Cap-and-trade Model for the Chilean Energy Market

Pía Amigo, Sebastián Cea, Felipe Feijoo

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1 Motivation

Cconsiderar un mercado de cap-and-trade como opción a incrementar el tax en chile[1]

2 Model

Let us consider a market of i producers, each of them minimizing the cost of operation and investment. We will assume perfect competition among the producers. Our model will assume two basic stages: operation at present (with investment for the future) and the future, at the spot market and trading of emission allowances.

At first stage, we consider the current operating plant and the cost of the operation C_i, and the quantity produce Q_i^τ with $\tau = 1$. The price of sale (that satisfies the demand) at this stage is $\pi_i^{d,\tau}$ with $\tau = 1$. The maximum quantity of production at current stage (initial) per producer is $\bar{Q}_i$. Each producer buys a certain amount of carbon A_i at a price π^a from an auctioner of regulatory agent. The auctioneer will try to maximize the revenue obtained from the sold allowances. At this stage, the producer i decides the amount x_i of capacity expansion at a cost I_i for the uncertain future.

At second stage, in the uncertain future, the producer wants to operate in the most efficient way at the spot market. In addition, we will consider the existence of a permit trading market, where producers can purchase permits from other producers if they need to surpass the emission allowances, or sell the unused permits if their emissions are below allowances. The producer will evaluate the cost of the uncertain future as risk neutral agents, i.e., as an expectation value and we will assume that the probability is a physical probability, which may be obtained from real world data.

The maximum budget for emissions is the parameter CAP, which in turn, follows a normal distribution, i.e., $CAP \sim N(\mu, \sigma^2)$. This CAP is determined by the auctioneer and accounts for the total emissions throughout the whole period considered, that is, does not depend of τ.

The amount of total allowances provided by the auctioneer θ is an endogenous random variable in our model, and it must satisfied the condition $\sum_i A_i = \theta$. Since θ must not surpass the budget for emissions, we consider a low margin (or risk) R for this to happen, i.e.,

$$P(\theta \geq CAP) \leq R \tag{1}$$

and since the variable CAP follows a normal distribution with mean μ and variance σ^2, we can write the equation above as

$$\theta \leq \phi^{-1}(R)\sigma^2 + \mu \tag{2}$$

[1] pag 64. https://mma.gob.cl/wp-content/uploads/2015/09/hojaderuta.pdf

where ϕ^{-1} is the inverse of the cumulative function of the normal distribution.

2.1 A slightly more complicated model

We make things a little more general by considering more detailed aspects:

- We consider that at first stage, the plant is operating and fulfiling the demand at first stage, at a price $\pi^{d,1}$
- Producers can invest in capacity expansion not only at first stage, but also at any point during second stage.
- Producers can change their plant's technology to maintain the production

2.2 Producer's problem

Each producer i represents one and only one technology in the economy. The choice set of the producer i over a time horizon of $\bar{t}$ years is given by: i) an amount of capacity expansion $x_i \in \mathbb{R}_+$ over an installed capacity amount of $\bar{Q}_i$, ii) a production plan $Q_i := \left(Q_i(0), (Q_i(t,\omega)_{(t,\omega)\in T:=\{1,\dots,\bar{t}\}\times\Omega}\right) \in \mathbb{R}_+ \times \mathbb{R}_+^{T\times\Omega}$, iii) allowances bought in the first period $A_i \in \mathbb{R}_+$, iv) allowances bought in the second period $P_i \in \mathbb{R}_+^{\Omega}$ and v) allowances sold in the second period $V_i \in \mathbb{R}_+^{\Omega}$.

We define the cost function of the first period by

$$C_i(0,\pi^d(0),Q_i(0)) = \underbrace{\left(a_iQ_i(0) + \frac{b}{2}Q_i(0)^2\right) - \pi_d(0)Q_i(0)}_{\left(C_t^{comb}\cdot C^{esp} + CVnC\right)Q_{i,t} + \underbrace{FixedCost_{i,t}}_{\frac{\alpha\cdot I\cdot x}{algo}}}$$

and for the rest of the periods

$$C_i(t,\pi^d(t),Q_i(t,\omega)) = \left(a_iQ_i(t) + \frac{b}{2}Q_i(t)^2\right) - \pi_d(t)Q_i(t,\omega)$$

$$\begin{aligned}\min_{(x_i,Q_i,A_i,P_i,V_i)\in(\mathbb{R}_+^3\times\mathbb{R}_+^{T\times\Omega}\times\mathbb{R}_+^{2\Omega})} & C_i(0) + A_i\pi_a + I_ix_i^1 + \sum_{\Omega} prob_\omega\Bigg(\sum_{t>1}\frac{1}{(1+R)^t}\Big[(TC_{i,t}C_i(t) \\ & + \sum_{t>1} TCR_{i,t}\cdot I_ix_{i,w}^t\Big] - V_{i,\omega}\pi_{v,i,w} + \sum_{j\neq i} P_{i,j,\omega}\pi_{v,j,w}\Bigg) \qquad (3) \\ & \text{s.t}\end{aligned}$$

