Vibrational isolation of pump unit

Project description

A pump unit on an industrial floor causes vibrations that is spreading to the building's structure and disturb nearby equipment. The assignment is to model the system as a 1-DOF mass-spring system assuming a machine weight of 150 kg and a rotational speed of 1800 rpm. The goal is to reduce the force transmission to the structure by at least 70%. Identify critical operating modes and dimension a new isolating mount (spring constant and potential damping).

Assumptions

1. Pump is not in the isolated region, meaning the force transmissibility is ≥ 1. This will act as our reference in order to reduce the force by at least 70%. In reality we might want to experimentally or theoretically determine the natural frequency and thereby also the actual force transmissibility.
2. The original pump mount (spring) is replaced by a new mount and therefore the original spring constant doesn't affect the new system.
3. The rotating unbalance is the dominating vibration, and other vibrations from the environment can be neglected.

Data

In[◦]:=
```
Remove["Global`*"]
m = 150; (*kg*)
N = 1800; (*rpm*)
g = 981 × 10^-2; (*m/s^2*)
```

Force frequency

In[◦]:=

$$\omega = \mathcal{N}\,\frac{2\,\pi}{60};\ (*\text{rad/s}*)$$

Force transmissibility

$$\text{Tf} = r^2\sqrt{\frac{1+(2\,\xi\,r)^2}{\left(1-r^2\right)^2+(2\,\xi\,r)^2}};$$

(*For rotating unbalance. r = Frequency ratio, ξ = Damping ratio*)

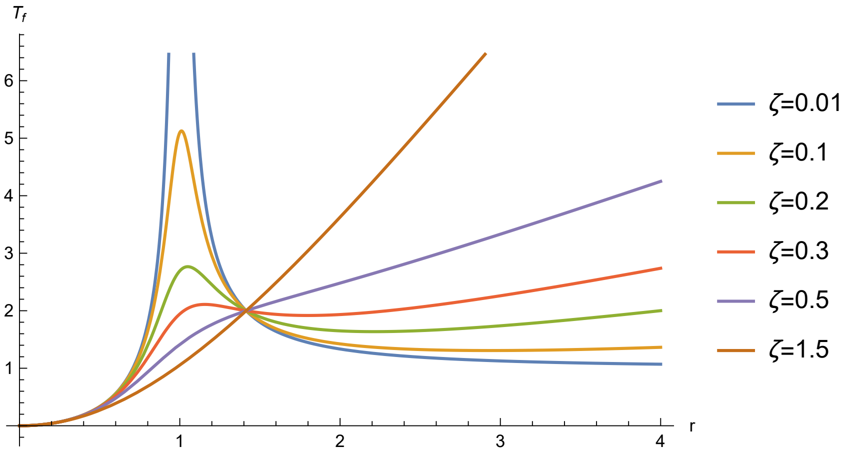
In[◦]:=
```
$Assumptions = {r > 0, ξ > 0};
```

Damping ratio

Although damping is not required in this case, some damping is recommended as to not result in extremely high force transmissibility in the case that the pump is subject to its resonance frequency. Surrounding machines and other vibration sources in the operating environment should be taken into account. For this example we choose a moderate damping ratio of $\zeta = 0.3$.

In[∘]:=
```
Plot[{Tf /. ζ → 0.01, Tf /. ζ → 0.1, Tf /. ζ → 0.2,
   Tf /. ζ → 0.3, Tf /. ζ → 0.5, Tf /. ζ → 1.5}, {r, 0, 4},
 PlotLegends → {"ζ=0.01", "ζ=0.1", "ζ=0.2", "ζ=0.3", "ζ=0.5", "ζ=1.5"},
 AxesLabel → {"r", "Tf"}]
```

Out[∘]=



In[∘]:=
```
ζ = 0.3;
```

Required frequency ratio

In[∘]:=
```
solr = Solve[Tf == 0.3, r, PositiveReals] // Normal // Flatten // Quiet
```

Out[∘]=

```
{r → 0.485396}
```

Frequency ratio

In[∘]:=

$$\omega n = \frac{\omega}{r} \text{ /. solr;}$$

Spring constant

In[∘]:=

$$k = \omega n^2 \, m$$

Out[∘]=

$$2.26205 \times 10^7$$

Damping constant

Critical

In[∘]:=

$$cc = 2 \sqrt{k\, m};$$

Actual

In[∘]:=
```
C = ξ Cc
```
Out[∘]=
```
34950.
```

Static displacement

In[∘]:= $\delta st = \frac{m\,g}{k}$ // EngineeringForm

Out[∘]//EngineeringForm=

65.0517×10^{-6}

(≈ 0)

Resonance speed

$r = 1$ gives

In[∘]:= solωr = Solve$\left[1 == \frac{\omega r}{\omega n}, \omega r\right]$ // Flatten

Out[∘]=
```
{ωr → 388.334}
```

In rpm

In[∘]:= $\omega r \frac{60}{2\,\pi}$ /. solωr

Out[∘]=
```
3708.31
```

Solution

Spring constant: $k \geq 22.6$ MN/m
Damping constant: $c = 35.0$ kNs/m
Resonance speed: ~ 3710 rpm

If our assumptions are applicable, we will see ≥ 70 % reduction in the force transmitted into the surroundings. The static deflection is almost zero as not to exceed elastic limits of the new springs. The pump should always operate well below 3700 rpm.