(a) Test for consistency and solve:- | Dinkar Ass-1

$2x - 3y + 7z = 5 \qquad 3x + y - 3z = 13$

$2x + 19y - 47z = 32$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 32 \\ 13 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad A:B = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 32 \\ 2 & 19 & 47 & : & 13 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 32 \\ 2 & 19 & 47 & : & 13 \end{bmatrix}$$

$R_1 \to -R_1$

$$A:B = \begin{bmatrix} -2 & +3 & -7 & : & -5 \\ 3 & 1 & -3 & : & 32 \\ 2 & 19 & 47 & : & 13 \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$A:B = \begin{bmatrix} 1 & 4 & -10 & : & 27 \\ 3 & 1 & -3 & : & 32 \\ 2 & 19 & 47 & : & 13 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$ Dinkar Thakur B2

$$= \begin{bmatrix} 1 & 4 & -10 & : & 27 \\ 0 & -11 & -27 & : & -39 \\ 2 & 19 & 47 & : & 13 \end{bmatrix}$$

$$\begin{array}{r} 71 \\ \hline 71 \\ 32 \\ \hline 39 \end{array}$$

$R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 4 & -10 & : & 27 \\ 0 & -11 & -27 & : & -39 \\ 0 & -11 & 67 & : & 11 \end{bmatrix}$$

$$\begin{array}{r} 13- \\ -54 \\ \hline 54 \\ 13 \\ \hline 41 \end{array}$$

$$\begin{array}{r} 67 \\ 27 \\ \hline 94 \end{array}$$

$R_3 \to R_3 - R_2$

$$\begin{array}{r} 11 \\ 39 \\ \hline 50 \end{array}$$

$$= \begin{bmatrix} 1 & 4 & -10 & : & 27 \\ 0 & -11 & -27 & : & -39 \\ 0 & 0 & 94 & : & 50 \end{bmatrix}$$

$\rho(A) = \rho(A:B) \to$ consistent

iii) $2x - y + 3z = 8 \quad -x + 2y + z = 4, \quad 3x + y - 4z = 0$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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$$A:B = \begin{bmatrix} 2 & -1 & 3 & : & 8 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}$$

$$R_1 \rightarrow R_1 + R_2$$

$$A:B = \begin{bmatrix} 1 & 1 & 4 & : & 12 \\ -1 & 2 & 1 & : & 4 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 + R_1$$

$$A:B = \begin{bmatrix} 1 & 1 & 4 & : & 12 \\ 0 & 3 & 5 & : & 16 \\ 3 & 1 & -4 & : & 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 1 & 4 & : & 12 \\ 0 & 3 & 5 & : & 16 \\ 0 & 0 & -16 & : & -36 \end{bmatrix}$$

$$\boxed{\begin{array}{c} \rho(A:B = \rho(A) \\ \text{consistent} \end{array}}$$

iii.) $4x - y = 12$ $\quad -2x + 4z = 8$

$x + 5y - 2z = 0$

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$-1 - 15$

$0 - (-6)$

6

$-4 - (-4)$

$16x = -$

$x = -$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -2 & 0 & 4 \\ 1 & 5 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 12 \\ 8 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc:c} 4 & -1 & 0 & 12 \\ -2 & 0 & 4 & 8 \\ 1 & 5 & -2 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 - 3R_3} \left[\begin{array}{ccc:c} 1 & 16 & 6 & 12 \\ -2 & 0 & 4 & 8 \\ 1 & 5 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to -1R_2} \left[\begin{array}{ccc:c} 1 & 16 & 6 & 12 \\ 2 & 0 & -4 & -8 \\ 1 & 5 & -2 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_3} \left[\begin{array}{ccc:c} 1 & 16 & 6 & 12 \\ 0 & -15 & 0 & -8 \\ 1 & 5 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{ccc:c} 1 & 16 & 6 & 12 \\ 0 & -15 & 0 & -8 \\ 0 & -11 & -8 & -12 \end{array}\right] \xrightarrow{R_3 \to R_3 - \frac{11}{15}R_2}$$

$$\left[\begin{array}{ccc:c} 1 & 16 & 6 & 12 \\ 0 & -15 & 0 & -8 \\ 0 & 0 & -8 & -12 \end{array}\right]$$

