DAT320: Compulsory assignment 1

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Task 1

Importing necessary variables

```
library(ggplot2)
library(dplyr)
```

1 a)

Load the data set and convert categorical variables to factors and all other variables to an appropriate type. Print a summary of the variables in the data.frame.

Loading the data

```
df <- read.csv(file = "gapminder.csv", header = TRUE, sep = ",")</pre>
```

Converting all variables to either factors, numeric or integers

```
# Convert categorical variables to factors
df[, c("country", "continent")] <- lapply(df[,c("country", "continent")], as.factor)

# Convert other variables to appropriate data types
df[, c("lifeExp", "gdpPercap")] <- lapply(df[,c("lifeExp", "gdpPercap")], as.numeric)
df[, c("X", "year", "pop")] <- lapply(df[,c("X", "year", "pop")], as.integer)</pre>
```

Summary of the variables in the data.frame:

```
#Summary
summary(df)
```

```
##
         X
                           country
                                         continent
                                                          year
##
              1.0
                    Afghanistan: 12
                                      Africa :624
                                                     Min.
                                                            :1952
##
   1st Qu.: 426.8
                    Albania
                               : 12
                                      Americas:300
                                                     1st Qu.:1966
  Median : 852.5
                    Algeria
                               : 12
                                      Asia
                                              :396
                                                     Median:1980
         : 852.5
                               : 12
                                      Europe :360
##
  Mean
                    Angola
                                                     Mean
                                                            :1980
##
   3rd Qu.:1278.2
                    Argentina : 12
                                      Oceania: 24
                                                     3rd Qu.:1993
                                                            :2007
##
   Max.
          :1704.0
                    Australia : 12
                                                     Max.
##
                    (Other)
                               :1632
##
      lifeExp
                        pop
                                        gdpPercap
  Min.
                        :6.001e+04
##
          :23.60
                                            :
                                                 241.2
                  Min.
                                      Min.
  1st Qu.:48.20
                   1st Qu.:2.794e+06
                                      1st Qu.: 1202.1
## Median :60.71
                   Median :7.024e+06
                                      Median :
                                                3531.8
                                      Mean : 7215.3
## Mean
          :59.47
                   Mean :2.960e+07
## 3rd Qu.:70.85
                   3rd Qu.:1.959e+07
                                      3rd Qu.: 9325.5
```

```
## Max. :82.60 Max. :1.319e+09 Max. :113523.1
```

All countries

levels(df\$country)

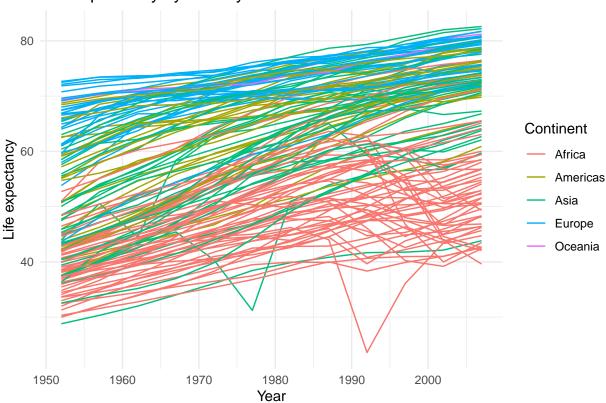
```
##
     [1] "Afghanistan"
                                      "Albania"
##
     [3] "Algeria"
                                      "Angola"
##
     [5] "Argentina"
                                      "Australia"
                                      "Bahrain"
##
     [7] "Austria"
##
     [9] "Bangladesh"
                                      "Belgium"
                                      "Bolivia"
##
    [11] "Benin"
                                      "Botswana"
##
    [13] "Bosnia and Herzegovina"
    [15] "Brazil"
                                      "Bulgaria"
    [17] "Burkina Faso"
                                      "Burundi"
##
    [19] "Cambodia"
                                      "Cameroon"
##
   [21] "Canada"
                                      "Central African Republic"
##
   [23] "Chad"
                                      "Chile"
##
                                      "Colombia"
##
   [25] "China"
##
    [27] "Comoros"
                                      "Congo, Dem. Rep."
                                      "Costa Rica"
##
   [29] "Congo, Rep."
   [31] "Cote d'Ivoire"
##
                                      "Croatia"
    [33] "Cuba"
                                      "Czech Republic"
##
##
    [35] "Denmark"
                                      "Djibouti"
##
   [37] "Dominican Republic"
                                      "Ecuador"
   [39] "Egypt"
                                      "El Salvador"
##
                                      "Eritrea"
    [41] "Equatorial Guinea"
##
##
   [43] "Ethiopia"
                                      "Finland"
##
   [45] "France"
                                      "Gabon"
##
   [47] "Gambia"
                                      "Germany"
##
    [49] "Ghana"
                                      "Greece"
##
   [51] "Guatemala"
                                      "Guinea"
   [53] "Guinea-Bissau"
                                      "Haiti"
                                      "Hong Kong, China"
   [55] "Honduras"
##
##
    [57] "Hungary"
                                      "Iceland"
   [59] "India"
                                      "Indonesia"
##
                                      "Iraq"
##
   [61] "Iran"
    [63] "Ireland"
                                      "Israel"
##
                                      "Jamaica"
##
    [65] "Italy"
##
    [67] "Japan"
                                      "Jordan"
    [69] "Kenya"
                                      "Korea, Dem. Rep."
##
##
    [71] "Korea, Rep."
                                      "Kuwait"
   [73] "Lebanon"
                                      "Lesotho"
##
                                      "Libya"
##
   [75] "Liberia"
##
   [77] "Madagascar"
                                      "Malawi"
##
    [79] "Malaysia"
                                      "Mali"
##
                                      "Mauritius"
    [81] "Mauritania"
   [83] "Mexico"
                                      "Mongolia"
                                      "Morocco"
   [85] "Montenegro"
##
                                      "Myanmar"
##
    [87] "Mozambique"
##
   [89] "Namibia"
                                      "Nepal"
##
  [91] "Netherlands"
                                      "New Zealand"
  [93] "Nicaragua"
                                      "Niger"
##
## [95] "Nigeria"
                                      "Norway"
## [97] "Oman"
                                      "Pakistan"
```

```
## [99] "Panama"
                                     "Paraguay"
## [101] "Peru"
                                     "Philippines"
## [103] "Poland"
                                     "Portugal"
## [105] "Puerto Rico"
                                     "Reunion"
                                     "Rwanda"
## [107] "Romania"
## [109] "Sao Tome and Principe"
                                     "Saudi Arabia"
## [111] "Senegal"
                                     "Serbia"
## [113] "Sierra Leone"
                                     "Singapore"
## [115] "Slovak Republic"
                                     "Slovenia"
## [117] "Somalia"
                                     "South Africa"
## [119] "Spain"
                                     "Sri Lanka"
## [121] "Sudan"
                                     "Swaziland"
## [123] "Sweden"
                                     "Switzerland"
## [125] "Syria"
                                     "Taiwan"
## [127] "Tanzania"
                                     "Thailand"
## [129] "Togo"
                                     "Trinidad and Tobago"
## [131] "Tunisia"
                                     "Turkey"
## [133] "Uganda"
                                     "United Kingdom"
## [135] "United States"
                                     "Uruguay"
## [137] "Venezuela"
                                     "Vietnam"
## [139] "West Bank and Gaza"
                                     "Yemen, Rep."
## [141] "Zambia"
                                     "Zimbabwe"
#All continents
levels(df$continent)
## [1] "Africa"
                  "Americas" "Asia"
                                         "Europe"
                                                    "Oceania"
1 b)
```

 ${\it Plot the time series of each country~(Year~versus~life Exp),~colored~by~continent.}$

Plotting using ggplot2:

Life expectancy by country



1 c)

Give a summary of the lifeExp for each continent and for each year. Compute minimum, median, mean, maximum, and standard deviations. Display this as a data.frame

Group data by continent and year, and summarise across life expectancy:

```
grp_df <- df %>% group_by(continent, year) %>%
            # summarise on grouped data across life expectancy
            summarise(across(lifeExp,
                             list(min
                                        = min,
                                 median = median,
                                 mean
                                        = mean,
                                 max
                                         = max,
                                  sd
                                         = sd)
                              )
                      ) %>%
            # convert tbl to data frame
            as.data.frame()
```

Display grouped data:

```
print(grp_df)
```

```
## continent year lifeExp_min lifeExp_median lifeExp_mean lifeExp_max
## 1 Africa 1952 30.000 38.8330 39.13550 52.724
```

