

Stanford University ICPC Team Notebook (2015-16)

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1 Combinatorial optimization

1.1 Sparse max-flow

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
//   O(|V|^2 |E|)
//
// INPUT:
//   - graph, constructed using AddEdge()
//   - source and sink
//
// OUTPUT:
```

```
//   - maximum flow value
//   - To obtain actual flow values, look at edges with capacity > 0
//     (zero capacity edges are residual edges).

#include<cstdio>
#include<vector>
#include<queue>
using namespace std;
typedef long long LL;

struct Edge {
    int u, v;
    LL cap, flow;
    Edge() {}
    Edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};

struct Dinic {
    int N;
    vector<Edge> E;
    vector<vector<int>>> g;
    vector<int> d, pt;

    Dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}

    void AddEdge(int u, int v, LL cap) {
        if (u != v) {
            E.emplace_back(Edge(u, v, cap));
            g[u].emplace_back(E.size() - 1);
            E.emplace_back(Edge(v, u, 0));
            g[v].emplace_back(E.size() - 1);
        }
    }

    bool BFS(int S, int T) {
        queue<int> q({S});
        fill(d.begin(), d.end(), N + 1);
        d[S] = 0;
        while(!q.empty()) {
            int u = q.front(); q.pop();
            if (u == T) break;
            for (int k: g[u]) {
                Edge &e = E[k];
                if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
                    d[e.v] = d[e.u] + 1;
                    q.emplace(e.v);
                }
            }
        }
        return d[T] != N + 1;
    }

    LL DFS(int u, int T, LL flow = -1) {
        if (u == T || flow == 0) return flow;
        for (int &i = pt[u]; i < g[u].size(); ++i) {
            Edge &e = E[g[u][i]];
            Edge &oe = E[g[u][i]^1];
            if (d[e.v] == d[e.u] + 1) {
                LL amt = e.cap - e.flow;
                if (flow != -1 && amt > flow) amt = flow;
                if (LL pushed = DFS(e.v, T, amt)) {
                    e.flow += pushed;
                    oe.flow -= pushed;
                    return pushed;
                }
            }
        }
        return 0;
    }

    LL MaxFlow(int S, int T) {
        LL total = 0;
        while (BFS(S, T)) {
            fill(pt.begin(), pt.end(), 0);
            while (LL flow = DFS(S, T))
                total += flow;
        }
        return total;
    }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)

int main()
{
    int N, E;
    scanf("%d%d", &N, &E);
    Dinic dinic(N);
    for(int i = 0; i < E; i++)
    {
        int u, v;
```

```

LL cap;
scanf("%d%d%d", &u, &v, &cap);
dinic.AddEdge(u - 1, v - 1, cap);
dinic.AddEdge(v - 1, u - 1, cap);
}
printf("%lld\n", dinic.MaxFlow(0, N - 1));
return 0;
}

// END CUT

```

1.2 Min-cost max-flow

```

// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972). This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]). For a regular max flow, set all edge costs to 0.
//
// Running time:  $O(|V|^2)$  cost per augmentation
// max flow:  $O(|V|^3)$  augmentations
// min cost max flow:  $O(|V|^4 * \text{MAX\_EDGE\_COST})$  augmentations
//
// INPUT:
// - graph, constructed using AddEdge()
// - source
// - sink
//
// OUTPUT:
// - (maximum flow value, minimum cost value)
// - To obtain the actual flow, look at positive values only.

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
    int N;
    VVL cap, flow, cost;
    VI found;
    VL dist, pi, width;
    VPII dad;

    MinCostMaxFlow(int N) :
        N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}

    void AddEdge(int from, int to, L cap, L cost) {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    }

    void Relax(int s, int k, L cap, L cost, int dir) {
        L val = dist[s] + pi[s] - pi[k] + cost;
        if (cap && val < dist[k]) {
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
        }
    }

    L Dijkstra(int s, int t) {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);
        fill(width.begin(), width.end(), 0);
        dist[s] = 0;
        width[s] = INF;

        while (s != -1) {
            int best = -1;
            found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
                Relax(s, k, flow[k][s], -cost[k][s], -1);
                if (best == -1 || dist[k] < dist[best]) best = k;
            }
        }
    }
}

```

```

    s = best;
}

for (int k = 0; k < N; k++)
    pi[k] = min(pi[k] + dist[k], INF);
return width[t];
}

pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
        totflow += amt;
        for (int x = t; x != s; x = dad[x].first) {
            if (dad[x].second == 1) {
                flow[dad[x].first][x] += amt;
                totcost += amt * cost[dad[x].first][x];
            } else {
                flow[x][dad[x].first] -= amt;
                totcost -= amt * cost[x][dad[x].first];
            }
        }
        return make_pair(totflow, totcost);
    }
};

// BEGIN CUT
// The following code solves UVA problem #10594: Data Flow

int main() {
    int N, M;

    while (scanf("%d%d", &N, &M) == 2) {
        VVL v(M, VL(3));
        for (int i = 0; i < M; i++)
            scanf("%Ld%Ld%Ld", &v[i][0], &v[i][1], &v[i][2]);
        L D, K;
        scanf("%Ld%Ld", &D, &K);

        MinCostMaxFlow mcmf(N+1);
        for (int i = 0; i < M; i++) {
            mcmf.AddEdge(int(v[i][0]), int(v[i][1]), K, v[i][2]);
            mcmf.AddEdge(int(v[i][1]), int(v[i][0]), K, v[i][2]);
        }
        mcmf.AddEdge(0, 1, D, 0);

        pair<L, L> res = mcmf.GetMaxFlow(0, N);

        if (res.first == D) {
            printf("%Ld\n", res.second);
        } else {
            printf("Impossible.\n");
        }
    }

    return 0;
}

// END CUT

```

1.3 Push-relabel max-flow

```

// Adjacency list implementation of FIFO push re-label maximum flow
// with the gap relabeling heuristic. This implementation is
// significantly faster than straight Ford-Fulkerson. It solves
// random problems with 10000 vertices and 1000000 edges in a few
// seconds, though it is possible to construct test cases that
// achieve the worst-case.
//
// Running time:
//  $O(|V|^3)$ 
//
// INPUT:
// - graph, constructed using AddEdge()
// - source
// - sink
//
// OUTPUT:
// - maximum flow value
// - To obtain the actual flow values, look at all edges with
// capacity > 0 (zero capacity edges are residual edges).

#include <cmath>
#include <vector>
#include <iostream>
#include <queue>

using namespace std;

```

```

typedef long long LL;

struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct PushRelabel {
    int N;
    vector<vector<Edge>> > G;
    vector<LL> excess;
    vector<int> dist, active, count;
    queue<int> Q;

    PushRelabel(int N) : N(N), G(N), excess(N), dist(N), active(N), count(2*N) {}

    void AddEdge(int from, int to, int cap) {
        G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
        if (from == to) G[from].back().index++;
        G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
    }

    void Enqueue(int v) {
        if (!active[v] && excess[v] > 0) { active[v] = true; Q.push(v); }
    }

    void Push(Edge &e) {
        int amt = int(min(excess[e.from], LL(e.cap - e.flow)));
        if (dist[e.from] <= dist[e.to] || amt == 0) return;
        e.flow += amt;
        G[e.to][e.index].flow -= amt;
        excess[e.to] += amt;
        excess[e.from] -= amt;
        Enqueue(e.to);
    }

    void Gap(int k) {
        for (int v = 0; v < N; v++) {
            if (dist[v] < k) continue;
            count[dist[v]]--;
            dist[v] = max(dist[v], N+1);
            count[dist[v]]++;
            Enqueue(v);
        }
    }

    void Relabel(int v) {
        count[dist[v]]--;
        dist[v] = 2*N;
        for (int i = 0; i < G[v].size(); i++)
            if (G[v][i].cap - G[v][i].flow > 0)
                dist[v] = min(dist[v], dist[G[v][i].to] + 1);
        count[dist[v]]++;
        Enqueue(v);
    }

    void Discharge(int v) {
        for (int i = 0; excess[v] > 0 && i < G[v].size(); i++) Push(G[v][i]);
        if (excess[v] > 0) {
            if (count[dist[v]] == 1)
                Gap(dist[v]);
            else
                Relabel(v);
        }
    }

    LL GetMaxFlow(int s, int t) {
        count[0] = N-1;
        count[N] = 1;
        dist[s] = N;
        active[s] = active[t] = true;
        for (int i = 0; i < G[s].size(); i++) {
            excess[s] += G[s][i].cap;
            Push(G[s][i]);
        }

        while (!Q.empty()) {
            int v = Q.front();
            Q.pop();
            active[v] = false;
            Discharge(v);
        }

        LL totflow = 0;
        for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;
        return totflow;
    }
};

// BEGIN CUT

```

// The following code solves SPOJ problem #4110: Fast Maximum Flow (FASTFLOW)

```

int main() {
    int n, m;
    scanf("%d%d", &n, &m);