2.3 Equilibrium

2.4 Computation

Set	Explanation
i	Producers (Technologies)
ω	Possible scenarios
t	Periods

For this model, each producer corresponds to a different technology present in the chilean electricity generation system, i.e., Biomasa, Carbon, Eolica, Gas, Geotermica, HidroEmbalse, HidroPasada, HidroMiniPasada, PetroleoDiesel, Solar.

Parameter	**Suggested**	**Units**	**Explanation**
I_i		USD/MW	Expansion cost per technology i
$\bar{Q}_i$	$\bar{X}_i$	MW	Maximum operation capacity per technology i
C_i		USD/MWh	Operation cost per technology i
D^1		MWh	Total demand in first stage
D^t_ω		MWh	Total demand in period $t > 1$
ε_i		tCO_2/MWh	Emission factor of technology i
Prob_ω	Π_ω		Physical probability of scenario ω
μ		tCO_2	Mean of the normal distribution of the CAP
σ		tCO_2	Standard deviation of the normal distribution of the CAP
ϵ			Margin for total emission allowances
R	**df**		Discount rate
TC_i			Change in the cost of operation per technology
TCR_i			Change in the investment cost per technology
CF_i			Capacity factor per technology
τ		hours	Number of hours in a year, $\tau = 8760$ hours
RP_i		MW	Resource potential per technology

Variable	**Suggested**	**Units**	**Explanation**
Q^1_i	Y^1_i	MWh	produced quantity in period $t = 1$ for producer i
$Q^t_{i,\omega}$	$Y^t_{i,\omega}$	MWh	produced quantity in period $t > 1$ for producer i in scenario ω
A_i	θ_i	tCO_2	Emission allowances purchased by producer i
$P_{i,j,\omega}$	B_i	tCO_2	Purchased permits in trading market by producer i
$V_{i,\omega}$		tCO_2	Sold permits in trading markets
π^a	P^a	USD/tCO_2	Price of the allowances offered by the auctioneer (dual to CAP constrai
$\pi^v_{i,\omega}$	$P^v_{i,\omega}$	USD/tCO_2	Price of the permits in trading market (dual to trading equilibrium)
$\pi^{d,1}$	$P^{d,1}$	USD/MWh	Price of electricity (dual to fulfilment of the demand –first stage)
$\pi^{d,t}_\omega$	$P^{d,t}$	USD/MWh	Price of electricity (dual to fulfilment of the demand)
x^1_i		MW	Capacity expansion decision for period $t = 1$ of producer i
$x^t_{i,\omega}$		MW	Capacity expansion decision for period t of producer i in scenario ω
θ	Θ	tCO_2	Emission allowances available in the market by the auctioneer

Producer's problem (each producer is a different technology):

$$\min_{x,Q,A,P,V} \underbrace{\left(a_iQ_{1,i}+\frac{b}{2}Q_{1,i}^2\right)-\pi^{d,1}Q_{1,i}}_{(C_t^{comb}\cdot C^{esp}+CVnC)Q_{i,t}+\underbrace{FixedCost_{i,t}}_{\frac{\alpha\cdot I\cdot x}{algo}}} +A_i\pi^a+I_ix_i^1+\sum_{\Omega}prob_\omega\Bigg(\sum_{t>1}\frac{1}{(1+R)^t}\Big[(TC_{i,\tau}\cdot(a_iQ_{i,t,\omega}+\frac{b}{2}Q_{i,t,\omega}^2)$$

$$-\pi_\omega^{d,2}Q_{i,t,\omega}+\sum_{t>1}TCR_i\cdot I_ix_{i,w}^t\Big]-V_{i,\omega}\pi_{i,w}^v+\sum_{j\neq i}P_{i,j,\omega}\pi_{j,w}^v\Bigg) \tag{4}$$

