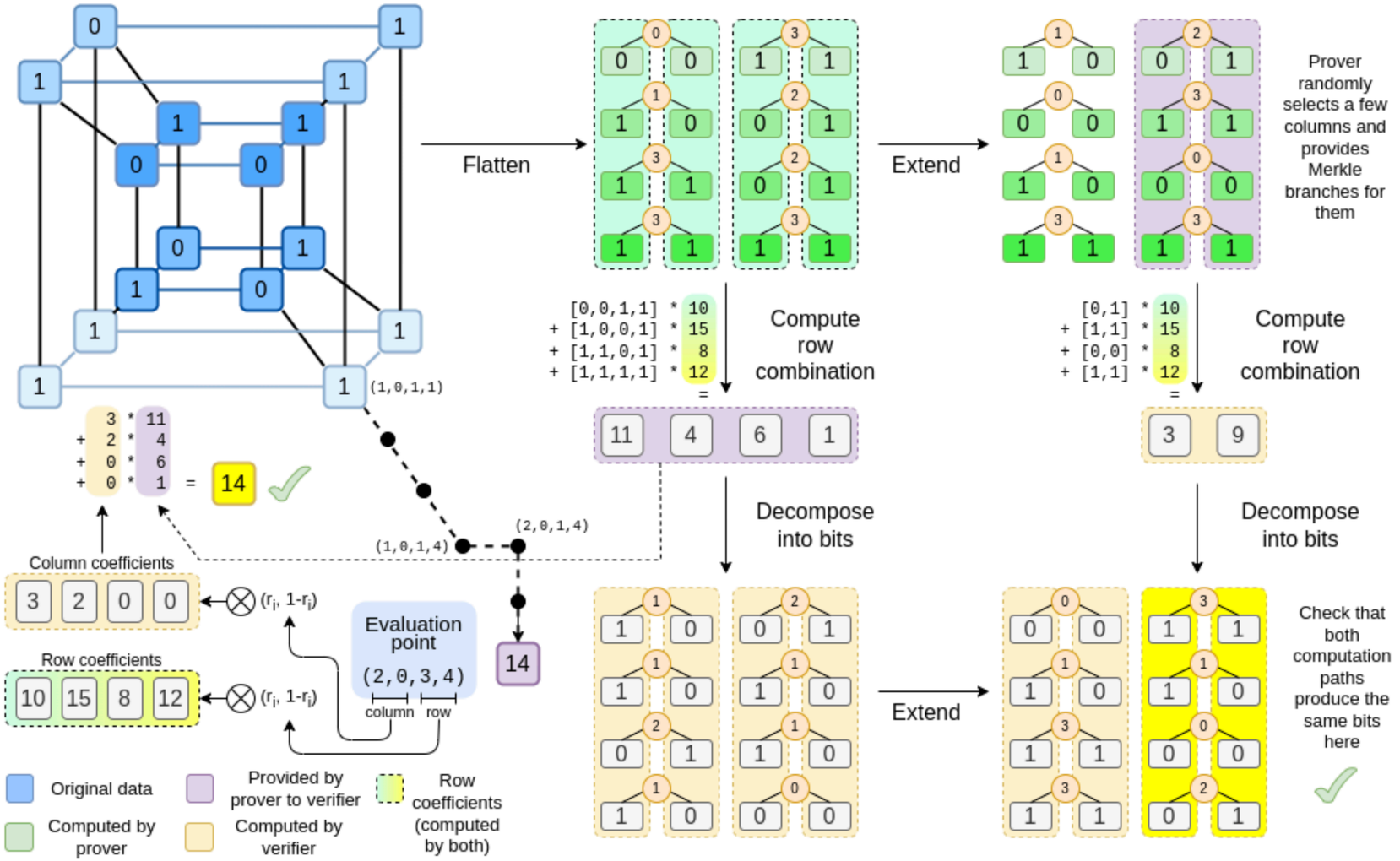
Binius

Dana Ben Porath



Based on [Vitalik's blog on Binius]

Intro - calculating $F(r_1, r_2, r_3, r_4)$

Using tensor products

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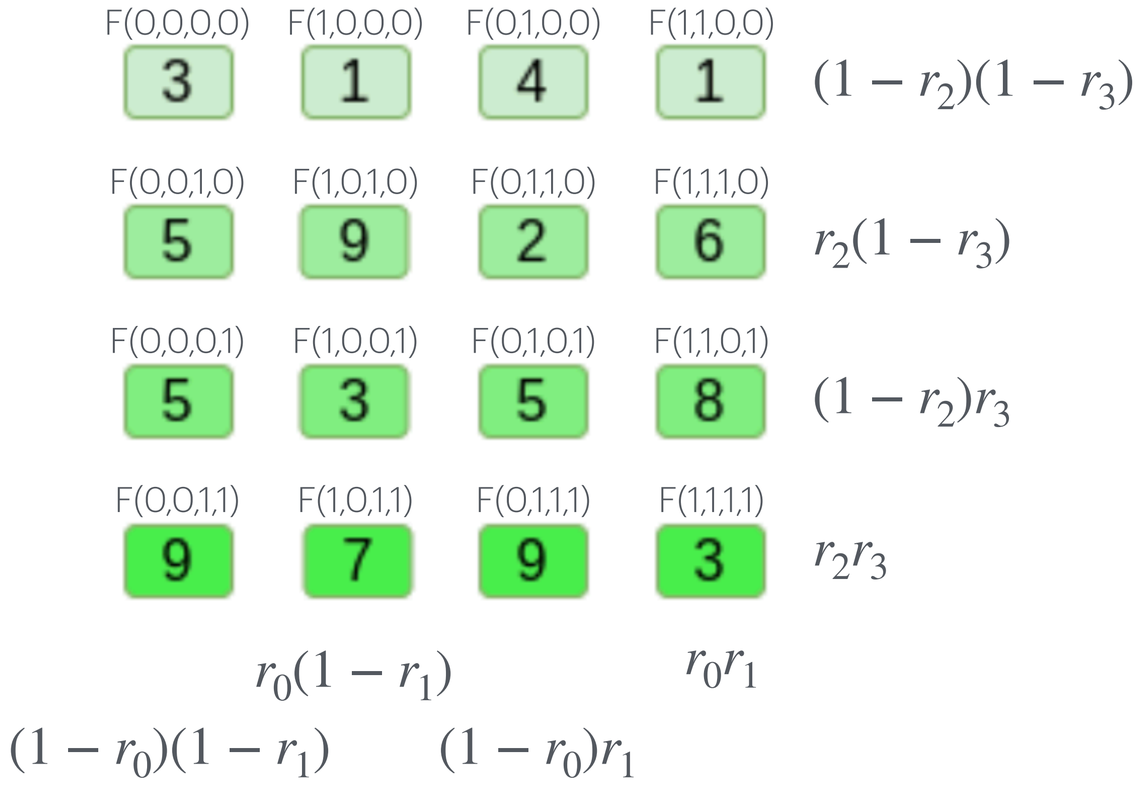
Using tensor products

$$F(r_1, r_2, r_3, r_4) = F(0,0,0,0)(1-r_0)(1-r_1)(1-r_2)(1-r_3)$$
$$+F(1,0,0,0)r_0(1-r_1)(1-r_2)(1-r_3) + \dots$$