##		Africa		31.570	40.5925	41.26635	58.089
##	3	Africa		32.767	42.6305	43.31944	60.246
##	4	Africa	1967	34.113	44.6985	45.33454	61.557
##	5	Africa	1972	35.400	47.0315	47.45094	64.274
##	6	Africa	1977	36.788	49.2725	49.58042	67.064
##	7	Africa	1982	38.445	50.7560	51.59287	69.885
##	8	Africa	1987	39.906	51.6395	53.34479	71.913
##	9	Africa	1992	23.599	52.4290	53.62958	73.615
##	10	Africa	1997	36.087	52.7590	53.59827	74.772
##	11	Africa	2002	39.193	51.2355	53.32523	75.744
##	12	Africa	2007	39.613	52.9265	54.80604	76.442
##	13	Americas	1952	37.579	54.7450	53.27984	68.750
##	14	Americas	1957	40.696	56.0740	55.96028	69.960
##	15	Americas	1962	43.428	58.2990	58.39876	71.300
##	16	Americas	1967	45.032	60.5230	60.41092	72.130
##	17	Americas	1972	46.714	63.4410	62.39492	72.880
##	18	Americas	1977	49.923	66.3530	64.39156	74.210
##	19	Americas	1982	51.461	67.4050	66.22884	75.760
##	20	Americas	1987	53.636	69.4980	68.09072	76.860
##	21	Americas	1992	55.089	69.8620	69.56836	77.950
##	22	Americas	1997	56.671	72.1460	71.15048	78.610
##	23	Americas	2002	58.137	72.0470	72.42204	79.770
##	24	Americas	2007	60.916	72.8990	73.60812	80.653
##	25	Asia	1952	28.801	44.8690	46.31439	65.390
##	26	Asia	1957	30.332	48.2840	49.31854	67.840
	27		1962	31.997	49.3250	51.56322	69.390
##	28		1967	34.020	53.6550	54.66364	71.430
	29		1972	36.088	56.9500	57.31927	73.420
	30		1977	31.220	60.7650	59.61056	75.380
	31		1982	39.854	63.7390	62.61794	77.110
	32		1987	40.822	66.2950	64.85118	78.670
	33		1992	41.674	68.6900	66.53721	79.360
	34		1997	41.763	70.2650	68.02052	80.690
	35		2002	42.129	71.0280	69.23388	82.000
	36		2007	43.828	72.3960	70.72848	82.603
	37	Europe		43.585	65.9000	64.40850	72.670
##		Europe		48.079	67.6500	66.70307	73.470
##		Europe		52.098	69.5250	68.53923	73.680
	40	Europe		54.336	70.6100	69.73760	74.160
##		Europe		57.005	70.8850	70.77503	74.720
	42	Europe		59.507	72.3350	71.93777	76.110
	43	Europe		61.036	73.4900	72.80640	76.990
	44	Europe		63.108	74.8150	73.64217	77.410
	45	Europe		66.146	75.4510	74.44010	78.770
	46	Europe		68.835	76.1160	75.50517	79.390
	47	Europe		70.845	77.5365	76.70060	80.620
	48	Europe		71.777	78.6085	77.64860	81.757
	49	Oceania		69.120	69.2550	69.25500	69.390
	50	Oceania		70.260	70.2950	70.29500	70.330
##		Oceania		70.200	70.2950	71.08500	70.330
	52	Oceania		71.100	71.3100	71.08500	71.520
	53	Oceania		71.100	71.9100	71.31000	71.520
	54	Oceania					
				72.220	72.8550	72.85500	73.490
##	55	Oceania	1302	73.840	74.2900	74.29000	74.740

```
## 56
        Oceania 1987
                           74.320
                                          75.3200
                                                       75.32000
                                                                      76.320
## 57
        Oceania 1992
                           76.330
                                          76.9450
                                                                      77.560
                                                       76.94500
        Oceania 1997
## 58
                           77.550
                                          78.1900
                                                       78.19000
                                                                      78.830
        Oceania 2002
## 59
                           79.110
                                          79.7400
                                                       79.74000
                                                                      80.370
##
   60
        Oceania 2007
                           80.204
                                          80.7195
                                                       80.71950
                                                                      81.235
##
       lifeExp_sd
## 1
       5.15158143
## 2
       5.62012285
       5.87536393
## 3
## 4
       6.08267263
## 5
       6.41625832
## 6
       6.80819741
## 7
       7.37594009
## 8
       7.86408911
## 9
       9.46107099
## 10
       9.10338658
## 11
       9.58649585
## 12
       9.63078067
## 13
       9.32608188
## 14
       9.03319228
## 15
       8.50354374
## 16
       7.90917104
       7.32301680
## 17
## 18
       7.06949562
## 19
       6.72083382
## 20
       5.80192884
## 21
       5.16710381
## 22
       4.88758390
## 23
       4.79970550
## 24
       4.44094763
## 25
       9.29175070
## 26
       9.63542862
## 27
       9.82063194
## 28
       9.65096458
## 29
       9.72270004
## 30 10.02219698
## 31
       8.53522141
## 32
       8.20379188
## 33
       8.07554897
## 34
       8.09117061
## 35
       8.37459539
## 36
       7.96372447
##
  37
       6.36108825
## 38
       5.29580539
## 39
       4.30249956
       3.79972850
## 40
## 41
       3.24057637
## 42
       3.12102998
## 43
       3.21826030
## 44
       3.16968034
## 45
       3.20978109
## 46
       3.10467655
## 47
       2.92217958
```

48 2.97981266

```
## 49 0.19091883
## 50 0.04949747
## 51 0.21920310
## 52 0.29698485
## 53 0.02828427
## 54 0.89802561
## 55 0.63639610
## 56 1.41421356
## 57 0.86974134
## 58 0.90509668
## 59 0.89095454
## 60 0.72902709
```

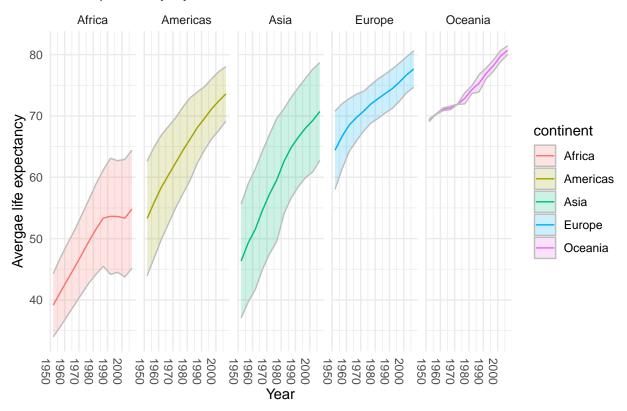
We observe that the life expectancy in general grows larger for each continent over the years. We will visualize the data for better interpretability in the next exercise.

1 d)

For each continent, plot the average lifeExp across all countries per year (Year versus lifeExp) as a line plot. Add errorbars or ribbons indicating \pm 1 standard deviation.

```
ggplot(data=grp_df,
       mapping=aes(x=year,
                   y=lifeExp_mean,
                   color=continent,
                   group=continent)) +
      # create lineplot
      geom_line() +
      # add error bars
      geom_ribbon(aes(ymin = lifeExp_mean - lifeExp_sd,
                      ymax = lifeExp_mean + lifeExp_sd,
                      fill = continent),
                  alpha=0.2, # transparency of sd
                  color='grey' # color of outline border
                  ) +
      labs(title = "Life expectancy by continent",
                 = "Year",
           X
                 = "Avergae life expectancy",
           у
           ) +
      theme_minimal() +
      theme(axis.text.x = element_text(angle = 270, hjust = 1)) +
      # create grid
      facet_grid( .~ continent)
```

Life expectancy by continent



We observe that Africa has the lowest life expectancy over the years. Compared to the other continents, Africa has an interesting "flattening" and small reduction in life expectancy around years 1990-2000. There might be great explanations for this, but we will not make a guess here.

America has somewhat higher life expectancy than Asia, in addition to a smaller standard deviation. Oceania has the highest life expectancy, followed by Europe. Interestingly, Oceania has a very low standard deviation compared to all other continents. We speculate whether this might be explained by factors such as few countries being part of Oceania, or the effect of living more isolated from the rest of the world when on an island.

Task 2

Import necessary libraries:

```
library(ggplot2)  # for plotting
library(patchwork)  # creates a patchwork of all our plots
library(fastDummies) # for dummy encoding
library(lubridate)  # for date/time handling
library(dplyr)
library(caret)  # test-train splitting
```

2 a)

Import the file weather History.csv. Look at the first few rows of the data. Discuss what types of variables you have. Which variables are categorical and which are numerical? Visualize the data (use bar plots and histograms with kernel density estimates). Discuss the data.

Read the dataset as a data.frame

```
df <- read.csv("weatherHistory.csv")</pre>
```

Display the first few rows of the data

```
head(df)
```

```
Summary Precip. Type Temperature..C.
##
                    Formatted.Date
## 1 2006-04-01 00:00:00.000 +0200 Partly Cloudy
                                                                       9.472222
                                                          rain
## 2 2006-04-01 01:00:00.000 +0200 Partly Cloudy
                                                                       9.355556
                                                          rain
## 3 2006-04-01 02:00:00.000 +0200 Mostly Cloudy
                                                          rain
                                                                       9.377778
## 4 2006-04-01 03:00:00.000 +0200 Partly Cloudy
                                                          rain
                                                                       8.288889
## 5 2006-04-01 04:00:00.000 +0200 Mostly Cloudy
                                                                       8.755556
                                                          rain
## 6 2006-04-01 05:00:00.000 +0200 Partly Cloudy
                                                          rain
                                                                       9.22222
     Apparent.Temperature..C. Humidity Wind.Speed..km.h. Wind.Bearing..degrees.
##
## 1
                      7.388889
                                   0.89
                                                   14.1197
                                                                               251
## 2
                      7.227778
                                   0.86
                                                   14.2646
                                                                               259
## 3
                      9.377778
                                   0.89
                                                    3.9284
                                                                               204
## 4
                      5.944444
                                   0.83
                                                   14.1036
                                                                               269
## 5
                      6.977778
                                   0.83
                                                   11.0446
                                                                               259
## 6
                     7.111111
                                   0.85
                                                   13.9587
                                                                               258
##
     Visibility..km. Loud.Cover Pressure..millibars.
## 1
             15.8263
                               0
                                               1015.13
                                               1015.63
## 2
             15.8263
                               0
## 3
             14.9569
                               0
                                               1015.94
                               0
## 4
             15.8263
                                               1016.41
## 5
             15.8263
                               0
                                               1016.51
## 6
             14.9569
                               0
                                               1016.66
                          Daily.Summary
##
## 1 Partly cloudy throughout the day.
## 2 Partly cloudy throughout the day.
## 3 Partly cloudy throughout the day.
## 4 Partly cloudy throughout the day.
## 5 Partly cloudy throughout the day.
## 6 Partly cloudy throughout the day.
```

Observing and editing variable classes

We observe dates, text and numbers. By investigating further we find that the column "Loud.Cover" consistently holds a value of 0, and does not actually contribute with any information. We believe this column is actually meant to be named "Cloud cover".

