Supersymmetric QCD at the LHC

Ingrid A V Holm

May 4, 2017

Outline

This project will contain the following

- A motivation for supersymmetry; what *is* supersymmetry and what is the motivation for it? An introduction.

- A calculation of the QCD production of squarks at the LHC, i.e. $qq' \to \tilde{q}\tilde{q}'$.

- Calculate the NLO terms using Prospino [1] and compare to the analytic calculation. What does this mean for the error in the cross section?

- Compare with data (from ATLAS?) to look for jets or some process with final state leptons (not decided yet).

$$\tilde{q} \to q\tilde{\chi}_0^1$$
$$\tilde{q} \to q'\tilde{\chi}_0^+ \to q'l\tilde{\chi}_0^1$$

1 Why supersymmetry?

Hierarchy problem.

2 QCD squark production

The feynman diagram for the process

$$qq \to \tilde{q}\tilde{q},$$

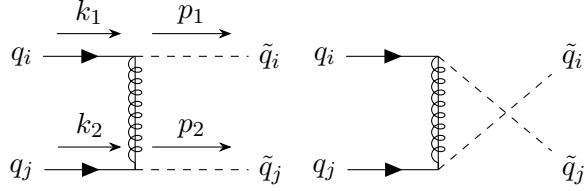is given in Fig. (1). The total matrix element gets a contribution from the t- and u-channel.

Figure 1: Feynman diagrams for squark pair production in quark quark collisions, both t and u diagram. Notice that the u-channel (right diagram) is only possible for $i = j$.

The momentum of the gluino is denoted p in the calculations, and defined as $p = k_2 - p_2$ for the t-channel and $p = k_2 - p_1$ for the u-channel. The chiralitites are decided later, and are simply denoted as P and P' for now. The expression for the matrix element becomes (reading direction is from q_j to q_i)

$$i\mathcal{M} = \bar{v}(k_1)\left(-i\sqrt{2}gP(t_a)^{ij}\right) \times \delta^{ab}\frac{i}{\not{p}-m_{\tilde{g}}} \times \left(-i\sqrt{2}gP'(t_b)^{lk}\right) \times u(k_2)$$

$$= -\frac{i2g^2}{p^2 - m_{\tilde{g}}^2 + i\epsilon}\left(\bar{v}(k_1)P(t_a)^{ij}(\not{p}+m_{\tilde{g}})P'(t^a)^{lk}u(k_2)\right)$$

$$= -(t_a)^{ij}(t^a)^{lk} \times \frac{i2g^2}{t_g^2} \times \bar{v}(k_1)P(\not{p}+m_{\tilde{g}})P'u(k_2),$$

where the color factor has been factored out. The charge conjugate becomes

$$i\bar{\mathcal{M}} = (t_a)^{ij}(t^a)^{lk} \times \frac{i2g^2}{t_g^2} \times \bar{u}(k_2)P(\not{p}+m_{\tilde{g}})P'v(k_1).$$

Some Mandelstam variables have been used to clean up the expresson; $t = p^2 = (k_2 - p_2)^2$, $t_g = t - m_{\tilde{g}}^2$. The matrix element squared is then

$$|\mathcal{M}_t|^2 = (t_a)^{ij}(t^a)^{lk}(t_b)^{mn}(t^b)^{op} \times \frac{4g^4}{t_g^2}\left(\bar{v}(k_1)P(\not{p}+m_{\tilde{g}})P'u(k_2)\right)\left(\bar{u}(k_2)P(\not{p}+m_{\tilde{g}})P'v(k_1)\right)$$

Now average over spin

$$\sum|\mathcal{M}_t|^2 = A_{color,t} \times \frac{4g^4}{t_g^2}\text{tr}\left[(\not{k}_1 - m_q)P(\not{p}+m_{\tilde{g}})P'(\not{k}_2 + m_q)P(\not{p}+m_{\tilde{g}})P'\right],$$

where $A_{color,t} = (t_a)^{ij}(t^a)^{lk}(t_b)^{mn}(t^b)^{op}$. Since the quark mass is small compared to m_g, $m_{\tilde{q}}$ and $m_{\tilde{g}}$, we set $m_q = 0$ and obtain

$$\sum|\mathcal{M}|^2 = A_{color,t} \times \frac{4g^4}{t_g^2}\text{tr}\left[\not{k}_1 P(\not{p}+m_{\tilde{g}})P'\not{k}_2 P(\not{p}+m_{\tilde{g}})P'\right]$$