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Using tensor products

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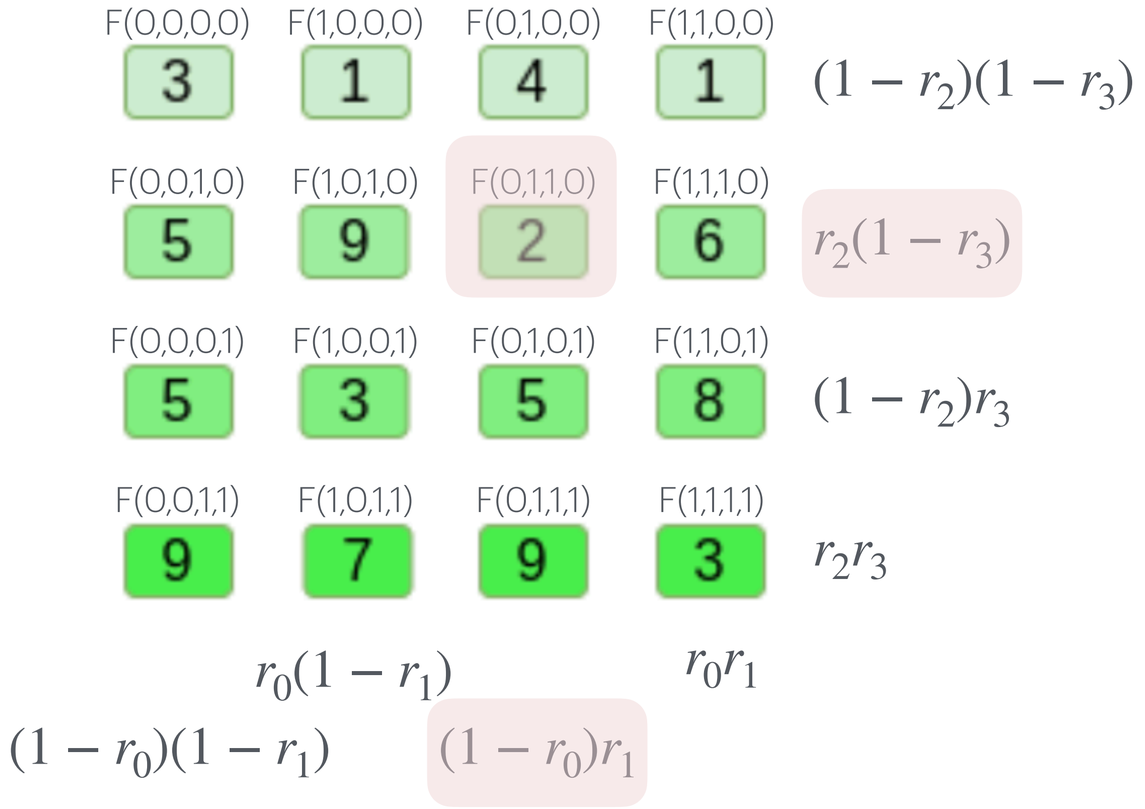

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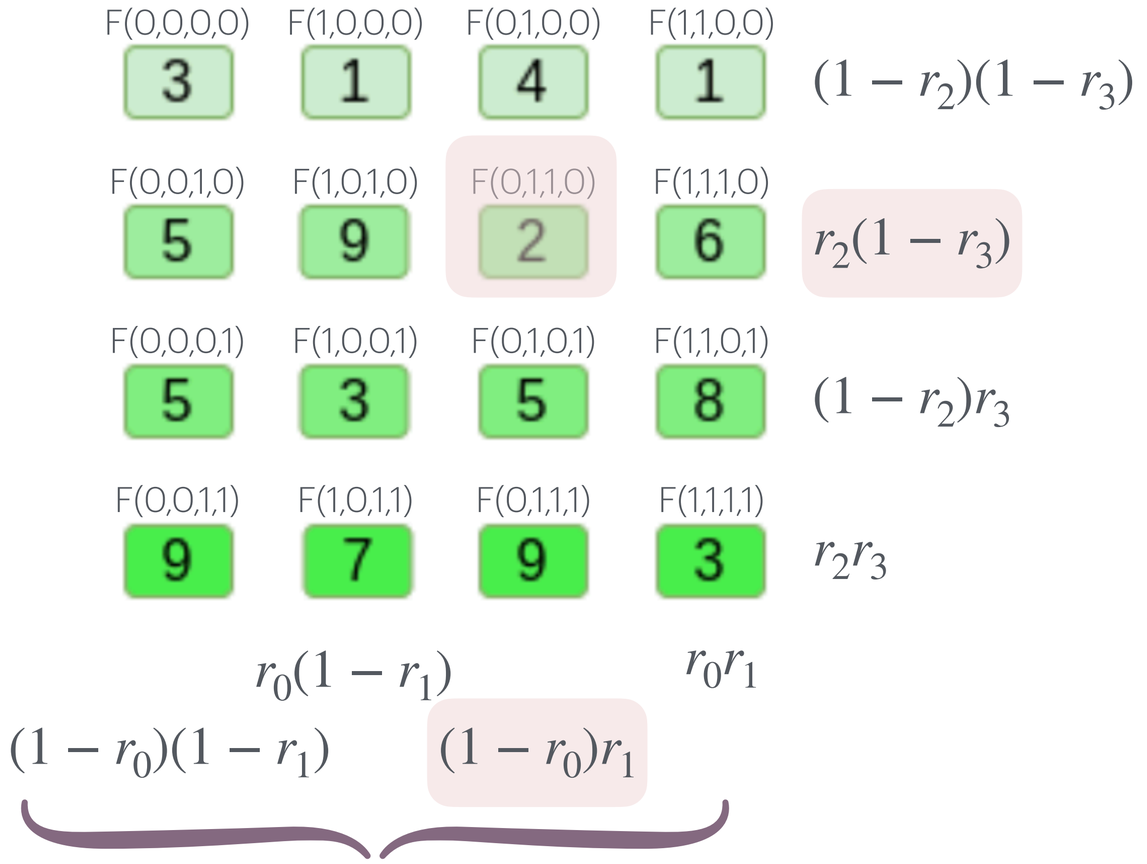
Using tensor products



$$F(r_1, r_2, r_3, r_4) = F(0,0,0,0)(1-r_0)(1-r_1)(1-r_2)(1-r_3) + F(1,0,0,0)r_0(1-r_1)(1-r_2)(1-r_3) + \dots$$

Example - calculating F(1,2,3,4)

$$\begin{array}{llllll} & 3 & 1 & 4 & 1 & \cdot 6 \\ + & 5 & 9 & 2 & 6 & \cdot (-9) \\ + & 5 & 3 & 5 & 8 & \cdot (-8) \\ + & 9 & 7 & 9 & 3 & \cdot 12 \end{array}$$

$$F(1,2,3,4) = \underbrace{[0, -1, 0, 2]}_{} \cdot [\ 41\ ,\ -15\ ,\ 74\ ,\ -76\ ] = -137$$