    PushRelabel pr(n);
    for (int i = 0; i < m; i++) {
        int a, b, c;
        scanf("%d%d%d", &a, &b, &c);
        if (a == b) continue;
        pr.AddEdge(a-1, b-1, c);
        pr.AddEdge(b-1, a-1, c);
    }
    printf("%d\n", pr.GetMaxFlow(0, n-1));
    return 0;
}

```

// END CUT

1.4 Min-cost matching

```

////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
// second.
//
// cost[i][j] = cost for pairing left node i with right node j
// Lmate[i] = index of right node that left node i pairs with
// Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[i][j] matrix.
//
////////////////////

```

```

#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>

using namespace std;

typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
    int n = int(cost.size());

    // construct dual feasible solution
    VD u(n);
    VD v(n);
    for (int i = 0; i < n; i++) {
        u[i] = cost[i][0];
        for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
    }
    for (int j = 0; j < n; j++) {
        v[j] = cost[0][j] - u[0];
        for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
    }

    // construct primal solution satisfying complementary slackness
    Lmate = VI(n, -1);
    Rmate = VI(n, -1);
    int mated = 0;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (Rmate[j] != -1) continue;
            if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
                Lmate[i] = j;
                Rmate[j] = i;
                mated++;
                break;
            }
        }
    }

    VD dist(n);
    VI dad(n);
    VI seen(n);

    // repeat until primal solution is feasible
    while (mated < n) {

```

```

// find an unmatched left node
int s = 0;
while (Lmate[s] != -1) s++;

// initialize Dijkstra
fill(dad.begin(), dad.end(), -1);
fill(seen.begin(), seen.end(), 0);
for (int k = 0; k < n; k++)
    dist[k] = cost[s][k] - u[s] - v[k];

int j = 0;
while (true) {

    // find closest
    j = -1;
    for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
    }
    seen[j] = 1;

    // termination condition
    if (Rmate[j] == -1) break;

    // relax neighbors
    const int i = Rmate[j];
    for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
            dist[k] = new_dist;
            dad[k] = j;
        }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
        if (k == j || !seen[k]) continue;
        const int i = Rmate[k];
        v[k] += dist[k] - dist[j];
        u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
        const int d = dad[j];
        Rmate[j] = Rmate[d];
        Lmate[Rmate[j]] = j;
        j = d;
    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
}

double value = 0;
for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

return value;
}

```

1.5 Max bipartite machine

```

// This code performs maximum bipartite matching.
//
// Running time: O(|E| |V|) -- often much faster in practice
//
// INPUT: w[i][j] = edge between row node i and column node j
// OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//         mc[j] = assignment for column node j, -1 if unassigned
//         function returns number of matches made

#include <vector>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
    for (int j = 0; j < w[i].size(); j++) {
        if (w[i][j] && !seen[j]) {
            seen[j] = true;

```

```

            if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
                mr[i] = j;
                mc[j] = i;
                return true;
            }
        }
    }
    return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
    mr = VI(w.size(), -1);
    mc = VI(w[0].size(), -1);

    int ct = 0;
    for (int i = 0; i < w.size(); i++) {
        VI seen(w[0].size());
        if (FindMatch(i, w, mr, mc, seen)) ct++;
    }
    return ct;
}

```

1.6 Global min-cut

```

// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
// O(|V|^3)
//
// INPUT:
// - graph, constructed using AddEdge()
//
// OUTPUT:
// - (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights) {
    int N = weights.size();
    VI used(N), cut, best_cut;
    int best_weight = -1;

    for (int phase = N-1; phase >= 0; phase--) {
        VI w = weights[0];
        VI added = used;
        int prev, last = 0;
        for (int i = 0; i < phase; i++) {
            prev = last;
            last = -1;
            for (int j = 1; j < N; j++)
                if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
            if (i == phase-1) {
                for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
                for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
                used[last] = true;
                cut.push_back(last);
                if (best_weight == -1 || w[last] < best_weight) {
                    best_weight = cut;
                    best_weight = w[last];
                }
            } else {
                for (int j = 0; j < N; j++)
                    w[j] += weights[last][j];
                added[last] = true;
            }
        }
    }
    return make_pair(best_weight, best_cut);
}

// BEGIN CUT
// The following code solves UVA problem #10989: Bomb, Divide and Conquer
int main() {
    int N;
    cin >> N;
    for (int i = 0; i < N; i++) {
        int n, m;
        cin >> n >> m;
        VVI weights(n, VI(n));

```

```

for (int j = 0; j < m; j++) {
    int a, b, c;
    cin >> a >> b >> c;
    weights[a-1][b-1] = weights[b-1][a-1] = c;
}
pair<int, VI> res = GetMinCut(weights);
cout << "Case #" << i+1 << ": " << res.first << endl;
}
// END CUT

```

1.7 Graph cut inference

```

// Special-purpose {0,1} combinatorial optimization solver for
// problems of the following by a reduction to graph cuts:
//
//      minimize      sum_i psi_i(x[i])
//      x[1]...x[n] in {0,1}      + sum_{i < j} phi_{ij}(x[i], x[j])
//
// where
//      psi_i : {0, 1} --> R
//      phi_{ij} : {0, 1} x {0, 1} --> R
//
// such that
//      phi_{ij}(0,0) + phi_{ij}(1,1) <= phi_{ij}(0,1) + phi_{ij}(1,0)  (*)
//
// This can also be used to solve maximization problems where the
// direction of the inequality in (*) is reversed.
//
// INPUT: phi -- a matrix such that phi[i][j][u][v] = phi_{ij}(u, v)
//        psi -- a matrix such that psi[i][u] = psi_i(u)
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution
//
// To use this code, create a GraphCutInference object, and call the
// DoInference() method. To perform maximization instead of minimization,
// ensure that #define MAXIMIZATION is enabled.

```

```

#include <vector>
#include <iostream>

```

```
using namespace std;

```

```

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;
typedef vector<VVVI> VVVVI;

```

```
const int INF = 1000000000;

```

```

// comment out following line for minimization
#define MAXIMIZATION

```

```

struct GraphCutInference {
    int N;
    VVI cap, flow;
    VI reached;

    int Augment(int s, int t, int a) {
        reached[s] = 1;
        if (s == t) return a;
        for (int k = 0; k < N; k++) {
            if (reached[k]) continue;
            if (int aa = min(a, cap[s][k] - flow[s][k])) {
                if (int b = Augment(k, t, aa)) {
                    flow[s][k] += b;
                    flow[k][s] -= b;
                    return b;
                }
            }
        }
        return 0;
    }

    int GetMaxFlow(int s, int t) {
        N = cap.size();
        flow = VVI(N, VI(N));
        reached = VI(N);

        int totflow = 0;
        while (int amt = Augment(s, t, INF)) {
            totflow += amt;
            fill(reached.begin(), reached.end(), 0);
        }
        return totflow;
    }
}

```

```

int DoInference(const VVVVI &phi, const VVI &psi, VI &x) {
    int M = phi.size();
    cap = VVI(M+2, VI(M+2));
    VI b(M);
    int c = 0;

    for (int i = 0; i < M; i++) {
        b[i] += psi[i][1] - psi[i][0];
        c += psi[i][0];
        for (int j = 0; j < i; j++)
            b[i] += phi[i][j][1][1] - phi[i][j][0][1];
        for (int j = i+1; j < M; j++) {
            cap[i][j] = phi[i][j][0][1] + phi[i][j][1][0] - phi[i][j][0][0] - phi[i][j][1][1];
            b[i] += phi[i][j][1][0] - phi[i][j][0][0];
            c += phi[i][j][0][0];
        }
    }

#ifdef MAXIMIZATION
    for (int i = 0; i < M; i++) {
        for (int j = i+1; j < M; j++)
            cap[i][j] *= -1;
        b[i] *= -1;
    }
    c *= -1;
#endif

    for (int i = 0; i < M; i++) {
        if (b[i] >= 0) {
            cap[M][i] = b[i];
        } else {
            cap[i][M+1] = -b[i];
            c += b[i];
        }
    }

    int score = GetMaxFlow(M, M+1);
    fill(reached.begin(), reached.end(), 0);
    Augment(M, M+1, INF);
    x = VI(M);
    for (int i = 0; i < M; i++) x[i] = reached[i] ? 0 : 1;
    score += c;
#ifdef MAXIMIZATION
    score *= -1;
#endif

    return score;
}

int main() {
    // solver for "Cat vs. Dog" from NWERC 2008

    int numcases;
    cin >> numcases;
    for (int caseno = 0; caseno < numcases; caseno++) {
        int c, d, v;
        cin >> c >> d >> v;

        VVVVI phi(c+d, VVVI(c+d, VVI(2, VI(2))));
        VVI psi(c+d, VI(2));
        for (int i = 0; i < v; i++) {
            char p, q;
            int u, v;
            cin >> p >> u >> q >> v;
            u--; v--;
            if (p == 'C') {
                phi[u][c+v][0][0]++;
                phi[c+v][u][0][0]++;
            } else {
                phi[v][c+u][1][1]++;
                phi[c+u][v][1][1]++;
            }
        }

