FYS4560 Project 1

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1 Standard model and beyond

- Allowed, forbidden and discovery processes

We consider some processes to state whether they are allowed or not. Quantities that should be conserved include

- Lepton number
- Baryon number
- Energy and momentum
- Isospin
- Parity
- Charge parity
- Strong interactions are only between quarks and gluons.
- The Higgs boson interacts through the weak interaction, $Z, W^{\pm}$, or fermionic coupling?

Electron-positron collisions

1.1 $e^+e^- \to q\bar{q}gg$

Can interact through a combination of electroweak and strong interaction, but requires 4 vertices, so the crossection is small

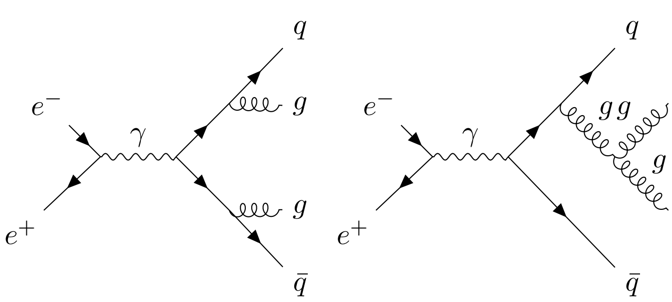


Figure 1: Possible diagrams for $e^+e^- \to q\bar{q}gg$. The diagram on the left can also have both gluons on one 'arm'.

1.2 $e^+e^- \to \tilde{l}^+\tilde{l}^-$

Is this a supersymmetric lepton? This decay can happen through the electromagnetic interaction QED. *Allowed.*

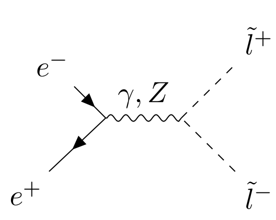


Figure 2: Diagram for $e^+e^- \to \tilde{l}^-\tilde{l}^+$.

1.3 $e^+e^- \to HH\gamma$

1.4 $e^+e^- \to ZZZ$

1.5 $e^+e^- \to H \to gg$

Doesn't Higgs decay through the weak interaction? Is color conserved here?

1.6 $e^+e^- \to \nu\bar{\nu}\gamma\gamma$

Maybe possible with a four-jet.

1.7 $e^+e^- \to Y(3s) \to B^0\bar{B}^0$

1.8 $e^+e^- \to Z^0t\bar{t}$

Gluon-gluon collisions

1.9 $gg \to e^+e^-$

This interaction is possible, but requires four vertices so its cross section must be small. It is a combination of the electroweak and strong interaction.

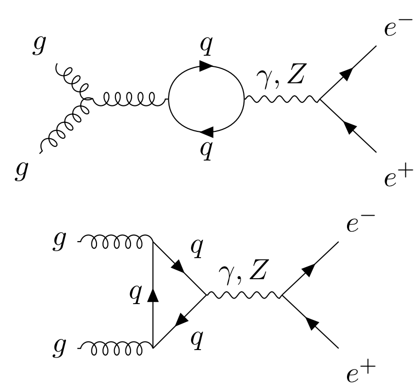


Figure 3: Diagrams for $gg \to e^-e^+$.

1.10 $gg \to t\bar{t}HH$

1.11 $gg \to H \to Z\gamma$

Higgs boson doesn't interact via the strong interaction. *Not allowed.*

Quark-antiquark collisions

1.12 $q\bar{q} \to W^+W^-Z$

1.13 $q\bar{q} \to gge^+e^-$

Proton-proton and proton-antiproton collisions

1.14 $p\bar{p} \to l^+l^-X$

Quark and antiquark from proton and antiproton can annihilate and become lepton and antilepton through a virtual photon. *Allowed.*

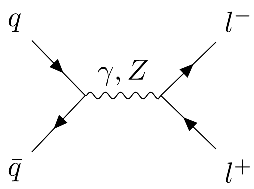


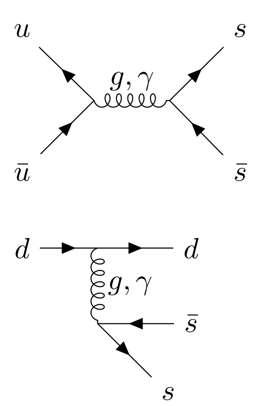
Figure 4: Diagram for $p\bar{p} \to l^+l^-$.