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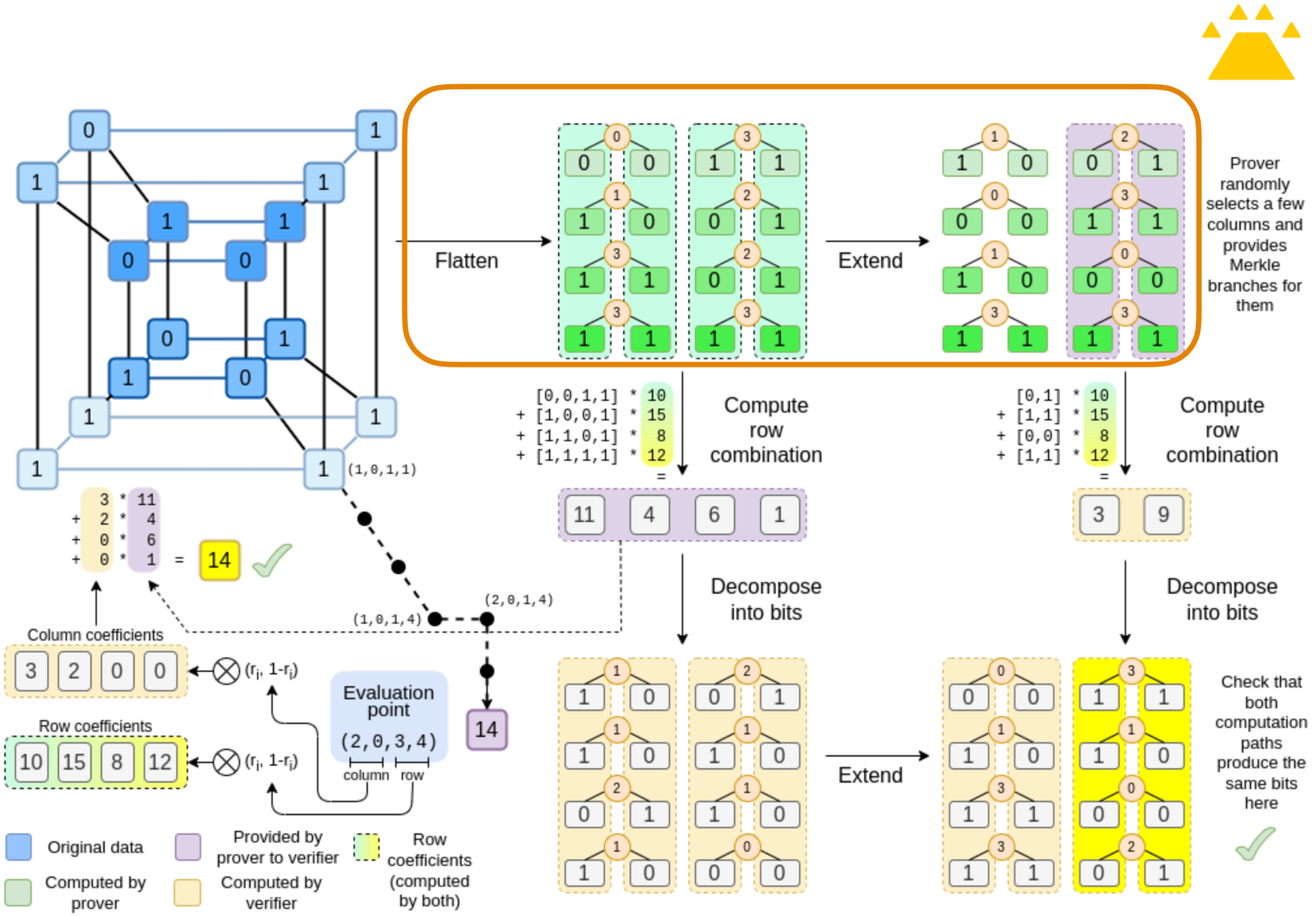
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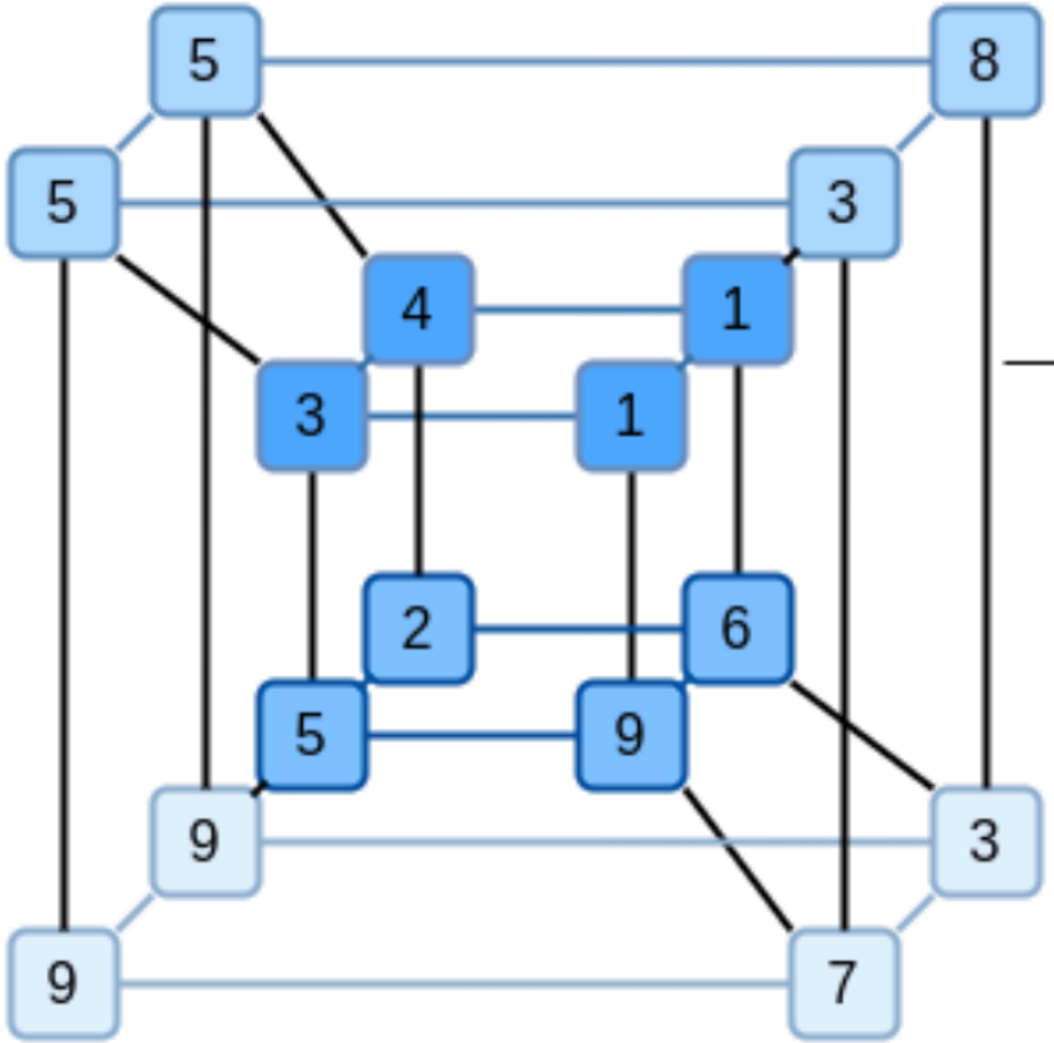

Flatten

F(0,0,0,0)	F(1,0,0,0)	F(0,1,0,0)	F(1,1,0,0)
3	1	4	1
F(0,0,1,0)	F(1,0,1,0)	F(0,1,1,0)	F(1,1,1,0)
5	9	2	6
F(0,0,0,1)	F(1,0,0,1)	F(0,1,0,1)	F(1,1,0,1)
5	3	5	8
F(0,0,1,1)	F(1,0,1,1)	F(0,1,1,1)	F(1,1,1,1)
9	7	9	3

Extend

Lagrange polynomials / Reed-Solomon extension

$$(0,3),(1,1),(2,4),(3,1) - - - - > (4,-19),(5,-67),(6,-154),(7,-291)$$

Extend

Lagrange polynomials / Reed-Solomon extension

$$(0,3),(1,1),(2,4),(3,1) - - - - > (4,-19),(5,-67),(6,-154),(7,-291)$$

The commitment is the root of the Merkle tree of the columns

Extend

Lagrange polynomials / Reed-Solomon extension

Given $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$, the Lagrange interpolating polynomial is:

$$L(x) = \sum_{j=0}^{n} y_j l_j(x), \text{ where } l_j(x) = \prod_{\substack{0 \leqslant m \leqslant n \\ m \neq j}} \frac{x - x_m}{x_j - x_m}$$

Extend

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Barycentric form:

$$L(x) = l(x) \sum_{j=0}^{n} \frac{w_j}{x - x_j} y_j, \text{ where}$$

$$l(x) = \prod_{0 \leqslant m \leqslant n} (x - x_m) \;\&\; w_j(x) = \prod_{\substack{0 \leqslant m \leqslant n \\ m \neq j}} \frac{1}{x_j - x_m}$$

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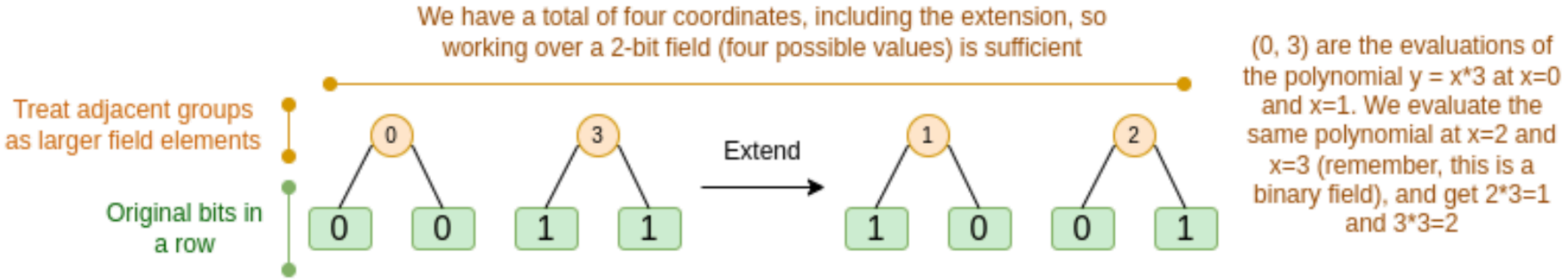
A limitation - if you are extending n values to kn values, you need to be working in a field that has kn different values that you can use as coordinates.

