COT 6938 Network Science Assignment 2

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Q1

Repeat Q1 of HW1 with N = 100,000 and d = 5. Also generate the degree distribution using Poisson approximation (basically Q2 of HW1) with d = 5. Draw the degree distributions for both on top of each other. Do not use the entire range for the x-axis. Use your judgement to bound the x-axis appropriately.

ii) What is the probability that the same node has exactly 1 link when d = 5?

$$\text{p}\{\text{single node falls within d and 99,999 do not}\} = \left(\frac{\pi(r^2)}{A}\right)^1 \left(1 - \frac{\pi(r^2)}{A}\right)^{99{,}999}$$

$$= \left(\frac{25\pi}{10000}\right)^1 \left(1 - \frac{25\pi}{10000}\right)^{99{,}999} = (.007854)^1(0.992146)^{99999} = \boxed{2.87e^{-345}}$$

iv) repeat this process for 3, 4, 5, 6... links. Essentially what you obtain is the degree distribution. Just use a 'for' loop. v) What is the expected degree of that node?

$$(.007854)^2(0.992146)^{99998} = \boxed{2.267e^{-347}}$$

$$(.007854)^3(0.992146)^{99997} = \boxed{1.795e^{-349}}$$

$$(.007854)^4(0.992146)^{99996} = \boxed{1.421e^{-351}}$$

$$(.007854)^5(0.992146)^{99995} = \boxed{1.125e^{-353}}$$

$$(.007854)^6(0.992146)^{99994} = \boxed{8.906e^{-356}}$$

$$\text{if p}\{1\} = 2.87e^{-345},$$

$$\text{the average degree} < k >= p(n-1) = 2.87e^{-345}(100,000 - 1) = \boxed{2.86e^{-340}}$$

In the experiment, 790 nodes saw 1193 links. This was the average. This number does not correlate with the aforementioned calculations. This question should be solved by 1) calculating the probability a link will form (basically the probability a node lands within d). 2) Using that probability to determine the average degree using:

$$< k >= p(n-1)$$

Then 3) Use the average degree to calculate the Poisson Distribution:

$$p\{k\} = \frac{e^{-<k>} < k >^k}{k!}$$

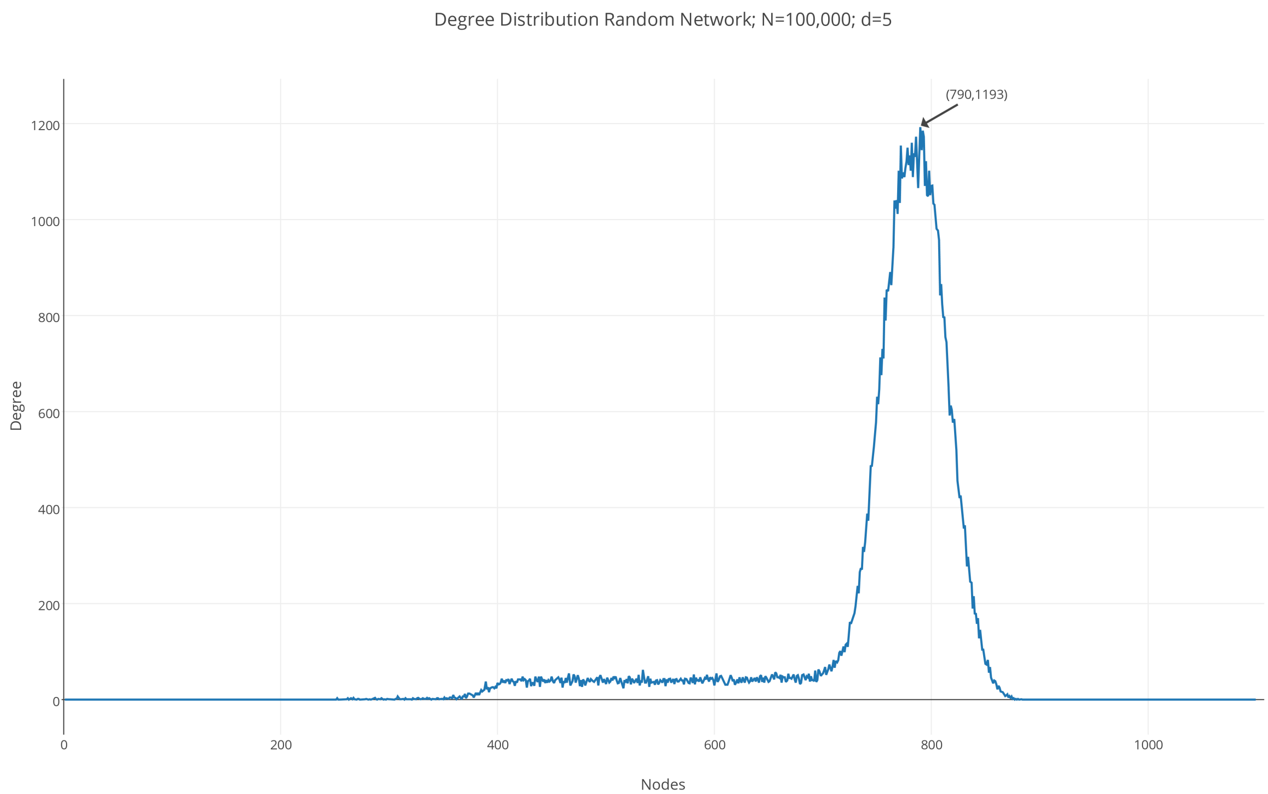

Figure 1: Degree Distribution for N=100,000; d=5

Q2

In HW 1, you generated a random network. Now, you will generated a scale-free network using the Barabasi-Albert model that we discussed in class. Submit hardcopy of the program for 2(b)

2(a): Create a network with 4 nodes that are connected to each other i.e., a complete graph with 4 nodes and the degree for all is 3. Add a new node (the 5th one) to this existing network such that the new node randomly attaches to 3 nodes of the existing network with probabilities that are proportional to the degrees of the nodes already in the network. Make sure that there are no multiple edges (the same node must not be linked more than once). Now, you have a 5-node network. Update the degrees of all the nodes. Add the 6th node (with 3 links) in a similar fashion. Continue to add 5000 nodes. (The final network has 5004 nodes with (6 + 15000) links.) Find and plot the degree distribution in i) linear scale and ii) log-log scale.