        GraphCutInference graph;
        VI x;
        cout << graph.DoInference(phi, psi, x) << endl;
    }

    return 0;
}

```

2 Geometry

2.1 Convex hull

```
// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm. Eliminate redundant points from the hull if REMOVE_REDUNDANT is
// #defined.
//
// Running time: O(n log n)
//
// INPUT:  a vector of input points, unordered.
// OUTPUT: a vector of points in the convex hull, counterclockwise, starting
//         with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>
// BEGIN CUT
#include <map>
// END CUT

using namespace std;

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
    T x, y;
    PT() {}
    PT(T x, T y) : x(x), y(y) {}
    bool operator<(const PT &rhs) const { return make_pair(y,x) < make_pair(rhs.y,rhs.x); }
    bool operator==(const PT &rhs) const { return make_pair(y,x) == make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
    return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end()), pts.end());
    vector<PT> up, dn;
    for (int i = 0; i < pts.size(); i++) {
        while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >= 0) up.pop_back();
        while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <= 0) dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
    if (pts.size() <= 2) return;
    dn.clear();
    dn.push_back(pts[0]);
    dn.push_back(pts[1]);
    for (int i = 2; i < pts.size(); i++) {
        if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
        dn.push_back(pts[i]);
    }
    if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
        dn[0] = dn.back();
        dn.pop_back();
    }
    pts = dn;
#endif
}

// BEGIN CUT
// The following code solves SPOJ problem #26: Build the Fence (BSHEEP)

int main() {
    int t;
    scanf("%d", &t);
    for (int caseno = 0; caseno < t; caseno++) {
        int n;
        scanf("%d", &n);
        vector<PT> v(n);
```

```
for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);
vector<PT> h(v);
map<PT,int> index;
for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
ConvexHull(h);

double len = 0;
for (int i = 0; i < h.size(); i++) {
    double dx = h[i].x - h[(i+1)%h.size()].x;
    double dy = h[i].y - h[(i+1)%h.size()].y;
    len += sqrt(dx*dx+dy*dy);
}

if (caseno > 0) printf("\n");
printf("%.2f\n", len);
for (int i = 0; i < h.size(); i++) {
    if (i > 0) printf(" ");
    printf("%d", index[h[i]]);
}
printf("\n");
}

// END CUT
```

2.2 Miscellaneous geometry

```
// C++ routines for computational geometry.

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
    PT operator * (double c) const { return PT(x*c, y*c); }
    PT operator / (double c) const { return PT(x/c, y/c); }
};

double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << ", " << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
```

```

        double a, double b, double c, double d)
    {
        return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
    }

    // determine if lines from a to b and c to d are parallel or collinear
    bool LinesParallel(PT a, PT b, PT c, PT d) {
        return fabs(cross(b-a, c-d)) < EPS;
    }

    bool LinesCollinear(PT a, PT b, PT c, PT d) {
        return LinesParallel(a, b, c, d)
            && fabs(cross(a-b, a-c)) < EPS
            && fabs(cross(c-d, c-a)) < EPS;
    }

    // determine if line segment from a to b intersects with
    // line segment from c to d
    bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
        if (LinesCollinear(a, b, c, d)) {
            if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
                dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
            if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
                return false;
            return true;
        }
        if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
        if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
        return true;
    }

    // compute intersection of line passing through a and b
    // with line passing through c and d, assuming that unique
    // intersection exists; for segment intersection, check if
    // segments intersect first
    PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
        b=b-a; d=c-d; c=c-a;
        assert(dot(b, b) > EPS && dot(d, d) > EPS);
        return a + b*cross(c, d)/cross(b, d);
    }

    // compute center of circle given three points
    PT ComputeCircleCenter(PT a, PT b, PT c) {
        b=(a+b)/2;
        c=(a+c)/2;
        return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90(a-c));
    }

    // determine if point is in a possibly non-convex polygon (by William
    // Randolph Franklin); returns 1 for strictly interior points, 0 for
    // strictly exterior points, and 0 or 1 for the remaining points.
    // Note that it is possible to convert this into an "exact" test using
    // integer arithmetic by taking care of the division appropriately
    // (making sure to deal with signs properly) and then by writing exact
    // tests for checking point on polygon boundary
    bool PointInPolygon(const vector<PT> &p, PT q) {
        bool c = 0;
        for (int i = 0; i < p.size(); i++){
            int j = (i+1)%p.size();
            if ((p[i].y <= q.y && q.y < p[j].y ||
                p[j].y <= q.y && q.y < p[i].y) &&
                q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[i].y))
                c = !c;
        }
        return c;
    }

    // determine if point is on the boundary of a polygon
    bool PointOnPolygon(const vector<PT> &p, PT q) {
        for (int i = 0; i < p.size(); i++)
            if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
                return true;
        return false;
    }

    // compute intersection of line through points a and b with
    // circle centered at c with radius r > 0
    vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
        vector<PT> ret;
        b = b-a;
        a = a-c;
        double A = dot(b, b);
        double B = dot(a, b);
        double C = dot(a, a) - r*r;
        double D = B*B - A*C;
        if (D < -EPS) return ret;
        ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
        if (D > EPS)
            ret.push_back(c+a+b*(-B-sqrt(D))/A);
        return ret;
    }

    // compute intersection of circle centered at a with radius r
    // with circle centered at b with radius R
    vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
        vector<PT> ret;
        double d = sqrt(dist2(a, b));
        if (d > r+R || d+min(r, R) < max(r, R)) return ret;
        double x = (d*d-R*R+r*r)/(2*d);
        double y = sqrt(r*r-x*x);
        PT v = (b-a)/d;
        ret.push_back(a+v*x + RotateCCW90(v)*y);
        if (y > 0)
            ret.push_back(a+v*x - RotateCCW90(v)*y);
        return ret;
    }

    // This code computes the area or centroid of a (possibly nonconvex)
    // polygon, assuming that the coordinates are listed in a clockwise or
    // counterclockwise fashion. Note that the centroid is often known as
    // the "center of gravity" or "center of mass".
    double ComputeSignedArea(const vector<PT> &p) {
        double area = 0;
        for(int i = 0; i < p.size(); i++) {
            int j = (i+1) % p.size();
            area += p[i].x*p[j].y - p[j].x*p[i].y;
        }
        return area / 2.0;
    }

    double ComputeArea(const vector<PT> &p) {
        return fabs(ComputeSignedArea(p));
    }

    PT ComputeCentroid(const vector<PT> &p) {
        PT c(0,0);
        double scale = 6.0 * ComputeSignedArea(p);
        for (int i = 0; i < p.size(); i++){
            int j = (i+1) % p.size();
            c = c + (p[i].x*p[j].y - p[j].x*p[i].y);
        }
        return c / scale;
    }

    // tests whether or not a given polygon (in CW or CCW order) is simple
    bool IsSimple(const vector<PT> &p) {
        for (int i = 0; i < p.size(); i++) {
            for (int k = i+1; k < p.size(); k++) {
                int j = (i+1) % p.size();
                int l = (k+1) % p.size();
                if (i == 1 || j == k) continue;
                if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                    return false;
            }
        }
        return true;
    }

    inline bool cw(const PT &from, const PT &to) { return cross(from, to) < -EPS; }
    inline bool ccw(const PT &from, const PT &to) { return cross(from, to) > EPS; }

    // cw
    inline bool isInsideTriangle(const PT &point, const PT triangle[])
    {
        const int n = 3;
        for (int i = 0; i < n; ++i)
        {
            if (cw(point - triangle[i], triangle[(i+1) % n] - triangle[i]))
            {
                return false;
            }
        }
        return true;
    }

    // cw
    inline bool isInsideHull(const PT &point, const int hullSize, const PT hull[])
    {
        int bottomNeighbourIndex = (int)(lower_bound(hull + 2, hull + hullSize, point, [&](const PT &
            current, const PT &needle) {
                return ccw(needle - hull[0], current - hull[0]);
            }) - hull);
        if (bottomNeighbourIndex >= hullSize)
        {
            return false;
        }
        const PT triangle[] = { hull[0], hull[bottomNeighbourIndex-1], hull[bottomNeighbourIndex] };
        return isInsideTriangle(point, triangle);
    }

```

```

int main() {
    // expected: (-5,2)
    cerr << RotateCCW90(PT(2,5)) << endl;

    // expected: (5,-2)
    cerr << RotateCW90(PT(2,5)) << endl;

    // expected: (-5,2)
    cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

    // expected: (5,2)
    cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

    // expected: (5,2) (7.5,3) (2.5,1)
    cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
        << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

    // expected: 6.78903
    cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

    // expected: 1 0 1
    cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 0 0 1
    cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
        << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

    // expected: 1 1 1 0
    cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
        << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;

    // expected: (1,2)
    cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << endl;