At this point we need to consider the different combinations of chiralities. The traces for the different cases are

Different chiralities $P = P_{R/L}$, $P' = P_{L/R}$

$$\begin{aligned}\text{tr}\left[\not{k}_1 P_{R/L}(\not{p}+m_{\tilde{g}})P_{L/R}\not{k}_2 P_{R/L}(\not{p}+m_{\tilde{g}})P_{L/R}\right] =& \text{tr}\left[\not{k}_1 P_{R/L}(\not{p}+m_{\tilde{g}})P_{L/R}\not{k}_2(\not{p}+m_{\tilde{g}})P_{L/R}\right]\\ =& \text{tr}\left[\not{k}_1 P_{R/L}\not{p}P_{L/R}\not{k}_2\not{p}P_{L/R} + m_{\tilde{g}}\not{k}_1 P_{R/L}\not{p}P_{L/R}\not{k}_2 P_{L/R}\right.\\ &\left.+ m_{\tilde{g}}\not{k}_1 P_{R/L}P_{L/R}\not{k}_2\not{p}P_{L/R} + m_{\tilde{g}}^2\not{k}_1 P_{R/L}P_{L/R}\not{k}_2 P_{L/R}\right]\\ =& \text{tr}\left[P_{L/R}\not{k}_1\not{p}\not{k}_2\not{p}\right]\\ =& \text{tr}\left[P_{R/L}[2p\cdot k\not{k}_1\not{p} - p^2\not{k}_1\not{k}_2]\right]\\ =& \frac{1}{2}\text{tr}\left[2p\cdot k_2\not{k}_1\not{p} - p^2\not{k}_1\not{k}_2\right]\\ =& 2\left(2(p\cdot k_2)(k_1 \cdot p) - p^2(k_1 \cdot k_2)\right)\end{aligned}$$

where the middle terms dissappear because they contain an odd number of γ^μ, and the terms with an even number of γ^μ in the last term cancel out.

Equal chiralities $P = P_{R/L}$, $P' = P_{R/L}$

2

$$\text{tr}\big[\not{k}_1 P_{R/L}(\not{p}+m_{\tilde{g}})P_{R/L}\not{k}_2 P_{R/L}(\not{p}+m_{\tilde{g}})P_{R/L}\big] = \text{tr}\big[\not{k}_1 P_{R/L}\not{p}P_{R/L}\not{k}_2 P_{R/L}\not{p}P_{R/L} + m_{\tilde{g}}\not{k}_1 P_{R/L}\not{p}P_{R/L}\not{k}_2 P_{R/L}P_{R/L}$$
$$+ m_{\tilde{g}}\not{k}_1 P_{R/L}P_{R/L}\not{k}_2 P_{R/L}\not{p}P_{R/L} + m_{\tilde{g}}^2 \not{k}_1 P_{R/L}P_{R/L}\not{k}_2 P_{R/L}P_{R/L}\big]$$
$$= \frac{1}{4}\text{tr}\big[m_{\tilde{g}}^2 \not{k}_1 (1\pm\gamma^5)\not{k}_2(1\pm\gamma^5)\big]$$
$$= \frac{1}{2}\text{tr}\big[m_{\tilde{g}}^2\not{k}_1\not{k}_2\big] = 2m_{\tilde{g}}^2(k_1\cdot k_2)$$

where we've used that $P_{R/L}P_{R/L} = P_{R/L}$, $(\gamma^5)^2 = 1$ and $\text{tr}[\gamma^5\gamma^\mu\gamma^\nu] = 0$. Now clean up using the following Mandelstam variables

