Ingrid Chan
998067871
g5chanin

Assignment 1

CSC418H

1. A. Tangent (Find Derivative):

$\vec{t}$ = (x'(t), y'(t))
= (a, -gt + b)

Normal:

$\vec{n}$ = (y'(t), -x'(y))
= (-gt + b, -a)

B. Time of Impact t_i is when y(t) = 0:
$0 = 1/2g(t_i)^2 + bt_i + h$

Location at Time of Impact
$(at_i, 1/2g(t_i)^2 + bt_i + h)$ y(t) should be 0 at time of impact
= $(at_i, 0)$

Velocity at time of impact, using the tangent from A
$(a, -gt_i + b)$

2. Let p0 be our starting point and p1 the transformed point

A. Translation and Shear are **not** commutative

Proof:
Let $\vec{t}$ be $\begin{bmatrix}2\\2\end{bmatrix}$ and s = $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$:

Translation + Shear:
p1 $= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}(p0 + \begin{bmatrix}2\\2\end{bmatrix})$ Distributive property
$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}p0 + \begin{bmatrix}5\\2\end{bmatrix}$

Shear + Translation
p1 $= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}p0 + \begin{bmatrix}2\\2\end{bmatrix}$

$\begin{bmatrix}5\\2\end{bmatrix} \neq \begin{bmatrix}2\\2\end{bmatrix}$, Hence Translation and Shear are not commutative

B. Multiple rotations are commutative

Proof:
Rotation A then Rotation B
p11 $= \begin{bmatrix}cosA & -sinA\\sinA & cosA\end{bmatrix}\begin{bmatrix}cosB & -sinB\\sinB & cosB\end{bmatrix}p0$
$= \begin{bmatrix}cosAcosB - sinAsinB & -cosAsinB - sinAcosB\\sinAcosB + cosAsinB & -sinAsinB + cosAcosB\end{bmatrix}p0$

Rotation B then Rotation A
p12 $= \begin{bmatrix}cosB & -sinB\\sinB & cosB\end{bmatrix}\begin{bmatrix}cosA & -sinA\\sinA & cosA\end{bmatrix}p0$
$= \begin{bmatrix}cosAcosB - sinAsinB & -cosAsinB - sinAcosB\\sinAcosB + cosAsinB & -sinAsinB + cosAcosB\end{bmatrix}p0$

p11 = p12, Hence rotations are commutative

C. Uniform Scaling and Rotation are commutative

Proof:
Uniform Scaling + Rotation

p11 $= \begin{bmatrix} cosB & -sinB \\ sinB & cosB \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$p0

$= \begin{bmatrix} xcosB & -xsinB \\ xsinB & xcosB \end{bmatrix}$p0

Rotation + Uniform Scaling

p12 $= \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} \begin{bmatrix} cosB & -sinB \\ sinB & cosB \end{bmatrix}$p0

$= \begin{bmatrix} xcosB & -xsinB \\ xsinB & xcosB \end{bmatrix}$p0

p11 = p12, Hence Rotation and Uniform Scaling are commutative

D. Non-Uniform scaling and Rotation are **not** commutative

Proof:
Non-Uniform Scaling + Rotation

p11 $= \begin{bmatrix} cosB & -sinB \\ sinB & cosB \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$p0

$= \begin{bmatrix} xcosB & -ysinB \\ xsinB & ycosB \end{bmatrix}$p0

Rotation + Non-Uniform Scaling

p12 $= \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \begin{bmatrix} cosB & -sinB \\ sinB & cosB \end{bmatrix}$p0

$= \begin{bmatrix} xcosB & -xsinB \\ ysinB & ycosB \end{bmatrix}$p0

$\begin{bmatrix} xcosB & -xsinB \\ ysinB & ycosB \end{bmatrix} \neq \begin{bmatrix} xcosB & -ysinB \\ xsinB & ycosB \end{bmatrix}$,
Hence Non-Uniform Scaling and Rotation are not commutative

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3. A. Point = $(\overline{P_i} + \overline{P_{i+1}})$ / 2
Slope of Normal = (y_{i+1} - y_i, x_i - x_{i+1})
$\vec{n}$ = $(\overline{P_i} + \overline{P_{i+1}})$ / 2 + t(y_{i+1} - y_i, x_i - x_{i+1}) By the implicit definition of a line

B. Use the Determinant to check whether the a point is on either side of a given line. Let Li be the line between Pi and Pi+1.

If $det \begin{bmatrix} Li & \bar{q} & \begin{matrix}1\\1\\1\end{matrix} \end{bmatrix}$ = 0, $\bar{q}$ is on the line.

For determinant greater or less than 0, it is on the right or left side of the line. Thus, to check if $\vec{n}$ is on the same side as point $\bar{q}$:

$$det \begin{bmatrix} Li & \bar{q} & \begin{matrix}1\\1\\1\end{matrix} \end{bmatrix} * det \begin{bmatrix} Li & \vec{n} & \begin{matrix}1\\1\\1\end{matrix} \end{bmatrix} > 0$$

Since the signs of the determinants must be the same for both $\bar{q}$ and $\vec{n}$ to be on the same side, its product must NOT be negative.