    // expected: (1,1)
    cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

    vector<PT> v;
    v.push_back(PT(0,0));
    v.push_back(PT(5,0));
    v.push_back(PT(5,5));
    v.push_back(PT(0,5));

    // expected: 1 1 1 0 0
    cerr << PointInPolygon(v, PT(2,2)) << " "
        << PointInPolygon(v, PT(2,0)) << " "
        << PointInPolygon(v, PT(0,2)) << " "
        << PointInPolygon(v, PT(5,2)) << " "
        << PointInPolygon(v, PT(2,5)) << endl;

    // expected: 0 1 1 1 1
    cerr << PointOnPolygon(v, PT(2,2)) << " "
        << PointOnPolygon(v, PT(2,0)) << " "
        << PointOnPolygon(v, PT(0,2)) << " "
        << PointOnPolygon(v, PT(5,2)) << " "
        << PointOnPolygon(v, PT(2,5)) << endl;

    // expected: (1,6)
    // (5,4) (4,5)
    // blank line
    // (4,5) (5,4)
    // blank line
    // (4,5) (5,4)
    vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
    u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
    for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;

    // area should be 5.0
    // centroid should be (1.1666666, 1.1666666)
    PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
    vector<PT> p(pa, pa+4);
    PT c = ComputeCentroid(p);
    cerr << "Area: " << ComputeArea(p) << endl;
    cerr << "Centroid: " << c << endl;
}

```

```

    return 0;
}

```

2.3 3D geometry

```

public class Geom3D {
    // distance from point (x, y, z) to plane aX + bY + cZ + d = 0
    public static double ptPlaneDist(double x, double y, double z,
        double a, double b, double c, double d) {
        return Math.abs(a*x + b*y + c*z + d) / Math.sqrt(a*a + b*b + c*c);
    }

    // distance between parallel planes aX + bY + cZ + d1 = 0 and
    // aX + bY + cZ + d2 = 0
    public static double planePlaneDist(double a, double b, double c,
        double d1, double d2) {
        return Math.abs(d1 - d2) / Math.sqrt(a*a + b*b + c*c);
    }

    // distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2)
    // (or ray, or segment; in the case of the ray, the endpoint is the
    // first point)
    public static final int LINE = 0;
    public static final int SEGMENT = 1;
    public static final int RAY = 2;
    public static double ptLineDistSq(double x1, double y1, double z1,
        double x2, double y2, double z2, double px, double py, double pz,
        int type) {
        double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);

        double x, y, z;
        if (pd2 == 0) {
            x = x1;
            y = y1;
            z = z1;
        } else {
            double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1)) / pd2;
            x = x1 + u * (x2 - x1);
            y = y1 + u * (y2 - y1);
            z = z1 + u * (z2 - z1);
            if (type != LINE && u < 0) {
                x = x1;
                y = y1;
                z = z1;
            }
            if (type == SEGMENT && u > 1.0) {
                x = x2;
                y = y2;
                z = z2;
            }
        }
        return (x-px)*(x-px) + (y-py)*(y-py) + (z-pz)*(z-pz);
    }

    public static double ptLineDist(double x1, double y1, double z1,
        double x2, double y2, double z2, double px, double py, double pz,
        int type) {
        return Math.sqrt(ptLineDistSq(x1, y1, z1, x2, y2, z2, px, py, pz, type));
    }
}

```

2.4 Slow Delaunay triangulation

```

// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
// Running time: O(n^4)
// INPUT:  x[] = x-coordinates
//         y[] = y-coordinates
// OUTPUT: triples = a vector containing m triples of indices
//               corresponding to triangle vertices

```

```

#include<vector>
using namespace std;

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
}

```



```

    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
    int n = x.size();
    vector<T> z(n);
    vector<triple> ret;

    for (int i = 0; i < n; i++)
        z[i] = x[i] * x[i] + y[i] * y[i];

    for (int i = 0; i < n-2; i++) {
        for (int j = i+1; j < n; j++) {
            for (int k = i+1; k < n; k++) {
                if (j == k) continue;
                double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])*(z[j]-z[i]);
                double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])*(z[k]-z[i]);
                double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])*(y[j]-y[i]);
                bool flag = zn < 0;
                for (int m = 0; flag && m < n; m++)
                    flag = flag && ((x[m]-x[i])*xn +
                                     (y[m]-y[i])*yn +
                                     (z[m]-z[i])*zn <= 0);
                if (flag) ret.push_back(triple(i, j, k));
            }
        }
    }
    return ret;
}

int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);

    //expected: 0 1 3
    //           0 3 2

    int i;
    for(i = 0; i < tri.size(); i++)
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}

```

3 Numerical algorithms

3.1 Number theory (modular, Chinese remainder, linear Diophantine)

```

// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.

```

```

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
    return ((a%b) + b) % b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
    while (b) { int t = a%b; a = b; b = t; }
    return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
    return a / gcd(a, b)*b;
}

// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
{

```

```

    int ret = 1;
    while (b)
    {
        if (b & 1) ret = mod(ret*a, m);
        a = mod(a*a, m);
        b >>= 1;
    }
    return ret;
}

// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
    int xx = y = 0;
    int yy = x = 1;
    while (b) {
        int q = a / b;
        int t = b; b = a%b; a = t;
        t = xx; xx = x - q*xx; x = t;
        t = yy; yy = y - q*yy; y = t;
    }
    return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
    int x, y;
    VI ret;
    int g = extended_euclid(a, n, x, y);
    if (!(b%g)) {
        x = mod(x*(b / g), n);
        for (int i = 0; i < g; i++)
            ret.push_back(mod(x + i*(n / g), n));
    }
    return ret;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
    int x, y;
    int g = extended_euclid(a, n, x, y);
    if (g > 1) return -1;
    return mod(x, n);
}

// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2. Here, z is unique modulo M = lcm(m1, m2).
// Return (z, M). On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
    int s, t;
    int g = extended_euclid(m1, m2, s, t);
    if (r1%g != r2%g) return make_pair(0, -1);
    return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g);
}

// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i. Note that the solution is
// unique modulo M = lcm_i (m[i]). Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
    PII ret = make_pair(r[0], m[0]);
    for (int i = 1; i < m.size(); i++) {
        ret = chinese_remainder_theorem(ret.second, ret.first, m[i], r[i]);
        if (ret.second == -1) break;
    }
    return ret;
}

// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
    if (!(a && !b))
    {
        if (c) return false;
        x = 0; y = 0;
        return true;
    }
    if (!a)
    {
        if (c % b) return false;
        x = 0; y = c / b;
        return true;
    }
    if (!b)
    {
        if (c % a) return false;
        x = c / a; y = 0;
        return true;
    }
    int g = gcd(a, b);
    if (c % g) return false;
    x = c / g * mod_inverse(a / g, b / g);

```

```

    y = (c - a*x) / b;
    return true;
}

int main() {
    // expected: 2
    cout << gcd(14, 30) << endl;

    // expected: 2 -2 1
    int x, y;
    int g = extended_euclid(14, 30, x, y);
    cout << g << " " << x << " " << y << endl;

    // expected: 95 451
    VI sols = modular_linear_equation_solver(14, 30, 100);
    for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";
    cout << endl;

    // expected: 8
    cout << mod_inverse(8, 9) << endl;

    // expected: 23 105
    //          11 12
    PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2, 3, 2 }));
    cout << ret.first << " " << ret.second << endl;
    ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
    cout << ret.first << " " << ret.second << endl;

    // expected: 5 -15
    if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" << endl;
    cout << x << " " << y << endl;
    return 0;
}

```

3.2 Systems of linear equations, matrix inverse, determinant

```

// Gauss-Jordan elimination with full pivoting.
//
// Uses:
// (1) solving systems of linear equations (AX=B)
// (2) inverting matrices (AX=I)
// (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:   a[]{} = an nxn matrix
//          b[]{} = an nxm matrix
//
// OUTPUT:  X      = an nxm matrix (stored in b[][])
//          A^-1    = an nxn matrix (stored in a[][])
//          returns determinant of a[]{}

```

```

#include <iostream>
#include <vector>
#include <cmath>

```

```
using namespace std;
```

```
const double EPS = 1e-10;
```

```

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

```

```

T GaussJordan(VVT &a, VVT &b) {
    const int n = a.size();
    const int m = b[0].size();
    VI irow(n), icol(n), ipiv(n);
    T det = 1;

    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
        if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl; exit(0); }
        ipiv[pj]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        if (pj != pk) det *= -1;
        irow[i] = pj;
        icol[i] = pk;

        T c = 1.0 / a[pk][pk];

```

```

        det *= a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
        }

        for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
            for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
        }

        return det;
    }

int main() {
    const int n = 4;
    const int m = 2;
    double A[n][n] = { { 1,2,3,4 }, { 1,0,1,0 }, { 5,3,2,4 }, { 6,1,4,6 } };
    double B[n][m] = { { 1,2 }, { 4,3 }, { 5,6 }, { 8,7 } };
    VVT a(n), b(n);
    for (int i = 0; i < n; i++) {
        a[i] = VT(A[i], A[i] + n);
        b[i] = VT(B[i], B[i] + m);
    }

    double det = GaussJordan(a, b);