$$s = (k_1 + k_2)^2 = 2k_1\cdot k_2 \tag{1}$$
$$t = (k_2 - p_2)^2 = m_{\tilde{q}}^2 - 2(k_2\cdot p_2), \qquad\qquad t_1 = t - m_{\tilde{q}}^2 \tag{2}$$
$$u = (k_1 - p_2)^2 = m_{\tilde{q}}^2 - 2(k_1\cdot p_2), \qquad\qquad u_1 = u - m_{\tilde{q}}^2 \tag{3}$$

$$2(2(p\cdot k_2)(k_1\cdot p) - p^2(k_1\cdot k_2)) + 2m_{\tilde{g}}^2(k_1\cdot k_2)$$
$$= 4((k_2 - p_2)\cdot k_2)(k_1\cdot(k_2 - p_2)) - 2t(k_1\cdot k_2) + 2m_{\tilde{g}}^2(k_1\cdot k_2)$$
$$= -4(p_2\cdot k_2)(k_1\cdot k_2 - k_1\cdot p_2) - 2t(k_1\cdot k_2) + 2m_{\tilde{g}}^2(k_1\cdot k_2)$$
$$= -4(p_2\cdot k_2)(k_1\cdot k_2) + 4(p_2\cdot k_2)(k_1\cdot p_2) - 2t(k_1\cdot k_2) + 2m_{\tilde{g}}^2(k_1\cdot k_2)$$
$$= 4\frac{1}{2}(t - m_{\tilde{q}}^2)\cdot\frac{1}{2}s + 4\frac{1}{2}(t - m_{\tilde{q}}^2)\cdot\frac{1}{2}(u - m_{\tilde{q}}^2) - ts + m_{\tilde{g}}^2 s$$
$$= t_1 s + t_1 u_1 - ts$$
$$= t_1 u_1 - s(t - t_1)m_{\tilde{g}}^2 s = 2\big[t_1 u_1 - s(m_{\tilde{q}}^2 - m_{\tilde{g}}^2)\big].$$

Now put this back into the expression

$$\sum |\mathcal{M}|^2 = (t_a)^{ij}(t^a)^{lk}(t_b)^{mn}(t^b)^{op} \times \frac{4g^4}{t_g^2}\big[t_1 u_1 - s(m_{\tilde{q}}^2 - m_{\tilde{g}}^2)\big].$$

We still need to sum over colors, and we use the relation

$$\sum_a (t^a)_{ij}(t^a)_{lk} = \frac{1}{2}\Big(\delta_{ik}\delta_{lj} - \frac{1}{N}\delta_{ij}\delta_{lk}\Big).$$

Combine the factors

$$(t^a)^{ij}(t_a)^{kl}(t^b)_{ij}(t_b)^{kl} = \frac{1}{4}\Big(\delta_{ik}\delta_{lj} - \frac{1}{N}\delta_{ij}\delta_{lk}\Big)\Big(\delta^{ik}\delta^{lj} - \frac{1}{N}\delta^{ij}\delta^{lk}\Big)$$
$$= \frac{1}{4}\Big(\delta_{ik}\delta_{lj}\big(\delta^{ik}\delta^{lj} - \frac{1}{N}\delta^{ij}\delta^{lk}\big) - \frac{1}{N}\delta_{ij}\delta_{lk}\big(\delta^{ik}\delta^{lj} - \frac{1}{N}\delta^{ij}\delta^{lk}\big)\Big)$$
$$= \frac{1}{4}\Big(\big(NN - \frac{1}{N}N\big) - \frac{1}{N}\big(N - \frac{1}{N}NN\big)\Big)$$
$$= \frac{1}{4}(N^2 - 1) = \frac{1}{2}NC_F,$$

where we have defined $C_F = \frac{N^2-1}{2}$. Adding a factor 2 because we are summing over the different chiralities, the full expression for the t-channel is then

$$\sum |\mathcal{M}_t|^2 = NC_F\frac{4g^4}{t_g^2}\big[t_1 u_1 - s(m_{\tilde{q}}^2 - m_{\tilde{g}}^2)\big]$$