Let us see what type of class R has stored these values as:

```
lapply(df, class) # printing the class for each column in df
```

```
## $Formatted.Date
## [1] "character"
##
## $Summary
## [1] "character"
##
## $Precip.Type
## [1] "character"
## $Temperature..C.
## [1] "numeric"
##
## $Apparent.Temperature..C.
## [1] "numeric"
##
## $Humidity
## [1] "numeric"
##
## $Wind.Speed..km.h.
## [1] "numeric"
## $Wind.Bearing..degrees.
## [1] "numeric"
##
## $Visibility..km.
## [1] "numeric"
##
## $Loud.Cover
## [1] "numeric"
##
## $Pressure..millibars.
## [1] "numeric"
##
## $Daily.Summary
## [1] "character"
```

We want to transform all variables of type character (expect date) to categorical values, and make "Loud.Cover" and "Wind.Bearing..degrees." integers.

```
# Categorical
df[,c("Summary", "Precip.Type", "Daily.Summary")] <-
    lapply(df[,c("Summary", "Precip.Type", "Daily.Summary")], as.factor)

# Integers
df[,c("Wind.Bearing..degrees.", "Loud.Cover")] <-
    lapply(df[,c("Wind.Bearing..degrees.", "Loud.Cover")], as.integer)</pre>
```

Also, we choose to format the date to contain only the necessary information.

```
# Convert 'Formatted.Date' column to POSIXIt format with specified date and time format
df$Formatted.Date <- as.POSIXIt(df$Formatted.Date, format="%Y-%m-%d %H:%M:%OS %z")

# Reformat date to a simpler format (year-month-day hour:minute)
df$Formatted.Date <- format(df$Formatted.Date, format="%Y-%m-%d %H:%M")</pre>
```

Now we can produce a better summary of the data:

summary(df)

```
Formatted.Date
                                      Summary
                                                   Precip.Type
                                                                Temperature..C.
## Length:96453
                       Partly Cloudy
                                                   null: 517
                                                                        :-21.822
                                          :31733
                                                                Min.
   Class : character
                       Mostly Cloudy
                                                   rain:85224
                                                                1st Qu.: 4.689
                                          :28094
                       Overcast
##
  Mode :character
                                          :16597
                                                   snow:10712
                                                                Median : 12.000
##
                       Clear
                                          :10890
                                                                       : 11.933
                                                                Mean
##
                       Foggy
                                          : 7148
                                                                3rd Qu.: 18.839
##
                                                                       : 39.906
                       Breezy and Overcast: 528
                                                                Max.
##
                       (Other)
                                          : 1463
##
   Apparent.Temperature..C.
                                Humidity
                                              Wind.Speed..km.h.
          :-27.717
##
   Min.
                             Min.
                                    :0.0000
                                              Min.
                                                     : 0.000
##
   1st Qu.: 2.311
                             1st Qu.:0.6000
                                              1st Qu.: 5.828
##
  Median : 12.000
                             Median :0.7800
                                              Median: 9.966
## Mean
          : 10.855
                             Mean
                                    :0.7349
                                              Mean
                                                     :10.811
   3rd Qu.: 18.839
##
                             3rd Qu.:0.8900
                                              3rd Qu.:14.136
## Max.
          : 39.344
                             Max.
                                    :1.0000
                                              Max.
                                                     :63.853
##
## Wind.Bearing..degrees. Visibility..km.
                                             Loud.Cover Pressure..millibars.
## Min.
          : 0.0
                           Min.
                                  : 0.00
                                           Min.
                                                  :0
                                                        Min.
##
  1st Qu.:116.0
                           1st Qu.: 8.34
                                                        1st Qu.:1012
                                           1st Qu.:0
## Median :180.0
                           Median :10.05
                                           Median:0
                                                        Median:1016
## Mean
          :187.5
                           Mean
                                 :10.35
                                                               :1003
                                           Mean
                                                  :0
                                                        Mean
   3rd Qu.:290.0
                           3rd Qu.:14.81
                                           3rd Qu.:0
                                                        3rd Qu.:1021
##
                                                                :1046
##
  Max.
           :359.0
                           Max.
                                  :16.10
                                           Max.
                                                  :0
                                                        Max.
##
##
                                               Daily.Summary
## Mostly cloudy throughout the day.
                                                       :20085
## Partly cloudy throughout the day.
                                                      : 9981
## Partly cloudy until night.
                                                       : 6169
## Partly cloudy starting in the morning.
                                                       : 5184
## Foggy in the morning.
                                                       : 4201
## Foggy starting overnight continuing until morning.: 3576
##
  (Other)
                                                       :47257
```

Visualizing the data

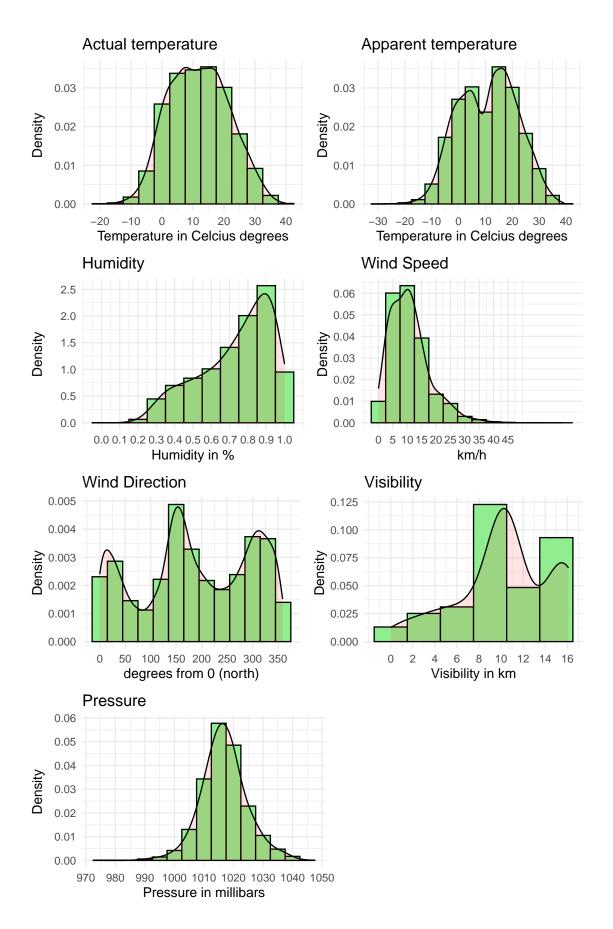
Create the histograms with kernel density estimate curve. Note that we first remove the rows where the pressure is recorded as 0. This is not a possible value, and has to be wrongful measurements.

```
sprintf("Number of rows with Pressure = 0 to delete: %s", sum(df$Pressure..millibars. == 0))
## [1] "Number of rows with Pressure = 0 to delete: 1288"
# Remove outliers/wrongful measurements of 0 pressure
df_filtered <- df[df$Pressure..millibars. != 0, ]</pre>
```

```
# Creating framework for the histogram with KDE plot
hist_KDE_plot <- function(data=df_filtered, col,</pre>
                          hist bw, density bw,
                          title, xlab,
                          xmin, xmax, xstep,
                          ymin, ymax, ystep){
        ggplot(data, aes(x=col)) +
    # Create histogram with density on the y-axis
        geom_histogram(aes(y=after_stat(density)), # density (not frequency) on y axis
                       binwidth=hist_bw, # set width of the bins
                       color="black",
                       fill="lightgreen") +
    # Create kernel density plot
        geom_density(alpha=0.1,
                     fill="red",
                     bw=density_bw) + # bw = smoothing bandwidth
    # Set title and axis-names
        labs(title=title,
           x = xlab,
            y = "Density") +
    # Set values for the span of the axes
        scale_x_continuous(breaks = seq(xmin, xmax, by = xstep)) +
        scale_y_continuous(breaks = seq(ymin, ymax, by = ystep)) +
        theme_minimal()
}
# Creating the different plots
tempC_plot <- hist_KDE_plot(col</pre>
                                        = df_filtered$Temperature..C.,
                             hist_bw = 5,
                             density_bw = 2,
                             title
                                      = "Actual temperature",
                             xlab
                                       = "Temperature in Celcius degrees",
                             xmin
                                       = -20
                                       = 40,
                             xmax
                             xstep
                                        = 10,
                                        = 0,
                             ymin
                             ymax
                                        = 0.04,
                             ystep
                                        = 0.01)
appTempC_plot <- hist_KDE_plot(col</pre>
                                        = df_filtered$Apparent.Temperature..C.,
                             hist bw
                                        = 5,
                             density_bw = 2,
                             title
                                       = "Apparent temperature",
                             xlab
                                       = "Temperature in Celcius degrees",
                             xmin
                                       = -30.
                             xmax
                                       = 40,
                                        = 10,
                             xstep
                             ymin
                                        = 0,
                             ymax
                                        = 0.04,
                                        = 0.01)
                             ystep
humid_plot <- hist_KDE_plot(col</pre>
                                        = df filtered$Humidity,
                             hist_bw
                                        = 0.1,
```

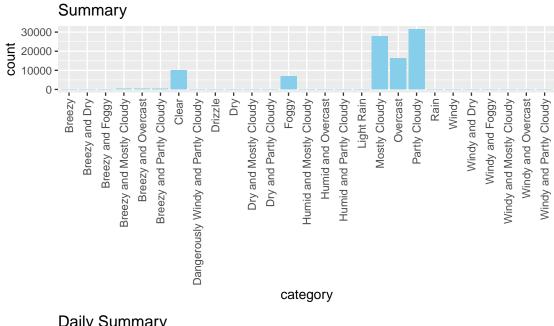
```
density_bw = 0.05,
                            title = "Humidity",
                                      = "Humidity in %",
                            xlab
                            xmin
                                      = 0,
                            xmax
                                      = 1,
                                      = 0.1,
                            xstep
                            ymin
                                      = 0,
                                      = 3,
                            ymax
                                      = 0.5)
                            ystep
windSpeed_plot <- hist_KDE_plot(col</pre>
                                      = df_filtered$Wind.Speed..km.h.,
                            hist_bw = 5,
                            density_bw = 2,
                                   = "Wind Speed",
                            title
                                      = "km/h",
                            xlab
                                      = 0.
                            xmin
                            xmax
                                      = 45,
                            xstep
                                      = 5,
                                      = 0,
                            ymin
                            ymax
                                      = 0.07,
                                      = 0.01)
                            ystep
windDirection_plot <- hist_KDE_plot(col = df_filtered$Wind.Bearing..degrees.,</pre>
                            hist_bw = 30,
                            density_bw = 10,
                            title = "Wind Direction",
                            xlab
                                     = "degrees from 0 (north)",
                            xmin
                                      = 0,
                            xmax
                                      = 400,
                                      = 50,
                            xstep
                                      = 0,
                            ymin
                                      = 0.005,
                            ymax
                            ystep
                                      = 0.001)
visibility_plot <- hist_KDE_plot(col = df_filtered$Visibility..km.,</pre>
                            hist_bw = 3,
                            density_bw = 1.5,
                            title = "Visibility",
                            xlab
                                     = "Visibility in km",
                            xmin
                                      = 0,
                            xmax
                                      = 17,
                            xstep
                                      = 2,
                                      = 0,
                            ymin
                                      = 0.125,
                            ymax
                                      = 0.025)
                            ystep
pressure_plot <- hist_KDE_plot(col</pre>
                                      = df_filtered$Pressure..millibars.,
                            hist_bw
                                      = 5,
                            density_bw = 2,
                                     = "Pressure",
                            title
                                     = "Pressure in millibars",
                            xlab
                            xmin
                                      = 970,
                                      = 1050,
                            xmax
```