    // expected: 60
    cout << "Determinant: " << det << endl;

    // expected: -0.233333 0.166667 0.133333 0.066667
    //          0.166667 0.166667 0.333333 -0.333333
    //          0.233333 0.833333 -0.133333 -0.066667
    //          0.05 -0.75 -0.1 0.2
    cout << "Inverse: " << endl;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            cout << a[i][j] << ' ';
        }
        cout << endl;
    }

    // expected: 1.63333 1.3
    //          -0.166667 0.5
    //          2.36667 1.7
    //          -1.85 -1.35
    cout << "Solution: " << endl;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++) {
            cout << b[i][j] << ' ';
        }
        cout << endl;
    }
}

```

3.3 Reduced row echelon form, matrix rank

```

// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxm matrix
//
// OUTPUT:  rref[][] = an nxm matrix (stored in a[][])
//          returns rank of a[][]

```

```

#include <iostream>
#include <vector>
#include <cmath>

```

```
using namespace std;
```

```
const double EPSILON = 1e-10;
```

```

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

```

```

int rref(VVT &a) {
    int n = a.size();
    int m = a[0].size();
    int r = 0;

```

```

for (int c = 0; c < m && r < n; c++) {
    int j = r;
    for (int i = r + 1; i < n; i++)
        if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
    if (fabs(a[j][c]) < EPSILON) continue;
    swap(a[j], a[r]);

    T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
    for (int i = 0; i < n; i++) if (i != r) {
        T t = a[i][c];
        for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
    }
    r++;
}
return r;
}

int main() {
    const int n = 5, m = 4;
    double A[n][m] = {
        {16, 2, 3, 13},
        { 5, 11, 10, 8},
        { 9, 7, 6, 12},
        { 4, 14, 15, 1},
        {13, 21, 21, 13}};
    VVT a(n);
    for (int i = 0; i < n; i++)
        a[i] = VT(A[i], A[i] + m);

    int rank = rref(a);

    // expected: 3
    cout << "Rank: " << rank << endl;

    // expected: 1 0 0 1
    //           0 1 0 3
    //           0 0 1 -3
    //           0 0 0 3.10862e-15
    //           0 0 0 2.22045e-15
    cout << "rref: " << endl;
    for (int i = 0; i < 5; i++) {
        for (int j = 0; j < 4; j++)
            cout << a[i][j] << " ";
        cout << endl;
    }
}

```

3.4 Fast Fourier transform

```

#include <cassert>
#include <cstdio>
#include <cmath>

struct cpx
{
    cpx() {}
    cpx(double aa):a(aa),b(0){}
    cpx(double aa, double bb):a(aa),b(bb){}
    double a;
    double b;
    double modsq(void) const
    {
        return a * a + b * b;
    }
    cpx bar(void) const
    {
        return cpx(a, -b);
    }
};

cpx operator +(cpx a, cpx b)
{
    return cpx(a.a + b.a, a.b + b.b);
}

cpx operator *(cpx a, cpx b)
{
    return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}

cpx operator /(cpx a, cpx b)
{
    cpx r = a * b.bar();
    return cpx(r.a / b.modsq(), r.b / b.modsq());
}

cpx EXP(double theta)

```

```

{
    return cpx(cos(theta), sin(theta));
}

const double two_pi = 4 * acos(0);

// in:    input array
// out:    output array
// step:   {SET TO 1} (used internally)
// size:   length of the input/output (MUST BE A POWER OF 2)
// dir:    either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size-1} in[j] * exp(dir * 2pi * i * j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
    if(size < 1) return;
    if(size == 1)
    {
        out[0] = in[0];
        return;
    }
    FFT(in, out, step * 2, size / 2, dir);
    FFT(in + step, out + size / 2, step * 2, size / 2, dir);
    for(int i = 0; i < size / 2; i++)
    {
        cpx even = out[i];
        cpx odd = out[i + size / 2];
        out[i] = even + EXP(dir * two_pi * i / size) * odd;
        out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) / size) * odd;
    }
}

// Usage:
// f[0...N-1] and g[0...N-1] are numbers
// Want to compute the convolution h, defined by
// h[n] = sum of f[k]g[n-k] (k = 0, ..., N-1).
// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product).
// To compute h[] in O(N log N) time, do the following:
// 1. Compute F and G (pass dir = 1 as the argument).
// 2. Get H by element-wise multiplying F and G.
// 3. Get h by taking the inverse FFT (use dir = -1 as the argument)
// and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.

int main(void)
{
    printf("If rows come in identical pairs, then everything works.\n");

    cpx a[8] = {0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0};
    cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};
    cpx A[8];
    cpx B[8];
    FFT(a, A, 1, 8, 1);
    FFT(b, B, 1, 8, 1);

    for(int i = 0; i < 8; i++)
    {
        printf("%7.21f%7.21f", A[i].a, A[i].b);
    }
    printf("\n");
    for(int i = 0; i < 8; i++)
    {
        cpx Ai(0,0);
        for(int j = 0; j < 8; j++)
        {
            Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
        }
        printf("%7.21f%7.21f", Ai.a, Ai.b);
    }
    printf("\n");

    cpx AB[8];
    for(int i = 0; i < 8; i++)
        AB[i] = A[i] * B[i];
    cpx aconvb[8];
    FFT(AB, aconvb, 1, 8, -1);
    for(int i = 0; i < 8; i++)
        aconvb[i] = aconvb[i] / 8;
    for(int i = 0; i < 8; i++)
    {
        printf("%7.21f%7.21f", aconvb[i].a, aconvb[i].b);
    }
    printf("\n");
    for(int i = 0; i < 8; i++)
    {
        cpx aconvbi(0,0);
        for(int j = 0; j < 8; j++)
        {
            aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
        }
        printf("%7.21f%7.21f", aconvbi.a, aconvbi.b);
    }
}

```

```
printf("\n");
return 0;
}
```

3.5 Simplex algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//      maximize    c^T x
//      subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//         b -- an m-dimensional vector
//         c -- an n-dimensional vector
//         x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
    int m, n;
    VI B, N;
    VVD D;

    LPSolver(const VVD &A, const VD &b, const VD &c) :
        m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
        for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] = A[i][j];
        for (int i = 0; i < m; i++) { B[i] = n + 1; D[i][n] = -1; D[i][n + 1] = b[i]; }
        for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
        N[n] = -1; D[m + 1][n] = 1;
    }

    void Pivot(int r, int s) {
        double inv = 1.0 / D[r][s];
        for (int i = 0; i < m + 2; i++) if (i != r)
            for (int j = 0; j < n + 2; j++) if (j != s)
                D[i][j] -= D[r][j] * D[i][s] * inv;
        for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
        for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }

    bool Simplex(int phase) {
        int x = phase == 1 ? m + 1 : m;
        while (true) {
            int s = -1;
            for (int j = 0; j < n; j++) {
                if (phase == 2 && N[j] == -1) continue;
                if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
            }
            if (D[x][s] > -EPS) return true;
            int r = -1;
            for (int i = 0; i < m; i++) {
                if (D[i][s] < EPS) continue;
                if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
                    (D[i][n + 1] / D[i][s] == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
            }
            if (r == -1) return false;
            Pivot(r, s);
        }
    }

    DOUBLE Solve(VD &x) {
        int r = 0;
        for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
        if (D[r][n + 1] < -EPS) {
```

```
Pivot(r, n);
if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<DOUBLE>::infinity();
for (int i = 0; i < m; i++) if (B[i] == -1) {
    int s = -1;
    for (int j = 0; j < n; j++)
        if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] < N[s]) s = j;
    Pivot(i, s);
}
}
if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
x = VD(n);
for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
return D[m][n + 1];
};

int main() {
    const int m = 4;
    const int n = 3;
    DOUBLE _A[m][n] = {
        { 6, -1, 0 },
        { -1, -5, 0 },
        { 1, 5, 1 },
        { -1, -5, -1 }
    };
    DOUBLE _b[m] = { 10, -4, 5, -5 };
    DOUBLE _c[n] = { 1, -1, 0 };

    VVD A(m);
    VD b(_b, _b + m);
    VD c(_c, _c + n);
    for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

    LPSolver solver(A, b, c);
    VD x;
    DOUBLE value = solver.Solve(x);

    cerr << "VALUE: " << value << endl; // VALUE: 1.29032
    cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
    for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
    cerr << endl;
    return 0;
}
```

4 Graph algorithms

4.1 Fast Dijkstra's algorithm

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
//
// Running time: O(|E| log |V|)

#include <queue>
#include <cstdio>

using namespace std;
const int INF = 2000000000;
typedef pair<int, int> PII;

int main() {
    int N, s, t;
    scanf("%d%d%d", &N, &s, &t);
    vector<vector<PII>> edges(N);
    for (int i = 0; i < N; i++) {
        int M;
        scanf("%d", &M);
        for (int j = 0; j < M; j++) {
            int vertex, dist;
            scanf("%d%d", &vertex, &dist);
            edges[i].push_back(make_pair(dist, vertex)); // note order of arguments here
        }
    }