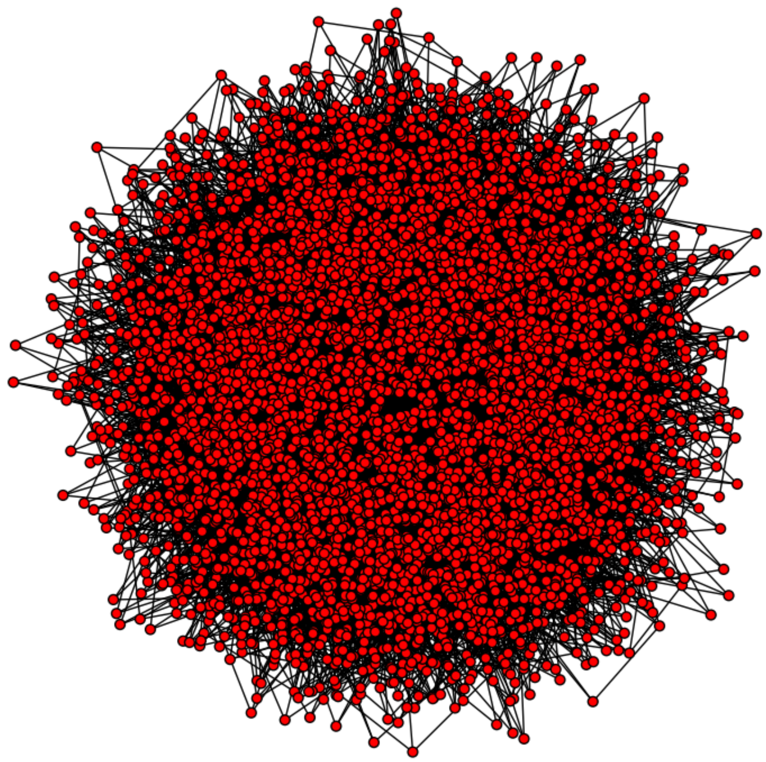

Figure 2: Visualization of 3 links formed for each new node N

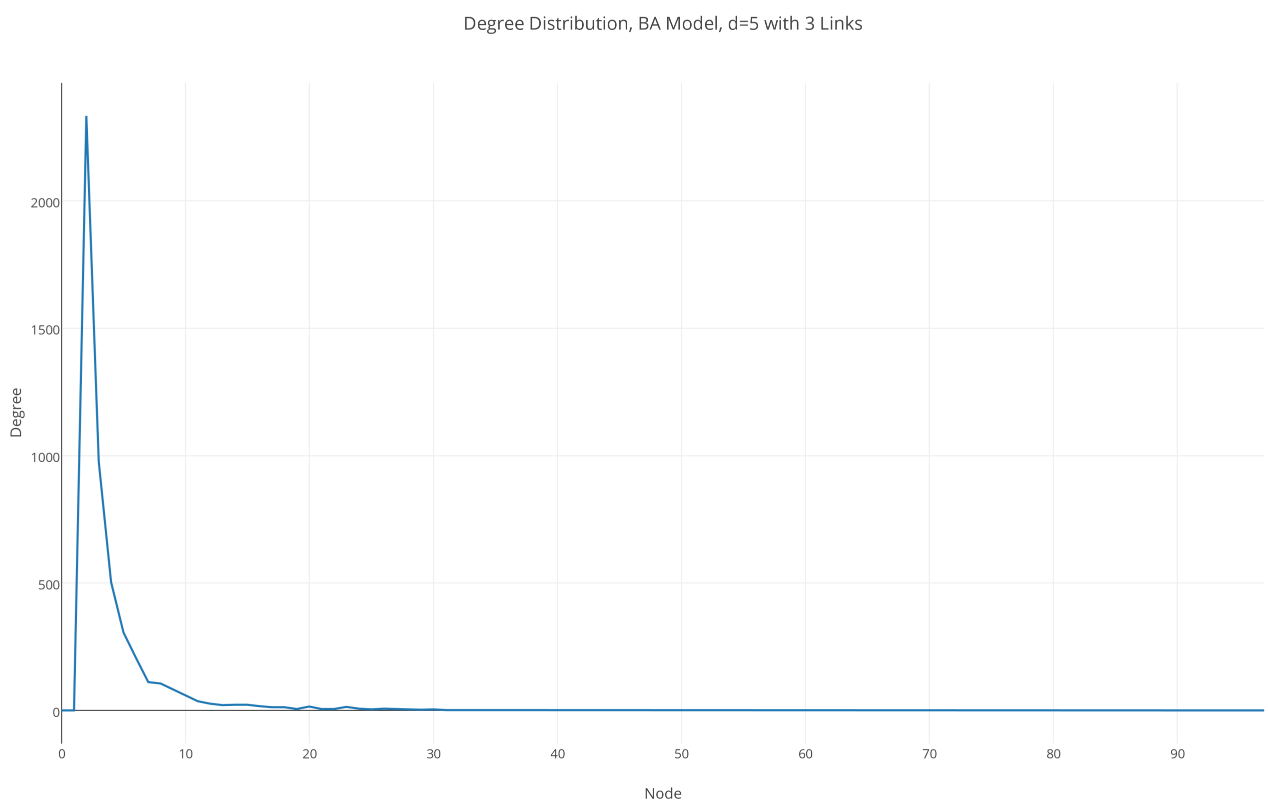

Figure 3: Linear Degree Distribution Using 3 Links

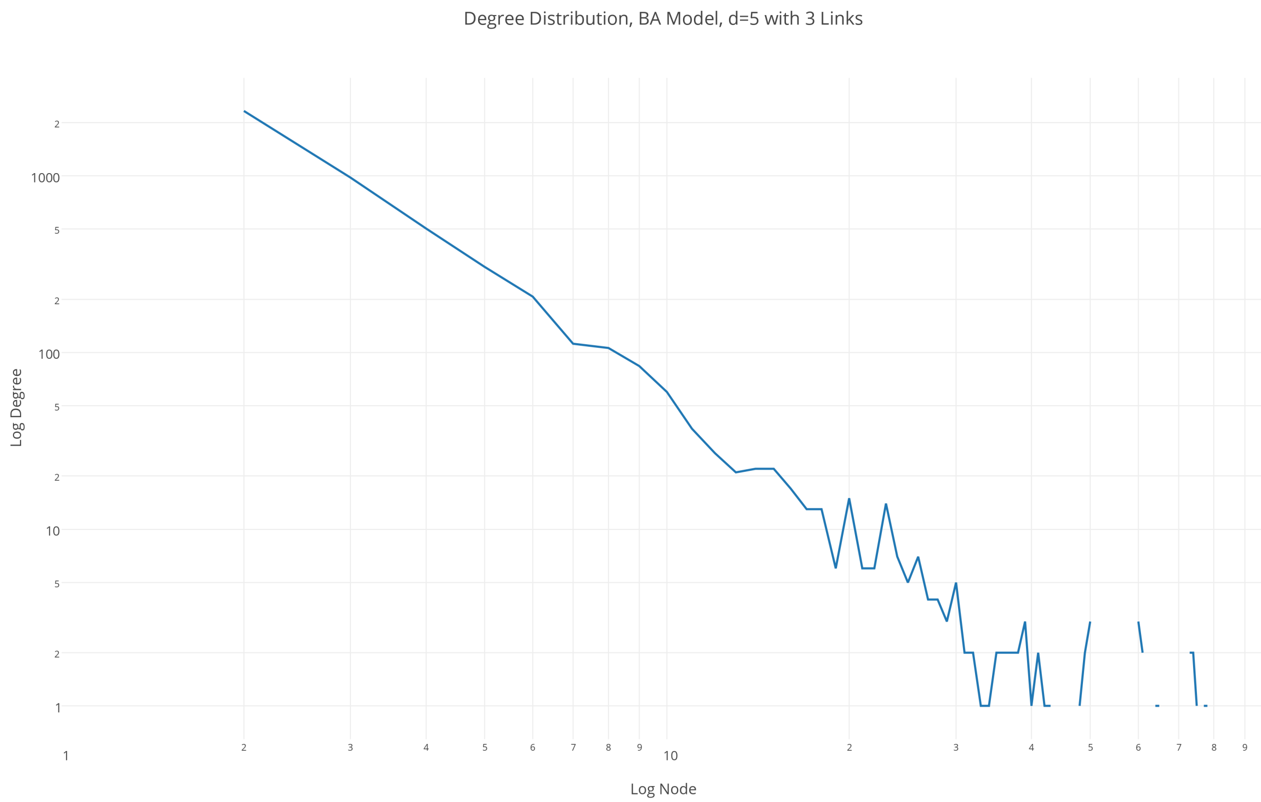

Figure 4: Log Degree Distribution Using 3 Links

2(b): Repeat 2(a) but instead of always adding 3 links for each new node, add either 1, 2, or 3 links with equal probability. That is, the new node links to 1 node with probability 1/3, links to 2 nodes with probability 1/3, and to 3 nodes with probability 1/3. How many links does the final network have? Plot the degree distribution using i) linear scale and ii) log-log scale.

The final network contains 5004 nodes, but only 9979 links. This makes sense because the maximum links available is 15006 links, but since a new node can generate less than 3 links (1 or 2), the total number of links should be less than 15006.

