Estimation of log-normal distribution from data

- Assume a known value for the mean (μ_x) of the sample and the coefficient of variation (cv). Use VALE values and the CV obtained by data conciliation
- Assume that the coefficient of variation of the random variable x is constant after the transformation into a random variable with a log-normal distribution $w = \ln(x)$. So, $cv_x = cv_w$
- Estimate the mean from the data or assume VALE mean as the arithmetic mean for the batch

$$\mu_x = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Estimate the standard deviation from the coefficient of variation of the data or assume a known cv for the batch

$$\sigma_x = cv_x \cdot \mu_x$$

- Estimate the mean of the log-normal distribution by using the equation:

$$\exp\left(\mu_w\right) = \frac{\mu_x^2}{\sqrt{\mu_x^2 + \sigma_x^2}}$$

- Estimate the standard deviation of the log-normal distribution by using the equation:

$$cv_w = \sqrt{\exp(\sigma_w^2) - 1} \rightarrow \sigma_w = \sqrt{\ln(1 + cv^2)}$$

- Compute the distribution:

$$f(x) = \frac{\exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_w}{\sigma_w}\right)^2\right]}{x\sigma_w\sqrt{2\pi}}$$

- Compute the other statistics as:
  - mean $= \exp(\mu_w + \frac{1}{2}\sigma_w^2)$
  - median $= \exp(\mu_w)$
  - mode $= \exp(\mu_w - \sigma_w^2)$
  - variance $= \left[\exp(\sigma_w^2) - 1\right] \cdot \exp(2\mu_w + \sigma_w^2)$