    // use priority queue in which top element has the "smallest" priority
    priority_queue<PII, vector<PII>, greater<PII>> Q;
    vector<int> dist(N, INF), dad(N, -1);
    Q.push(make_pair(0, s));
    dist[s] = 0;
    while (!Q.empty()) {
        PII p = Q.top();
        Q.pop();
        int here = p.second;
```

```

        if (here == t) break;
        if (dist[here] != p.first) continue;

        for (vector<PII>::iterator it = edges[here].begin(); it != edges[here].end(); it++) {
            if (dist[here] + it->first < dist[it->second]) {
                dist[it->second] = dist[here] + it->first;
                dad[it->second] = here;
                Q.push(make_pair(dist[it->second], it->second));
            }
        }

        printf("%d\n", dist[t]);
        if (dist[t] < INF)
            for (int i = t; i != -1; i = dad[i])
                printf("%d%c", i, (i == s ? '\n' : ' '));

        return 0;
    }

    /*
    Sample input:
    5 0 4
    2 1 2 3 1
    2 2 4 4 5
    3 1 4 3 3 4 1
    2 0 1 2 3
    2 1 5 2 1

    Expected:
    5
    4 2 3 0
    */

```

4.2 Strongly connected components

```

#include<memory.h>
struct edge{int e, nxt;};
int V, E;
edge e[MAXE], er[MAXE];
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];
int stk[MAXV];
void fill_forward(int x)
{
    int i;
    v[x]=true;
    for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
    stk[++stk[0]]=x;
}
void fill_backward(int x)
{
    int i;
    v[x]=false;
    group_num[x]=group_cnt;
    for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
}
void add_edge(int v1, int v2) //add edge v1->v2
{
    e[++E].e=v2; e[E].nxt=sp[v1]; sp[v1]=E;
    er[E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
}
void SCC()
{
    int i;
    stk[0]=0;
    memset(v, false, sizeof(v));
    for(i=1;i<=V;i++) if(!v[i]) fill_forward(i);
    group_cnt=0;
    for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++; fill_backward(stk[i]);}
}

```

4.3 Eulerian path

```

struct Edge;
typedef list<Edge>::iterator iter;

struct Edge
{
    int next_vertex;
    iter reverse_edge;

    Edge(int next_vertex)

```

```

        :next_vertex(next_vertex)
        { }
    };

    const int max_vertices = ;
    int num_vertices;
    list<Edge> adj[max_vertices]; // adjacency list

    vector<int> path;

    void find_path(int v)
    {
        while(adj[v].size() > 0)
        {
            int vn = adj[v].front().next_vertex;
            adj[vn].erase(adj[v].front().reverse_edge);
            adj[v].pop_front();
            find_path(vn);
        }
        path.push_back(v);
    }

    void add_edge(int a, int b)
    {
        adj[a].push_front(Edge(b));
        iter ita = adj[a].begin();
        adj[b].push_front(Edge(a));
        iter itb = adj[b].begin();
        ita->reverse_edge = itb;
        itb->reverse_edge = ita;
    }

```

5 Data structures

5.1 Suffix array

```

// Suffix array construction in O(L log^2 L) time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:   string s
//
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//          of substring s[i...L-1] in the list of sorted suffixes.
//          That is, if we take the inverse of the permutation suffix[],
//          we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
    const int L;
    string s;
    vector<vector<int>> > P;
    vector<pair<pair<int,int>,int>> > M;

    SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L, 0)), M(L) {
        for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
        for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
            P.push_back(vector<int>(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[level-1][i + skip] : -1000), i);
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ? P[level][M[i-1].second] : i;
        }
    }

    vector<int> GetSuffixArray() { return P.back(); }

    // returns the length of the longest common prefix of s[i...L-1] and s[j...L-1]
    int LongestCommonPrefix(int i, int j) {
        int len = 0;
        if (i == j) return L - i;
        for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
}

```

```

}
};

// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
    int T;
    cin >> T;
    for (int caseno = 0; caseno < T; caseno++) {
        string s;
        cin >> s;
        SuffixArray array(s);
        vector<int> v = array.GetSuffixArray();
        int bestlen = -1, bestpos = -1, bestcount = 0;
        for (int i = 0; i < s.length(); i++) {
            int len = 0, count = 0;
            for (int j = i+1; j < s.length(); j++) {
                int l = array.LongestCommonPrefix(i, j);
                if (l >= len) {
                    if (l > len) count = 2; else count++;
                    len = l;
                }
            }
            if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen) > s.substr(i, len)) {
                bestlen = len;
                bestcount = count;
                bestpos = i;
            }
        }
        if (bestlen == 0) {
            cout << "No repetitions found!" << endl;
        } else {
            cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
        }
    }
}

#else
// END CUT
int main() {

    // bobocel is the 0'th suffix
    // obocel is the 5'th suffix
    // bocel is the 1'st suffix
    // ocel is the 6'th suffix
    // cel is the 2'nd suffix
    // el is the 3'rd suffix
    // l is the 4'th suffix
    SuffixArray suffix("bobocel");
    vector<int> v = suffix.GetSuffixArray();

    // Expected output: 0 5 1 6 2 3 4
    //
    for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
    cout << endl;
    cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
// BEGIN CUT
#endif
// END CUT

```

5.2 Binary Indexed Tree

```

#include <iostream>
using namespace std;

#define LOGSZ 17

int tree[(1<<LOGSZ)+1];
int N = (1<<LOGSZ);

// add v to value at x
void set(int x, int v) {
    while(x <= N) {
        tree[x] += v;
        x += (x & -x);
    }
}

// get cumulative sum up to and including x
int get(int x) {
    int res = 0;
    while(x) {
        res += tree[x];
        x -= (x & -x);
    }
}

```

```

}
return res;
}

// get largest value with cumulative sum less than or equal to x;
// for smallest, pass x-1 and add 1 to result
int getind(int x) {
    int idx = 0, mask = N;
    while(mask && idx < N) {
        int t = idx + mask;
        if(x >= tree[t]) {
            idx = t;
            x -= tree[t];
        }
        mask >>= 1;
    }
    return idx;
}

```

5.3 Union-find set

```

// BEGIN CUT
#include <iostream>
#include <vector>
using namespace std;

// END CUT
struct UnionFind {
    vector<int> parent;
    vector<int> rank;
    UnionFind(int n) : parent(n), rank(n) {
        for (int i = 0; i < n; ++i) {
            parent[i] = i;
            rank[i] = 0;
        }
    }

    int find_set(int v) {
        if (v == parent[v])
            return v;
        return parent[v] = find_set(parent[v]);
    }

    void union_sets(int a, int b) {
        a = find_set(a);
        b = find_set(b);
        if (a != b) {
            if (rank[a] < rank[b])
                swap(a, b);
            parent[b] = a;
            if (rank[a] == rank[b])
                ++rank[a];
        }
    }
};

// BEGIN CUT

int main()
{
    int n = 5;
    UnionFind C(n);
    C.union_sets(0, 2);
    C.union_sets(1, 0);
    C.union_sets(3, 4);
    for (int i = 0; i < n; i++) cout << i << " " << C.find_set(i) << endl;
    return 0;
}

// END CUT

```

5.4 KD-tree

```

// -----
// A straightforward, but probably sub-optimal KD-tree implementation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
// - constructs from n points in O(n lg^2 n) time
// - handles nearest-neighbor query in O(lg n) if points are well
//   distributed
// - worst case for nearest-neighbor may be linear in pathological
//   case
//
// Sonny Chan, Stanford University, April 2009
// -----

```

```

#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>

using namespace std;

// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b)
{
    return a.x == b.x && a.y == b.y;
}

// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
{
    return a.x < b.x;
}

// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
{
    return a.y < b.y;
}

// squared distance between points
ntype pdist2(const point &a, const point &b)
{
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy;
}

// bounding box for a set of points
struct bbox
{
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x);    x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y);    y1 = max(y1, v[i].y);
        }
    }

    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
            if (p.y < y0) return pdist2(point(x0, y0), p);
            else if (p.y > y1) return pdist2(point(x0, y1), p);
            else return pdist2(point(x0, p.y), p);
        }
        else if (p.x > x1) {
            if (p.y < y0) return pdist2(point(x1, y0), p);
            else if (p.y > y1) return pdist2(point(x1, y1), p);
            else return pdist2(point(x1, p.y), p);
        }
        else {
            if (p.y < y0) return pdist2(point(p.x, y0), p);
            else if (p.y > y1) return pdist2(point(p.x, y1), p);
            else return 0;
        }
    }
};