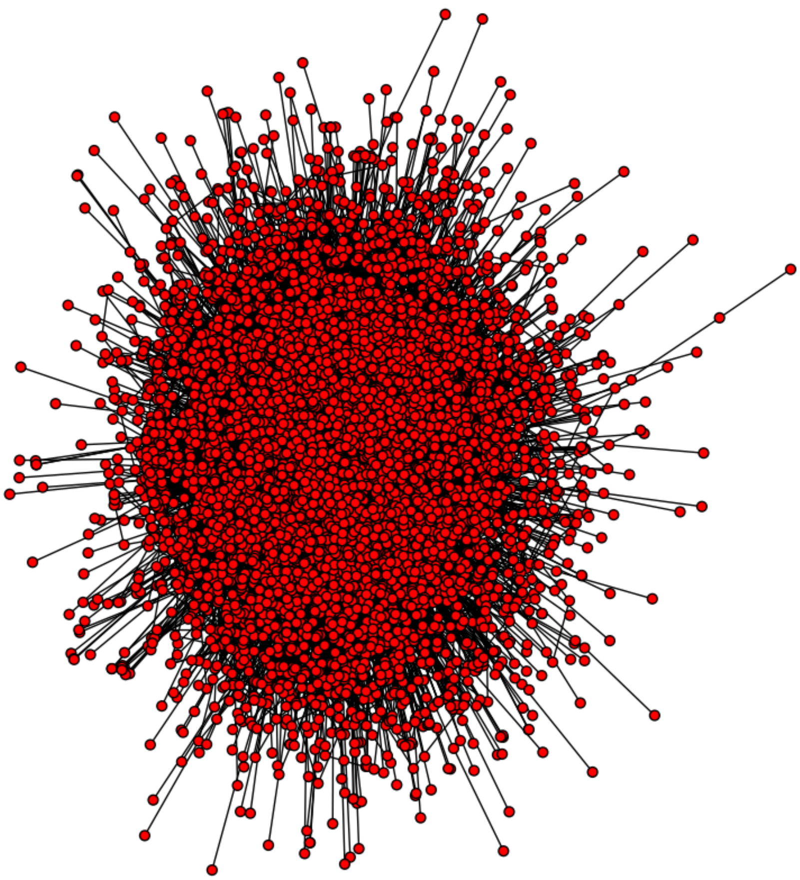

Figure 5: Visualization for variable links formed for each new node N

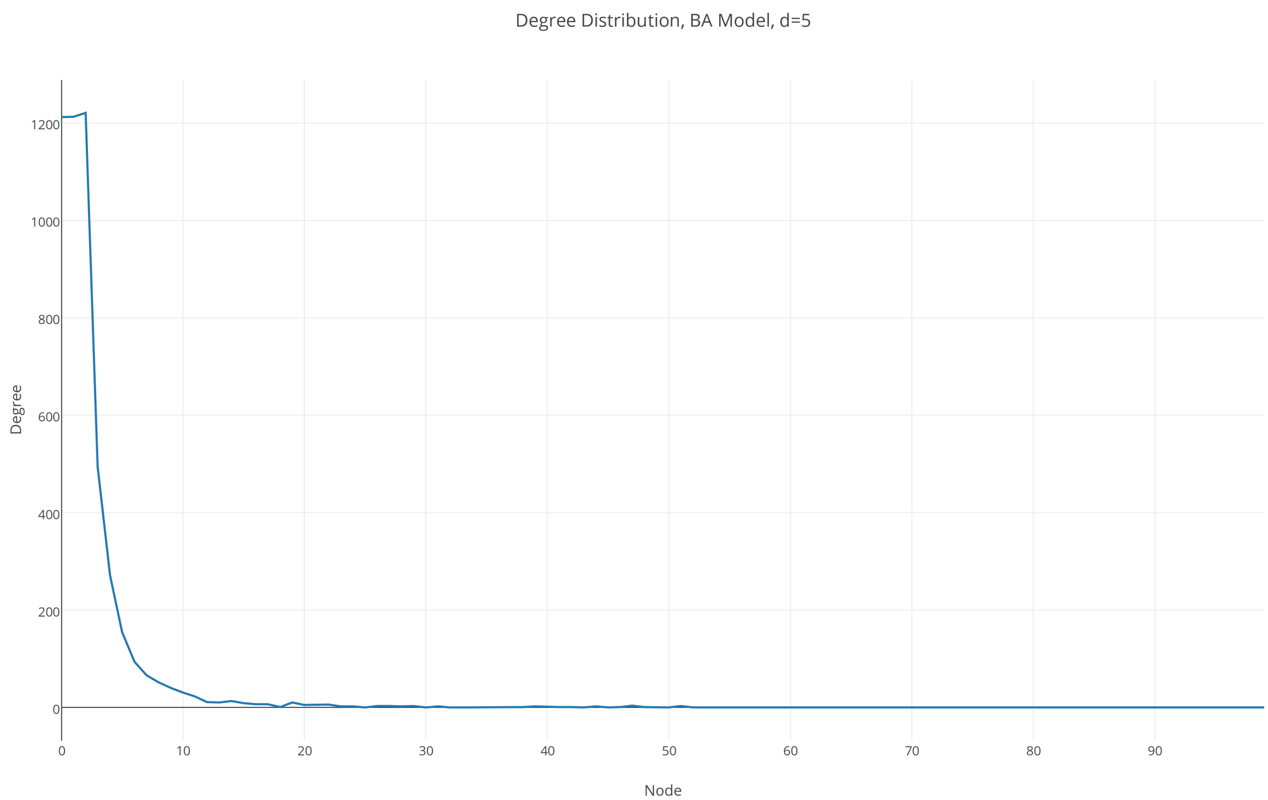

Figure 6: Linear Degree Distribution Using Variable Links

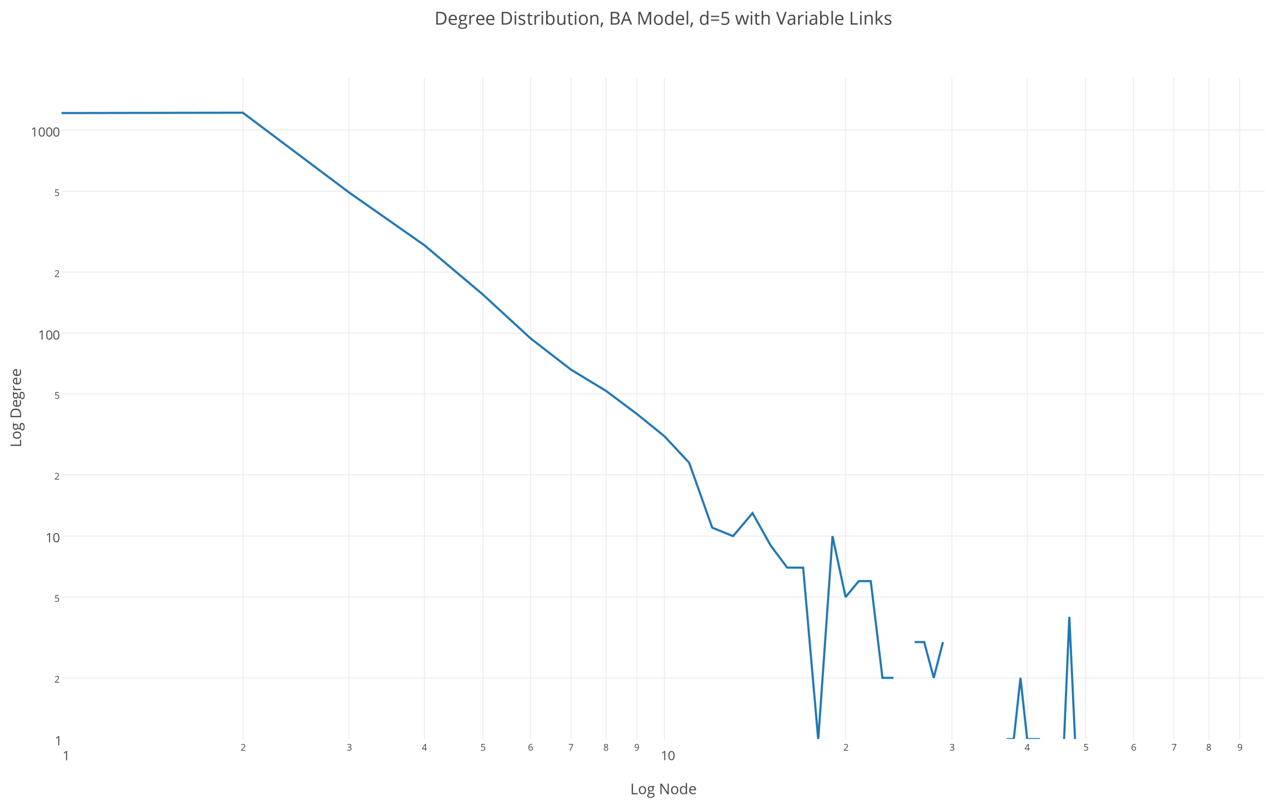

Figure 7: Log Degree Distribution Using Variable Links

Q3

Show that a Poisson distribution function is not scale-free, and hence that a random network is not a scale-free network.

A scale-free network is a network whose degree distribution follows a power law. When graphed on a log scale this exhibits a gradual peak followed by a sharp decline. The Poisson distribution, however, follows a bell shaped curve. These different shapes indicate that in a random network most nodes have degrees similar to the mean and hubs, or nodes with many links, do not form. Hubs, however, are expected in scale free networks. As a scale free network grows the number of hubs grow. This is unlike random networks that follow the Poisson distribution as the size of the largest node grows slower with N.

