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Assignment 2

CSC418H1

1. Given the camera position *e*, the up direction *t*, and the viewing direction:

w = Negative of viewing direction

$$= \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$$

v $= \dfrac{t \times w}{||t \times w||}$

$$= \frac{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}}{\left|\left|\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}\right|\right|}$$

$$= \frac{\begin{bmatrix} -3 \\ 0 \\ -1 \end{bmatrix}}{10}$$

$$= \begin{bmatrix} -0.3 \\ 0 \\ -0.1 \end{bmatrix}$$

u = Vector perpendicular to both v and w
= Cross product of v and w

$$= \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} \text{X} \begin{bmatrix} -0.3 \\ 0 \\ -0.1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.1 \\ 1 \\ 0.3 \end{bmatrix}$$

Mwc $= \begin{bmatrix} u^T & v^T & w^T & -[u^T v^T w^T]\, e \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -0.1 & 1 & 0.3 & 2.5 \\ -0.3 & 0 & 0.1 & -0.1 \\ 1 & 1 & -3 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. p' = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/f & 0 \end{bmatrix} \begin{bmatrix} px \\ py \\ pz \\ 1 \end{bmatrix} = \begin{bmatrix} px \\ py \\ -pz/f \end{bmatrix} = \begin{bmatrix} (-\frac{f}{pz})px \\ (-\frac{f}{pz})py \\ 1 \end{bmatrix}$

q' = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/f & 0 \end{bmatrix} \begin{bmatrix} qx \\ qy \\ qz \\ 1 \end{bmatrix} = \begin{bmatrix} qx \\ qy \\ -qz/f \end{bmatrix} = \begin{bmatrix} (-\frac{f}{qz})qx \\ (-\frac{f}{qz})qy \\ 1 \end{bmatrix}$

m' = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/f & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}(px + qx) \\ \frac{1}{2}(py + qy) \\ \frac{1}{2}(pz + qz) \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(px + qx) \\ \frac{1}{2}(py + qy) \\ \frac{-1}{2f}(pz + qz) \end{bmatrix} = \begin{bmatrix} \frac{-f}{(pz + qz)}(px + qx) \\ \frac{-f}{(pz + qz)}(py + qy) \\ 1 \end{bmatrix}$

0.5(p' + q') = 0.5($\begin{bmatrix} (-\frac{f}{qz})qx \\ (-\frac{f}{qz})qy \\ 1 \end{bmatrix} + \begin{bmatrix} (-\frac{f}{pz})px \\ (-\frac{f}{pz})py \\ 1 \end{bmatrix}$) = $\begin{bmatrix} \left(-\frac{f}{2}\right)(\frac{px}{pz} + \frac{qx}{qz}) \\ \left(-\frac{f}{2}\right)(\frac{py}{pz} + \frac{qy}{qz}) \\ 1 \end{bmatrix}$

Therefore, m' ≠ 0.5(p' + q')

For an Orthographic Projection, no f value is used to scale the x, y-coordinate

m' = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}(px + qx) \\ \frac{1}{2}(py + qy) \\ \frac{1}{2}(pz + qz) \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(px + qx) \\ \frac{1}{2}(py + qy) \end{bmatrix}$ = 0.5 (p' + q')

Hence, for an Orthographic projection, m' = 0.5 (p' + q')

3. The intersection of the lines can be described as the point where l1 and l2 meet after they have been scaled by the focal length and Z-coordinate. Let p = l1 and q = l2, Let p' and q' be their projected lines respectively.

$$p' = \begin{bmatrix} \frac{f}{pz}(px + udx) \\ \frac{f}{pz}(py + udy) \\ 1 \end{bmatrix} \qquad q' = \begin{bmatrix} \frac{f}{qz}(qx + udx) \\ \frac{f}{qz}(qy + udy) \\ 1 \end{bmatrix}$$

Vanishing point = Intersection of p' and q'

$$\begin{bmatrix} \frac{f}{pz}(px + udx) \\ \frac{f}{pz}(py + udy) \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{f}{qz}(qx + udx) \\ \frac{f}{qz}(qy + udy) \\ 1 \end{bmatrix}$$