// stores a single node of the kd-tree, either internal or leaf
struct kndnode
{
    bool leaf; // true if this is a leaf node (has one point)
    point pt; // the single point of this is a leaf
    bbox bound; // bounding box for set of points in children

    kndnode *first, *second; // two children of this kd-node

    kndnode() : leaf(false), first(0), second(0) {}
    ~kndnode() { if (first) delete first; if (second) delete second; }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
        return bound.distance(p);
    }
};

```

```

}

// recursively builds a kd-tree from a given cloud of points
void construct(vector<point> &vp)
{
    // compute bounding box for points at this node
    bound.compute(vp);

    // if we're down to one point, then we're a leaf node
    if (vp.size() == 1) {
        leaf = true;
        pt = vp[0];
    }
    else {
        // split on x if the bbox is wider than high (not best heuristic...)
        if (bound.x1-bound.x0 >= bound.y1-bound.y0)
            sort(vp.begin(), vp.end(), on_x);
        // otherwise split on y-coordinate
        else
            sort(vp.begin(), vp.end(), on_y);

        // divide by taking half the array for each child
        // (not best performance if many duplicates in the middle)
        int half = vp.size()/2;
        vector<point> vl(vp.begin(), vp.begin()+half);
        vector<point> vr(vp.begin()+half, vp.end());
        first = new kndnode(); first->construct(vl);
        second = new kndnode(); second->construct(vr);
    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kndnode *root;

    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kndnode();
        root->construct(v);
    }
    ~kdtree() { delete root; }

    // recursive search method returns squared distance to nearest point
    ntype search(kndnode *node, const point &p)
    {
        if (node->leaf) {
            // commented special case tells a point not to find itself
            if (p == node->pt) return sentry;
            else
                return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        }
        else {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }

    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
    }
};

// -----
// some basic test code here

int main()
{
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    }
    kdtree tree(vp);
}

```

```

// query some points
for (int i = 0; i < 10; ++i) {
    point q(rand()%100000, rand()%100000);
    cout << "Closest squared distance to (" << q.x << ", " << q.y << ") "
          << " is " << tree.nearest(q) << endl;
}

return 0;
}

// -----

```

5.5 Splay tree

```

#include <cstdio>
#include <algorithm>
using namespace std;

const int N_MAX = 130010;
const int oo = 0x3f3f3f3f;
struct Node
{
    Node *ch[2], *pre;
    int val, size;
    bool isTurned;
} nodePool[N_MAX], *null, *root;

Node *allocNode(int val)
{
    static int freePos = 0;
    Node *x = &nodePool[freePos++];
    x->val = val, x->isTurned = false;
    x->ch[0] = x->ch[1] = x->pre = null;
    x->size = 1;
    return x;
}

inline void update(Node *x)
{
    x->size = x->ch[0]->size + x->ch[1]->size + 1;
}

inline void makeTurned(Node *x)
{
    if(x == null)
        return;
    swap(x->ch[0], x->ch[1]);
    x->isTurned ^= 1;
}

inline void pushDown(Node *x)
{
    if(x->isTurned)
    {
        makeTurned(x->ch[0]);
        makeTurned(x->ch[1]);
        x->isTurned ^= 1;
    }
}

inline void rotate(Node *x, int c)
{
    Node *y = x->pre;
    x->pre = y->pre;
    if(y->pre != null)
        y->pre->ch[y->pre->ch[1] == x] = x;
    y->ch[1-c] = x->ch[c];
    if(x->ch[c] != null)
        x->ch[c]->pre = y;
    x->ch[c] = y, y->pre = x;
    update(y);
    if(y == root)
        root = x;
}

void splay(Node *x, Node *p)
{
    while(x->pre != p)
    {
        if(x->pre->pre == p)
            rotate(x, x == x->pre->ch[0]);
        else
        {
            Node *y = x->pre, *z = y->pre;
            if(y == z->ch[0])
            {
                if(x == y->ch[0])

```

```

                rotate(y, 1), rotate(x, 1);
            else
                rotate(x, 0), rotate(x, 1);
        }
    }
    if(x == y->ch[1])
        rotate(y, 0), rotate(x, 0);
    else
        rotate(x, 1), rotate(x, 0);
}

update(x);
}

void select(int k, Node *fa)
{
    Node *now = root;
    while(1)
    {
        pushDown(now);
        int tmp = now->ch[0]->size + 1;
        if(tmp == k)
            break;
        else if(tmp < k)
            now = now->ch[1], k -= tmp;
        else
            now = now->ch[0];
    }
    splay(now, fa);
}

Node *makeTree(Node *p, int l, int r)
{
    if(l > r)
        return null;
    int mid = (l + r) / 2;
    Node *x = allocNode(mid);
    x->pre = p;
    x->ch[0] = makeTree(x, l, mid - 1);
    x->ch[1] = makeTree(x, mid + 1, r);
    update(x);
    return x;
}

int main()
{
    int n, m;
    null = allocNode(0);
    null->size = 0;
    root = allocNode(0);
    root->ch[1] = allocNode(oo);
    root->ch[1]->pre = root;
    update(root);

    scanf("%d%d", &n, &m);
    root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
    splay(root->ch[1]->ch[0], null);

    while(m --)
    {
        int a, b;
        scanf("%d%d", &a, &b);
        a ++, b ++;
        select(a - 1, null);
        select(b + 1, root);
        makeTurned(root->ch[1]->ch[0]);
    }

    for(int i = 1; i <= n; i ++){
        select(i + 1, null);
        printf("%d ", root->val);
    }
}

```

5.6 Fast segment tree

```

//BEGIN CUT
#include <cstdio>
//END CUT

struct SegmentTree{
    static const int N = 1e5; // limit for array size
    int n; // array size
    int t[2 * N];

```



```

void build() { // build the tree
    for (int i = n - 1; i > 0; --i) t[i] = t[i<<1] + t[i<<1|1];
}

void modify(int p, int value) { // set value at position p
    for (t[p += n] = value; p > 1; p >>= 1) t[p>>1] = t[p] + t[p^1];
}

int query(int l, int r) { // sum on interval [l, r]
    int res = 0;
    for (l += n, r += n; l < r; l >>= 1, r >>= 1) {
        if (l&1) res += t[l++];
        if (r&1) res += t[--r];
    }
    return res;
}

};

// BEGIN CUT

SegmentTree st;

int main() {
    scanf("%d", &st.n);
    for (int i = 0; i < st.n; ++i) scanf("%d", st.t + st.n + i);
    st.build();
    st.modify(0, 1);
    printf("%d\n", st.query(3, 11));
    return 0;
}

// END CUT

```

5.7 Lazy segment tree

```

public class SegmentTreeRangeUpdate {
    public long[] leaf;
    public long[] update;
    public int origSize;
    public SegmentTreeRangeUpdate(int[] list) {
        origSize = list.length;
        leaf = new long[4*list.length];
        update = new long[4*list.length];
        build(1, 0, list.length-1, list);
    }

    public void build(int curr, int begin, int end, int[] list) {
        if (begin == end)
            leaf[curr] = list[begin];
        else {
            int mid = (begin+end)/2;
            build(2 * curr, begin, mid, list);
            build(2 * curr + 1, mid+1, end, list);
            leaf[curr] = leaf[2*curr] + leaf[2*curr+1];
        }
    }

    public void update(int begin, int end, int val) {
        update(1, 0, origSize-1, begin, end, val);
    }

    public void update(int curr, int tBegin, int tEnd, int begin, int end, int val) {
        if (tBegin >= begin && tEnd <= end)
            update[curr] += val;
        else {
            leaf[curr] += (Math.min(end, tEnd) - Math.max(begin, tBegin) + 1) * val;
            int mid = (tBegin+tEnd)/2;
            if (mid >= begin && tBegin <= end)
                update(2*curr, tBegin, mid, begin, end, val);
            if (tEnd >= begin && mid+1 <= end)
                update(2*curr+1, mid+1, tEnd, begin, end, val);
        }
    }

    public long query(int begin, int end) {
        return query(1, 0, origSize-1, begin, end);
    }

    public long query(int curr, int tBegin, int tEnd, int begin, int end) {
        if (tBegin >= begin && tEnd <= end) {
            if (update[curr] != 0) {
                leaf[curr] += (tEnd-tBegin+1) * update[curr];
                if (2*curr < update.length) {
                    update[2*curr] += update[curr];
                    update[2*curr+1] += update[curr];
                }
                update[curr] = 0;
            }
            return leaf[curr];
        }
        else {

```

```

            leaf[curr] += (tEnd-tBegin+1) * update[curr];
            if (2*curr < update.length) {
                update[2*curr] += update[curr];
                update[2*curr+1] += update[curr];
            }
            update[curr] = 0;
            int mid = (tBegin+tEnd)/2;
            long ret = 0;
            if (mid >= begin && tBegin <= end)
                ret += query(2*curr, tBegin, mid, begin, end);
            if (tEnd >= begin && mid+1 <= end)
                ret += query(2*curr+1, mid+1, tEnd, begin, end);
            return ret;
        }
    }
}

```

5.8 Lowest common ancestor

