Exercises for the course
Deep Learning 1
Winter Semester 2022/23

Machine Learning Group
Faculty IV – Electrical Engineering and Computer Science
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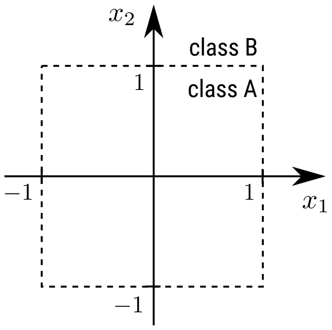
Exercise Sheet 1–2

Exercise 1: Building a Neural Network (10 P)

We would like to implement a neural network that classifies data points in $\mathbb{R}^2$ according to decision boundary given in the figure below.



We consider as an elementary computation the *threshold neuron* whose relation between inputs $(a_i)_i$ and output a_j is given by

$$z_j = \textstyle\sum_i a_i w_{ij} + b_j \qquad\qquad a_j = I(z_j > 0).$$

where I is an indicator function that outputs 1 when the statement given as input is true, and 0 otherwise.

(a) *Design* at hand a neural network composed of two layers of parameters, that takes x_1 and x_2 as input and produces the output "1" if the input belongs to class A, and "0" if the input belongs to class B. *Draw* the neural network model and *write down* the weights w_{ij} and bias b_j of each neuron.

Exercise 2: Backpropagation in the Error Function (5 + 5 + 5 + 5 P)

Let y be the prediction of a neural network for some data point $\boldsymbol{x}$. The true target value that the network should predict is given by t. We define the error function to be

$$E = \log\cosh(y - t).$$

This error function can be used as an alternative to the square error and is more robust to outliers.

(a) *Compute* the gradient of the error with respect to the output y of the neural network.

(b) Assume we have a dataset composed of neural network inputs $\boldsymbol{x}_1, \dots, \boldsymbol{x}_N$ and associated targets $t_1, \dots, t_N$. We denote by $y_1, \dots, y_N$ the predictions of the neural network for these points. We define the error

$$E = \frac{1}{N}\sum_{k=1}^{N} E_k \qquad \text{with} \qquad E_k = \log\cosh(y_k(\boldsymbol{x}_k, \boldsymbol{w}) - t_k)$$

State the chain rule for transmitting the gradient from the output of the neural network to the model parameters.

(c) Assume that $y_k(\boldsymbol{x}_k, \boldsymbol{w}) = \sum_{i=1}^{d} w_i x_i^{(k)}$ where $x_i^{(k)}$ denotes the ith element of the vector $\boldsymbol{x}_k$. Compute the gradient of the error function w.r.t. the parameter w_i, i.e. compute $\partial E / \partial w_i$.

Exercise 3: Backpropagation in a Multilayer Network (10 + 10 P)

We consider a neural network that takes two inputs x_1 and x_2 and produces an output y based on the following set of computations:

$$
\begin{aligned}
z_3 &= x_1 \cdot w_{13} + x_2 \cdot w_{23} & z_5 &= a_3 \cdot w_{35} + a_4 \cdot w_{45} & y &= a_5 + a_6 \\
a_3 &= \tanh(z_3) & a_5 &= \tanh(z_5) & E &= (y - t)^2 \\
\\
z_4 &= x_1 \cdot w_{14} + x_2 \cdot w_{24} & z_6 &= a_3 \cdot w_{36} + a_4 \cdot w_{46} \\
a_4 &= \tanh(z_4) & a_6 &= \tanh(z_6)
\end{aligned}
$$

(a) *Draw* the neural network graph associated to this set of computations.

(b) *Write* the set of backward computations that leads to the evaluation of the partial derivative $\partial E / \partial w_{13}$. Your answer should avoid redundant computations. Hint: $\tanh'(t) = 1 - (\tanh(t))^2$.

Exercise 4: Backpropagation with Shared Parameters ($5 + 5 + 5 + 5$ P)

Let x_1, x_2 be two observed variables. Consider the two-layer neural network that takes these two variables as input and builds the prediction y by computing iteratively:

$$z_3 = x_1 w_{13}, \quad z_4 = x_2 w_{24}, \qquad a_3 = 0.5\, z_3^2, \quad a_4 = 0.5\, z_4^2, \qquad y = a_3 + a_4.$$

(a) *Draw* the neural network graph associated to these computations.

(b) We now consider the error $E = (y - t)^2$ where t is a target variable that the neural network learns to approximate. Using the rules for backpropagation, compute the derivatives $\partial E / \partial w_{13}$ and $\partial E / \partial w_{24}$.

(c) Let us now assume that w_{13} and w_{24} cannot be adapted freely, but are a function of the same shared parameter v:

$$w_{13} = \log(1 + \exp(v)) \qquad \text{and} \qquad w_{24} = -\log(1 + \exp(-v))$$

State the multivariate chain rule that links the derivative $\partial E / \partial v$ to the partial derivatives you have computed above.

(d) Using the computed $\partial E / \partial w_{13}$ and $\partial E / \partial w_{24}$, write an analytic expression of $\partial E / \partial v$.

Exercise 5: Programming (30 P)

Download the programming files on ISIS and follow the instructions.