Hence the vanishing point as a homogeneous coordinate is:

$$\begin{bmatrix} \frac{f}{pz}(px + udx) - \frac{f}{qz}(qx + udx) \\ \frac{f}{pz}(py + udy) - \frac{f}{qz}(qy + udy) \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

4. Canonical View Transformation Matrix:

$$Mc = \begin{bmatrix} \frac{2f}{L-R} & 0 & \frac{R+L}{L-R} & 0 \\ 0 & \frac{2f}{B-T} & \frac{B+T}{B-T} & 0 \\ 0 & 0 & \frac{f+F}{f+F} & \frac{2Ff}{F-f} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Point A:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Pseudodepth = -1

Point B:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -8.018 \\ -10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.8018 \\ 1 \end{bmatrix}$$

Pseudodepth = 0.8018

Point C:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -100 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -98.198 \\ -100 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.98198 \\ 1 \end{bmatrix}$$

Pseudodepth = 0.98198

Point D:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1000 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -999.998 \\ -1000 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.999998 \\ 1 \end{bmatrix}$$

Pseudodepth = 0.999998

The relationship between the depth and pseudodepth is not linear but the "further" the depth is, the closer it gets to the far end of the canonical view

5. A. Cross-section at z=0:
p(t, u) = (a*cos(t/2π)sin(u/2π), b*sin(t/2π)sin(u/2π), 0)

Cross-section at y=0:
q(t, u) = (a*cos(t/2π)sin(u/2π), 0, c*sin(u/2π))

B. Partial Derivatives of t and u:

$$P_u(t,u) = (\frac{a}{2\pi} \times \cos\left(\frac{t}{2\pi}\right)\cos\left(\frac{u}{2\pi}\right), \frac{b}{2\pi} \times \sin\left(\frac{t}{2\pi}\right)\cos\left(\frac{u}{2\pi}\right), 0)$$

$$Q_u(t,u) = (\frac{a}{2\pi} \times \cos\left(\frac{t}{2\pi}\right)\cos\left(\frac{u}{2\pi}\right), 0, c \times cos(\frac{u}{2\pi}))$$

$$P_t(t,u) = (\frac{-a}{2\pi} \times \sin\left(\frac{t}{2\pi}\right)\sin\left(\frac{u}{2\pi}\right), \frac{b}{2\pi} \times \cos\left(\frac{t}{2\pi}\right)\sin\left(\frac{u}{2\pi}\right), 0)$$

$$Q_t(t,u) = (\frac{-a}{2\pi} \times \sin\left(\frac{t}{2\pi}\right)\sin\left(\frac{u}{2\pi}\right), 0, \frac{-c}{2\pi} \times sin(\frac{u}{2\pi}))$$

Using the cross product on $P_{u/t}$ and $Q_{u/t}$ to find the normal:

$$\begin{bmatrix} p_2q_3 - p_3q_2 \\ p_3q_1 - p_1q_3 \\ p_1q_2 - p_2q_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{ac}{2\pi}\cos\left(\frac{t}{2\pi}\right)\cos^2\left(\frac{u}{2\pi}\right) \\ -\frac{ab}{4\pi^2}\sin\left(\frac{t}{2\pi}\right)\cos^2\left(\frac{u}{2\pi}\right)\cos\left(\frac{t}{2\pi}\right) \end{bmatrix}$$

$$\begin{bmatrix} p_2q_3 - p_3q_2 \\ p_3q_1 - p_1q_3 \\ p_1q_2 - p_2q_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{-ac}{4\pi^2} \times \sin\left(\frac{t}{2\pi}\right)\sin^2\left(\frac{u}{2\pi}\right) \\ \frac{ab}{4\pi^2}\cos\left(\frac{t}{2\pi}\right)\sin\left(\frac{t}{2\pi}\right)\sin^2\left(\frac{u}{2\pi}\right) \end{bmatrix}$$

