

Venn Diagram

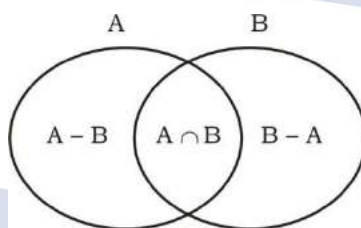
Introduction

Pictorial representation of sets gives most of the ideas about sets and their properties in a much easier way than the representation of sets given in language form. This pictorial representation is done by means of diagrams, known as Venn diagram.

The objects in a set are called the **members** or **elements** of the set.

If $A = \{1, 2, 3, 4, 5, 6\}$, then 1, 2, 3, 4, 5 and 6 are the members or elements of the set A.

If $B = \{x : x \text{ is a positive integer divisible by 5 and } x < 25\}$ or, $B = \{5, 10, 15, 20\}$, then 5, 10, 15 and 20 are the elements of the set B.



$A \cap B$ (read as set A intersection set B) is the set having the common elements of both the sets A and B.

$A \cup B$ (read as set A union set B) is the set having all the elements of the sets A and B.

$A - B$ (read as set A minus set B) is the set having those elements of set A which are not in set B.

In other words, $A - B$ represents the set A exclusively, i.e. $A - B$ have the elements which are only in A.

Similarly, $B - A$ represents the set B exclusively. We keep it in mind that $n(A \cup B) = n(B \cup A)$ and $n(A \cap B) = n(B \cap A)$.

The number of elements of a set A is represented by $n(A)$, but $n(A - B) \neq n(B - A)$

Now, by the above Venn diagram it is obvious that

$$n(A) = n(A - B) + n(A \cap B) \dots\dots\dots (1)$$

$$n(B) = n(B - A) + n(A \cap B) \dots\dots\dots (2)$$

$$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A) \dots\dots\dots (i)$$

Adding (1) and (2) we get,

$$n(A) + n(B) = n(A - B) + n(B - A) + n(A \cap B) + n(A \cap B)$$

$$\text{or, } n(A) + n(B) - n(A \cap B) = n(A - B) + n(B - A) + n(A \cap B) \dots\dots\dots (ii)$$

From (i) and (ii), we have

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \quad (3)$$

Let us see some worked out examples given below:

Solved Examples

Ex. 1: In a class of 70 students, 40 like a certain magazine and 37 like another certain magazine. Find the number of students who like both the magazines simultaneously.

Sol: We have, $n(A \cup B) = 70$,
 $n(A) = 40$, $n(B) = 37$
 Now, $70 = 40 + 37 - n(A \cap B)$
 $\therefore n(A \cap B) = 77 - 70 = 7$.

Ex. 2: In a group of 64 persons, 26 drink tea but not coffee and 34 drink tea. Find how many drink (i) tea and coffee both, (ii) coffee but not tea.

Sol: (i) $n(T \cup C) = 64$, $n(T - C) = 26$, $n(T) = 34$
 We have, $n(T) = n(T - C) + n(T \cap C)$
 or, $34 = 26 + n(T \cap C)$



$$\therefore n(T \cap C) = 34 - 26 = 8$$

(ii) Again, we have

$$n(T \cup C) = n(T) + n(C) - n(T \cap C) \text{ or,}$$

$$64 = 34 + n(C) - 8$$

$$\therefore n(C) = 38$$

$$\text{Now, } n(C) = n(C - T) + n(T \cap C) \text{ or,}$$

$$38 = n(C - T) + 8$$

$$\therefore n(C - T) = 38 - 8 = 30$$

Ex. 3: In a class of 30 students, 16 have opted Mathematics and 12 have opted Mathematics but not Biology. Find the number of students who have opted Biology but not Mathematics.

Sol: $n(M \cup B) = 30$, $n(M) = 16$, $n(M - B) = 12$, $n(B - M) = ?$

We have, $n(M) = n(M - B) + n(M \cap B)$ or,

$$16 = 12 + n(M \cap B)$$

$$\therefore n(M \cap B) = 16 - 12 = 4$$

Again, we have, $n(M \cup B) = n(M) + n(B) - n(M \cap B)$

$$\text{or, } 30 = 16 + n(B) - 4$$

$$\text{or, } n(B) = 30 - 12 = 18$$

Now, $n(B) = n(B - M) + n(M \cap B)$ or,

$$18 = n(B - M) + 4$$

$$\therefore n(B - M) = 18 - 4 = 14$$

Ex. 4: In a class of 70 students, 40 like a certain magazine and 37 like another while 7 like neither.

(i) Find the number of students who like at least one of the two magazines.

(ii) Find the number of students who like both the magazines simultaneously.

Sol: We have, total number of students = 70 in which 7 do not like any of the magazines.

For our consideration regarding liking of magazines, we are left with $(70 - 7 =) 63$ students.

$$\text{Thus, } n(A \cup B) = 63, n(A) = 40, n(B) = 37$$

(i) The number of students who like at least one of the two magazines = $n(A \cup B) = 63$.

(ii) The number of students who like both the magazines simultaneously = $n(A \cap B) = ?$

$$\text{We have, } n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\text{or, } 63 = 40 + 37 - n(A \cap B)$$

$$\therefore n(A \cap B) = 77 - 63 = 14$$

Ex. 5: In a school, 45% of the students play cricket, 30% play hockey and 15% play both. What percent of the students play neither cricket nor hockey?

Sol: $n(C) = 45$, $n(H) = 30$, $n(C \cap H) = 15$

$$\therefore n(C \cup H) = 45 + 30 - 15 = 60$$

ie, 60% of the students play either cricket or hockey or both.

So, the remaining $(100 - 60 =) 40\%$ students play neither cricket nor hockey.

Ex. 6: out of a total of 360 musicians in a club 15% can play all the three instruments — guitar, violin and flute. The number of musicians who can play two and only two of the above instruments is 75. The number of musicians who can play the guitar alone is 73.

(i) Find the total number of musicians who can play violin alone and flute alone.

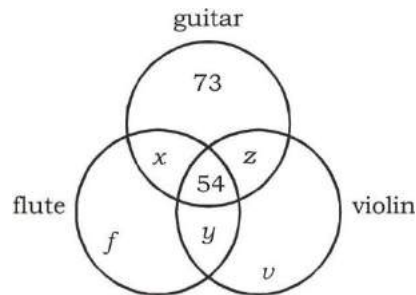
(ii) If the number of musicians who can play violin alone be the same as the number of musicians who can play guitar alone, then find the number of musicians who can play flute.

Sol: (i) Total number of musicians = 360

15% of 360 = 54 musicians can play all the three instruments.



Given that $x + y + z = 75$



Now, $73 + f + v + (x + y + z) = 75 + 54 = 360$
 $\therefore v + f = 360 - (73 + 75 + 54) = 158$

(ii) Now we have $v = 73$

The number of musicians, who can play flute alone,

$$f = (v + f) - v = 158 - 73 = 85$$

and the number of musicians who can play flute =

$$f + x + y + 54 = 85 + 54 + (x + y) \text{ we have } x + y + z = 75, x + y = 75 - z.$$

As either $x + y$ or z is unknown, we cannot find out the number of musicians who can play flute.

Hence, data is inadequate.

LEARNIZY