The expression for the u-channel diagram is identical, except the exchange of $t_g^2 \to u_g^2$ and a different color factor

$$\sum |\mathcal{M}_u|^2 = A_{color,u}^2 \frac{4g^4}{u_g^2}\big[u_1 t_1 - s(m_{\tilde{q}}^2 - m_{\tilde{g}}^2)\big].$$

The color factor becomes

$$\sum_{a,b}(t^a)^{ik}(t_a)^{jl}(t^b)_{ik}(t_b)_{jl} = \frac{1}{4}(\delta_{ij}\delta_{lk} - \frac{1}{N}\delta_{ik}\delta_{jl})(\delta^{ij}\delta^{lk} - \frac{1}{N}\delta^{ik}\delta^{jl})$$
$$= \frac{1}{4}\big(\delta_{ij}\delta_{lk}(\delta^{ij}\delta^{lk} - \frac{1}{N}\delta^{ik}\delta^{jl}) - \frac{1}{N}\delta_{ik}\delta_{jl}(\delta^{ij}\delta^{lk} - \frac{1}{N}\delta^{ik}\delta^{jl})\big)$$
$$= \frac{1}{4}\big(\delta_{ij}\delta^{ij}\delta_{kl}\delta^{kl} - \frac{1}{N}\delta^{jk}\delta_{kj} - \frac{1}{N}\delta^{jk}\delta_{jk} + \frac{1}{N^2}\delta_{ik}\delta^{ik}\delta_{jl}\delta^{jl}\big)$$
$$= \frac{1}{4}(N^2 - 1).$$

Since we are summing over the different parity combinations we add a factor of 2, and the total u-channel contribution becomes

$$\sum |\mathcal{M}_u|^2 = NC_F 4g^4\Big(\frac{u_1 t_1 - s(m_{\tilde{q}}^2 - m_{\tilde{g}}^2)}{t_g^2}\Big)$$

Cross term

The cross term comes from

$$i\mathcal{M}_{ut} = A_{color,ut}\frac{-i2g^2}{t_g}\big(\bar{v}(k_1)P(\not{p}+m_{\tilde{g}})P'u(k_2)\big)\frac{i2g^2}{u_g}\big(\bar{u}(k_2)P(\not{p}+m_{\tilde{g}})P'v(k_1)\big)$$

$$\sum\mathcal{M}_{ut} = A_{color,ut}\frac{4g^4}{u_gt_g}\big(\not{k}_1P(\not{k}_2-\not{p}_2+m_{\tilde{g}})P'\not{k}_2P(\not{k}_2-\not{p}_1+m_{\tilde{g}})P'\big)$$

Average over spins

$$\sum\mathcal{M}_{ut} = A_{color,ut}\frac{4g^4}{u_gt_g}\big(\mathrm{tr}\big[\not{k}_1P_{L/R}(\not{k}_2-\not{p}_2)P_{R/L}\not{k}_2(\not{k}_2-\not{p}_1)\big] + \mathrm{tr}\big[\not{k}_1m_{\tilde{g}}\not{k}_2m_g^2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\mathrm{tr}\big[\frac{1}{2}\big(1-\gamma^5\big)\not{k}_1(\not{k}_2-\not{p}_2)\not{k}_2(\not{k}_2-\not{p}_1)\big] + \mathrm{tr}\big[\frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\frac{1}{2}\mathrm{tr}\big[\big(1-\gamma^5\big)\big(\not{k}_1\not{k}_2\not{k}_2\not{p}_2 - \not{k}_1\not{k}_2\not{k}_2\not{k}_1 - \not{k}_1\not{p}_2\not{k}_2\not{p}_2 + \not{k}_1\not{p}_2\not{k}_2\not{k}_1\big)\big] + \mathrm{tr}\big[\frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\frac{1}{2}\mathrm{tr}\big[\big(1-\gamma^5\big)\big(k_2^2(\not{k}_1\not{p}_2 - \not{k}_1\not{k}_1) - \not{k}_1(2p_2\cdot k_2 - \not{k}_2\not{p}_2)\not{p}_2 + (2k_1\cdot p_2 - \not{p}_2\not{k}_1)(2k_2\cdot k_1 - \not{k}_1\not{k}_2)\big)\big] + \mathrm{tr}\big[$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\frac{1}{2}\mathrm{tr}\big[\big(1-\gamma^5\big)\big(-2p_2\cdot k_2\not{k}_1\not{p}_2 + p_2^2\not{k}_1\not{k}_2$$