1.15 $pp \to l^+l^-l^+l^-X$

Proton-proton collisions contain no antiquarks, so they can be no annihilation to form the photon for the leptons. *Not allowed.*

Collisions

1.16 $K^-p \to \Omega^-K^+K^0$



1.17 $\nu_e e^- \to \nu_e e^- \gamma$

Allowed.

1.18 $ep \to J/\psi + X$

Decays

1.19 $\tau^+ \to \mu^+\nu_e\bar{\nu}_\tau$

This process is not allowed because it violates lepton flavour conservation. If we changed ν_e by ν_μ it would be allowed.

1.20 $D^0 \leftrightarrow \bar{D}^0$

Allowed. Weak current or something.

2 Top quark and W-boson

CKM-matrix

The electroweak interaction does not preserve quark flavour. Quarks can change flavour by emitting a $W^{\pm}$-boson. The flavour-changes occur according to the classification of quarks into upper and lower, and the doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}, \begin{pmatrix} t \\ b' \end{pmatrix},$$

where d',s' and b' are the weak eigenstates constructed from the quarks d, s, b

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

The matrix V_{CKM} relates the eigenstates to the asymptotic state quarks, and is called the *Cabbibo-Kobayashi-Maskawa matrix.* The current best approximation is given by V_{CKM}

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.97427 & 0.22534 & 0.00351 \\ 0.22520 & 0.97344 & 0.0412 \\ 0.00867 & 0.0400 & 0.999146 \end{pmatrix}$$

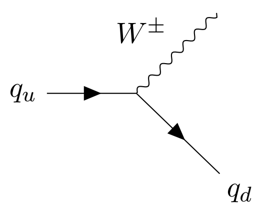


Figure 5: Flavour-changing weak interaction with quarks.

The interactions of quarks and leptons with the $W^{\pm}$ bosons are called charged currents. Parity and charge conjugation are broken, but CP is usually conserved. The top quark mass $m_t = 173$ GeV, however, is so large that it's production with the bottom quark

$$W^- \to tb',$$

is kinematically forbidden. We will consider some ways the top quark is produced.

$B^0 - \bar{B}^0$-oscillations

An example of weak and electroweak neutral currents, where Z and γ couple to a fermion. Neutral currents are, unlike charged ones, flavour-conserving.

Top quark decay

The top quark is so heavy and shortlived that it does not have time to form hadrons before it decays. It decays through the weak interaction to a W-boson and a down-type quark, i.e. down, strange or bottom.

$$t \to W^+ b, s, d$$

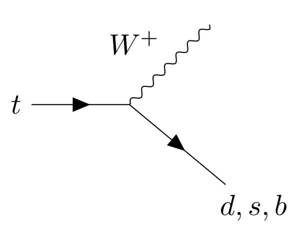


Figure 6: Top quark decay

3 Gauge theories

Standard model

The particles in the Standard model are organized in a threefold family structure

$$\begin{pmatrix} \nu_e & u \\ e^- & d' \end{pmatrix}, \begin{pmatrix} \nu_\mu & c \\ \mu^- & s' \end{pmatrix}, \begin{pmatrix} \nu_\tau & t \\ \tau^- & b' \end{pmatrix}.$$

Gauge and symmetries

The gauge principle is the requirement that a Lagrangian, e.g. the free Dirac fermion Lagrangian

$$\mathcal{L}_0 = \bar{\psi}(i\partial_\mu - m)\psi,$$

which is symmetric under $U(1)$ *global* transformations also be *symmetric under local transformations.*

The standard model is based on a non-Abelian group of transformations, called the Lorentz group, or $SU(3) \otimes U(1)$. The spinors are symmetric under local transformations

$$\psi \to \psi' = e^{iQ\theta}\Psi,$$

and the theory is gauged by promoting these symmetries to local symmetries, i.e.

$$\theta \to \theta(x).$$

In order to preserve symmetries we must introduce the covariant derivative

$$D_\mu(x) = \partial_\mu - ieQA_\mu(x) \tag{1}$$

Symmetries are related to conserved quantities through Noether's theorem, which claims that for every symmetry there must be a conserved quantity.