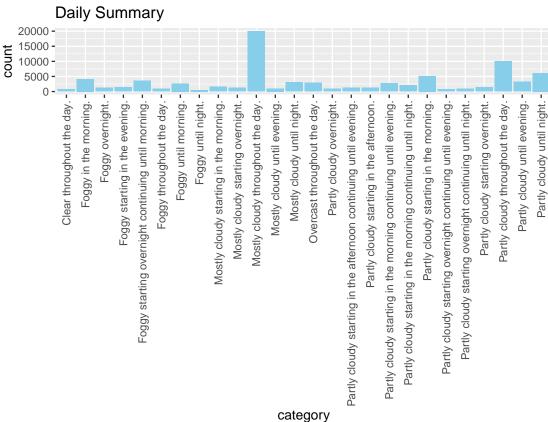
Displaying the histograms

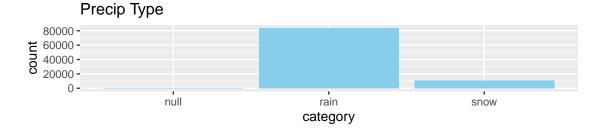


Creating bar plots for categorical values. For "Daily.Summary" we only show categories with above 500 observations.

```
### Summary
summary_plot = ggplot(df_filtered, aes(x=Summary)) +
  geom bar(fill = "skyblue") +
  labs(title ="Summary",
            x = "category",
            y = "count") +
 theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1)) #adjust label angle
### Daily Summary
# length(unique(filtered_df$Daily.Summary)) = 214 levels
# count category observations
category_count <- table(df_filtered$Daily.Summary)</pre>
# find categories with observations above threshold
categories_above_threshold <- names(category_count[category_count>500])
# pick the correct observations
categories_to_plot <- df_filtered[df_filtered$Daily.Summary %in% categories_above_threshold,]
dailySummary_plot = ggplot(categories_to_plot, aes(x=Daily.Summary)) +
  geom_bar(fill = "skyblue") +
  labs(title = "Daily Summary",
            x = "category",
            y = "count") +
  theme(axis.text.x = element_text(angle = 90, vjust = 0.5, hjust=1))
### Precip Type
precip_plot = ggplot(df_filtered, aes(x=Precip.Type)) +
  geom_bar(fill="skyblue") +
   labs(title = "Precip Type",
           x = "category",
            y = "count")
### Arrange the plots in a grid layout
combined_plot <- (summary_plot) /</pre>
                 (dailySummary_plot) /
                 (precip_plot) +
                 plot_layout(ncol=1)
print(combined_plot)
```







Discussion: As mentioned we have unvalid measurements of pressure, and hence we choose to exclude this data. Also, the column "Loud.Cover" is consistently 0, and does not contribute with any information. We will remove this column later on in this exercise. We observe a tendency to a bimodal distribution in the apparent temperatures compared to the distribution for the actual temperatures, which seems to be normally distributed. As expected, the pressure also seems to follow a normal distribution. The wind distribution is right skewed, while the humidity has a left skewed distribution. The humidity also has interestigly high values. The wind direction has a multimodal distribution, and the visibility is relatively great on average.

For the categorical variables "Summary" and "Daily.Summary" we have quite many categories. The "Daily Summary" contains 214 levels, which is too many to obtain valuable information. In addition, the majority of the ledf_filvels contain relatively few observations, while only a few of the categories contain large amounts of observations. The same goes for "Summary" even though the number of categories are only 27. This uneven distribution between the categories is the reason for excluding these variables later on, when creating a linear regression model.

As for the precip type, we observe a majority of rain. It is unclear what the "null" values represents. This could represent missing information, but it could also represent dry days, without any sort of precipitation. However, the amount of rainy days does correspond greatly with the high level of humidity and relatively high temperatures.

2 b)

Perform preprocessing using dummy/onehot encoding and standard scaling. Divide the data into 75%-25% train-test split randomly.

We have already removed outliers of 0 pressure, so we continue with the following preprocessing:

- 1. Date formatting and converting date and time to numerical values using sinus and cosinus transformations.
- 2. Dummy encoding of selected features
- 3. Standard scaling
- 4. Train-test splitting

For simplicity, we start by renaming our dataframe

```
lr df = df filtered
```

Date formatting

We extract the different elements of the date into separate columns

```
# Format as a date
lr_df$Formatted.Date <- as.POSIXlt(lr_df$Formatted.Date, format="%Y-%m-%d %H:%M")
# Extract features and create new columns
lr_df$day <- yday(lr_df$Formatted.Date) # day of year
lr_df$month <- month(lr_df$Formatted.Date)
lr_df$year <- year(lr_df$Formatted.Date)
lr_df$hour <- hour(lr_df$Formatted.Date)</pre>
```

We transform the day of year, month and hour to cyclical features using cosinus and sinus transformations. This way, january represented as number 1 and december represented as number 12, will no longer be "far" from each other when in reality they are closer than say, january and july. We keep the year as it is.

```
cycle <- function(col, div, sin=TRUE) {
  if (sin) {
    return(sin(2*pi * col/div))
  }</pre>
```

```
else {
    return(cos(2*pi * col/div))
}

# turn date and time into cyclical features
nMonths = 12
nHours = 24
nDays = 365 # ignore leap years

# Create new features
lr_df$hour_sin <- cycle(lr_df$hour, nHours)
lr_df$hour_cos <- cycle(lr_df$hour, nHours, FALSE)

lr_df$day_sin <- cycle(lr_df$day, nDays)
lr_df$day_cos <- cycle(lr_df$day, nDays, FALSE)

lr_df$month_sin <- cycle(lr_df$month, nMonths)
lr_df$month_cos <- cycle(lr_df$month, nMonths, FALSE)</pre>
```

Now, let's remove the redundant columns. Note, we also remove "Loud.Cover" as it contains no useful information, and "Daily.Summary" as it contains too many categories with uneven distribution to provide useful information.

Dummy-encoding

We perform dummy encoding on the columns "summary" and "precip type", and delete the original columns. We use dummy encoding rather than OneHot encoding to get fewer new features, and prevent issues arising from perfect correlation when doing linear regression later on.

Scaling

Scaling all the numerical columns, except the target, Apparent Temperature.

Test-train splits

Create random test-train splits

2 c)

Train a linear regression model with all the predictors you find logical to include based on your analysis in A) and use ApparentTemperature (C) as the target. Calculate the metrics below and interpret the hypotheses tests for the model parameters.

- 1. RMSE (root mean squared error)
- 2. MAE (mean absolute error)
- 3. coefficient of determination (R2 score)

Training a linear regression model; We choose to keep all the numerical values, but remove the dummy-encoded values for the "Summary"-groups. This is due to the same reason as excluding "Daily.Summary" earlier.

Calculating metrics

```
RMSE(pred, test$Apparent.Temperature..C.)

## [1] 1.056845

MAE(pred, test$Apparent.Temperature..C.)

## [1] 0.8270956

summary(lm)$r.squared
```

```
## [1] 0.9902131
```

The R-squared value indicated that 99% of the variation in the data is explained by the predictors. We also have a Mean Absolute Error of 0.8, meaning our absolute errors is less than 1 degree celcius off. This indicated a relatively good model.

Let's interpret the hypotheses tests for the model parameters. The hypotheses are:

- H0: beta = 0 (not a linear relationship between the target and the predictor)
- H1: beta!= 0 (a linear relationship between the target and predictor exists)

We observe the p-values:

```
summary(lm)
```

```
##
## Call:
  lm(formula = Apparent.Temperature..C. ~ ., data = train %% select(-starts_with("Summary")))
##
##
  Residuals:
##
       Min
                1Q Median
                                 3Q
                                        Max
   -4.1375 -0.7043 -0.1137
                             0.6323
                                     5.2357
##
## Coefficients:
##
                            Estimate Std. Error
                                                 t value Pr(>|t|)
## (Intercept)
                           11.296669
                                       0.054251
                                                  208.230
                                                           < 2e-16 ***
## Temperature..C.
                                                  912.529
                                                           < 2e-16 ***
                           10.491381
                                       0.011497
## Humidity
                            0.198972
                                       0.007043
                                                   28.251
                                                           < 2e-16 ***
## Wind.Speed..km.h.
                                                           < 2e-16 ***
                           -0.624294
                                       0.004549 - 137.228
## Wind.Bearing..degrees.
                            0.048700
                                       0.004021
                                                   12.110
                                                           < 2e-16 ***
## Visibility..km.
                           -0.039261
                                       0.005032
                                                   -7.803 6.14e-15 ***
## Pressure..millibars.
                            0.127198
                                       0.004714
                                                   26.983
                                                           < 2e-16 ***
## year
                           -0.006757
                                       0.004048
                                                   -1.669 0.095068 .
## hour_sin
                           -0.014972
                                       0.004098
                                                   -3.654 0.000259 ***
## hour cos
                           -0.052666
                                       0.005382
                                                   -9.786
                                                           < 2e-16 ***
## day_sin
                           -0.016655
                                       0.026312
                                                   -0.633 0.526740
## day cos
                           -0.301277
                                                           < 2e-16 ***
                                       0.027461
                                                  -10.971
## month_sin
                            0.060171
                                       0.026188
                                                    2.298 0.021583 *
## month cos
                            0.045421
                                       0.026296
                                                    1.727 0.084119 .
## Precip.Type_rain
                           -0.358696
                                       0.054479
                                                   -6.584 4.61e-11 ***
## Precip.Type_snow
                           -0.904626
                                       0.055935
                                                 -16.173 < 2e-16 ***
##
                     '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## Residual standard error: 1.057 on 71358 degrees of freedom
## Multiple R-squared: 0.9902, Adjusted R-squared: 0.9902
## F-statistic: 4.813e+05 on 15 and 71358 DF, p-value: < 2.2e-16
```

Not significant predictors have a p-value >0.05, which in our case are only the columns "year", "day_sin" and "month_cos". A possible interpretation could be that the global warming over the few years we are investigating are not significant enough to have an effect on our target variable. As for the cos/sin transformation of day and month: We assume that the cosinus variable are able to explain the yearly seasonal variances better than the sinus variable. As they are both representing seasonal variances over the year, the effect on the target explained by day_sin might correspond greatly to the effect already explained by month_sin, and vica verca for month_cos and day_cos. This model might therefore see day_sin and month_cos as "redundant". As opposed to this, hour_sin and hour_cos are explaining seasonal variances on a daily basis, explaining why both variables are considered significant.

We reject the null hypothesis for significant variables. The p-value for the overall model also indicates that we can reject the null hypothesis, telling us that we have a significant model.

Task 3

Import libraries

```
library(ggplot2)
library(dplyr)
```

3 a)

1) Linearity

Linear relationship between the independent variables (predictors) and the dependent variable (response). Lin.reg. models assume the relationship between predictors and the response is linear, and makes it straightforward to interpret the coefficients.