$\rho(A) = \rho(A:B)$

consistent

(5) $x+y+z=6$ $\quad$ $x+2y+3z=10$ $\quad$ $x+2y+\lambda z=\mu$

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$$A=\begin{bmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{bmatrix}\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix}\quad B=\begin{bmatrix}6\\10\\\mu\end{bmatrix}$$

$$A:B=\begin{bmatrix}1&1&1&:&6\\2&2&3&:&10\\1&2&\lambda&:&\mu\end{bmatrix}$$

$R_3 \to R_3-R_1$

$$\begin{bmatrix}1&1&1&:&6\\2&2&3&:&10\\0&1&\lambda-1&:&\mu-6\end{bmatrix}$$

$$\cancel{\begin{bmatrix}1&1&1&:&6\\0&0&1&:&-2\\0&1&\lambda-1&:&\mu-6\end{bmatrix}}$$

$R_2 \to R_2-2R_1$

$$\begin{bmatrix}1&1&1&:&6\\0&0&1&:&-2\\0&1&\lambda-1&:&\mu-6\end{bmatrix}$$

$R_2 \to R_2+R_3$

$$\begin{bmatrix}1&1&1&:&6\\0&1&\lambda-2&:&\mu-8\\0&1&\lambda-1&:&\mu-6\end{bmatrix} \to R_3\to R_3-R_2$$

$$\begin{bmatrix}1&1&1&:&6\\0&1&\lambda-1&:&\mu-8\\0&0&-4&:&-14\end{bmatrix}$$

here $\to$
$\rho(A)=\rho(A:B)$
Consistent

• No - solution → does not exist.

• unique solution $(\lambda, \mu) \in R$ (Dinkar Thakur)

• Infinite soln. does not exist.

ⓒ $x+y+z=1$ $\quad x+4y+10z=\lambda^2.$

$x+2y+4z=\lambda$

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & 4 & 10\\ 1 & 2 & 4\end{bmatrix} \quad B=\begin{bmatrix}1\\ \lambda\\ \lambda^2\end{bmatrix} \quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$A:B=\left[\begin{array}{ccc:c}1 & 1 & 1 & 1\\ 1 & 4 & 10 & \lambda\\ 1 & 2 & 4 & \lambda^2\end{array}\right] \xrightarrow{R_2\to R_2-R_1} \left[\begin{array}{ccc:c}1 & 1 & 1 & 1\\ 0 & 3 & 9 & \lambda-1\\ 1 & 2 & 4 & \lambda^2\end{array}\right]$$

$$R_3\to R_3-R_1 \left[\begin{array}{ccc:c}1 & 1 & 1 & 1\\ 0 & 3 & 9 & \lambda-1\\ 0 & 1 & 3 & \lambda^2-1\end{array}\right] \xrightarrow{R_3\to 3R_3} \left[\begin{array}{ccc:c}1 & 1 & 1 & 1\\ 0 & 3 & 9 & \lambda-1\\ 0 & 3 & 9 & 3\lambda^2-3\end{array}\right]$$

$$R_3\to R_3-R_2 \left[\begin{array}{ccc:c}1 & 1 & 1 & 1\\ 0 & 3 & 9 & \lambda-1\\ 0 & 0 & 0 & 3\lambda^2-3-\lambda+1\end{array}\right]$$

$$A:B.=\left[\begin{array}{ccc:c}1 & 1 & 1 & 1\\ 0 & 3 & 9 & \lambda-1\\ 0 & 0 & 0 & 3\lambda^2-\lambda-2\end{array}\right]$$

$\rho(A) = 2.$

$\rho(A:B)$ if $3\lambda^2 - \lambda - 2 = 0 \Rightarrow 2$

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then consistent.

for consistency $= .\ 3\lambda^2 - \lambda - 2 = 0$

$(\lambda - 1)(\lambda + \frac{2}{3}) = 0.$

for consistency $\lambda = 1, -\frac{2}{3}$

$\boxed{\text{Unique, Infinite sol}^n}$

For

for inconsistency.

$$\boxed{\lambda \in R - \{1, -\tfrac{2}{3}\}}$$

(d) $x + 3y - 2z = 0,\ 2x - y + 4z = 0,\ x - 11y + 14z = 0$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 3 & -2 & : & 0 \\ 2 & -1 & 4 & : & 0 \\ 1 & -11 & 14 & : & 0 \end{bmatrix}$$

$$A:B = \left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right] \quad R_2 \to R_2 - 2R_1$$