3.1 QCD

From experiments we know that hadronic matter is made up of quarks – mesons are $q\bar{q}$ and baryons are qqq. Because of Fermi statistics we need another quantum number, which we call colour. There are three colors

$$\alpha, \beta, \gamma = \text{ red, green, blue},$$

and all asymptotic states are colorless. We take color to be the quantum number of strong interactions.

QCD Lagrangian

Consider the free Lagrangian for quarks of color α and flavor f

$$\mathcal{L}_0 = \sum_f \bar{q}_f^\alpha (i\partial\!\!\!/ - m_f) q_f^\alpha.$$

This Lagrangian is symmetric under global $SU(3)_C$ transformations in color space, and we wish to promote these to local symmetries. The derivative is then no longer invariant, so we must derive a *covariant derivative* which transforms in the same way as q_f^α under local transformations. The transformations are given by

$$U = \exp\left(i\frac{\lambda^a}{2}\theta_a\right),$$

where λ^a are the 8 generators of the group. Since this is a non-Abelian group the generators don't commute

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = if^{abc}\frac{\lambda^c}{2},$$

where f^{abc} are the structure constants. The covariant derivative is

$$D^\mu q_f = \left[\partial^\mu + ig_s\frac{\lambda^a}{2}G_a^\mu(x)\right]qf \equiv [\partial^\mu + ig_s G^\mu(x)]q_f,$$

where G_a^μ are the eight gluons, or gauge bosons, corresponding to the eight generators. Because we want D^μ to transform exactly like the quarks, the transformation of the gluons is fixed

$$\begin{aligned}
&D^\mu \to D^{\mu'} = UD^\mu U^\dagger \\
&G^\mu \to G^{\mu'} = UG^\mu U^\dagger + \frac{i}{g_s}(\partial^\mu U)U^\dagger.
\end{aligned}$$

For infinitesimal $SU(3)_C$ transformations we therefore get

$$\begin{aligned}
q_f^\alpha &\to q_f^{\alpha'} = q_f^\alpha + i\left(\frac{\lambda^a}{2}\right)_{\alpha\beta}\delta\theta_a q_f^\beta \\
G_a^\mu &\to G_a^{\mu'} = G_a^\mu - \frac{1}{g_s}\partial^\mu(\delta\theta_a) - f^{abc}\delta\theta_b G_c^\mu.
\end{aligned}$$

In order to construct a Lagrangian we must see which terms are allowed. Only terms symmetric under local $SU(3)_c$ transformations are allowed, to there is no mass term e.g.

$$\begin{aligned}
G_\mu G^\mu \to (G_\mu)'(G^\mu)' &= (UG_\mu U^\dagger + \frac{i}{g_s}(\partial_\mu U)U^\dagger) \\
&\times (UG^\mu U^\dagger + \frac{i}{g_s}(\partial^\mu U)U^\dagger) \\
&= UG_\mu U^\dagger UG^\mu U^\dagger + \frac{i}{g_s}UG_\mu U^\dagger(\partial^\mu U)U^\dagger \\
&+ \frac{i}{g_s}(\partial_\mu U)U^\dagger UG^\mu U^\dagger \\
&- \frac{1}{g_s^2}(\partial_\mu U)U^\dagger(\partial^\mu U)U^\dagger \\
&= UG_\mu G^\mu U^\dagger + \frac{i}{g_s}UG_\mu U^\dagger(\partial^\mu U)U^\dagger \\
&+ \frac{i}{g_s}(\partial_\mu U)G^\mu U^\dagger - \frac{1}{g_s^2}(\partial_\mu U)U^\dagger(\partial^\mu U)U^\dagger \neq G_\mu G^\mu
\end{aligned}$$

since $U^\dagger U = U^\dagger U = 1$. So no mass term is allowed, and the gluons are massless.

