FYS4150: Project 3

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Abstract

This is an abstract

1 Introduction

We wish to solve coupled ordinary differential equations using the Verlet algorithm. From Newton's law of gravitation we get the force between the sun and the earth due to gravitation

$$F_G = \frac{GM_{sun}M_{earth}}{r^2} = \frac{M_{earth}v^2}{r} \tag{1}$$

where r is the distance between the earth and the sun. We can write this as two separate equations

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_{earth}} \tag{2}$$

$$\frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_{earth}} \tag{3}$$

which gives the differential equations

$$\frac{dv_x}{dt} = -\frac{G_{sun}M_{earth}}{r^3}x \tag{4}$$

$$\frac{dx}{dt} = v_x \tag{5}$$

$$\frac{dv_y}{dt} = -\frac{GM_{sun}}{r^3}y \tag{6}$$

$$\frac{dy}{dt} = v_y \tag{7}$$

2 The Euler method

The algorithm for the Euler method is

$$y_{n+1}^{(1)} = y_n^{(1)} + hy_{n+1}^{(2)} + \mathcal{O}(h^2) \tag{8}$$

$$y_{n+1}^{(2)} = y_n^{(2)} + ha_n + \mathcal{O}(h^2) \tag{9}$$

where $h = \frac{b-a}{N}$ and $t_{i+1} = t_i + h$. a_n is the acceleration, which also needs to be calculated. For our project this gives:

$$v_{x,i+1} = v_{x,i} - h\frac{4\pi^2}{r_i^3}x_i \tag{10}$$

$$x_{i+1} = x_i + hv_{x,i} \tag{11}$$

$$v_{y,i+1} = v_{y,i} - h\frac{4\pi^2}{r_i^3}y_i \tag{12}$$

$$y_{i+1} = y_i + hv_{y,i} \tag{13}$$

3 Velocity Verlet

The equations are

$$x_{i+1} = x_i + hv_i + \frac{h^2}{2}v_i^{(1)} + \mathcal{O}(h^3) \tag{14}$$

$$v_{i+1} = v_i + \frac{h}{2}\left(v_{i+1}^{(1)} + v_i^{(1)}\right) + \mathcal{O}(h^3) \tag{15}$$