Dinkar

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right] \quad R_3 \to R_3 - R_1 \quad \left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & +4 & 6 & 0 \end{array}\right]$$

$$R_3 \to R_3 + R_2 \quad \left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -3 & 14 & 0 \end{array}\right] \quad R_3 \to R_3 - \frac{3}{7} R_2$$

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & 0 & 10.5 & 0 \end{array}\right] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\boxed{z = 10.5}$

$x + 3y - 2z = 0$

$-7y + 8z = 0$

$x + 36 - 21 = 0$

$\boxed{x = -15}$

$-7y + 84 = 0$

$+\cancel{7}y = +84$

$\boxed{y = 12}$

Assignment - II

Are following sets of vectors linearly indepent OR linearly dependent.

(Dinkar)

i) $\underset{\hookrightarrow v_1}{[1, 0, 0]}$, $\underset{\hookrightarrow v_2}{[1, 1, 0]}$, $\underset{\hookrightarrow v_3}{[1, 1, 1]}$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1(1, 0, 0) + c_2(1, 1, 0) + c_3(1, 1, 1) = (0, 0, 0)$

$c_1 + c_2 + c_3 = 0$

$0 + c_2 + c_3 = 0$

$0 + 0 + c_3 = 0$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) = n \rightarrow$ unique soln

lineary Indepedent.

② $\underset{\hookrightarrow v_1}{[7\ -3\ 11\ -6]}$ $\underset{\hookrightarrow v_2}{[-56\ 24\ -88\ 48]}$

$c_1 v_1 + c_2 v_2 = 0$

$c_1(7, -3, 11, -6) + c_2(-56\ 24\ -88\ 48)$

(3.) $[-1, 5, 0]\ [16, 8, -3]\ [-64, 56, 9]$

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$$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$$

$$c_1(-1, 5, 0) + c_2(16, 8, -3) + c_3(-64, 56, 9) = (0, 0, 0)$$

$$(-c_1, 5c_1, 0) + (16c_2, 8c_2, -3c_2)$$
$$+ (-64c_3, 56c_3, 9c_3) = (0, 0, 0)$$

$$-c_1 + 16c_2 - 64c_3 = 0$$
$$5c_1 + 8c_2 + 56c_3 = 0$$
$$0 + -3c_2 + 9c_3 = 0$$

$$A = \begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}$$

$$R_2 \to R_2 + 5R_1$$

$$A = \begin{bmatrix} -1 & 16 & -64 \\ 0 & 88 & -264 \\ 0 & -3 & 9 \end{bmatrix}$$

(4.) $[1,-1,-1]\ [1,1,-1]\ [1,1,1],\ [0,1,1]$

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$\Rightarrow c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = [0,0,0,0]$

$\Rightarrow c_1(1,-1,-1) + c_2(1,1,-1) + c_3(1,1,1) + c_4(0,1,1)$
$= (0,0,0,0)$

$= (c_1, -c_1, -c_1) + (c_2, c_2, -c_2) + (c_3, c_3, c_3) +$
$(0, c_4, c_4) \equiv (0,0,0,0)$

$$\begin{cases} c_1 + c_2 + c_3 + 0 = 0 \\ -c_1 + c_2 + c_3 + c_4 = 0 \\ -c_1 - c_2 + c_3 + c_4 = 0 \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \Rightarrow R_2 \to R_2 + R_1$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 2 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} R_3 \to R_3 + R_1 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$\rho(A) = 3 \neq n \to$ infinite soln.

linearly dependent.

(5) $[2,-4], [1,9], [3,5]$

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$c_1v_1 + c_2v_2 + c_3v_3 = 0$

$c_1(2,-4) + c_2(1,9) + c_3(3,5) = 0$

$2c_1 + 2c_2 + 3c_3 = 0$ No-solution.

