. Every $n \geq 6$ satisfies that $4.n < n^2 - 7$. (Then, in Dafny)