Each coefficient represents the change in the response variable for a one-unit change in the predictor. If the relationship is not linear, estimated coefficients in least squares may not accurately represent the relationship between predictors and the response.

Diagnosed by: Residuals. Plotting the residuals against the fitted values or against each predictor. In a well-fitted linear model, residuals should be randomly scattered around 0 without any discernible pattern.

2) Homoscedasticity

The variance of the residuals is constant across all levels of the independent variables, shown by the spread across all levels of the independent variables. It should be the same for all predictors.

Diagnosed by: Using a residual vs.fitted plot, where the residuals are on the y-axis and the fitted values are on the x-axis. If the residuals show a random scatter with constant spread across the range of fitted values, homoscedasticity likely holds. If the plot shows a pattern, specifically we are looking for a fanning out or fanning in pattern, meaning that residuals are spreading out as predicted values change, then homoscedasticity likely does not hold.

Can be diagnosed with a Scale-Location plot. Homoscedasticity is indicated if the points are evenly distributed horizontally with no discernible pattern. A trend in the red line indicates non-constant variance.

Residuals vs. Leverage: If the spread of residuals changes with leverage, it can suggest heteroscedasticity.

3) Independence

Observations are independent of each other. The residuals for one observation should not be correlated with the residuals for another observation. This assumption is crucial because if observations are not independent, it can bias the regression estimates, affect standard errors and lead to incorrect statistical inferences.

It is likely to be violated in Time-Series Data, because observations here are often dependent as data points are recorded over time. In cases where data are clustered or grouped, observations within the same group may be more similar to each other than to observations from different groups.

Diagnosed with residual autocorrelation: how much a residual at time t depends on the residual at time t-1. Diagnosed by: plotting residuals. Plotting the residuals in the order of data collection(against time for time-series).

If the residuals show a pattern; trends, cycles or clustering, it suggests autocorrelation and thus a violation of the independence assumption.

4) Normality

Assumption that residuals are normally distributed.

Diagnosed by: Histogram of residuals. Visually check if they resemble a normal distribution. The histogram should appear symmetric, centered around 0 and bell-shaped.

Diagnosed by: QQ-Plot. If the residuals are normally distributed, the points on the QQ-plot will lie approximately on a straight diagonal line. If the points deviate from the line(e.g., forming an "S" shape), this suggests a violation of the normality assumption, such as skewness or heavy tails (kurtosis).

Diagnosed by: A Shapiro-Wilk test, checking whether the residuals follow a normal distribution. The Null-Hypothesis here is that the residuals are normally distributed. The Alternative Hypothesis is that the residuals are not normally distributed. A significant p-value, less than 0.05, indicates that the residuals are not normally distributed.

3 b)

Residuals vs. Fitted: Plots the residuals (errors): The distance of the observed points to the predicted values from the regression line, vs. the Fitted: The fitted line going through the points generated by the model. The coefficients of the terms in this line are called the fitted values. Fitted values are on the x-axis. Residuals are on the y-axis. Residuals vs. Fitted shows the distance between the residuals and the fitted line.

Normal QQ: Quantiles of the residuals plotted against the quantiles of a normal distribution. If the residuals are normally distributed, the points on the QQ-plot will lie approximately on a straight diagonal line. If the points deviate from the line, this suggests a violation of the normality assumption, such as skewness or heavy tails (kurtosis).

Scale-Location (or Spread-Location): Scale-Location plot, is transformation of the residual plot. It plots the square root of the standardized residuals on the y-axis against the fitted values on the x-axis.

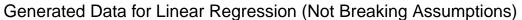
Residuals vs Leverage: The influence and quality of individual data points in a fitted model. Helps in identifying potential outliers, influential points, or cases where the model is poorly fitted. The vertical axis represents the residuals, showing how far off each observation is from the regression line. Leverage measures how far an observation's independent variable values (i.e. the predictor values) are from the mean of the predictor values. Points falling far outside the Cook's distance curves could be influential.

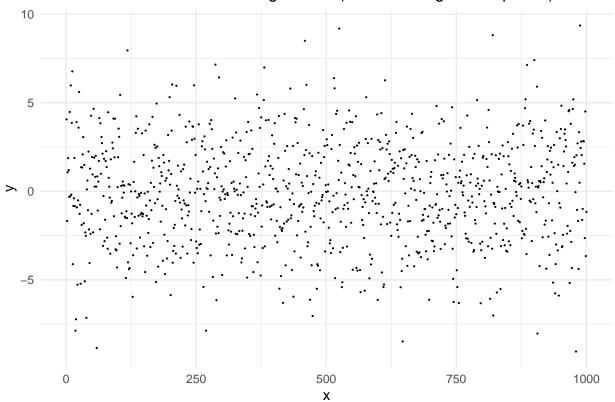
3 c)

Generating data and creating a linear regression that: - Holds to all the assumptions

```
#A model that Holds to all the assumptions:
set.seed(42)
n <- 1000
x \leftarrow 1:n
constant <- 0
trend <- 0 #A trend will indicate nonconstant variance among the residuals,
#so for homoscedasticity to hold it is set to O.
#If the variance of the residuals is not constant, the residuals will fan out
#or contract as the fitted values increase. So both trend<0 and trend>0
#might contribute to non-constant
#variance.
curve_magnitude <- 0 #If a relationship between variables is curved (non-linear)
#and a linear model is fitted, the linear model will be a poor fit.
#In this case, the linearity assumption of a linear regression is violated
#When curve_magnitude = 0, it suggests there is no curvature, and the relationship
#between X and Y is linear.
#This would fit the assumptions of a linear model.
#A curve magnitude > 0 would leave a non-linear coefficient among the residuals.
#Also, as the relationship becomes more non-linear, which can lead to unequal variance
```

```
#in the residuals across the range of x. So it can also contribute to heteroscedasticity.
curve_period <- 100</pre>
curve shift <- 0 #There is no shift in the curve, essential for a linear model.
#If the QQ-plot is not a straight line (has a shift), normality
#assumption will break. So it is essential for this to hold.
normal_noise_magnitude <- 1 #The magnitude of the noise in the normal distribution.
#Too much noise may make it harder to interpret accurate results, and
#will not accurately represent normally distributed residuals.
#So this is essential for not breaking the normality assumption.
norm_noise_periode <- 10000 #If residuals are not independent, we may see patterns
#over time, such as cycles or periodicity. This can be violated by adding periodic noise
#components, and setting this parameter to a small value can add periodic noise components.
#So if this is low, independence can break.
shift_norm_noise <- 500</pre>
non_normal_noise_magnitude <- 0 #Adding this (setting a value >0) here would add a
#non-normal component to the residuals, breaking the normality assumption.
#Increasing it can add more variability, so it could also
#contribute to heteroscedasticity.
non_norm_noise_periode <- 10000 #Setting this to a small value can also add periodic noise components.
shift_non_norm_noise <- 500.</pre>
y.gen <- constant +
  trend * x +
  curve_magnitude * sin((x/curve_period + curve_shift) * pi) +
  normal_noise_magnitude * cos((x/norm_noise_periode + shift_norm_noise/norm_noise_periode) * pi) * rno
  non_normal_noise_magnitude * cos((x/non_norm_noise_periode + shift_non_norm_noise/non_norm_noise_peri
# Use qqplot2 for the scatter plot
p \leftarrow ggplot(data = data.frame(x = x, y = y.gen), aes(x = x, y = y)) +
  geom_point(size = 0.1) +
  labs(title = "Generated Data for Linear Regression (Not Breaking Assumptions)", y = "y") +
  theme_minimal()
print(p)
```

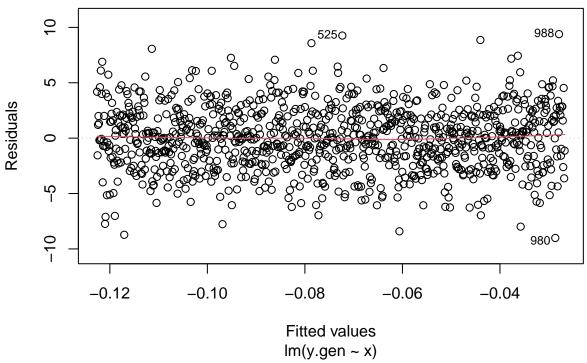




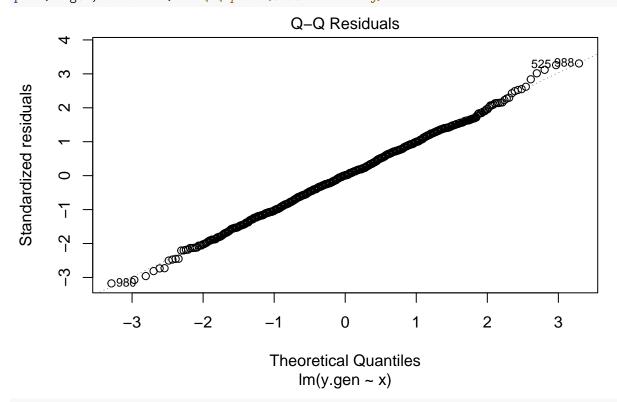
```
#Diagnostics of model that Holds to all assumptions:
lm.gen <- lm(y.gen ~ x)

# Plot diagnostics
plot(lm.gen, which = 1) # Residuals vs. Fitted (check linearity, homoscedasticity)</pre>
```

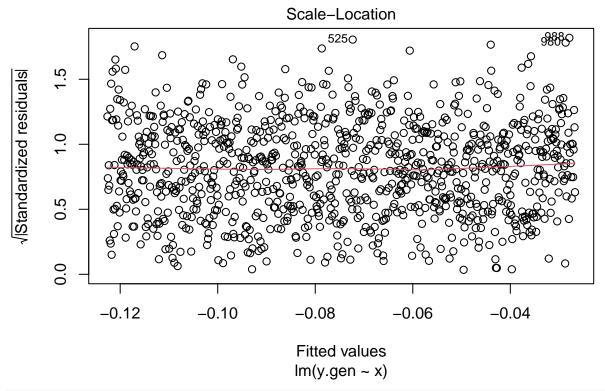




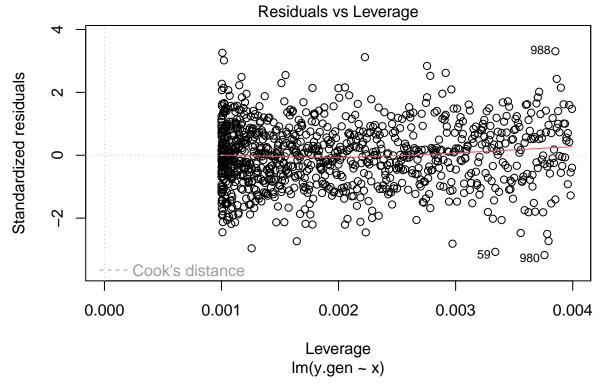
plot(lm.gen, which = 2) # Q-Q plot (check normality)



plot(lm.gen, which = 3) # Scale-Location (check homoscedasticity)

