FYS4560 Project 2

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1 Di-lepton production in e^+e^- in the Standard Model

Possible feynman diagrams for the process

$$e^-e^+ \to l^-l^+$$

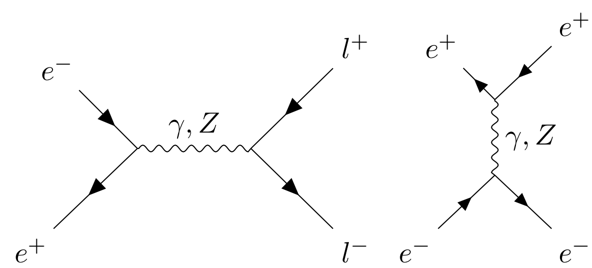

Figure 1: Possible diagrams for di-lepton production.

1.1 $e^+e^- \to l^+l^-$

The feynman diagrams that contribute are shown in Fig. (2).

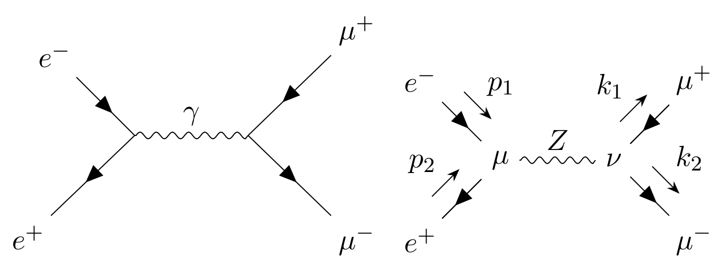

Figure 2: Feynman diagrams for $e^+e^- \to \mu^+\mu^-$.

First calculate the Z-diagram, using the expressions

$$g_v^f = \frac{1}{2}T_f^3 - Q_f \sin^2\theta_W, \tag{1}$$

$$g_A^f = \frac{1}{2}T_f^3. \tag{2}$$

$$\begin{aligned}
i\mathcal{M} &= \bar{v}^s(p_2)\big[i\frac{g}{\cos\theta_W}\gamma_\mu(g_v^f - g_A^f\gamma_5)\big]u^s(p_1)\Big(-\frac{ig_{\mu\nu}}{k^2 - M_Z^2 + i\epsilon}\Big)\bar{u}^s(k_2)\big[i\frac{g}{\cos\theta_W}\gamma_\nu(g_v^f - g_A^f\gamma_5)\big]v^s(k_1)\\
&= \frac{ig^2}{(k^2 - M_Z^2 + i\epsilon)\cos^2\theta_W}\bar{v}^s(p_2)\gamma_\mu(g_v^f - g_A^f\gamma_5)u^s(p_1)\bar{u}^s(k_2)\gamma^\mu(g_v^f - g_A^f\gamma_5)v^s(k_1)
\end{aligned}$$

$$\begin{aligned}
|\mathcal{M}|^2 = &\frac{g^4}{(k^2 - M_Z^2)^2\cos^4\theta_W}\Big(\bar{v}^s(p_2)\gamma_\mu(g_v^f - g_A^f\gamma_5)u^s(p_1)\bar{u}^s(p_1)\gamma_\nu(g_v^f - g_A^f\gamma_5)v^s(p_2)\Big)\\
&\times\Big(\bar{u}^s(k_2)\gamma^\mu(g_v^f - g_A^f\gamma_5)v^s(k_1)\bar{v}^s(k_1)\gamma^\nu(g_v^f - g_A^f\gamma_5)u^s(k_2)\Big)
\end{aligned}$$

Take the trace and get (where $A^2 = \frac{g^4}{(k^2-M_Z^2)^2\cos^4\theta_W}$), and since $M_z >> m_e, m_\mu$, set $m_e = m_\mu \simeq 0$