Extend - binary fields



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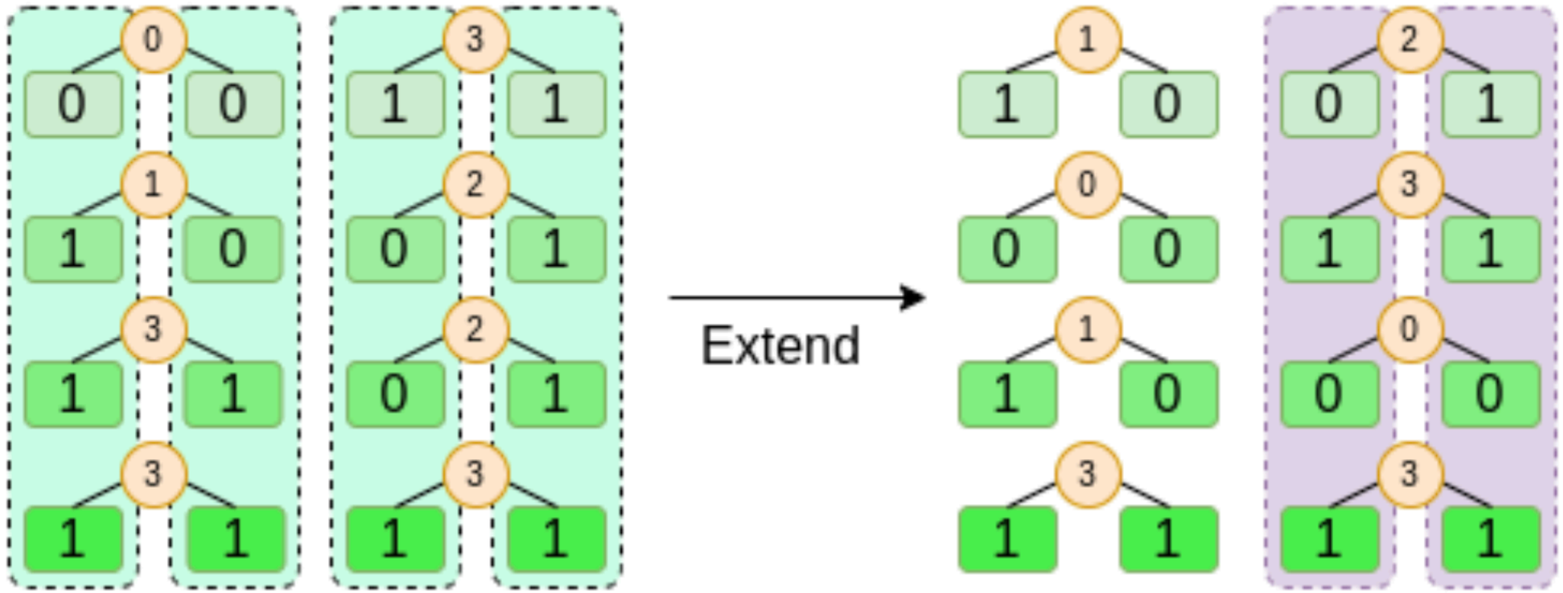

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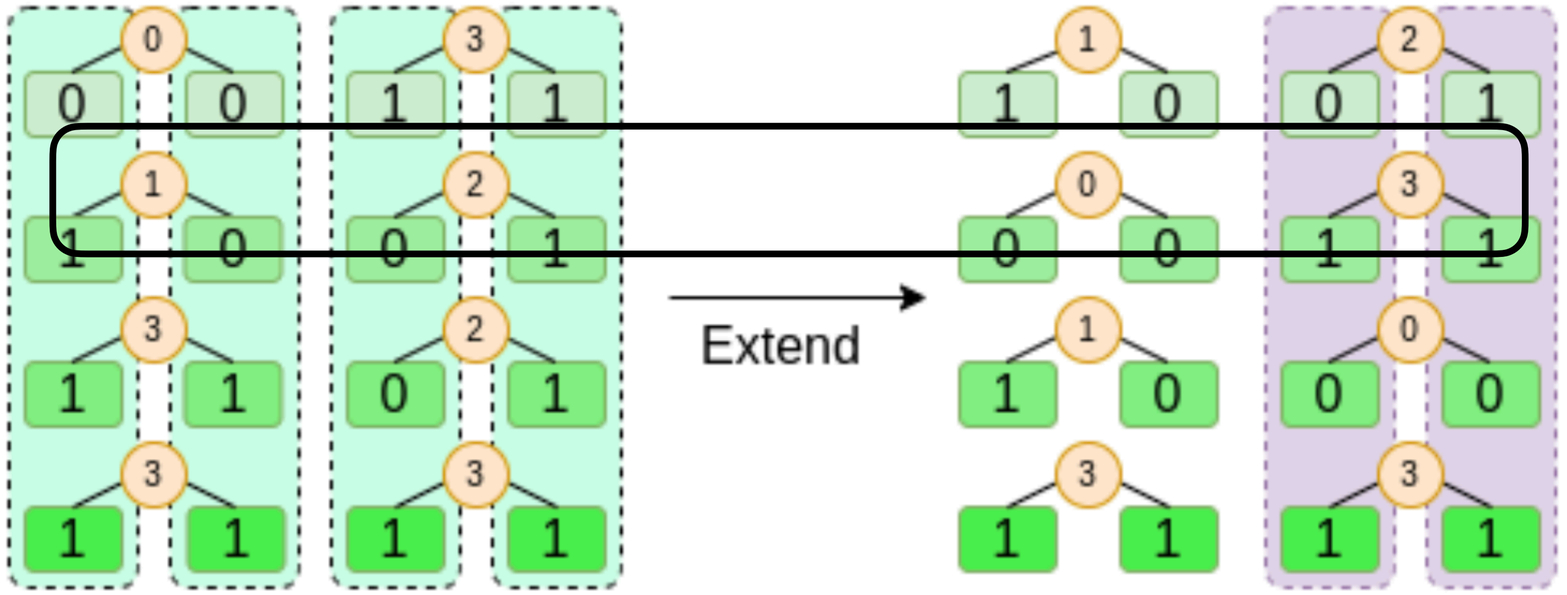
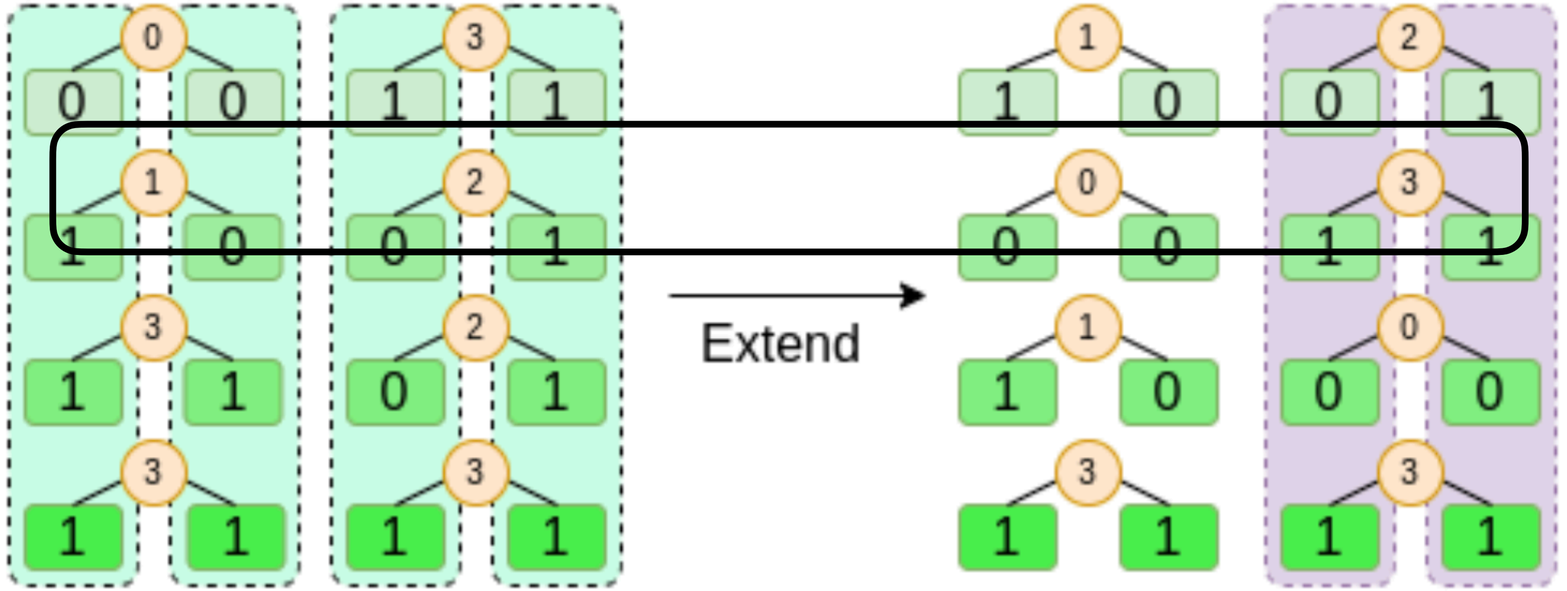