Q4

What is Degree Correlation? Draw (hand sketch) a connected network with at least 10 nodes. Identify two nodes (say A and B) and find their degree correlation. Categorize the following networks into Neutral, Assortative, or Disassortative networks—Citation, Facebook, Email, WWW, Internet, IMDB, Random graph, Scientific coauthorship.

Degree correlation indicates how connected each degree-k node is connected to the same degree-k nodes. Neutral networks exhibit chance connections between hubs while Assortative Networks tend toward connections between hubs. Disassortative Networks show connections between hub and low-degree nodes which resembles a hub and spoke.

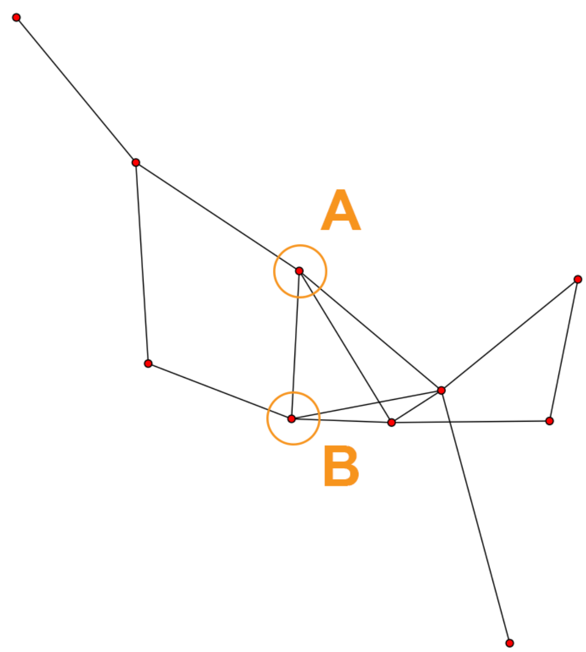

Figure 8: Connected network with 10 nodes

For Node A:

$$k_{nn}(k_i) = k_{nn}(4) = \frac{(3+4+4+5)}{4} = 4$$

For Node B:

$$k_{nn}(k_i) = k_{nn}(4) = \frac{(2+4+5+4)}{4} = 3.75$$

Similarly, C (not shown) that has 4 links:

$$k_{nn}(k_i) = k_{nn}(4) = \frac{(4+4+5+2)}{4} = 3.75$$

This shows an associative network tendency as the three nodes with 4 links have neighbors with a similar amount of links.

Network	Categorization	Notes
Citation	Extreme Assortative	Exhibits mixed behavior
Facebook	Assortative	Assortative nature is not rooted in the degree distribution
Email	Disassortative	Structural disassortativity
WWW	Disassortative	The disassortative nature of the WWW is not fully explained by its degree distribution
Internet	Assortative	For small degrees ($k < 30$)
IMDB	Assortative	Assortative nature is not rooted in the degree distribution
Random Graph	Neutral	No preferential attachment between hubs/nodes
Scientific Coauthorship	Assortative	Assortative nature is not rooted in the degree distribution

Q5

Find the degree distribution of the Barabasi-Albert model analytically. That is, each time a new node arrives and connects with m nodes proportional to the existing nodes' degree (which is nothing but the preferential attachment model).

Calculate the number of nodes with degree smaller than k. Nodes are added at equal time intervals.

$$t_i < t\left(\frac{m}{k}\right)^{\frac{1}{\beta}} \rightarrow t\left(\frac{m}{k}\right)^{\frac{1}{\beta}}$$

There are N = m_0 + t nodes. The probability that a randomly chosen node has degree k or smaller:

$$P(k) = 1 - \left(\frac{m}{k}\right)^{\frac{1}{\beta}}$$

The derivative of p(k) yields the degree distribution.

$$p_k = \frac{\delta P(k)}{\delta k} = \frac{1}{\beta}\frac{m^{\frac{1}{\beta}}}{k^{\frac{1}{\beta+1}}} = \boxed{2m^2k^{-3}}$$

This is equation 5.9 in the Barabasi book. For large values of k, p(k) $\propto k^{-\gamma}$