$$\frac{1}{4}\sum_{spins}|\mathcal{M}|^2 = A^2\text{tr}\Big[(\not{p}_2 - m_e)\gamma_\mu(g_v^e - g_A^e\gamma_5)(\not{p}_1 + m_e)\gamma_\nu(g_v^e - g_A^e\gamma_5)\Big]$$
$$\text{tr}\Big[(\not{k}_2 + m_\mu)\gamma^\mu(g_v^l - g_A^l\gamma_5)(\not{k}_1 - m_\mu)\gamma^\nu(g_v^l - g_A^l\gamma_5)\Big]$$
$$(\text{set mass to zero}) = A^2\text{tr}\Big[\not{p}_2\gamma_\mu(g_v^e - g_A^e\gamma_5)\not{p}_1\gamma_\nu(g_v^e - g_A^e\gamma_5)\Big]$$
$$\text{tr}\Big[\not{k}_2\gamma^\mu(g_v^l - g_A^l\gamma_5)\not{k}_1\gamma^\nu(g_v^l - g_A^l\gamma_5)\Big]$$
$$= A^2\text{tr}\Big[(g_v^e)^2\not{p}_2\gamma_\mu\not{p}_1\gamma_\nu + (g_A^e)^2\not{p}_2\gamma_\mu\gamma_5\not{p}_1\gamma_\nu\gamma_5 - g_v^e g_A^e\not{p}_2\gamma_\mu\not{p}_1\gamma_\nu\gamma_5 - g_v^e g_A^e\not{p}_2\gamma_\mu\gamma_5\not{p}_1\gamma_\nu g_v^e\gamma_5)\Big]$$
$$\times\,\text{tr}\Big[(g_v^l)^2\not{k}_2\gamma^\mu\not{k}_1\gamma^\nu + (g_A^l)^2\not{k}_2\gamma^\mu\gamma_5\not{k}_1\gamma^\nu\gamma_5 - g_v^l g_A^l\not{k}_2\gamma_\mu\not{k}_1\gamma_\nu\gamma_5 - g_v^l g_A^l\not{k}_2\gamma_\mu\gamma_5\not{k}_1\gamma_\nu\gamma_5)\Big]$$
$$= A^2\text{tr}\Big[((g_A^e)^2 + (g_v^e)^2)\not{p}_2\gamma_\mu\gamma_5\not{p}_1\gamma_\nu\gamma_5 - 2g_v^e g_A^e\not{p}_2\gamma_\mu\not{p}_1\gamma_\nu\gamma_5\Big]$$
$$\times\,\text{tr}\Big[((g_v^l)^2 + (g_A^l)^2)\not{k}_2\gamma^\mu\not{k}_1\gamma^\nu - 2g_v^l g_A^l\not{k}_2\gamma_\mu\not{k}_1\gamma_\nu\gamma_5\Big]$$

$$= A^2\text{tr}\Big[((g_v^e)^2 + (g_A^e)^2)p_2^\rho p_1^\sigma\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu - 2g_v^e g_A^e p_2^\rho p_1^\sigma\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu\gamma_5\Big]$$
$$\times\,\text{tr}\Big[((g_v^l)^2 + (g_A^l)^2)k_{2\rho}k_{1\sigma}\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu - 2g_v^l g_A^l k_{2\rho}k_{1\sigma}\gamma^\rho\gamma_\mu\gamma^\sigma\gamma_\nu\gamma_5\Big]$$

Now use the trace identities for the gamma matrices

$$\text{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})$$
$$\text{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) = -4i\epsilon^{\mu\nu\rho\sigma}$$

$$= 16A^2\Big[((g_v^e)^2 + (g_A^e)^2)p_2^\rho p_1^\sigma\big(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\sigma}g_{\mu\nu} + g_{\rho\nu}g_{\mu\sigma}\big) - 2g_v^e g_A^e p_2^\rho p_1^\sigma\epsilon_{\rho\mu\sigma\mu}\Big]$$
$$\times\Big[((g_v^l)^2 + (g_A^l)^2)k_{2\rho}k_{1\sigma}\big(g^{\rho\mu}g^{\sigma\nu} - g^{\rho\sigma}g^{\mu\nu} + g^{\rho\nu}g^{\mu\sigma}\big) - 2g_v^l g_A^l k_{2\rho}k_{1\sigma}\epsilon^{\rho\mu\sigma\nu}\Big]$$
$$= 16A^2\Big[\{((g_v^e)^2 + (g_A^e)^2)((g_v^l)^2 + (g_A^l)^2)\}\cdot p_2^\rho p_1^\sigma\big(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\sigma}g_{\mu\nu} + g_{\rho\nu}g_{\mu\sigma}\big)k_{2\rho}k_{1\sigma}\big(g^{\rho\mu}g^{\sigma\nu} - g^{\rho\sigma}g^{\mu\nu} + g^{\rho\nu}g^{\mu\sigma}\big)$$
$$- \{((g_v^e)^2 + (g_A^e)^2)2g_v^l g_A^l\}\cdot p_2^\rho p_1^\sigma\big(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\sigma}g_{\mu\nu} + g_{\rho\nu}g_{\mu\sigma}\big)k_{2\rho}k_{1\sigma}\epsilon^{\rho\mu\sigma\nu})$$
$$- \{2((g_v^l)^2 + (g_A^l)^2)g_v^e g_A^e\}\cdot p_2^\rho p_1^\sigma\epsilon_{\rho\mu\sigma\mu}k_{2\rho}k_{1\sigma}\big(g^{\rho\mu}g^{\sigma\nu} - g^{\rho\sigma}g^{\mu\nu} + g^{\rho\nu}g^{\mu\sigma}\big)$$
$$+ \{4g_v^e g_A^e g_v^l g_A^l\}\cdot p_2^\rho p_1^\sigma\epsilon_{\rho\mu\sigma\mu}k_{2\rho}k_{1\sigma}\epsilon^{\rho\mu\sigma\nu}\Big]$$
$$= 16A^2\Big[\{((g_v^e)^2 + (g_A^e)^2)((g_v^l)^2 + (g_A^l)^2)\}\cdot\big(p_{2\mu}p_{1\nu} - p_{2\sigma}p_1^\sigma g_{\mu\nu} + p_{2\nu}p_{1\mu}\big)\Big(k_2^\mu k_1^\nu - k_2^\sigma k_{1\sigma}g^{\mu\nu} + k_2^\nu k_1^\mu\Big)$$
$$- \{2((g_v^e)^2 + (g_A^e)^2)g_v^l g_A^l\}\cdot\big(p_{2\mu}p_{1\nu} - p_{2\sigma}p_1^\sigma g_{\mu\nu} + p_{2\nu}p_{1\mu}\big)k_{2\rho}k_{1\sigma}\epsilon^{\rho\mu\sigma\nu})$$
$$- \{2((g_v^l)^2 + (g_A^l)^2)g_v^e g_A^e\}\cdot p_2^\rho p_1^\sigma\epsilon_{\rho\mu\sigma\mu}\big(k_2^\mu k_1^\nu - k_2^\sigma k_{1\sigma}g^{\mu\nu} + k_2^\nu k_1^\mu\big)$$
$$+ 24\{4g_v^e g_A^e g_v^l g_A^l\}\cdot p_2^\rho k_{2\rho}p_1^\sigma k_{1\sigma}\Big]$$