Extend - binary fields



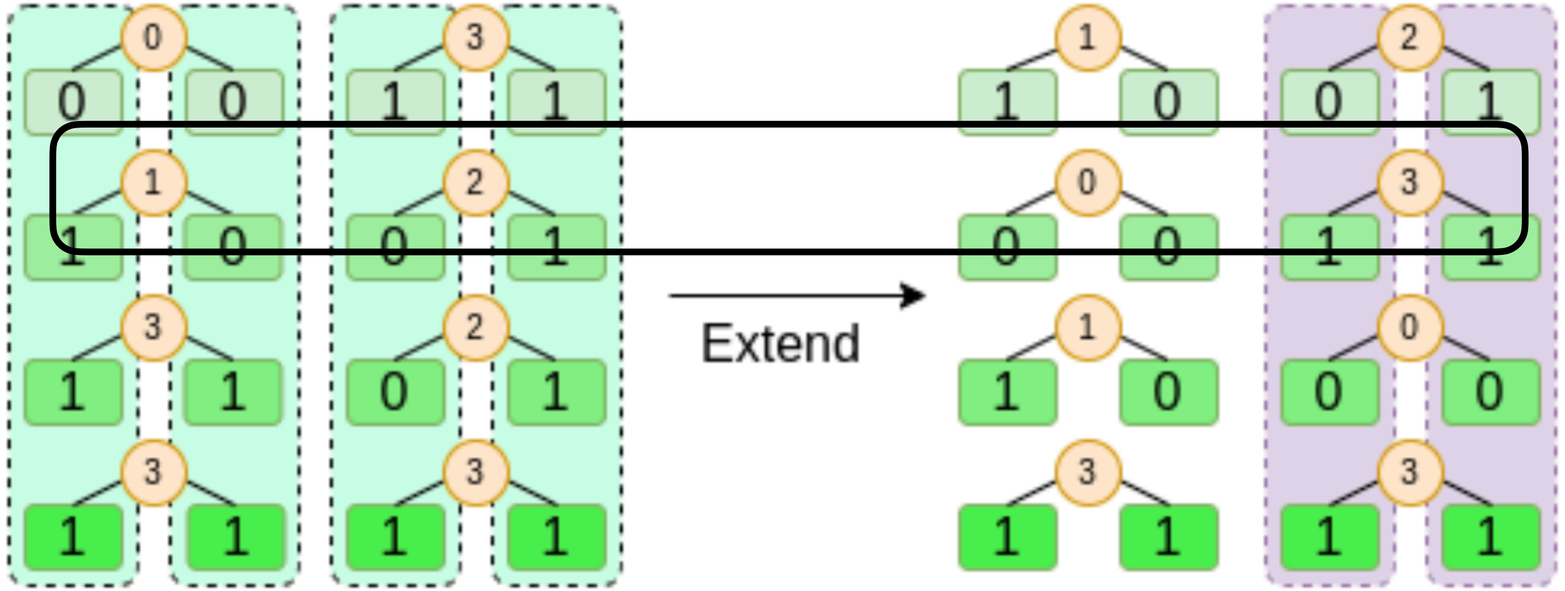
+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

*	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

$$L(x) = l(x)\sum_{j=0}^{n}\frac{w_j}{x-x_j}y_j, \text{ where } l(x) = \prod_{0\leqslant m\leqslant n}(x-x_m) \;\&\; w_j(x) = \prod_{\substack{0\leqslant m\leqslant n \\ m\neq j}}\frac{1}{x_j-x_m}$$

Extend - binary fields

$(0,1),(1,2) - - - - - > (2,0),(3,3)$



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0	0	1	2	3
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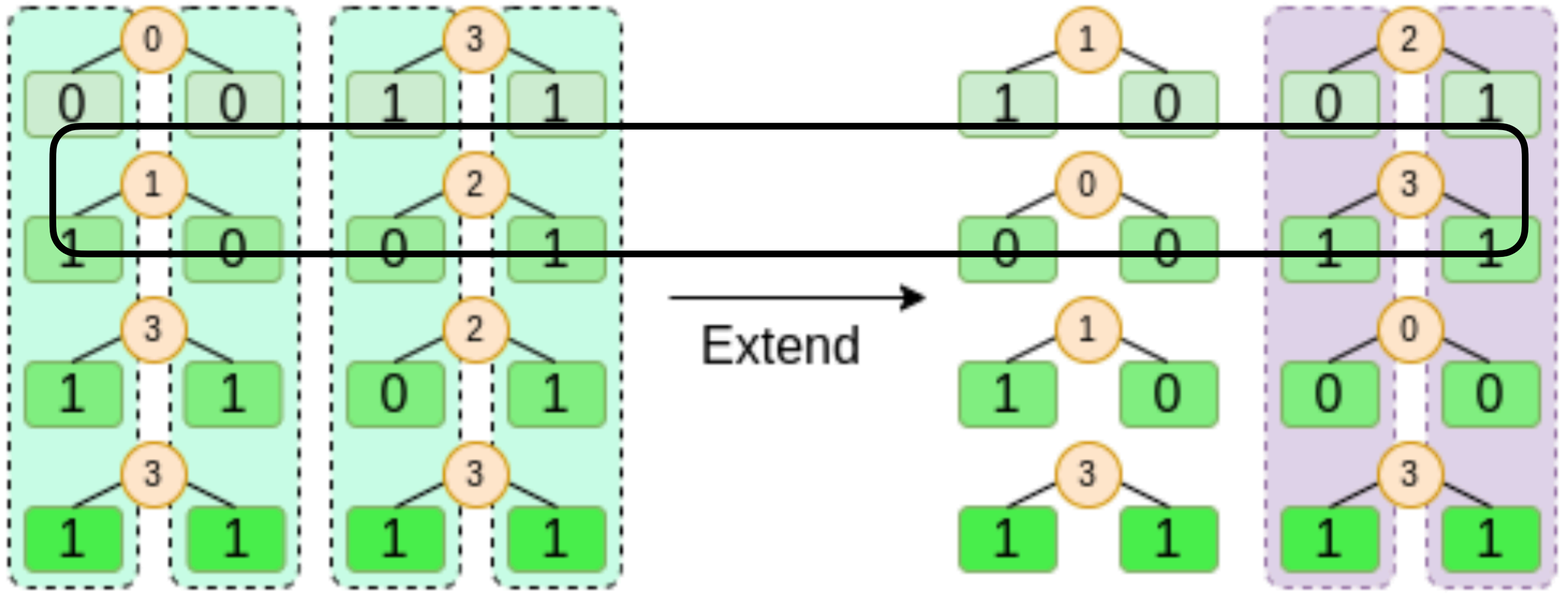
$$\Rightarrow \quad l(x) = x(x-1), w_0 = \frac{1}{0-1} = 1, w_1 = \frac{1}{1-0} = 1$$