```

const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;

vector<int> children[max_nodes]; // children[i] contains the children of node i
int A[max_nodes][log_max_nodes+1]; // A[i][j] is the 2^j-th ancestor of node i, or -1 if that
// ancestor does not exist
int L[max_nodes]; // L[i] is the distance between node i and the root

// floor of the binary logarithm of n
int lb(unsigned int n)
{
    if (n==0)
        return -1;
    int p = 0;
    if (n >= 1<<16) { n >>= 16; p += 16; }
    if (n >= 1<< 8) { n >>= 8; p += 8; }
    if (n >= 1<< 4) { n >>= 4; p += 4; }
    if (n >= 1<< 2) { n >>= 2; p += 2; }
    if (n >= 1<< 1) { p += 1; }
    return p;
}

void DFS(int i, int l)
{
    L[i] = l;
    for (int j = 0; j < children[i].size(); j++)
        DFS(children[i][j], l+1);
}

int LCA(int p, int q)
{
    // ensure node p is at least as deep as node q
    if (L[p] < L[q])
        swap(p, q);

    // "binary search" for the ancestor of node p situated on the same level as q
    for (int i = log_num_nodes; i >= 0; i--)
        if (L[p] - (1<<i) >= L[q])
            p = A[p][i];

    if (p == q)
        return p;

    // "binary search" for the LCA
    for (int i = log_num_nodes; i >= 0; i--)
        if (A[p][i] != -1 && A[p][i] != A[q][i])
        {
            p = A[p][i];
            q = A[q][i];
        }

    return A[p][0];
}

int main(int argc, char* argv[])
{
    // read num_nodes, the total number of nodes
    log_num_nodes = lb(num_nodes);

    for (int i = 0; i < num_nodes; i++)
    {
        int p;
        // read p, the parent of node i or -1 if node i is the root
        A[i][0] = p;
        if (p != -1)
            children[p].push_back(i);
        else
            root = i;
    }
}

```

```

    }

    // precompute A using dynamic programming
    for(int j = 1; j <= log_num_nodes; j++)
        for(int i = 0; i < num_nodes; i++)
            if(A[i][j-1] != -1)
                A[i][j] = A[A[i][j-1]][j-1];
            else
                A[i][j] = -1;

    // precompute L
    DFS(root, 0);

    return 0;
}

```

6 Miscellaneous

6.1 Longest increasing subsequence

```

// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
// INPUT: a vector of integers
// OUTPUT: a vector containing the longest increasing subsequence

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPPII;

#define STRICTLY_INCREASNG

VI LongestIncreasingSubsequence(VI v) {
    VPPII best;
    VI dad(v.size(), -1);

    for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
        PII item = make_pair(v[i], 0);
        VPPII::iterator it = lower_bound(best.begin(), best.end(), item);
        item.second = i;
#else
        PII item = make_pair(v[i], i);
        VPPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
        if (it == best.end()) {
            dad[i] = (best.size() == 0 ? -1 : best.back().second);
            best.push_back(item);
        } else {
            dad[i] = it == best.begin() ? -1 : prev(it)->second;
            *it = item;
        }
    }

    VI ret;
    for (int i = best.back().second; i >= 0; i = dad[i])
        ret.push_back(v[i]);
    reverse(ret.begin(), ret.end());
    return ret;
}

```

6.2 Dates

```

// Routines for performing computations on dates. In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.

#include <iostream>
#include <string>

```

```

using namespace std;

string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
    return
        1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
        d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
    int x, n, i, j;

    x = jd + 68569;
    n = 4 * x / 146097;
    x -= (146097 * n + 3) / 4;
    i = (4000 * (x + 1)) / 1461001;
    x -= 1461 * i / 4 - 31;
    j = 80 * x / 2447;
    d = x - 2447 * j / 80;
    x = j / 11;
    m = j + 2 - 12 * x;
    y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
    return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
    int jd = dateToInt (3, 24, 2004);
    int m, d, y;
    intToDate (jd, m, d, y);
    string day = intToDay (jd);

    // expected output:
    // 2453089
    // 3/24/2004
    // Wed
    cout << jd << endl
         << m << "/" << d << "/" << y << endl
         << day << endl;
}

```

6.3 Regular expressions

```

// Code which demonstrates the use of Java's regular expression libraries.
// This is a solution for
//
// Loglan: a logical language
// http://acm.uva.es/p/v1/134.html
//
// In this problem, we are given a regular language, whose rules can be
// inferred directly from the code. For each sentence in the input, we must
// determine whether the sentence matches the regular expression or not. The
// code consists of (1) building the regular expression (which is fairly
// complex) and (2) using the regex to match sentences.

import java.util.*;
import java.util.regex.*;

public class LogLan {

    public static String BuildRegex () {
        String space = " ";

        String A = "[aeiou]";
        String C = "[a-zA&[aeiou]]";
        String MOD = "(g" + A + ")";
        String BA = "(b" + A + ")";
        String DA = "(d" + A + ")";
        String LA = "(l" + A + ")";
        String NAM = "([a-z]" + C + ")";
        String PREDA = "((" + C + C + A + C + A + "|" + C + A + C + C + A + ")";

        String predstring = "(" + PREDA + "(" + space + PREDA + ")*)";
        String predname = "(" + LA + space + predstring + "|" + NAM + ")";
        String preds = "(" + predstring + "(" + space + A + space + predstring + ")*)";
        String predclain = "(" + predname + space + BA + space + preds + "|" + DA + space +
            preds + ")";
        String verbpred = "(" + MOD + space + predstring + ")";
        String statement = "(" + predname + space + verbpred + space + predname + "|" +

```



```

k++;
if(k == p.length() - 1) {
    // p matches t[i-m+1, ..., i]
    cout << "matched at index " << i-k << ": ";
    cout << t.substr(i-k, p.length()) << endl;
    k = (k == -1) ? -2 : pi[k];
}
}
return 0;
}

int main()
{
    string a = "AABAACAADAABAABA", b = "AABA";
    KMP(a, b); // expected matches at: 0, 9, 12
    return 0;
}

```

6.7 Latitude/longitude

```

/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/

#include <iostream>
#include <cmath>

using namespace std;

struct ll
{
    double r, lat, lon;
};

struct rect
{
    double x, y, z;
};

ll convert(rect& P)
{
    ll Q;
    Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
    Q.lat = 180/M_PI*asin(P.z/Q.r);
    Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

    return Q;
}

rect convert(ll& Q)
{
    rect P;
    P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
    P.z = Q.r*sin(Q.lat*M_PI/180);

    return P;
}

int main()
{
    rect A;
    ll B;

    A.x = -1.0; A.y = 2.0; A.z = -3.0;

    B = convert(A);
    cout << B.r << " " << B.lat << " " << B.lon << endl;

    A = convert(B);
    cout << A.x << " " << A.y << " " << A.z << endl;
}

```

6.8 Fast exponentiation

```

/*
Uses powers of two to exponentiate numbers and matrices. Calculates

```

n^k in $O(\log(k))$ time when n is a number. If A is an $n \times n$ matrix, calculates A^k in $O(n^3 \log(k))$ time.

```

*/

#include <iostream>
#include <vector>

using namespace std;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T power(T x, int k) {
    T ret = 1;

    while(k) {
        if(k & 1) ret *= x;
        k >>= 1; x *= x;
    }
    return ret;
}

VVT multiply(VVT& A, VVT& B) {
    int n = A.size(), m = A[0].size(), k = B[0].size();
    VVT C(n, VT(k, 0));

    for(int i = 0; i < n; i++)
        for(int j = 0; j < k; j++)
            for(int l = 0; l < m; l++)
                C[i][j] += A[i][l] * B[l][j];

    return C;
}

VVT power(VVT& A, int k) {
    int n = A.size();
    VVT ret(n, VT(n));
    for(int i = 0; i < n; i++) ret[i][i] = 1;

    while(k) {
        if(k & 1) ret = multiply(ret, A);
        k >>= 1; A = multiply(A, A);
    }
    return ret;
}

int main()
{
    /* Expected Output:
    2.37^48 = 9.72569e+17

    376 264 285 220 265
    550 376 529 285 484
    484 265 376 264 285
    285 220 265 156 264
    529 285 484 265 376 */
    double n = 2.37;
    int k = 48;

    cout << n << "^" << k << " = " << power(n, k) << endl;

    double At[5][5] = {
        { 0, 0, 1, 0, 0 },
        { 1, 0, 0, 1, 0 },
        { 0, 0, 0, 0, 1 },
        { 1, 0, 0, 0, 0 },
        { 0, 1, 0, 0, 0 } };

    vector<vector<double>> A(5, vector<double>(5));
    for(int i = 0; i < 5; i++)
        for(int j = 0; j < 5; j++)
            A[i][j] = At[i][j];

    vector<vector<double>> Ap = power(A, k);

    cout << endl;
    for(int i = 0; i < 5; i++) {
        for(int j = 0; j < 5; j++)
            cout << Ap[i][j] << " ";
        cout << endl;
    }
}

```