Where we've used

$$\epsilon_{\rho\mu\sigma\mu}\epsilon^{\rho\mu\sigma\mu} = n! = 4! = 24.$$

$$= 16A^2\Big[\{((g_v^e)^2 + (g_A^e)^2)((g_v^l)^2 + (g_A^l)^2)\}\cdot\Big[2(p_2\cdot k_2)(p_1\cdot k_1) + 2(p_1\cdot k_2)(p_2\cdot k_1)\Big]$$
$$- \{2((g_v^e)^2 + (g_A^e)^2)g_v^l g_A^l\}\cdot\Big(\big(p_{2\mu}p_{1\nu} - p_{2\sigma}p_1^\sigma g_{\mu\nu} + p_{2\nu}p_{1\mu}\big)k_{2\rho}k_{1\sigma}\epsilon^{\rho\mu\sigma\nu} + p_{2\rho}p_{1\sigma}\epsilon^{\rho\mu\sigma\nu}\big(k_{2\mu}k_{1\nu} - k_{2\sigma}k_{1\sigma}g_{\mu\nu} + k_{2\nu}k_{1\mu}\big)\Big)$$

$$+ 24\{4g_v^e g_A^e g_v^l g_A^l\}\cdot(p_2\cdot k_2)(p_1\cdot k_1)\Big]$$

Note:

$$p_2^\rho p_1^\sigma 4\big(g_{\rho\mu}g_{\sigma\nu} - g_{\rho\sigma}g_{\mu\nu} + g_{\rho\nu}g_{\mu\sigma}\big)4k_{2\rho}k_{1\sigma}\Big(g^{\rho\mu}g^{\sigma\nu} - g^{\rho\sigma}g^{\mu\nu} + g^{\rho\nu}g^{\mu\sigma}\Big)$$
$$= \Big(p_{2\mu}p_{1\nu} - p_{2\sigma}p_1^\sigma g_{\mu\nu} + p_{2\nu}p_{1\mu}\Big)\Big(k_2^\mu k_1^\nu - k_2^\sigma k_{1\sigma}g^{\mu\nu} + k_2^\nu k_1^\mu\Big)$$
$$= \Big[(p_2\cdot k_2)(p_1\cdot k_1) - (k_2\cdot k_1)(p_2\cdot p_1) + (p_1\cdot k_2)(p_2\cdot k_1)$$
$$- (p_2\cdot p_1)(k_2\cdot k_1) + 4(k_2\cdot k_1)(p_2\cdot p_1) - (p_2\cdot p_1)(k_2\cdot k_1)$$
$$+ (k_2\cdot p_1)(p_2\cdot k_1) - (k_2\cdot k_1)(p_2\cdot p_1) + (p_2\cdot k_2)(p_1\cdot k_1)\Big]$$
$$= \Big[2(p_2\cdot k_2)(p_1\cdot k_1) + 2(p_1\cdot k_2)(p_2\cdot k_1)\Big]$$

KInematics