$-2c_1 + 9c_2 + 5c_3 = 0$

Assignment-3

Dinkar

$|A - \lambda I| = 0$

(1) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$$\left| \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0.$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$-(2+\lambda)\left[\lambda^2 - \lambda - 12\right] - (2)\left[-2\lambda - 6\right] - (3)\left[-4 + (1-\right.$

$-(2\cancel{\lambda^2} - \cancel{2\lambda} + 24 + \cancel{\lambda^3} - \cancel{\lambda^2} - \cancel{12\lambda}) + 4\cancel{\lambda} + 12 + 12 - 3 + 3\lambda$

$-\lambda^3 - \lambda^2 + 21\lambda - 3 = 0 \Rightarrow \lambda^3 + \lambda^2 - 21\lambda + 3 = 0$

eigenvalues $\boxed{\lambda = -5.4, -0.14, 4}$

$\underline{\underline{\lambda = 4}}$

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$$\left\{ \begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\left\{ \begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right\} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -6 & 2 & 3 \\ 2 & -3 & -6 \\ -1 & -2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-6x + 2y + 3z = 0$ — (1)

$2x - 3y - 6z = 0$ — (11)

$-x - 2y - 4z = 0$

$-6\cancel{x} + 2y + 3z = 0$

$6\cancel{x} - 9y - 18z = 0$

$-7y - 15z = 0$

$-7y = -15z$

$y = \frac{-15}{7} z$

(Dinkar Thakur)

(2.) $A \Rightarrow \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$A - \lambda I \Rightarrow \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$

$\Rightarrow (4-\lambda)((1-\lambda)^2) + 1(2(1-\lambda)) = 0$

$\Rightarrow (4-\lambda)(1-\lambda)^2 + 2(1-\lambda) = 0$

$(1-\lambda)[(4-\lambda)(1-\lambda) + 2] = 0$

$(1-\lambda)[6 - 5\lambda + \lambda^2] = 0$

$(\lambda-1)(\lambda-2)(\lambda-3) = 0$

~~eg~~ eigen Values $= \lambda = 1, 2, 3.$

for $\lambda = 1$

$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$

$-2x = 0$ $\boxed{x = 0, z = 0, y = k}$

$3x + z = 0$

eigen Vectors $= k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda=2$

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$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

$-2x-y=0 \qquad -2x-z=0$

$2x=+y \qquad z \Rightarrow -2x.$

$x=k,\ y \Rightarrow -2k,\ z \Rightarrow -2k$

eigenvector $\Rightarrow k\begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$

for $\lambda=3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}$$

$-2x-2z \Rightarrow 0$

$x \Rightarrow -z$

$-2x-2y \Rightarrow 0$

$x=-y$

eigenvectors $= k\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

3.) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$ $\qquad A-\lambda I \Rightarrow \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$

$(5-\lambda)((-\lambda)(3-\lambda))=0$

$\lambda=5,0,3$

for $\lambda=5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

$-5y=0 \qquad -x-2z=0$

$y \Rightarrow 0 \qquad x=2z$

$x=2k,\ y=0,\ z=k$

Eigen vectors $= k\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ Ans

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for $\lambda=0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$x=0$
$-x+3z=0$
$z=0$

$x=0,\ y=k,\ z=0$

eigen vector $= k\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda=3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$y=0$
$x=0$
$z=k$

eigen vector $\Rightarrow k\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Linear Algebra

Assignment-4

Q.1 finding rank of the matrix.

Dinkar Thakur

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

$R_2 \to -\frac{1}{3} R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

$R_3 \to R_3 + 4R_2$

$R_4 \to R_4 + 4R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -4 & 1 \\ 0 & 0 & -1 & 3 \end{bmatrix}$$

$R_3 \to -\frac{1}{4} R_3$.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 1 & -1/4 \\ 0 & 0 & -1 & 3 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

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$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{12} \end{bmatrix}$$

Rank = no. of non-zero Rows in echleon for i.e = 4.

2. $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

find Rank & nullity of T.

Dinkar thakur

(3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

for eigen value $\Rightarrow A - \lambda I = 0$

$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$

$(2-\lambda)(2-\lambda) - 1 = 0$

$(2-\lambda)^2 - 1 = 0$

$(\lambda-1)(\lambda-3) = 0$ so, the eigen values

are $= 1 \,\&\, 3.$

* for $\lambda = 1$

$[A - I]\, x = 0$ $\quad \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$x_1 - x_2 = 0$

$-x_1 + x_2 = 0.$ $\quad k \begin{bmatrix} 1 \\ +1 \end{bmatrix}$

eigenvector

* for $\lambda = 3.$ $\quad |A - 3I|\, x = 0$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$-x_1 - x_2 = 0$

$-x_1 = x_2 = k$

$x_1 = -k$

$x = k$ $\quad = \begin{bmatrix} -k \\ k \end{bmatrix} = k \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

eigen vector.