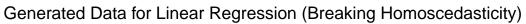


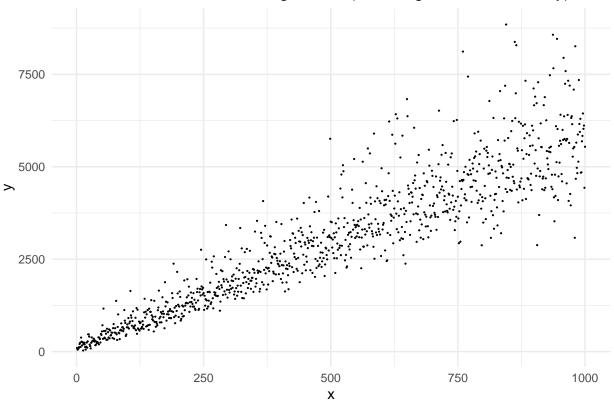




```
#A model breaking the Homoscedasticity assumption: set.seed(42) n <- 1000 \\ x <- 1:n
```

```
constant <- 0
trend <- 5 #A trend will indicate nonconstant variance among the residuals,
#so for homoscedasticity to hold it is set to 0.
#If the variance of the residuals is not constant, the residuals will fan out
#or contract as the fitted values increase. So both trend<0 and trend>0 will give non-constant
curve_magnitude <- 20 #Increased to create variability</pre>
curve period <- 100
curve shift <- 0
normal_noise_magnitude <- 15 + 0.2*x #Adjusting the noise to vary
#with x to get heteroscedasticity by getting a "fanning in" pattern
norm_noise_periode <- 10000</pre>
shift_norm_noise <- 100</pre>
non_normal_noise_magnitude <- 20 + 0.2*x #Adjust the noise to vary
#with x to get heteroscedasticity by getting a "fanning in" pattern
non_norm_noise_periode <- 10000</pre>
shift_non_norm_noise <- 100.</pre>
y.gen <- constant +
  trend * x +
  curve_magnitude * sin((x/curve_period + curve_shift) * pi) +
  normal_noise_magnitude * cos((x/norm_noise_periode + shift_norm_noise/norm_noise_periode) * pi) * rno
 non_normal_noise_magnitude * cos((x/non_norm_noise_periode + shift_non_norm_noise/non_norm_noise_peri
p \leftarrow ggplot(data = data.frame(x = x, y = y.gen), aes(x = x, y = y)) +
  geom_point(size = 0.1) +
  labs(title = "Generated Data for Linear Regression (Breaking Homoscedasticity)", y = "y") +
  theme_minimal()
print(p)
```

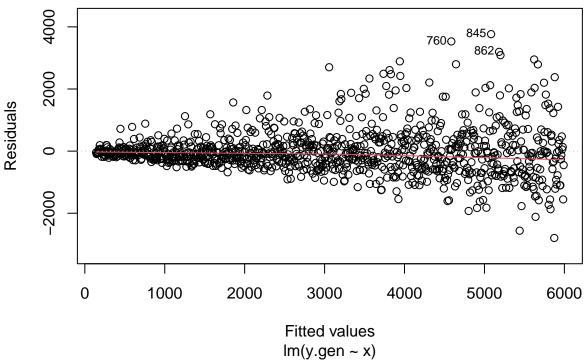




```
#Diagnostics of model:
lm.gen <- lm(y.gen ~ x)

# Plot diagnostics
plot(lm.gen, which = 1) # Residuals vs. Fitted (check linearity, homoscedasticity)</pre>
```



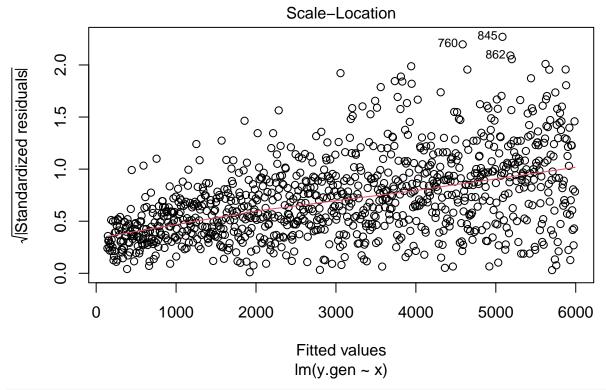


#Now, there are more anomalies and larger group of residuals further away from the mean, #which is no longer around the residual mid-values.:

#If the residuals show a random scatter with constant
#spread across the range of fitted values, homoscedasticity likely holds.
#If the plot shows a pattern, homoscedasticity likely does not hold.

#Here, there is not a random scatter with constant spread across the range of fitted values: #Spread is not constant. The variance of the residuals increases as fitted values increase, meaning the

plot(lm.gen, which = 3) # Scale-Location (check homoscedasticity)

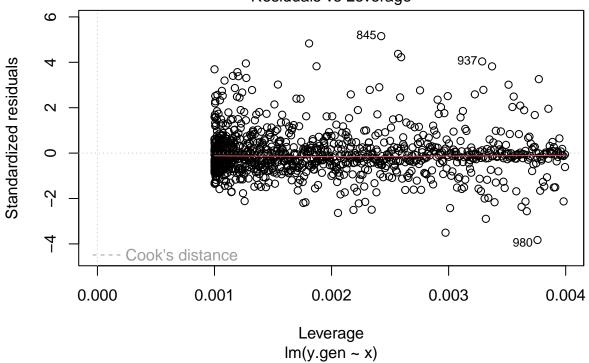


 $\hbox{\it\#Homoscedasticity is indicated if the points are evenly distributed}\\ \hbox{\it\#horizontally with no discernible pattern. A trend indicates non-constant \#variance}.}$

 $\#The\ scale-location\ plot\ has\ more\ scatter\ and\ more\ anomalies\ further\ as\ x\ increases,\ and\ the\ red\ line\ s$

plot(lm.gen, which = 5) # Residuals vs. Leverage (check influence)

Residuals vs Leverage

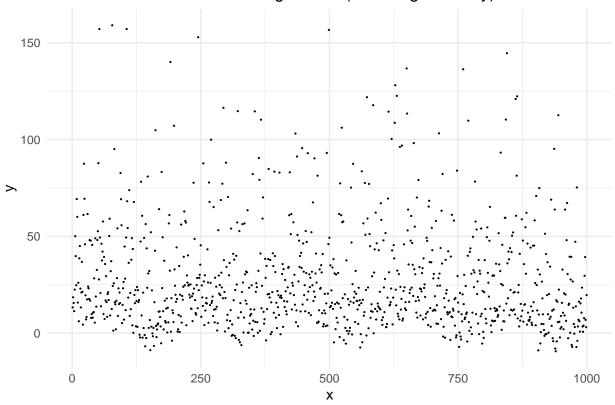


If the spread of residuals changes with leverage, it suggests heteroscedasticity. The spread seems to

```
#Here, A model that Breaks the Linearity assumption:
set.seed(42)
n <- 1000
x \leftarrow 1:n
constant <- 0
trend <- 0
curve_magnitude <- 6 #Increased to create non-linearity, also among the residuals
curve_period <- 100</pre>
curve_shift <- 4 # A linear model should not have a curve shift.
normal_noise_magnitude <- 1
norm_noise_periode <- 10000
shift_norm_noise <- 500</pre>
non_normal_noise_magnitude <- 6 #Increased to create variability
non_norm_noise_periode <- 10000
shift_non_norm_noise <- 500.</pre>
y.gen <- constant +
  trend * x +
  curve_magnitude * sin((x/curve_period + curve_shift) * pi) +
  normal_noise_magnitude * cos((x/norm_noise_periode + shift_norm_noise/norm_noise_periode) * pi) * rno
  non_normal_noise_magnitude * cos((x/non_norm_noise_periode + shift_non_norm_noise/non_norm_noise_peri
p \leftarrow ggplot(data = data.frame(x = x, y = y.gen), aes(x = x, y = y)) +
  geom_point(size = 0.1) +
  labs(title = "Generated Data for Linear Regression (Breaking Linearity)", y = "y") +
  theme_minimal()
```

print(p)

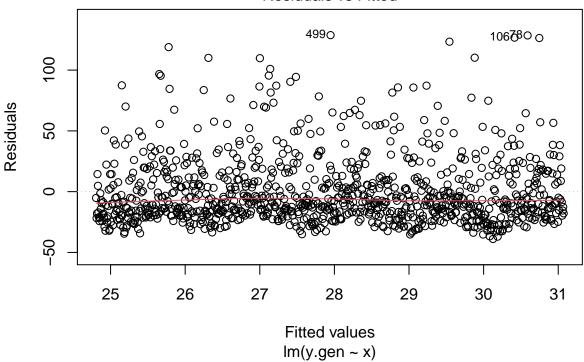
Generated Data for Linear Regression (Breaking Linearity)



```
#Diagnostics of model:
lm.gen <- lm(y.gen ~ x)

# Plot diagnostics
plot(lm.gen, which = 1)  # Residuals vs. Fitted (check linearity, homoscedasticity)</pre>
```