Extend - binary fields

$(0,1),(1,2) - - - - - > (2,0),(3,3)$



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0	0	1	2	3
1	1	0	3	2
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3	3	2	1	0

*	0	1	2	3
0	0	0	0	0
1	0	1	2	3
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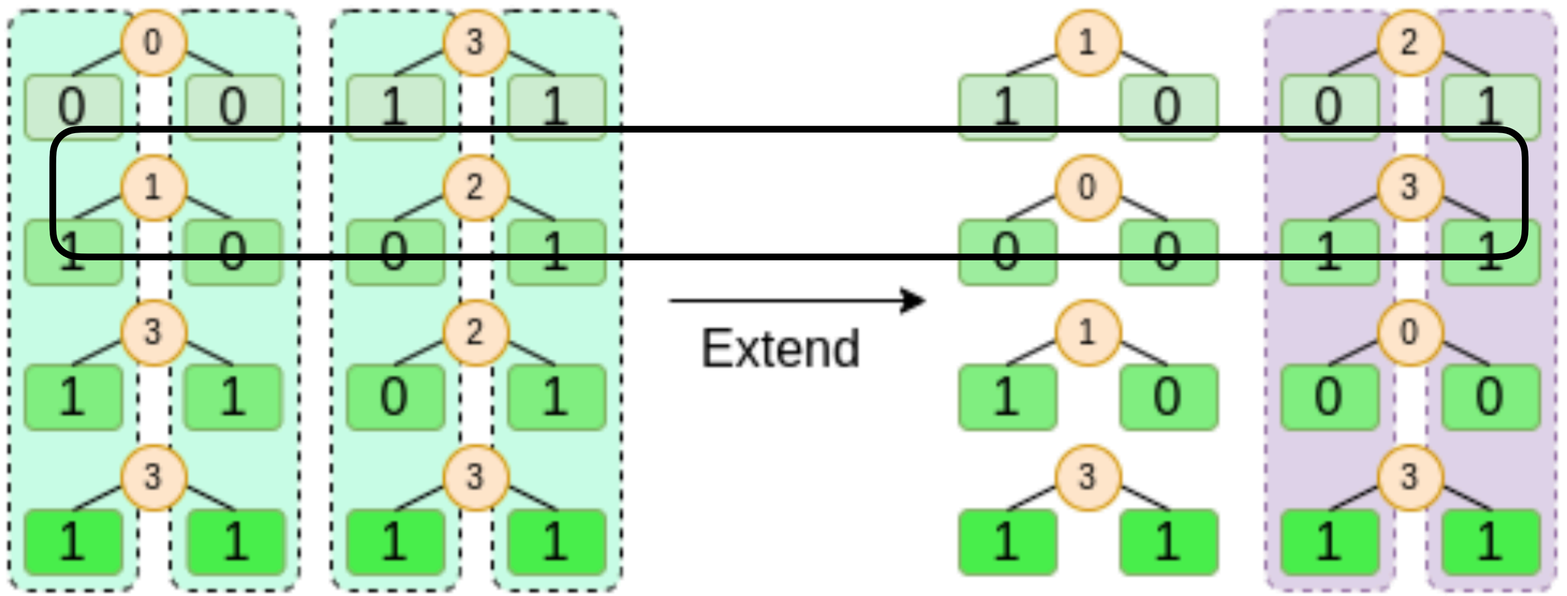
$$\Rightarrow \quad l(x) = x(x-1), w_0 = \frac{1}{0-1} = 1, w_1 = \frac{1}{1-0} = 1$$

$$\Rightarrow \quad L(x) = x(x-1)\left[\frac{1\cdot 1}{x} + \frac{1\cdot 2}{x-1}\right] = (x-1) + 2x = -1 + (1+2)x = 1 + 3x$$

Extend - binary fields



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$$\Rightarrow \quad L(2) = 0, L(3) = 3$$

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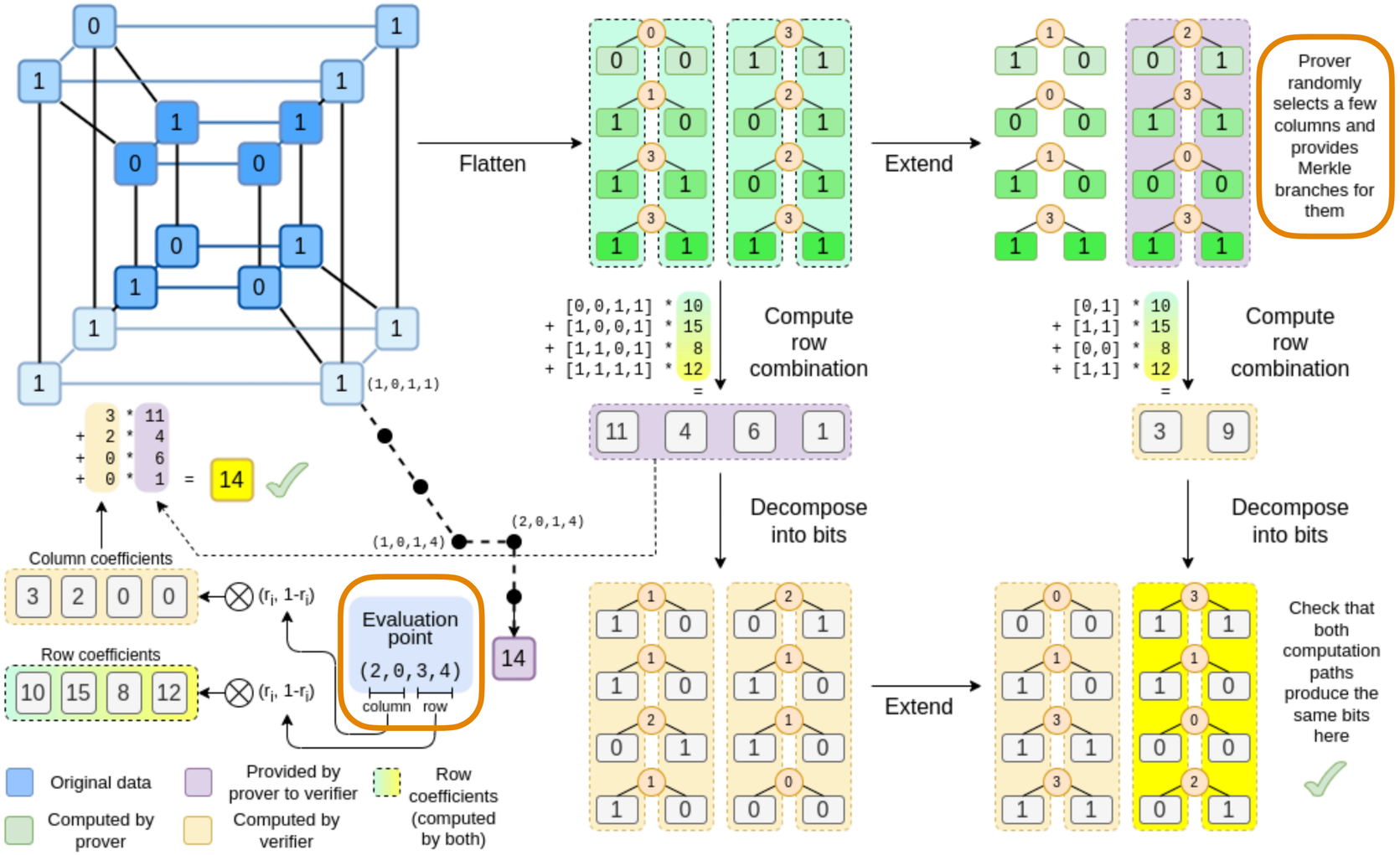
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Prover randomly selects a few columns and provides Merkle branches for them
Compute row combination
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Column coefficients
Row coefficients
Evaluation point
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column
row
14
Original data
Computed by prover
Provided by prover to verifier
Computed by verifier
Row coefficients (computed by both)