C. Following the hint, the point lies within the convex shape only if it is within the left side of each edge (Determinant < 0)

```
// Main function
function IsPointInShape(Point q)
       return pointNotInR(q) && pointInP(q)

// Check point is not located within whited out block
function pointNotInR(q)
       for all {Ri, Rj} of adjacent (Ri, Ri+1 …)
              Li = line containing Ri, Rj
              det = det[Li   q̄   1; 1; 1]

              if (det < 0)         // Check if point inside of R
                     return false

       return true   // Point lays outside R

// Check point is not located outside greyed block
function pointInP(q)
       for all {Pi, Pj} of adjacent (Pi, Pi+1 …)
              Li = line containing Pi, Pj
              det = det[Li   q̄   1; 1; 1]

              if (det >= 0)        // Check if point is outside of P
                     return false

       return true   // Point lays inside P
```

4. A. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$ $\quad$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -0.5 \\ 1 \end{bmatrix}$

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.5 \\ 1 \end{bmatrix}$ $\quad$ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

Simplifying these matrices gives us the system:

a = -1.5g + 2.5	e = -0.5h - 1.5
b = 0.5h + 4.5	f = 1
c = -4	a + b = 4
d = -0.5g - 1.5	d + e = g + h

Solving this system
(Using http://www.quickmath.com/webMathematica3/quickmath/equations/solve/advanced.jsp)

Equations $\qquad$ Exact

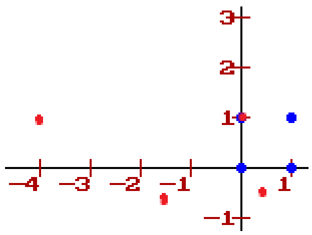
$$\left\{\begin{array}{l} a = (-1.5)\, g + 2.5 \\ b = 0.5\, h + 4.5 \\ c = -4 \\ d = (-0.5)\, g - 1.5 \\ e = (-0.5)\, h - 1.5 \\ f = 1 \\ a + b = 4 \\ d + e = g + h \end{array}\right. \qquad \left\{\begin{array}{l} a = 1 \\ b = 3 \\ c = -4 \\ d = -2 \\ e = 0 \\ f = 1 \\ g = 1 \\ h = -3 \end{array}\right.$$

Resulting in the Homography: $\begin{bmatrix} 1 & 3 & -4 \\ -2 & 0 & 1 \\ 1 & -3 & 1 \end{bmatrix}$

B. $= \begin{bmatrix} 1 & 3 & -4 \\ -2 & 0 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.75 \\ -0.25 \\ 1 \end{bmatrix}$

C. This homography does not represent an affine transformation. Affine transformations must preserve parallel lines which this homography doesn't, using the coordinates of A as an example:

(0,0) (0,1) (1,0) (1,1) -> (-4, 1) (-1.5, -0.5) (0.5, -0.5) (0,1)



5. Interpret Matrix as:

$$\begin{bmatrix} 8 & 3 & -7 \\ 6 & -4 & -24 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A & & b \\ & & \\ 0 & 0 & 1 \end{bmatrix}$$

Translation vector:

$$\vec{b} = \begin{bmatrix} -7 \\ -24 \end{bmatrix}$$

Scale then Rotation:

$$\begin{bmatrix} 8 & 3 \\ 6 & -4 \end{bmatrix} = \begin{bmatrix} cosx & -sinx \\ sinx & cosx \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

System of solutions:

acosx = 8
-bsinx = 3
asinx = 6
bcosx =-4

Solving this system (Using Wolfram Alpha), we get two solutions:

- a=-10, b=5, x=2(πn - $\tan^{-1}$(3)), for integer n
- a=10, b=-5, x=-2(πn + $\tan^{-1}$(1/3)), for integer n

Altogether, for point $\overline{p_0}$

$$\overline{p_1} = \begin{bmatrix} \cos\left(2\left(\pi n - \tan^{-1}(3)\right)\right) & -\sin\left(2\left(\pi n - \tan^{-1}(3)\right)\right) \\ \sin\left(2\left(\pi n - \tan^{-1}(3)\right)\right) & \cos\left(2\left(\pi n - \tan^{-1}(3)\right)\right) \end{bmatrix} \begin{bmatrix} -10 & 0 \\ 0 & 5 \end{bmatrix} \left(\overline{p_0} + \begin{bmatrix} -7 \\ -24 \end{bmatrix}\right)$$

or

$$\overline{p_1} = \begin{bmatrix} \cos\left(2\left(\pi n + \tan^{-1}(1/3)\right)\right) & -\sin\left(2\left(\pi n + \tan^{-1}(1/3)\right)\right) \\ \sin\left(2\left(\pi n + \tan^{-1}(1/3)\right)\right) & \cos\left(2\left(\pi n + \tan^{-1}(1/3)\right)\right) \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & -5 \end{bmatrix} \left(\overline{p_0} + \begin{bmatrix} -7 \\ -24 \end{bmatrix}\right)$$