Residuals vs Fitted



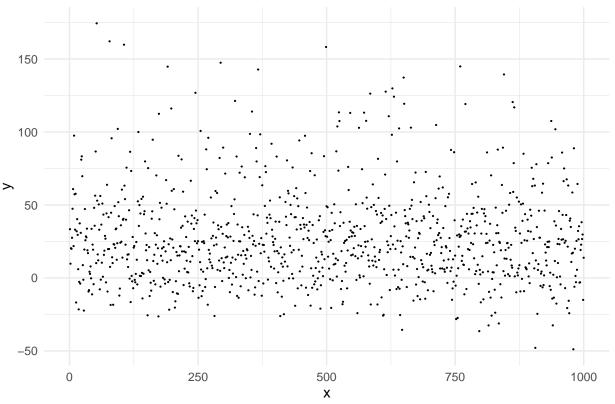
```
#Linearity:
##Diagnosed by: Residuals. Plotting the residuals against the fitted values
#or against each predictor. In a well-fitting linear model, residuals
#should be randomly scattered around 0 without any discernible pattern.

#More anomalies and larger group of residuals further away from the mean,
#which are no longer randomly scattered around 0.
```

```
#A model breaking the Normality assumption:
set.seed(42)
n <- 1000
x <- 1:n
constant <- 0
trend <- 0
curve_magnitude <- 0</pre>
curve_period <- 100</pre>
curve_shift <- 5 #There is no shift in the curve, essential for a linear model.
#If the QQ-plot is not a straight line (has a shift), normality
#assumption will break. So it is essential for this to hold.
normal_noise_magnitude <- 6 #The magnitude of the noise in the normal distribution.
#Too much noise may make it harder to interpret accurate results, and
#will not accurately represent normally distributed residuals.
#So this is essential for not breaking the normality assumption.
norm_noise_periode <- 10000</pre>
shift_norm_noise <- 500</pre>
non_normal_noise_magnitude <- 6 #Setting this non-zero would add a
#non-normal component to the residuals, breaking the normality assumption.
non_norm_noise_periode <- 10000</pre>
shift_non_norm_noise <- 500.</pre>
```

```
y.gen <- constant +
    trend * x +
    curve_magnitude * sin((x/curve_period + curve_shift) * pi) +
    normal_noise_magnitude * cos((x/norm_noise_periode + shift_norm_noise/norm_noise_periode) * pi) * rnonnormal_noise_magnitude * cos((x/non_norm_noise_periode + shift_non_norm_noise/non_norm_noise_periode)
p <- ggplot(data = data.frame(x = x, y = y.gen), aes(x = x, y = y)) +
    geom_point(size = 0.1) +
    labs(title = "Generated Data for Linear Regression (Breaking Normality)", y = "y") +
    theme_minimal()
print(p)</pre>
```

Generated Data for Linear Regression (Breaking Normality)



```
#If the residuals are normally distributed, the

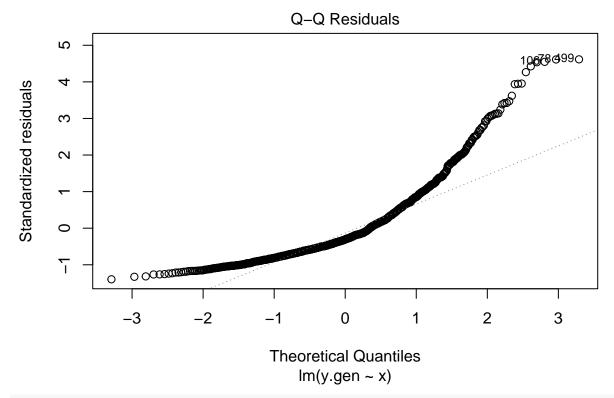
#points on the QQ-plot will lie approximately on a straight diagonal line.

#If the points deviate symmetrically from the line(e.g., forming an "S" shape),

#this suggests a violation of the normality assumption, such as

#skewness or heavy tails(kurtosis).

plot(lm.gen, which = 2) # Q-Q plot
```

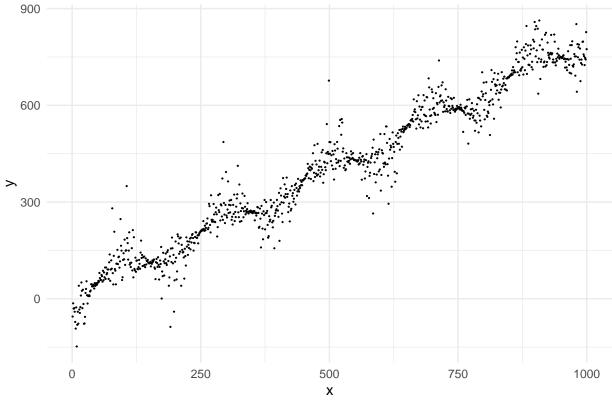


#Breaks normality as the line forms an S-shape, with kurtosis.

```
#A model breaking all assumptions including independence:
library(ggplot2)
set.seed(42)
n <- 1000
x \leftarrow 1:n
constant <- 0
trend <- 0.8 #A trend will indicate nonconstant variance among the residuals,
#so for homoscedasticity to hold it is set to 0.
#If the variance of the residuals is not constant, the residuals will fan out
#or contract as the fitted values increase. So both trend<0 and trend>0
#might contribute to non-constant
#variance.
curve_magnitude <- 10 #If a relationship between variables is curved (non-linear)
#and a linear model is fitted, the linear model will be a poor fit.
#In this case, the linearity assumption of a linear regression is violated
#When curve_magnitude = 0, it suggests there is no curvature, and the relationship
\#between\ X\ and\ Y\ is\ linear.
#This would fit the assumptions of a linear model.
#A curve magnitude > 0 would leave a non-linear coefficient among the residuals.
#Also, as the relationship becomes more non-linear, which can lead to unequal variance
#in the residuals across the range of x. So it can also contribute to heteroscedasticity.
curve_period <- 100</pre>
curve_shift <- 10 #There is no shift in the curve, essential for a linear model.
#If the QQ-plot is not a straight line (has a shift), normality
#assumption will break. So it is essential for this to hold.
normal_noise_magnitude <- 10 #The magnitude of the noise in the normal distribution.
#Too much noise may make it harder to interpret accurate results, and
```

```
#will not accurately represent normally distributed residuals.
#So this is essential for not breaking the normality assumption.
norm_noise_periode <- 100 #If residuals are not independent, we may see patterns
#over time, such as cycles or periodicity. This can be violated by adding periodic noise
#components, and setting this parameter to a small value can add periodic noise components.
#So if this is low, independence can break.
shift_norm_noise <- 500</pre>
non_normal_noise_magnitude <- 10 #Setting a value >0 here would add a
#non-normal component to the residuals, breaking the normality assumption.
#Increasing it can add more variability, so it could also
#contribute to heteroscedasticity.
non_norm_noise_periode <- 100 #Setting this to a small value can also add periodic noise
#components. So if this is low, independence can break.
shift_non_norm_noise <- 500.</pre>
y.gen <- constant +
  trend * x +
  curve_magnitude * sin((x/curve_period + curve_shift) * pi) +
  normal_noise_magnitude * cos((x/norm_noise_periode + shift_norm_noise/norm_noise_periode) * pi) * rno
 non_normal_noise_magnitude * cos((x/non_norm_noise_periode + shift_non_norm_noise/non_norm_noise_peri
# Use ggplot2 for the scatter plot
p \leftarrow ggplot(data = data.frame(x = x, y = y.gen), aes(x = x, y = y)) +
  geom_point(size = 0.1) +
  labs(title = "Generated Data for Linear Regression (Breaking all Assumptions)", y = "y") +
  theme minimal()
print(p)
```

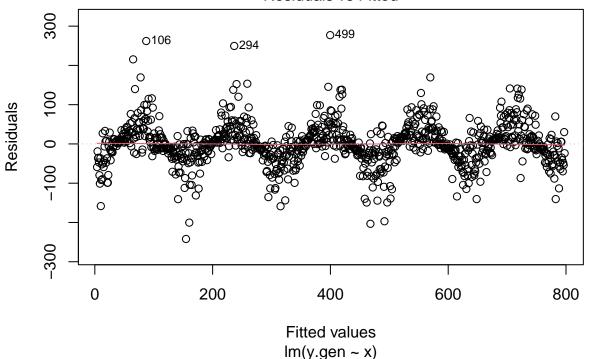
Generated Data for Linear Regression (Breaking all Assumptions)



```
#Diagnostics of model:
lm.gen <- lm(y.gen ~ x)

# Plot diagnostics
plot(lm.gen, which = 1)  # Residuals vs. Fitted (check linearity, homoscedasticity)</pre>
```

Residuals vs Fitted



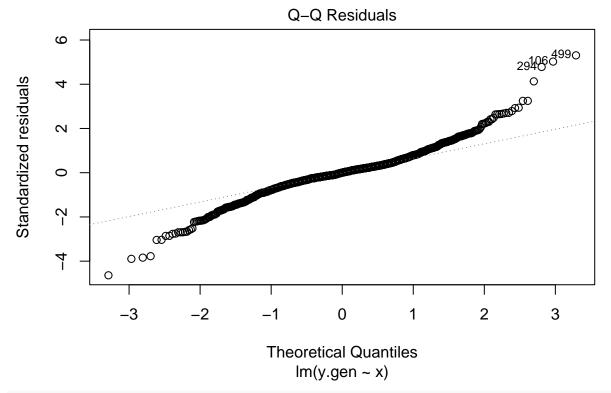
#Residuals are fairly placed around 0, but they are not randomly scattered as #there are some big outliers and anomalies, breaking linearity.

#If the residuals show a random scatter with constant
#spread across the range of fitted values, homoscedasticity likely holds.
#If the plot shows a pattern, homoscedasticity likely does not hold.

#There is a clear jigsaw pattern in the residuals, breaking homoscedasticity.

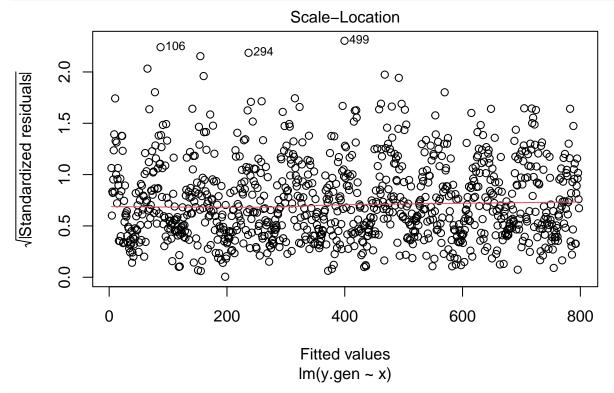
#If the residuals are normally distributed, the #points on the QQ-plot will lie approximately on a straight diagonal line. #If the points deviate symmetrically from the line(e.g., forming an "S" shape), #this suggests a violation of the normality assumption, such as $\#skewness\ or\ heavy\ tails(kurtosis).$

plot(lm.gen, which = 2) # Q-Q plot



#Breaks normality as the line is skewed.

plot(lm.gen, which = 3) # Scale-Location (check homoscedasticity)



#Homoscedasticity is indicated if the points are evenly distributed #horizontally with no discernible pattern. A trend indicates non-constant

#variance.