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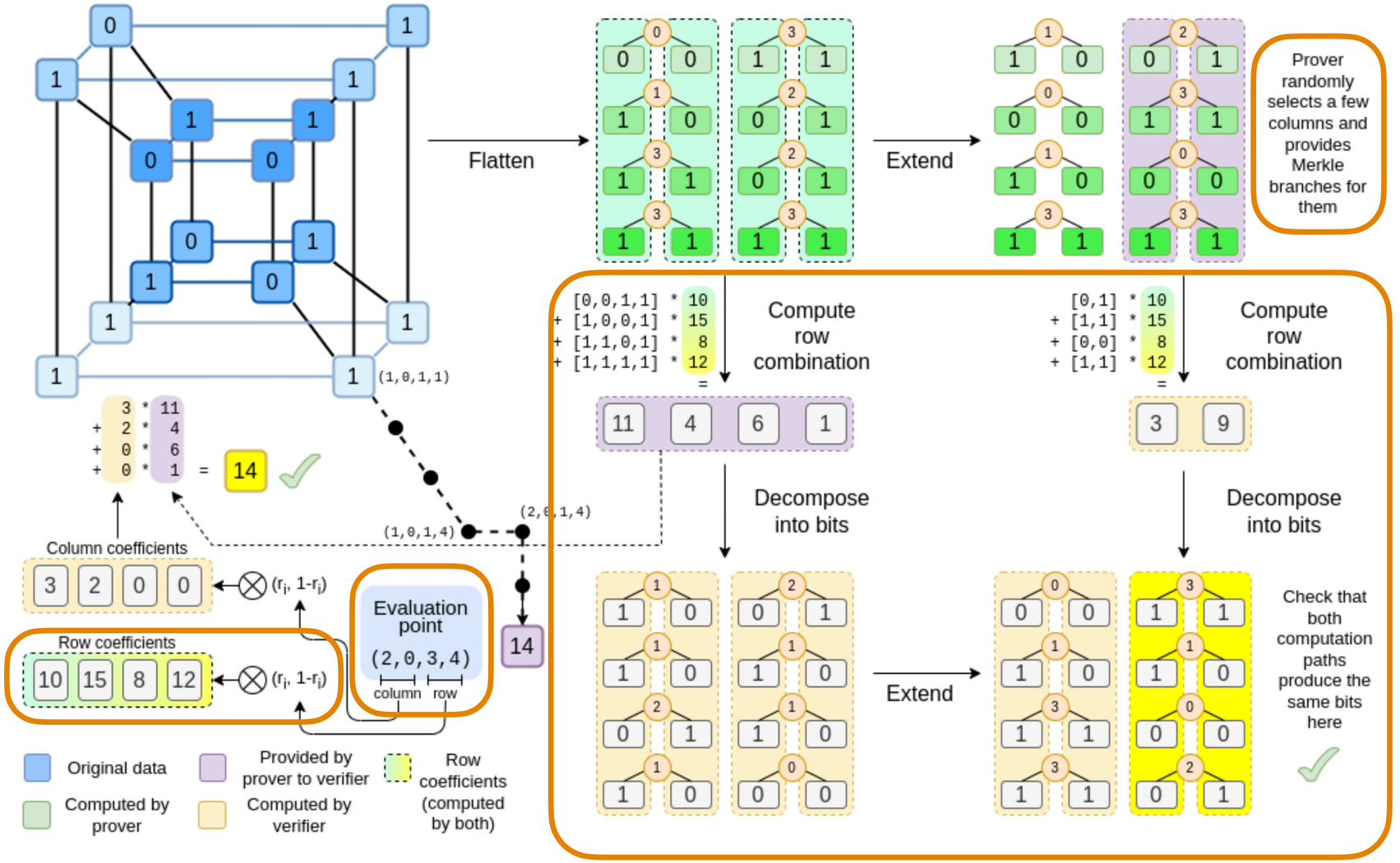
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Compute row combination

In the evaluation point $(r_o, r_1, r_2, r_3) = (2,0,3,4)$

$$\otimes_{i=2,3}(1-r_i, r_i) = [(1-r_2)\cdot(1-r_3), r_2\cdot(1-r_3), (1-r_2)\cdot r_3, r_2\cdot r_3] = [(1-3)\cdot(1-4), 3\cdot(1-4), (1-3)\cdot 4, 3\cdot 4]$$
$$= [2\cdot 5, 3\cdot 5, 2\cdot 4, 3\cdot 4] = [10,15,8,12]$$

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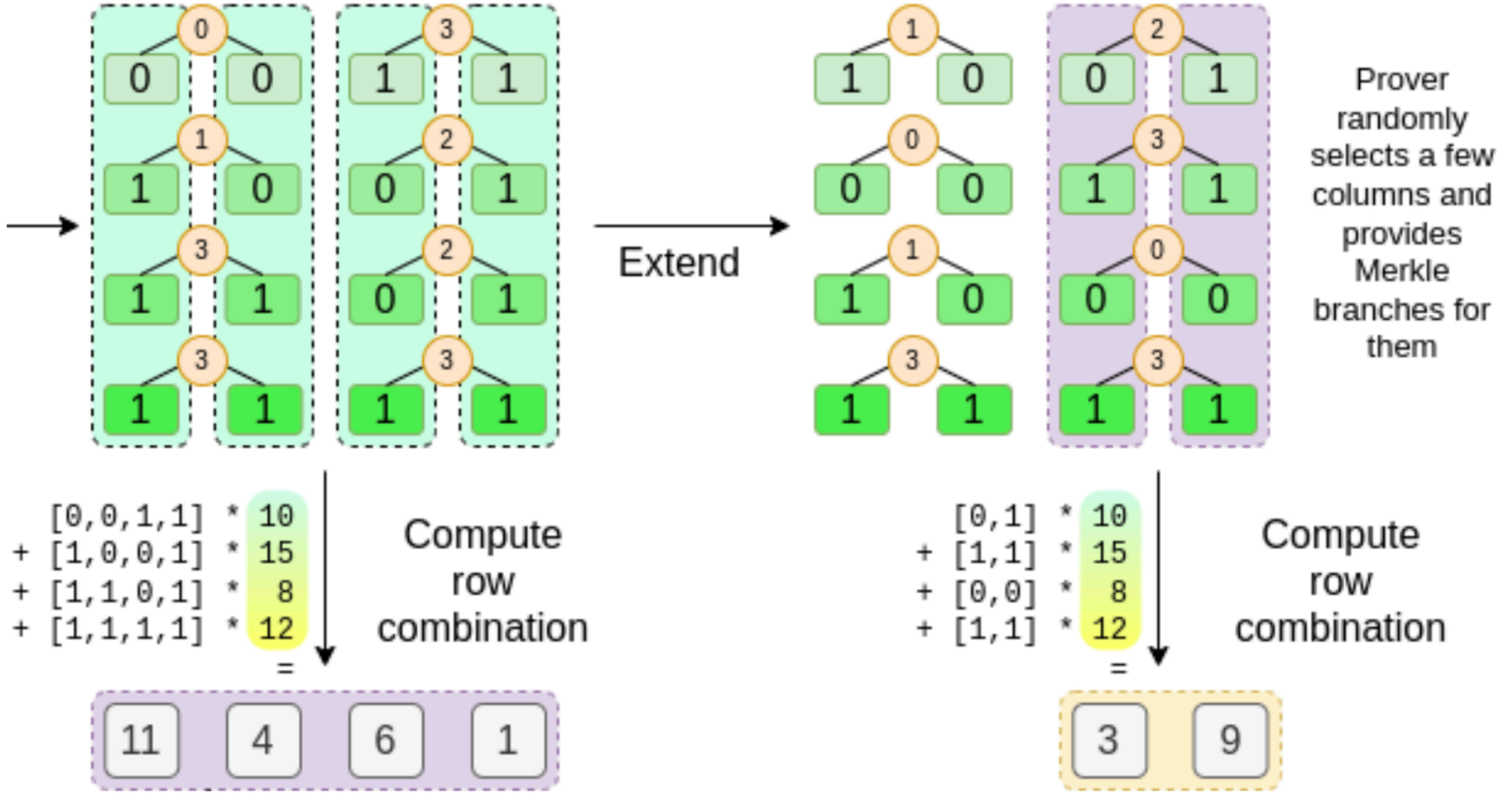
$$= [2\cdot 5, 3\cdot 5, 2\cdot 4, 3\cdot 4] = [10,15,8,12]$$