$$+ (2k_1\cdot p_2)(2k_2\cdot k_1) - (2k_1\cdot p_2)\not{k}_1\not{k}_2 - \not{p}_2\not{k}_1(2k_2\cdot k_1) + k_1^2\not{p}_2\not{k}_2)\big] + \frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(-(2p_2\cdot k_2)(2k_1\cdot p_2) + p_2^2(2k_1\cdot k_2)$$

$$+ 2(2k_1\cdot p_2)(2k_2\cdot k_1) - (2k_1\cdot p_2)(2k_1\cdot k_2) - (2p_2\cdot k_1)(2k_2\cdot k_1) + m_{\tilde{g}}^2(2k_1\cdot k_2)\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(-t_1u_1 + p_2^2s + 2(-u_1)s + u_1s + u_1s + m_{\tilde{g}}^2s\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(-t_1u_1 + m_{\tilde{q}}^2s + m_{\tilde{g}}^2s\big)$$

The other counter term is similar, but yields a different order

$$\sum i\mathcal{M}_{tu} = A_{color,tu}\frac{4g^4}{u_gt_g}\big(\not{k}_1P(\not{k}_2-\not{p}_1+m_{\tilde{g}})P'\not{k}_2P(\not{k}_2-\not{p}_2+m_{\tilde{g}})P'\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\mathrm{tr}\big[\frac{1}{2}\big(1-\gamma^5\big)\not{k}_1(\not{k}_2-\not{p}_1)\not{k}_2(\not{k}_2-\not{p}_2)\big] + \mathrm{tr}\big[\frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\frac{1}{2}\mathrm{tr}\big[\big(1-\gamma^5\big)\not{k}_1(\not{p}_2 - \not{k}_1 - \not{k}_2)\not{k}_2\not{p}_2\big] + \mathrm{tr}\big[\frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\frac{1}{2}\mathrm{tr}\big[\big(1-\gamma^5\big)\big(\not{k}_1\not{p}_2\not{k}_2\not{p}_2 - \not{k}_1\not{k}_1\not{k}_2\not{p}_2 - \not{k}_1\not{k}_2\not{k}_2\not{p}_2\big)\big] + \mathrm{tr}\big[\frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\frac{1}{2}\mathrm{tr}\big[\big(1-\gamma^5\big)\big(\not{k}_1\not{p}_2(2(k_2\cdot p_2) - \not{p}_2\not{k}_2))\big)\big] + \mathrm{tr}\big[\frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(\frac{1}{2}\mathrm{tr}\big[\not{k}_1\not{p}_2(2k_2\cdot p_2) - p_2^2\not{k}_1\not{k}_2)\big] + \mathrm{tr}\big[\frac{1}{2}m_{\tilde{g}}^2\not{k}_1\not{k}_2\big]\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big((2k_1\cdot p_2)(2k_2\cdot p_2) - p_2^2(2k_1\cdot k_2)\big] + m_{\tilde{g}}^2(2k_1\cdot k_2)\big)$$

$$= A_{color,ut}\frac{4g^4}{u_gt_g}\big(u_1t_1 - m_{\tilde{q}}^2s + m_{\tilde{g}}^2s\big).$$

Adding the counter terms then gives

$$\sum|\mathcal{M}_{ut+tu}| = A_{color,ut}\frac{4g^4}{u_gt_g}\big(2m_{\tilde{g}}^2s\big).$$

