e8y0b

1.
10
ptr (0x30)
20
ptr (nil)
30
ptr (0x28)

2. (please refer to the other file)

3. (a) Double linked list
(b) Hash table
(c) Array

Explanation: (a) is easy to reverse, since there are both the previous pointer and the next pointer. Traversing it from the rear side can form the reverse of the word.
(b) A hashing function can provide the code to unsubscribe. Since we don't know the maximum numbers, using hash table can give us more flexibility.
(c) the number of the residents is almost fixed, and we need to access their data by a 9-digits SIN (can be treated as an integer)

4. (a) Let the property $P(n)$ be

$$\sum_{i=1}^{n} \frac{1}{i \times (i+1)} = \frac{n}{n+1}$$

$$P(1): \frac{1}{1\times 2} = \frac{1}{1+1}$$

(b) Base case: (show $P(1)$ is true)

$$\frac{1}{1\times 2} = \frac{1}{1+1} = \frac{1}{2}$$

(c) Induction step:
Suppose $P(k)$ is true, where k is an particular but arbitrarily chose integer $(k \geq 1)$
(d) Show for all integers $k \geq 1$, if $P(k)$ is true then $P(k+1)$ is true.

(e) $$\sum_{i=1}^{k} \frac{1}{i\times(i+1)} = \frac{k}{k+1} \qquad \boxed{P(k)}$$

$$\frac{1}{(k+1)(k+2)} + \sum_{i=1}^{k} \frac{1}{i\times(i+1)} = \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

$$\boxed{P(k) + \frac{1}{(k+1)(k+2)}} = \frac{k(k+2)+1}{(k+1)(k+2)}$$

$$\downarrow \qquad = \frac{(k+1)^2}{(k+1)(k+2)}$$

$$\boxed{P(k+1)} \qquad = \frac{k+1}{k+2}$$

Since we have proved the basis step and the inductive step, we conclude that the property is true.

5. (see on the next page)

6. $(\log n)^4 < \sqrt{n} < n < \lg(n!) < n^2 \log n < 2^n < 2^{n^2} < 2^{2^n}$

5 (a) Suppose $f(x)$ is $\Omega(g(x))$

By def of Ω-notation, there exist a positive real number A and a nonnegative real number a such that:

$A|g(x)| \le |f(x)|$ for all real numbers $x > a$

Divide both sides by A:

$|g(x)| \le \frac{1}{A}|f(x)|$ for all real numbers $x > a$

Let $B = 1/A$ and $b = a$, then B is a positive real number and b is a nonnegative real number, and $|g(x)| \le B|f(x)|$ for all real numbers $x > b$.

so $g(x)$ is $O(f(x))$ by def. of O-notatition. (Epp. 729)

(b) Suppose $\sum_{i=1}^{n}(i \lg i)$ is $\Omega(n^2 \log n)$, there exists a positive real number A and a non-negative real number a such that $A|n^2 \log n| \le |\sum_{i=1}^{n}(i \cdot \lg i)|$ for all real numbers $x > a$

$\sum_{i=1}^{n}(i \cdot \lg(i)) = 1\lg 1 + 2\lg 2 + \cdots + n\lg n$

$\ge \frac{n}{2}\lg\frac{n}{2} + \cdots + n\lg n$

$\ge \frac{n}{2}\lg\frac{n}{2} + \cdots + \frac{n}{2}\lg\frac{n}{2}$

$\ge \frac{n}{2}(\frac{n}{2}\lg\frac{n}{2}) \ge \frac{n^2}{4}\lg\frac{n}{2}$

$\ge \frac{n^2}{4}\log\frac{n}{2} \ge n'\log n'$ $(n' = n/2)$

Similarly, suppose $\sum_{i=1}^{n}(i \lg i)$ is $O(n^2 \log n)$. There exist positive real number B and non-negative real number b such that $|\sum_{i=1}^{n}(i \lg i)| \le B|n^2 \log n|$ for all real numbers $x > b$

$\sum_{i=1}^{n}(i \cdot \lg(i)) = 1\lg 1 + 2\lg 2 + \cdots + n\lg n$

$\le n\lg n + \cdots + n\lg n \le n^2 \lg n$

$\le \underbrace{|(\log 2)|}_{B}(n^2 \log n)$

Thus, by def. of Θ-notation, $\sum_{i=1}^{n}(i \lg i)$ is $\Theta(n^2 \log n)$

7. (a) $T(0) \le b$ $(b = 3,\ c = 4)$

$T(n) \le c + T(n/2)$ if $n > 0$

$T(n) \le c + c + T(n/4)$

$= 2c + T(n/4)$

$\le 2c + c + T(n/8)$

$= 3c + T(n/8)$

$\le kc + T(n/2^k)$

$= T(0) + c\lg n$

$(2^k = n,\ k = \lg n)$

$T(n) \in O(\log n)$

$T(0) \geq b$ $\quad (b=0, c=2)$

$T(n) \geq c + T(n/2)$

$T(n) \geq c + c + T(n/4)$

$= 2c + T(n/4)$

$\geq kc + T(n/2^k)$

$= c \lg n + T(0)$

$T(n) \in \Omega(\log n)$

$\therefore T(n) \in \Theta(\log n)$

(b) $\left(\sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{k=j}^{i+j} 1\right) + 2 = T(n)$

$T(n) = \left(\sum_{i=0}^{n} \sum_{j=0}^{i} i\right) + 2 \quad (i+j-j = i)$

$T(n) = \left(\sum_{i=0}^{n} (0+i)^{i}/_{2}\right) + 2$

$T(n) = \sum_{i=0}^{n} \frac{i^2}{2} = \left(\frac{1}{2} \sum_{i=0}^{n} i^2\right) + 2$

$T(n) = \left(\frac{1}{2} \frac{(n+1)n(2n+1)}{6}\right) + 2$

$= \frac{1}{12} n(n+1)(2n+1) + 2$

$T(n) \geq \frac{n^3}{6}$ for $n > 0$

$T(n) \leq n^3$ for $n > \cancel{10}\ 2$

$T(n) \in \Theta(n^3)$

Q2.

```
Object DLL_Deque::popAnywhere
(Node *&left, Node *&right, Node *node)
  Node *lec = left, *rec = NULL;
  Object data = node->data;
  if (data == left->data) {
    left = left->next;
    left->prev = NULL;
    delete node;
    return data;
  } else if (data == right->data) {
    right = right->prev;
    right->next = NULL;
    delete node;
    return data;
  }
  while (lec && lec->next != node)
    lec = lec->next;
  if (lec->next == node) {
    rec = node->next;
    lec->next = rec;
    rec->prev = lec;
    delete node;
    return data;
  }
}
```