The linearity of the extension

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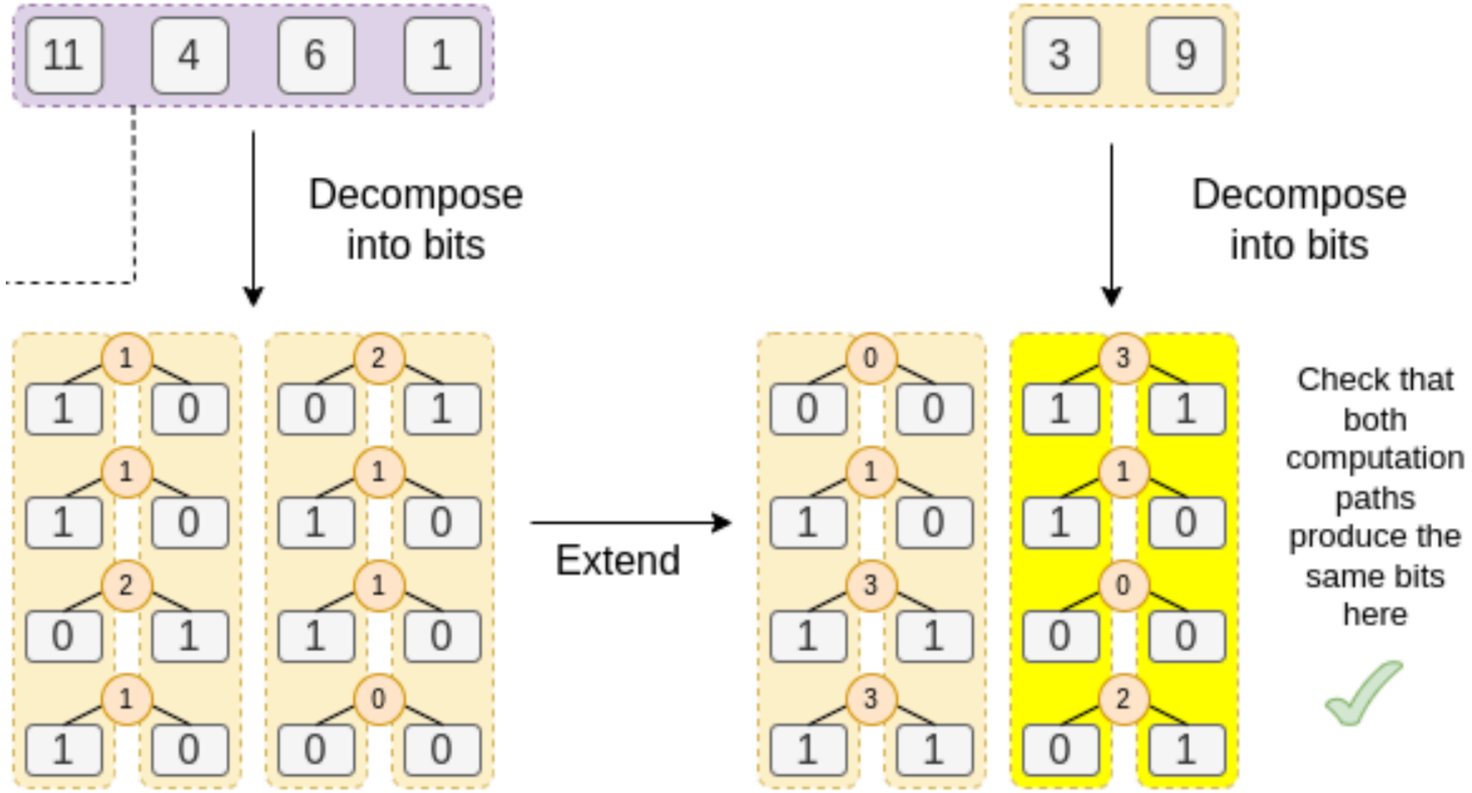

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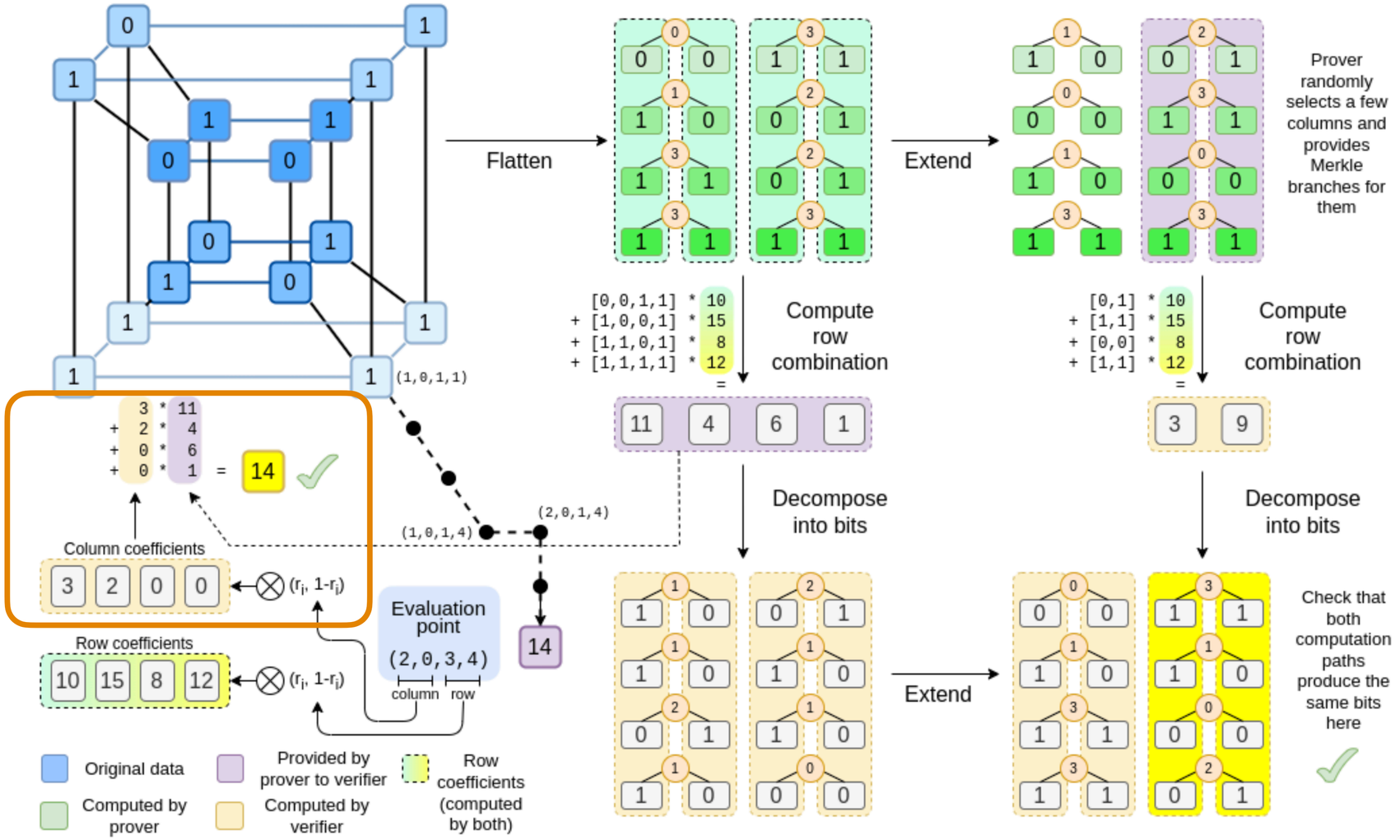
Verify that the answer is 14



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Thank you!