C. The standard equation of an ellipsoid:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Plugging in parametric equation variables:

$$\frac{\left(\mathrm{a}\times\cos\left(\frac{t}{2\pi}\right)\sin\left(\frac{u}{2\pi}\right)\right)^2}{a^2}+\frac{\left(\mathrm{b}\times\sin\left(\frac{t}{2\pi}\right)\sin\left(\frac{u}{2\pi}\right)\right)^2}{b^2}+\frac{\left(\mathrm{c}\times\cos\left(\frac{u}{2\pi}\right)\right)^2}{c^2}=1$$

$$(\cos\left(\frac{t}{2\pi}\right)\sin\left(\frac{u}{2\pi}\right))^2+(\sin\left(\frac{t}{2\pi}\right)\sin\left(\frac{u}{2\pi}\right))^2+(\cos\left(\frac{u}{2\pi}\right))^2=1$$

$$\cos^2\left(\frac{t}{2\pi}\right)\sin^2\left(\frac{u}{2\pi}\right)+\sin^2\left(\frac{t}{2\pi}\right)\sin^2\left(\frac{u}{2\pi}\right)+\cos^2\left(\frac{u}{2\pi}\right)=1$$

$$\sin^2\left(\frac{u}{2\pi}\right)(\cos^2\left(\frac{t}{2\pi}\right)+\sin^2\left(\frac{t}{2\pi}\right))+\cos^2\left(\frac{u}{2\pi}\right)=1$$ Trig law

$$\sin^2\left(\frac{u}{2\pi}\right)+\cos^2\left(\frac{u}{2\pi}\right)=1$$ Trig law

1=1

Therefore, the implicit equation for an ellipsoid is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0$

D. Given implicit equation: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0$

The normal is the gradient:

$$n=(\frac{2x}{a^2},\frac{2y}{b^2},\frac{2z}{c^2})$$

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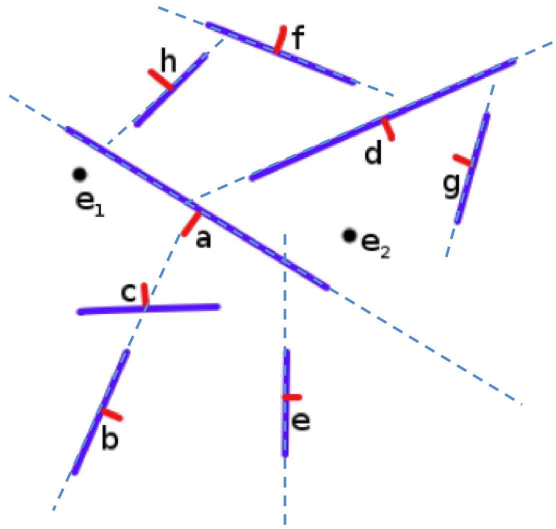
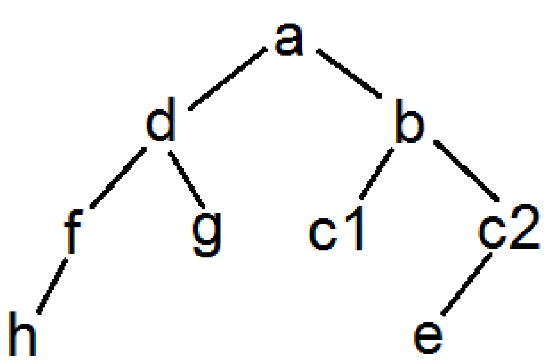
6. A.



BSP Tree corresponding to above scene:



B. Let Paint() be the function called to traverse the tree. Let Draw() be the function to render the segment

e1 Traversal:
Paint(a):
- Paint(a.inside):
  - Paint(d.outside):
    - Draw g
  - Draw d
  - Paint(d.inside):
    - paint(f.outside):
      - \-
    - Draw f
    - paint(f.inside):
      - Draw h
- Draw a
- Paint(a.outside):

```
        Paint(b.inside):
            Draw c1
        Draw b
        Paint(b.outside):
            Paint(c2.inside):
                Draw e
            Draw c2
            Paint(c2.outside):
                -
```

Order segments are drawn: g, d, f, h, a, c1, b, e, c2

e2 Traversal:

```
Paint(a):
    Paint(a.outside):
        Paint(b.inside):
            Draw c1
        Draw b
        Paint(b.outside):
            Paint (c2.inside):
                Draw e
            Draw c2
            Paint (c2.outside):
                -
    Draw a
    Paint (a.inside):
        Paint (d.inside):
            Paint (f.outside):
                -
            Draw f
            Paint (f.inside):
                Draw h
        Draw d
        Paint (d.outside):
            Draw g
```

Order segments are drawn: c1, b, e, c2, a, f, h, d, g