s.t

$$\Big(CF_i\cdot\tau\Big)\left[\bar{Q}_i+\sum_{t'<t_{y_i}}x_{i,w}^{t'}+x_i^1+\bar{Q}_{i,t}\right]-Q_{i,\omega}^t\geq 0 \qquad \forall\quad \omega,t>1 \quad (\alpha_{i,\omega,t}) \tag{5}$$

$$\Big(CF_i\cdot\tau\Big)\bar{Q}_i-Q_i^t\geq 0 \qquad t=1 \quad (\kappa_i) \tag{6}$$

$$A_i-V_{i,\omega}\geq 0 \qquad \forall\quad \omega \quad (\beta_{i,\omega}) \tag{7}$$

$$A_i+\sum_{j\neq i}P_{i,j,\omega}-V_{i,\omega}-\sum_{t>1}Q_{i,\omega}^t\varepsilon_i-Q_i^1\varepsilon_i\geq 0 \qquad \forall\quad \omega \quad (\gamma_{i,\omega}) \tag{8}$$

$$Q_i^1\geq 0 \qquad (\lambda_i) \tag{9}$$

$$Q_{i,\omega}^t\geq 0 \qquad \forall\quad \omega,t>1 \quad (\delta_{i,\omega}) \tag{10}$$

$$x_i^1\geq 0 \qquad (\xi_i) \tag{11}$$

$$x_{i,\omega}^t\geq 0 \qquad \forall\quad \omega,t>1 \quad (\varphi_{i,\omega,t}) \tag{12}$$

$$RP_i-\bar{Q}_i-\bar{Q}_{i,t}-x_i^1-\sum_{t>1}x_{i,w}^t\geq 0 \qquad \forall\quad i,\omega \quad (\psi_{i,\omega}) \tag{13}$$

(**FELIPE**: Note that $C_i(Q_i)$ represents the marginal cost function of producer i. We consider quadratic marginal cost functions of the form $C_i(Q_i)=a+\frac{b}{2}Q_i$. Hence, I have rewritten the model formulation to account for this change)

The units of each parameter and variable in Equation (4) must be suitable such that the objective function has units of cost (us\$). In Equations (5) and (6) the factor $CF_i\cdot\tau$ is added to take into account the real production of electricity.

Auctioneer's problem:

$$\min_{\theta} -\theta\pi^a \tag{14}$$

$$\text{s.t}\quad \phi^{-1}(\epsilon)\sigma^2+\mu-\theta\geq 0 \tag{15}$$

$$\theta\geq 0 \tag{16}$$

Market clearing constraints:

$$\text{(available allowances)}: \quad \sum_i A_i = \theta \qquad (\pi^a) \tag{17}$$

$$\text{(equilibrium in trading market)}: \quad \sum_{i,j\neq i} P_{j,i,\omega} = \sum_i V_{i,\omega} \qquad \forall\ \omega \quad (\pi^v) \tag{18}$$

$$\text{(fulfillment of the demand –first stage)}: \quad \sum_i Q_i^1 = D^1, \qquad (\pi^{d,1}) \tag{19}$$

$$\text{(fulfillment of the demand –second stage)}: \quad \sum_i Q_{i,\omega}^t = D_\omega^t, \qquad \forall\ \omega, \tau \quad (\pi_\omega^{d,t}) \tag{20}$$

The Lagrangian function for the producer's problem of each $i \in \{1, ..., n\}$ is:

