```
> restart;
> with(plots):
```

Problembeschreibung:

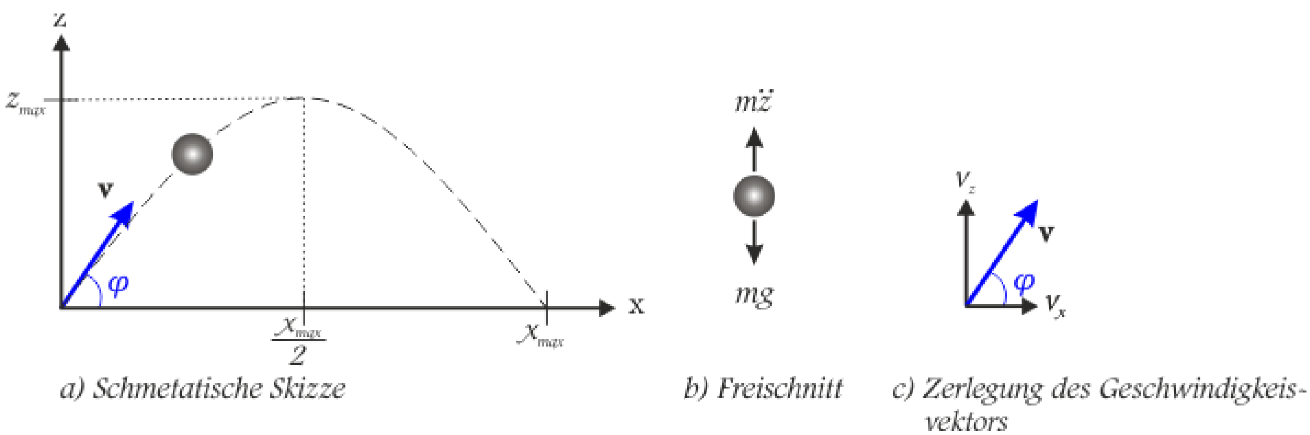


Abbildung 1: Graphische Darstellung

Geschwindigkeitsvektor (siehe Abbildung 1 *c*)):

$\mathbf{v} = \begin{bmatrix} v_x \\ v_z \end{bmatrix} = \begin{bmatrix} v_0 \cdot \cos(\varphi) \\ v_0 \cdot \sin(\varphi) \end{bmatrix}$

Kräftegleichgewicht (siehe Abbildung 1 *b*)):

$\Sigma F_{x,\, i} = 0 : m \cdot \frac{d^2}{d t^2} x = 0$

$\Sigma F_{z,\, i} = 0 : m \cdot \frac{d^2}{d t^2} z = -\ m \cdot g$

Randbedingungen:

#1) x(t=0) = 0

 # 2) z(t=0)=0

#3) $\frac{d}{d t} x\ (t=0) = v_0 \cdot \cos(\varphi)$

 # 4) $\frac{d}{d t} z\ (t=0) = v_0 \cdot \sin(\varphi)$

Differentialgleichung

```
> v := Vector[column]([ v__0 * cos(varphi) ,
                        v__0 * sin(varphi) ]);
```

(2.1)

$$v := \begin{bmatrix} v_0 \cos(\varphi) \\ v_0 \sin(\varphi) \end{bmatrix} \tag{2.1}$$

```
> dgl := Vector[column]([ m * diff(x(t),t,t) ,
                          m * diff(z(t),t,t)]) = Vector[column](
  [ 0 ,
                    - m * g ]);
```

$$dgl := \begin{bmatrix} m \left(\frac{d^2}{dt^2} x(t)\right) \\ m \left(\frac{d^2}{dt^2} z(t)\right) \end{bmatrix} = \begin{bmatrix} 0 \\ -m\,g \end{bmatrix} \tag{2.2}$$

```
> rb := x(0)=0, z(0) = 0, D(x)(0) = v[1],D(z)(0) = v[2];
```

$$rb := x(0) = 0, z(0) = 0, \mathrm{D}(x)(0) = v_0 \cos(\varphi), \mathrm{D}(z)(0) = v_0 \sin(\varphi) \tag{2.3}$$