A^{-1} for A, the eigen values are 1 & 3
the eigen values of A^{-1} are $\frac{1}{1}$ & $\frac{1}{3}$.
$\left(1, \frac{1}{3}\right)$ An

for $A+4I \rightarrow A+4I$ is $1+4=5$
$3+4=7$

(3, 4) An

Dinkar Thakur

(4.) $3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = 19.3$
$0.3x - 0.2y + 10z = 71.4.$

$$\boxed{x_0 = 0 \ , \ y_0 = 0 \ , \ z_0 = 0}$$

Iteration-1

$$3x_1 = 7.85 + 0.1y + 0.2z$$

$$x_1 = \frac{7.85 + 0.1\cancel{y}^{\,0} + 0.2\cancel{z}^{\,0}}{3}$$

$$x_1 = \frac{7.85}{3} = 2.6167$$

$$y_1 = \frac{19.3 + 0.3\cancel{z}^{\,0} - 0.1\cancel{x}^{\,0}}{7}$$

$$y_1 = \frac{19.3}{7} = 2.757$$

$$z_1 = \frac{71.4 - 0.3\cancel{x}^{\,0} + 0.2\cancel{y}^{\,0}}{10}$$

$$z_1 = 7.1$$

$$(x_1, y_1, z_1) = (2.61, 2.75, 7.1)$$

Iteration-2

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$$x_2 = \frac{7.85 + 0.1(2.75) - 0.2(7.1)}{3} \simeq 2.9$$

$$y_2 = \frac{19.3 - 0.1(2.6) + 0.3(7.1)}{7} = 3.01.$$

$$z_2 = \frac{71.4 - 0.3(2.61) + 0.2(2.75)}{10} \simeq 7.01$$

$$(x_2, y_2, z_2) \equiv (2.9, 3.01, 7.01)$$

Iteration-3

$$x_3 = \frac{7.85 + 0.1(2.9) - 0.2(7.01)}{3} = 3.00$$

$$y_3 = \frac{19.3 - 0.1(3.0032) - 0.3(7.01)}{7} = 3.001$$

$$z_3 = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10} = 7.00$$

$$(x_3, y_3, z_3) \equiv (3.0032, 3.0001, z = 7.00)$$

(5) check for consistent and inconsistent system of eqn and hence solve the following system of eqn (Dinkar Thakur)

$x+3y+2z=0$, $2x-y+3z=0$

$3x-5y+4z=0$

$x+17y+4z=0$

for inconsistent $\rho(A) \neq \rho(A:B)$

for consistent $\rho(A) = \rho(A:B)$

$Ax=0$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

After performing row reduction.

$$\left[\begin{array}{ccc:c} 1 & 3 & 2 & 0 \\ 0 & -7 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$, $\rho(A:B) = 2$

so, it is consistent

The corresponding system of eqn are:-

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$x+3y+2z=0$

$-7y-z=0$

let take $y=t$

$x=-3t$

$y=t$ consistent solupaul.

$z=-7t$

(7.) $S=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 1 & 3 \end{bmatrix}$

converting into echelon form:-

$R_2 \to R_2-2R_1 \quad R_3 \to R_3-3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$R_3 \leftrightarrow R_3+\frac{9}{5}R_2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

determinat of matrix = 0

So, no unique soln

~~there~~ ~~for~~ therefore not a LOR so,
can span R^3,

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(8) $3x - 6y + 2z = 23$

$-4x + y - z = -15$

$x - 3y + 7z = 16$

$(x_0, y_0, z_0) = (1, 1, 1)$

Itt-1 $x_1 = \dfrac{23 + 6y_0 - 2z_0}{3} = 8.0$

$y_1 = \dfrac{-15 + 4x_0 + z_0}{1} = -9.0$

$z_1 = \dfrac{16 - x_0 + 3y_0}{7} = 2.0$

Itt-2

$x_2 = \dfrac{23 + 6y_1 - 2z_1}{3} = 5$

$y_2 = \dfrac{-15 + 4x_1 + z_1}{1} = -5.0$

$z_2 = \dfrac{16 - x_1 + 3y_1}{7} = 3.0$

Itt-3

$x_3 = \dfrac{23 + 6y_2 - 2z_2}{3} = 6.0$

$y_3 = \dfrac{-15 + 4x_2 + z_2}{1} = -6.0$

$z_3 = \dfrac{16 - x_2 + 3y_2}{7} = 2.0$