$$\begin{aligned}
\mathcal{L}_i(x, Q, A, P, V) =& (a_i Q_{1,i} + \frac{b}{2} Q_{1,i}^2) - \pi^{d,1} Q_{1,i} + A_i \pi^a + I_i x_i^1 + \sum_\Omega prob_\omega \Bigg(\sum_{t>1} \frac{1}{(1+R)^t} \Big[(TC_i \cdot (a_i Q_{i,t,\omega} + \frac{b}{2} Q_{i,t,\omega}^2) \\
& - \pi_\omega^{d,2}) Q_{i,t,\omega} \\
& + \sum_{t>1} TCR_i \cdot I_i x_{i,\omega}^t \Big] - V_{i,\omega} \pi_{i,\omega}^v + \sum_{j\neq i} P_{i,j,\omega} \pi_{j,\omega}^v \Bigg) + \kappa_i \Big[Q_i^1 - \Big(CF_i \cdot \tau \Big) \bar{Q}_i \Big] \\
& + \sum_{\omega, t>1} \alpha_{i,\omega,t} \Bigg[Q_{i,\omega}^t - \Big(CF_i \cdot \tau \Big) \Big(\bar{Q}_i + \bar{Q}_{i,t} + \sum_{t'<=t} x_{i,w}^t + x_i^1 \Big) \Bigg] + \sum_\omega \beta_{i,\omega} \Big[V_{i,\omega} - A_i \Big] \\
& + \sum_\omega \gamma_{i,\omega} \Big[- A_i - \sum_{j\neq i} P_{i,j,\omega} + V_{i,\omega} + \sum_{t>1} Q_{i,\omega}^t \varepsilon_i + Q_i^1 \varepsilon_i \Big] \\
& - \sum_{\omega, t>1} \delta_{i,\omega,t} Q_{i,\omega}^t - \lambda_i \Big[Q_i^1 \Big] - \sum_{\omega, t>1} \varphi_{i,\omega,t} x_{i,\omega}^t - \xi_i x_i^1 + \sum_\omega \psi_{i,\omega} \Big[\bar{Q}_i + \bar{Q}_{i,t} + x_i^1 + \sum_{t>1} x_{i,w}^t - RP_i \Big]
\end{aligned} \tag{21}$$

where $\alpha_{i,\omega,\tau}$, κ_i, $\beta_{i,\omega}$, $\gamma_{i,\omega}$ and $\delta_{i,\omega}$ λ_i, $\varphi_{i,\omega,t}$ ad ξ_i are the lagrange multipliers of the constraints.

The KKT conditions of the producer's problem are then

$$I_i + \sum_{\omega} \psi_{i,\omega} - \sum_{\omega,\tau>1} \alpha_{i,\omega,\tau} - \xi_i = 0 \qquad \forall i \qquad (x_i^1) \tag{22}$$

$$prob_\omega \left[\frac{1}{(1+R)^t} TCR_i \cdot I_i - \varphi_{i,\omega,t} \right] - \sum_{t>t\prime} \alpha_{i,\omega,t}(CF_i \cdot \tau) + \psi_{i,\omega} = 0 \quad \forall i,\omega,t>1 \qquad (x_{i,\omega}^t) \tag{23}$$

$$(a_i + b_i Q_i^1) - (\pi^{d,1}) + \kappa_i - \lambda_i + \sum_{\omega} \gamma_{i,\omega}\varepsilon_i = 0 \qquad \forall i \qquad (Q_i^1) \tag{24}$$

$$prob_\omega \left(\frac{1}{(1+R)^t} \right) ((TC_i \cdot a_i + b_i Q_{i,\omega}^\tau) - \pi_\omega^{d,t}) + \alpha_{i,\omega,\tau} + \gamma_{i,\omega}\varepsilon_i - \delta_{i,\omega,\tau} = 0 \quad \forall i,\omega,\tau>1 \qquad (Q_{i,\omega}^\tau) \tag{25}$$

$$\pi^a - \sum_{\omega} \beta_{i,\omega} - \sum_{\omega} \gamma_{i,\omega} = 0 \qquad \forall i \qquad (A_i) \tag{26}$$

$$-prob_\omega \pi_{i,\omega}^v + \beta_{i,\omega} + \gamma_{i,\omega} = 0 \qquad \forall i,\omega \qquad (V_{i,\omega}) \tag{27}$$

$$prob_\omega \pi_{j,\omega}^v - \gamma_{i,\omega} = 0 \quad \forall i, j \neq i, \omega \qquad (P_{i,j,\omega}) \tag{28}$$

Primal feasibility

$$\Big(CF_i \cdot \tau\Big) \left[\bar{Q}_i + \sum_{t \le t'} x_{i,w}^t + x_i^1 \right] - Q_{i,\omega}^t \ge 0 \qquad \forall \quad \omega, t>1 \quad (\alpha_{i,\omega,t}) \tag{29}$$

$$\Big(CF_i \cdot \tau\Big) \bar{Q}_i - Q_i^t \ge 0 \qquad t=1 \quad (\kappa_i) \tag{30}$$

$$A_i - V_{i,\omega} \ge 0 \qquad \forall \quad \omega \quad (\beta_{i,\omega}) \tag{31}$$

$$A_i + \sum_{j \neq i} P_{i,j,\omega} - V_{i,\omega} - \sum_{t>1} Q_{i,\omega}^t \varepsilon_i - Q_i^1 \varepsilon_i \ge 0 \qquad \forall \quad \omega \quad (\gamma_{i,\omega}) \tag{32}$$