Berechnung

```
> lsg_weg := dsolve({dgl,rb})
```

$$lsg\_weg := \left\{x(t) = t\, v_0 \cos(\varphi), z(t) = -\frac{1}{2}\, g\, t^2 + v_0 \sin(\varphi)\, t\right\} \tag{3.1}$$

```
> lsg_geschw := diff(lsg_weg,t);
```

$$lsg\_geschw := \left\{\frac{d}{dt}\, x(t) = v_0 \cos(\varphi), \frac{d}{dt}\, z(t) = -g\, t + v_0 \sin(\varphi)\right\} \tag{3.2}$$

```
> t__max := 2 * solve(-g*t+v__0*sin(varphi)=0,t);
```

$$t_{max} := \frac{2\, v_0 \sin(\varphi)}{g} \tag{3.3}$$

```
> x__max := v__0 * cos(varphi)*t__max;
```

$$x_{max} := \frac{2\, v_0^2 \cos(\varphi) \sin(\varphi)}{g} \tag{3.4}$$

```
> # Mit der trigonometrischen Umformung:
> cos(varphi)*sin(varphi)=trigsubs(cos(varphi)*sin(varphi));
```

$$\cos(\varphi) \sin(\varphi) = \left[\frac{1}{2} \sin(2\,\varphi)\right] \tag{3.5}$$

```
> #folgt
> x__max := v__0^2/g*sin(2*varphi);
```

$$x_{max} := \frac{v_0^2 \sin(2\,\varphi)}{g} \tag{3.6}$$

```
> z__max := -(1/2)*g*t__max/2^2+v__0*sin(varphi)*t__max/2;
```

$$z_{max} := -\frac{1}{4}\, v_0 \sin(\varphi) + \frac{v_0^2 \sin(\varphi)^2}{g} \tag{3.7}$$

Numerische Berechnungsparameter

```
> m := 10:            # kg
```

```
> v__0 := 115:         # m/s
> g := 9.81:          # m/s^2
> varphi := Pi/6:     # rad
```

Lösung mit den Werten

```
> evalf(lsg_weg,5);
```

$$\left\{x(t) = 99.596\, t, z(t) = -4.9050\, t^2 + 57.500\, t\right\} \tag{5.1}$$

```
> evalf(lsg_geschw,5);
```

$$\left\{\frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = 99.596, \frac{\mathrm{d}}{\mathrm{d}t}\, z(t) = -9.81\, t + 57.500\right\} \tag{5.2}$$

```
> t__maxNum := evalf(t__max,3);
```

$$t_{maxNum} := 11.7 \tag{5.3}$$

```
> x__maxNum := evalf(x__max,3);
```

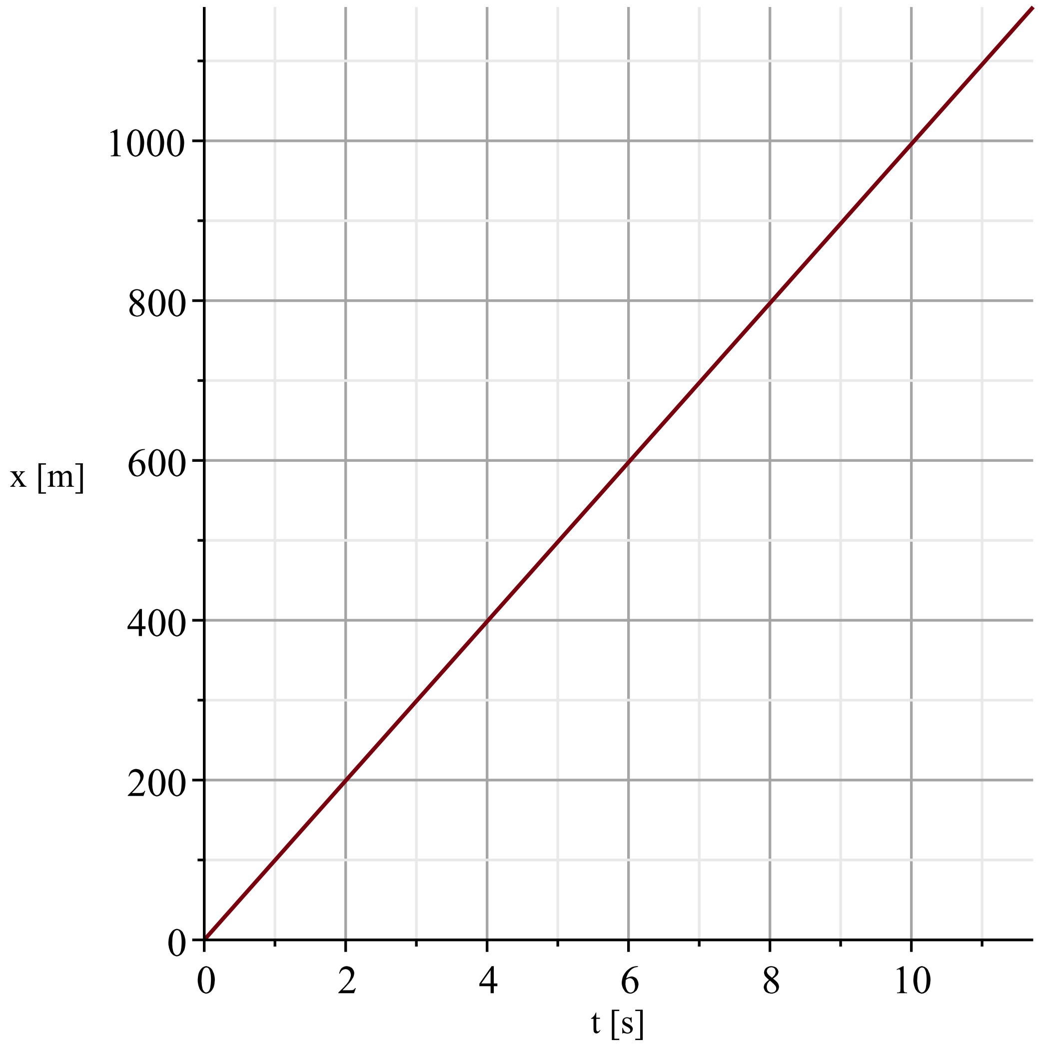
$$x_{maxNum} := 1170. \tag{5.4}$$

```
> z__maxNum := evalf(z__max,3);
```