We instead consider kinetic terms, analogous to QED

$$
\begin{aligned}
ig_s G_{\mu\nu} &= [D_\mu, D_\nu] = (\partial_\mu + ig_s G_\mu)(\partial_\nu + ig_s G_\nu) - (\partial_\nu + ig_s G_\nu)(\partial_\mu + ig_s G_\mu) \\
&= \partial_\mu \partial_\nu + ig_s \partial_\mu G_\nu + ig_s G_\mu \partial_\nu - g_s^2 G_\mu G_\nu - \partial_\nu \partial_\mu - ig_s \partial_\nu G_\mu - ig_s G_\nu \partial_\mu + g_s^2 G_\nu G_\mu \\
&= [\partial_\mu, \partial_\nu] + ig_s(\partial_\mu G_\nu - \partial_\nu G_\mu) + ig_s[G_\mu \partial_\nu, G_\nu \partial_\mu] + g_s^2 [G_\mu, G_\nu] \\
&= ig_s(\partial_\mu G_\nu - \partial_\nu G_\mu) + g_s^2 [G_\mu, G_\nu] \\
&\to \frac{\lambda^a}{2} G_a^{\mu\nu} \equiv \partial_\mu G_\nu - \partial_\nu G_\mu + g_s [G_\mu, G_\nu], \\
G_a^{\mu\nu}(x) &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f^{abc} G_b^\mu G_c^\nu
\end{aligned}
$$

where we've used that

$$[\partial_\mu, \partial_\nu] = 0, \; [G_\mu \partial_\nu, G_\nu \partial_\mu] = 0.$$

Under $SU(3)_C$ this term transforms in the desired way

$$G^{\mu\nu} \to G^{\mu\nu\prime} = U G^{\mu\nu} U^\dagger.$$

We now have the terms for the QCD Lagrangian

$$\mathcal{L}_{QCD} \equiv -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f. \qquad (2)$$

We now note a difference between QED and QCD. In QED the gauge bosons commute, and so the field strength only contains terms of te first order in A_μ. In QCD, however, the field strength contains a term $G_b^\mu G_c^\nu$, which means that the Lagrangian has terms that have third and fourth order terms in the gauge boson - meaning that we have self-interaction vertices with three and four gluons, as seen in figure (7).

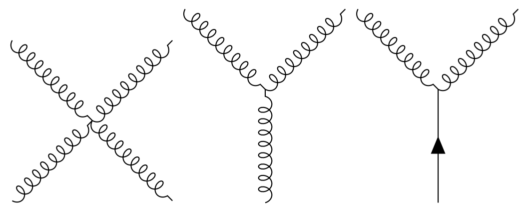


Figure 7: The interaction vertices from the QCD Lagrangian.

QCD and QED

QCD is a non-Abelian theory, while QED is an Abelian theory. As metnioned in the derivation of $\mathcal{L}_{QCD}$ this means that the generators of QCD don't commute, while the ones in QED do. This leads to the presence of self-interaction vertices in QCD, which could explain asymptotic freedom (antiscreening makes strong interactions weaker at short distances) and confinement (we can't observe free quarks, because strong forces become strong at long distances).

Electroweak unification

Appendix

Proof that $[G_\mu \partial_\nu, G_\nu \partial_\mu] = 0$. Use the identity

$$[AB, CD] = A[B, C]D + [A, C]BD + CA[B, D] + C[A, D]B. \tag{3}$$

Set in the expression

$$\begin{aligned}
[G_\mu \partial_\nu, G_\nu \partial_\mu] &= G_\mu[\partial_\nu, G_\nu]\partial_\mu + [G_\mu, G_\nu]\partial_\nu\partial_\mu + G_\nu G_\mu[\partial_\nu, \partial_\mu] + G_\nu[G_\mu, \partial_\mu]\partial_\nu \\
&= G_\mu[\partial_\nu, G_\nu]\partial_\mu + [G_\mu, G_\nu]\partial_\nu\partial_\mu + G_\nu[G_\mu, \partial_\mu]\partial_\nu \\
&= G_\mu(\partial_\nu G_\nu - G_\nu\partial_\nu)\partial_\mu + (G_\mu G_\nu - G_\nu G_\mu)\partial_\nu\partial_\mu + G_\nu(G_\mu\partial_\mu - \partial_\mu G_\mu)\partial_\nu \\
&= G_\mu\partial_\nu G_\nu\partial_\mu - G_\mu G_\nu\partial_\nu\partial_\mu + G_\mu G_\nu\partial_\nu\partial_\mu - G_\nu G_\mu\partial_\nu\partial_\mu + G_\mu G_\nu\partial_\nu\partial_\mu - G_\mu\partial_\nu G_\nu\partial_\mu \\
&= 0
\end{aligned}$$

because when indices are summed over they can be interchanged.