The color factor is

$$\sum_{a,b}(t^a)^{ij}(t_a)^{kl}(t^b)_{ik}(t_b)_{jl} = \frac{1}{4}(\delta_{ik}\delta_{lj} - \frac{1}{N}\delta_{ij}\delta_{kl})(\delta^{ij}\delta^{lk} - \frac{1}{N}\delta^{ik}\delta^{jl})$$

$$= \frac{1}{4}\big(\delta_{ik}\delta_{lj}(\delta^{ij}\delta^{lk} - \frac{1}{N}\delta^{ik}\delta^{jl}) - \frac{1}{N}\delta_{ij}\delta_{kl}(\delta^{ij}\delta^{lk} - \frac{1}{N}\delta^{ik}\delta^{jl})\big)$$

$$= \frac{1}{4}\big(N - \frac{1}{N}N^2 - \frac{1}{N}N^2 + \frac{1}{N^2}N\big)$$

$$= -\frac{1}{4}\big(N - \frac{1}{N}\big)$$

$$= -\frac{1}{2}C_F$$

Now, combining the terms, noting that the u-channel will only contribute for same-flavour quarks, we find

$$\sum |\mathcal{M}|^2 = \delta_{ij} 4g^4 \Big[N C_F (u_1 t_1 - s m_{\tilde{q}}^2)\Big(\frac{1}{t_g^2} + \frac{1}{u_g^2}\Big) + N C_F s m_{\tilde{g}}^2\Big(\frac{1}{t_g^2} + \frac{1}{u_g^2}\Big) - 2C_F \frac{s m_{\tilde{g}}^2}{u_g t_g}\Big]$$
$$+ (1 - \delta_{ij}) 4g^4 N C_F \Big[\frac{u_1 t_1 - s(m_{\tilde{g}}^2 - m_{\tilde{g}}^2)}{t_g^2}\Big].$$

Appendix A

Feynman rules

The following Feynman rules are from [3] and [2]. External quarks and squarks

$$--\blacktriangleright--\bullet = 1, \quad --\blacktriangleleft--\bullet = 1, \quad \xrightarrow{}\blacktriangleright\bullet = u(P), \quad \bullet\xleftarrow{} = \bar{v}(P).$$

Propagator

$$a \;\text{---}\; b \;=\; \delta^{ab}\frac{i}{\not{p} - m_{\tilde{g}}}, \qquad\qquad = -i\sqrt{2}gP_{L/R}(t_a)^{ij},$$

where the small arrows indicate the reading direction.
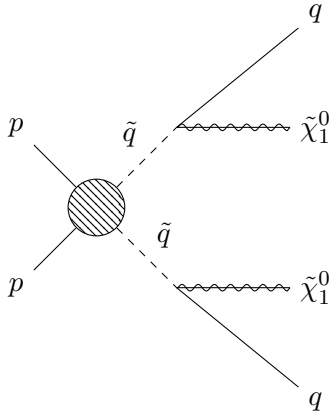
3 Prospino

Why NLO terms?

Calculating the NLO terms should reduce the dependence on the renormalization scale considerably, thereby making the predictions less uncertain. Cross sections have been calculated to NLO for the following squark masses.

4 Experimental tests

Squark production should be characterized by the production of jets and missing momentum, owing to the lightest supersymmetric particle χ_1^0 (neutralino). This is a possible candidate for dark matter. Possible signatures to look for at the LHC are

$$q_i q_j \to \tilde{q}_i \tilde{q}_j \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 q_i q_j, \tilde{\chi}_1^0 \tilde{\chi}_1^0 q_i q_j WW, \tilde{\chi}_1^0 \tilde{\chi}_1^0 q_i q_j WZ/h.$$

We consider final states containing only hadronic jets and large missing transverse momentum.

References

[1] W Beenakker, R Höpker, and M Spira. Prospino: A program for the production of supersymmetric particles in next-to-leading order qcd. *arXiv preprint hep-ph/9611232*, 1996.

[2] Wim Beenakker, R Höpker, Michael Spira, and PM Zerwas. Squark and gluino production at hadron colliders. *Nuclear Physics B*, 492(1-2):51–103, 1997.

[3] R Keith Ellis, W James Stirling, and Bryan R Webber. *QCD and collider physics*. Cambridge university press, 2003.