$$Q_i^1 \ge 0 \qquad (\lambda_i) \tag{33}$$

$$Q_{i,\omega}^t \ge 0 \qquad \forall \quad \omega, t>1 \quad (\delta_{i,\omega}) \tag{34}$$

$$x_i^1 \ge 0 \qquad (\xi_i) \tag{35}$$

$$x_{i,\omega}^t \ge 0 \qquad \forall \quad \omega, t>1 \quad (\varphi_{i,\omega,t}) \tag{36}$$

$$RP_i - \bar{Q}_i - x_i^1 - \sum_{t>1} x_{i,w}^t \ge 0 \qquad \forall \quad i,\omega \quad (\psi_{i,\omega}) \tag{37}$$

Complementary slackness

$$\left(\left(CF_i \cdot \tau\right)\left[\bar{Q}_i + \sum_{t \leq t'} x_{i,w}^t + x_i^1\right] - Q_{i,\omega}^t\right) \cdot \alpha_{i,\omega,\tau} = 0 \qquad \forall \quad \omega, t > 1 \tag{38}$$

$$\left(\left(CF_i \cdot \tau\right)\bar{Q}_i - Q_i^t\right) \cdot \kappa_i = 0 \qquad t = 1 \tag{39}$$

$$\left(A_i - V_{i,\omega}\right) \cdot \beta_{i,\omega} = 0 \qquad \forall \quad \omega \tag{40}$$

$$\left(A_i + \sum_{j \neq i} P_{i,j,\omega} - V_{i,\omega} - \sum_{t>1} Q_{i,\omega}^t \varepsilon_i - Q_i^1 \varepsilon_i\right) \cdot \gamma_{i,\omega} = 0 \qquad \forall \quad \omega \tag{41}$$

$$\left(Q_i^1\right) \cdot \lambda_i = 0 \tag{42}$$

$$\left(Q_{i,\omega}^\tau\right) \cdot \delta_{i,\omega} = 0 \qquad \forall \quad \omega, t > 1 \tag{43}$$

$$\left(x_i^1\right) \cdot \xi_i = 0 \tag{44}$$

$$\left(x_{i,\omega}^t\right) \cdot \varphi_{i,\omega,t} = 0 \qquad \forall \quad \omega, t > 1 \tag{45}$$

$$\left(RP_i - \bar{Q}_i - x_i^1 - \sum_{t>1} x_{i,w}^t\right) \cdot \psi_{i,\omega} = 0 \qquad \forall \quad i, \omega RP \tag{46}$$

Dual Feasibility

$$\alpha_{i,\omega,\tau} \geq 0 \tag{47}$$
$$\kappa_i \geq 0 \tag{48}$$
$$\beta_{i,\omega} \geq 0 \tag{49}$$
$$\gamma_{i,\omega} \geq 0 \tag{50}$$
$$\lambda_i \geq 0 \tag{51}$$
$$\delta_{i,\omega,t} \geq 0 \tag{52}$$
$$\xi_i \geq 0 \tag{53}$$
$$\varphi_{i,\omega,t} \geq 0 \tag{54}$$
$$\psi_{i,\omega} \geq 0 \tag{55}$$

For the Auctioneer, the Lagrangian function is

$$\mathcal{L}(\theta) = -\theta\pi^a + \eta(\phi^{-1}(\epsilon)\sigma^2 + \mu - \theta) - \zeta\theta \tag{56}$$

where η and ζ are the Lagrange multipliers. Thus, we can obtain the KKT conditions for the auctioneer problem.

$$-\pi^a + \eta - \zeta = 0 \tag{57}$$
$$\theta \geq 0 \tag{58}$$
$$(\phi^{-1}(R)\sigma^2 + \mu - \theta)\eta = 0 \tag{59}$$
$$\eta \geq 0 \tag{60}$$
$$\zeta \geq 0 \tag{61}$$

3 Chile's commitment in the Paris Agreement[2]

- By 2030: 30% below 2007 GHG emission (uncondicional target)
- By 2035: 60% electricity production from renewable energy
- By 2050: 70% electricity production from renewable energy
- By 2024: close eight of the oldest coal-fired power plants (20% of current coal electricity capacity)
- By 2040: phase-out coal

[2]https://climateactiontracker.org/countries/chile/2019-06-17/current-policy-projections/
https://mma.gob.cl/wp-content/uploads/2015/09/hojaderuta.pdf