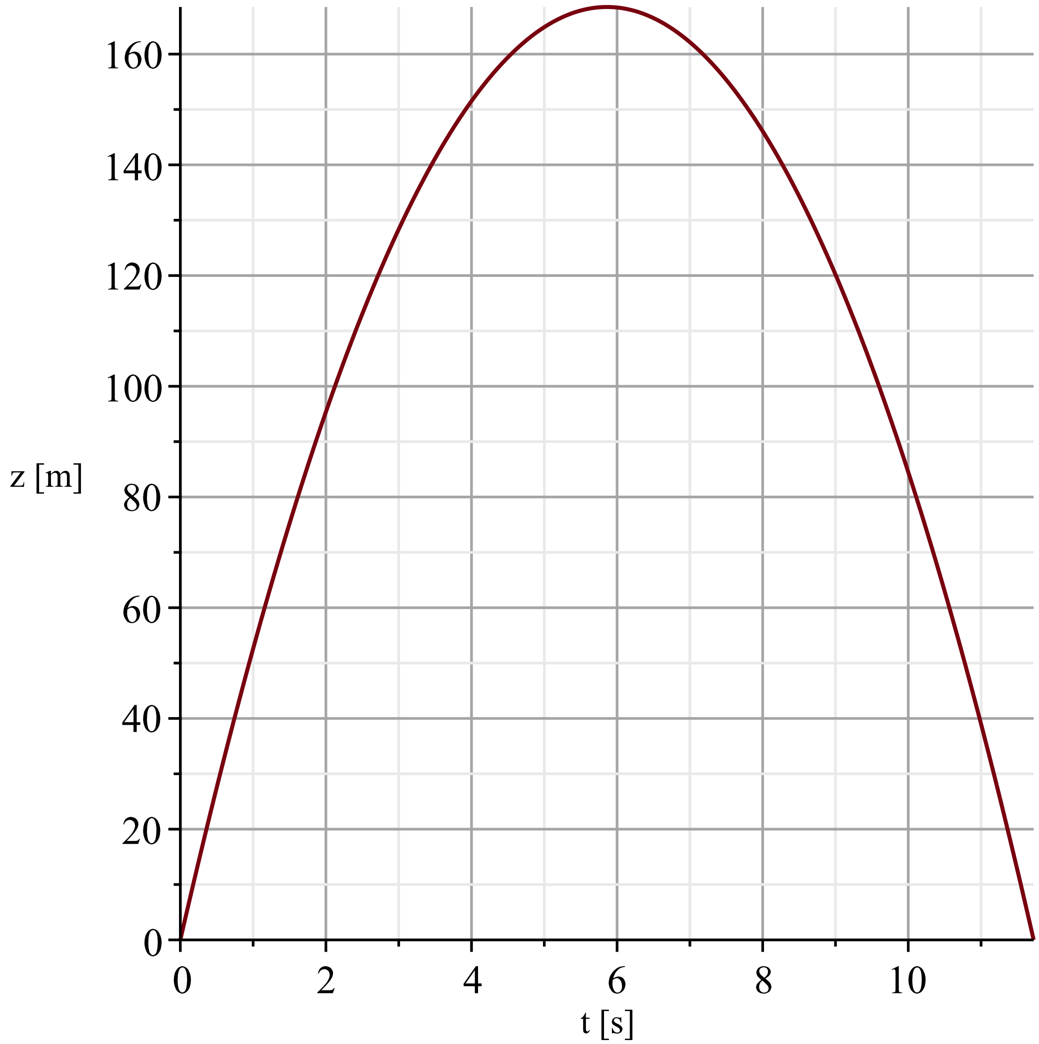
$$z_{maxNum} := 323. \tag{5.5}$$

Plots

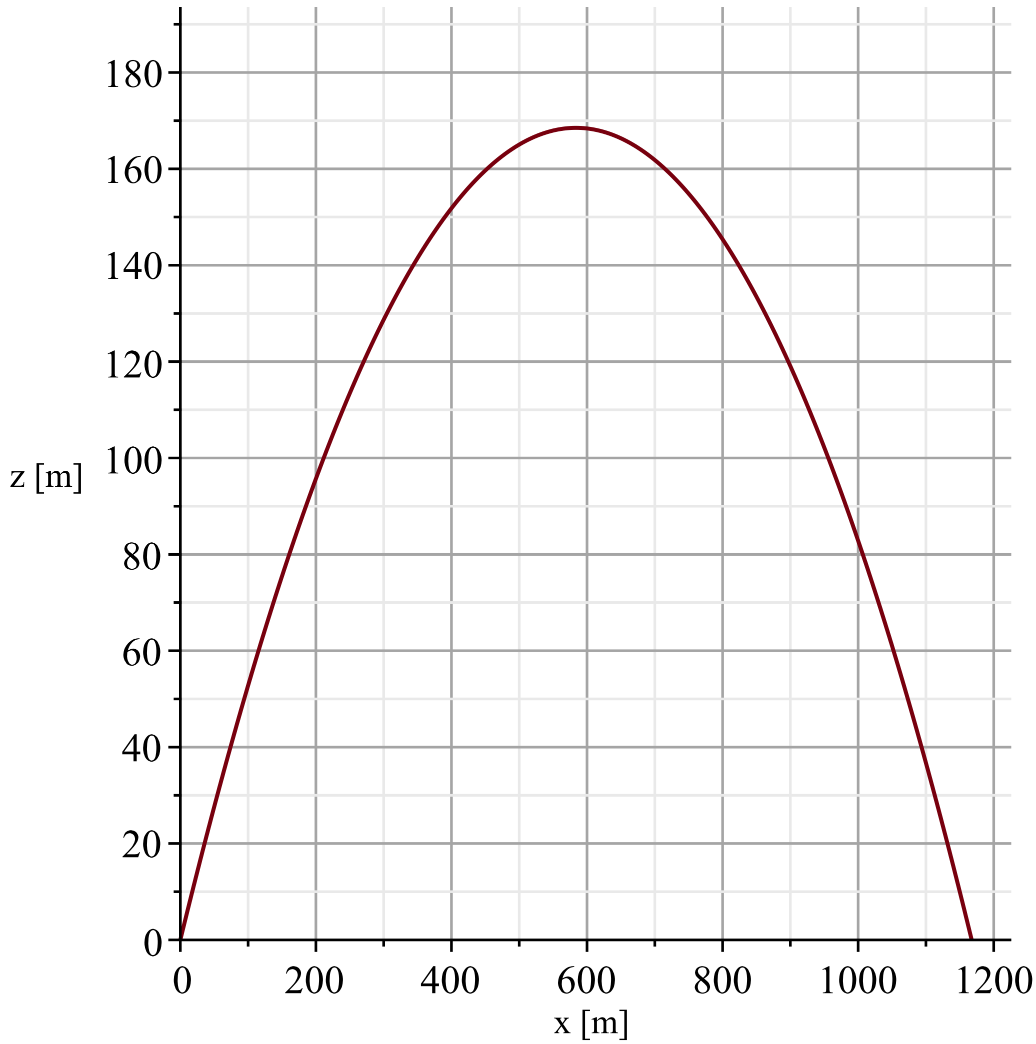
```
> plot(t*v__0*cos(varphi),t = 0..t__max,labels = ["t [s]","x [m]
"],gridlines);
```




```
> plot(-(1/2)*g*t^2+v__0*sin(varphi)*t,t=0..t__max, labels = ["t
  [s]","z [m]"],gridlines);
```




```
> x__vec := [seq(t*v__0*cos(varphi),t=0..t__max,0.01)]:
> z__vec := [seq(-(1/2)*g*t^2+v__0*sin(varphi)*t, t = 0..t__max,
  0.01)]:
> plot(x__vec, z__vec,labels = ["x [m] ","z [m]"],gridlines, view
  = [0..x__max*1.05,0..z__max*0.6]);
```



z [m]
x [m]
0
20
40
60
80
100
120
140
160
180
0
200
400
600
800
1000
1200