#The scale-location plot has less scatter and there are more anomalies further away from the mean, so t

Exercise 4

Task a

The concept of correlation

By standard definition, correlation measures the degree of association between two continuous variables, represented by the correlation coefficient ρ . In this assignment, the Pearson correlation coefficient will be used. For Pearson correlation, the correlation coefficient ranges from -1 to 1 and indicates the linear relationship's direction and strength. There are general guidelines for interpreting the values of the correlation coefficient, and for two continuous variables X and Y, the following applies to Pearson correlation ("Pearson's Correlation Coefficient," n.d.):

- $|\rho_{XY}| \leq 0.29$: Weak correlation
- $|\rho_{XY}| = 0.30$ to 0.49: Moderate correlation
- $|\rho_{XY}| \ge 0.50$: Strong correlation

Hence, $\rho_{XY} = 1$ indicates a strong positive correlation, $\rho_{XY} = -1$ indicates a strong negative correlation, and $\rho_{XY} = 0$ indicates no correlation. (Note that r is used instead of ρ when analyzing sample data, using Pearson correlation.)

Impact of scaling

According to the given formulas, $\rho_{W_1W_2}$ can be written as follows:

$$\begin{split} \rho_{W_1W_2} &= \frac{\text{cov}(\alpha_1 X, \alpha_2 Y)}{\sqrt{\text{var}(\alpha_1 X) \text{var}(\alpha_2 Y)}} \\ \rho_{W_1W_2} &= \text{sgn}(\alpha_1 \alpha_2) \rho_{XY} \end{split}$$

As the formulas indicate, scaling the different variables X and Y might change the sign of $\rho_{W_1W_2}$, but not the magnitude. The sign depends on the product $\alpha_1\alpha_2$, and $\operatorname{sgn}(\alpha_1\alpha_2)$ can flip from positive to negative or negative to positive, based on the signs of α_1 and α_2 . Hence, scaling of the different variables X and Y may affect the direction of the correlation, but not the magnitude.

Task b

By standard definition, partial correlation measures the degree of association between two continuous variables, while controlling for the influence of one or more other continuous variables. These variables are called confounding variables and are associated with either the independent variable, the dependent variable, or both. Usually, the confounding variable will influence both the dependent and the independent variable. Like regular correlation, partial correlation is represented by the correlation coefficient ρ (r is used for sample data), which ranges from -1 to 1. The interpretation of ρ is the same for partial correlation as it is for regular correlation. In the formula provided, Z acts as a confounding variable, while X and Y serve as the main variables. If regular correlation suggests that X and Y are correlated, partial correlation can be used to determine whether this correlation is driven by the confounding variable Z. Thus, using the regular correlation coefficient may produce misleading results, and partial correlation addresses this issue by removing the influence of such confounding variables. The formula for partial correlation is:

$$\rho_{XY|Z} = \frac{\rho_{XY} - \rho_{XZ}\rho_{ZY}}{\sqrt{(1 - \rho_{XZ}^2)(1 - \rho_{ZY}^2)}}$$

The formula computes the correlation between X and Y after controlling for the effect of Z. The numerator adjusts the correlation between X and Y by subtracting the influence of Z on both variables. The denominator

ensures that $\rho_{XY|Z}$ is properly scaled (remains within the bounds of a correlation coefficient) and adjusts for the amount of correlation between X and Y that cannot be explained by Z.

Examples of scenarios to investigate partial correlations

Scenario 1 - Causes of Lung Cancer as the dependent variable: When studying the causes of cancer, one might find a correlation between alcohol consumption and lung cancer. However, this relation might be confounded by smoking, as smoking is associated with alcohol consumption and is a well-known risk factor for lung cancer. In this case, smoking serves as a confounder that could disrupt the relationship between alcohol and lung cancer. By setting x = alcohol consumption, y = lung cancer, and z = number of cigarettes per day, one can calculate the partial correlation which adjusts for the effect of smoking on both variables.

Scenario 2 - Exam Performance as the dependent variable: When analyzing the factors that influence exam performance, intelligence may act as a confounding variable. Intelligence could influence both study hours and exam performance, thereby affecting the true relationship between the two. By setting x = study hours, y = exam performance, and z = intelligence (e.g. measured by IQ), one can use the given formula to calculate the partial correlation which adjusts for intelligence's influence on both variables.

Scenario 3 - Heart health as the dependent variable: In a study investigating the relationship between heart health and exercise, age could act as a confounder because it affects both exercise habits and heart health. By setting x = heart health, y = hours of exercise and z = age, one can use the given formula to calculate. This way, one can better isolate the effect of exercise on heart health.

Task c

The property from part (a) indicates that scaling variables X and Y by constants does not affect the magnitude of the correlation, only the sign. To investigate whether this property holds for partial correlation, one can apply the scaling factors α_1 for X, α_2 for Y, and α_3 for Z from part (a) in the equation given in part (b), see equation *.

(*)
$$\rho_{XY|Z} = \frac{\rho_{XY} - \rho_{XZ}\rho_{ZY}}{\sqrt{1 - \rho_{XZ}^2}\sqrt{1 - \rho_{ZY}^2}}$$

Adjusting the formula given in (*) to account for these scaling factors yields the following:

$$\rho_{(\alpha_1 X)(\alpha_2 Y)} = \operatorname{sgn}(\alpha_1 \alpha_2) \rho_{XY} \tag{1}$$

$$\rho_{(\alpha_1 X)(\alpha_3 Z)} = \operatorname{sgn}(\alpha_1 \alpha_3) \rho_{XZ} \tag{2}$$

$$\rho_{(\alpha_2 Y)(\alpha_3 Z)} = \operatorname{sgn}(\alpha_2 \alpha_3) \rho_{ZY} \tag{3}$$

After scaling, the partial correlation formula becomes:

$$\begin{split} \rho_{(\alpha_1 X)(\alpha_2 Y)|(\alpha_3 Z)} &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) \rho_{XY} - \operatorname{sgn}(\alpha_1 \alpha_3) \rho_{XZ} \cdot \operatorname{sgn}(\alpha_2 \alpha_3) \rho_{ZY}}{\sqrt{1 - (\operatorname{sgn}(\alpha_1 \alpha_3) \rho_{XZ})^2} \sqrt{1 - (\operatorname{sgn}(\alpha_2 \alpha_3) \rho_{ZY})^2}} \\ &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) \rho_{XY} - \operatorname{sgn}(\alpha_1) \cdot \operatorname{sgn}(\alpha_3) \rho_{XZ} \cdot \operatorname{sgn}(\alpha_2) \cdot \operatorname{sgn}(\alpha_3) \rho_{ZY}}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}} \\ &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) \rho_{XY} - \operatorname{sgn}(\alpha_3) \cdot \operatorname{sgn}(\alpha_3) \cdot \operatorname{sgn}(\alpha_1) \rho_{XZ} \cdot \operatorname{sgn}(\alpha_2) \cdot \rho_{ZY}}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}} \\ &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) \rho_{XY} - 1 \cdot \operatorname{sgn}(\alpha_1) \cdot \rho_{XZ} \cdot \operatorname{sgn}(\alpha_2) \cdot \rho_{ZY}}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}} \\ &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) \rho_{XY} - \operatorname{sgn}(\alpha_1) \cdot \rho_{XZ} \cdot \operatorname{sgn}(\alpha_2) \cdot \rho_{ZY}}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}} \\ &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) \rho_{XY} - \operatorname{sgn}(\alpha_1 \alpha_2) \cdot \rho_{XZ} \cdot \rho_{ZY}}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}} \\ &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) (\rho_{XY} - \rho_{XZ} \cdot \rho_{ZY})}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}} \\ &= \frac{\operatorname{sgn}(\alpha_1 \alpha_2) (\rho_{XY} - \rho_{XZ} \cdot \rho_{ZY})}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}} \end{split}$$

Which can be simplified to:

$$\rho_{(\alpha_1 X)(\alpha_2 Y)|(\alpha_3 Z)} = \operatorname{sgn}(\alpha_1 \alpha_2) \cdot \frac{\rho_{XY} - \rho_{XZ} \cdot \rho_{ZY}}{\sqrt{1 - \rho_{XZ}^2} \sqrt{1 - \rho_{ZY}^2}}$$

Hence, the principle from task (a) also applies to partial correlation. The formula for partial correlation is based on correlations ρ_{XY} , ρ_{XZ} , ρ_{ZY} , which are unaffected by scaling. Therefore, scaling X or Y in the presence of a third variable Z might change the sign, but not the magnitude of the partial correlation.

Task d

```
# Load necessary packages
library(dplyr)
library(ppcor)
# Reading the data from the weatherHistory csv-file
data <- read.csv("weatherHistory.csv")</pre>
# Selecting the required features: Temperature (C), Apparent Temperature (C), Humidity
selected_data <-
data %>% dplyr::select(Temperature..C., Apparent.Temperature..C., Humidity)
# Compute the pairwise Pearson correlation of each pair of features
pairwise_correlation <- cor(selected_data)</pre>
# Display the pairwise Pearson correlation
cat("Pairwise Pearson correlation")
## Pairwise Pearson correlation
print(pairwise_correlation)
##
                            Temperature..C. Apparent.Temperature..C.
                                                                         Humidity
## Temperature..C.
                                   1.0000000
                                                            0.9926286 -0.6322547
## Apparent.Temperature..C.
                                   0.9926286
                                                             1.0000000 -0.6025710
```

```
print(ppcor_partial_corr_results)
```

```
## $`Partial Pearson Correlation (Temp & App Temp | Humidity)`
## [1] 0.9892298
##
## $`Partial Pearson Correlation (Temp & Humidity | App Temp)`
## [1] -0.3528185
## $`Partial Pearson Correlation (App Temp & Humidity | Temp)`
## [1] 0.2664916
```

The relationship between the variables: Do the variables have any confounding effect on each other?

```
When z = Humidity:
```

Both the pairwise correlation and the partial correlation between the temperature and the apparent temperature are strong. Hence, the humidity variable does not confound this relationship significantly.

When z = Apparent temperature:

Both the pairwise and the partial correlations are negative. However, the absolute value of the pairwise correlation is higher than the absolute value of the partial correlation. The pairwise correlation can be classified as strong negative, while the partial correlation is weak negative, indicating that the apparent temperature variable has a confounding effect.

When z = Temperature:

The pairwise correlation shows a strong negative relationship, while the partial correlation shows a weak positive relationship. The change from a strong negative pairwise correlation to a weak positive partial correlation indicates that the temperature variable is confounding the relationship between the humidity and the apparent temperature.

References

Pearson's correlation coefficient: A comprehensive overview. (n.d.). In *Statistics Solutions*. Retrieved September 27, 2024, from https://www.statisticssolutions.com/free-resources/directory-of-statistical-analyses/pearsons-correlation-coefficient/