FYS4150: Project 3

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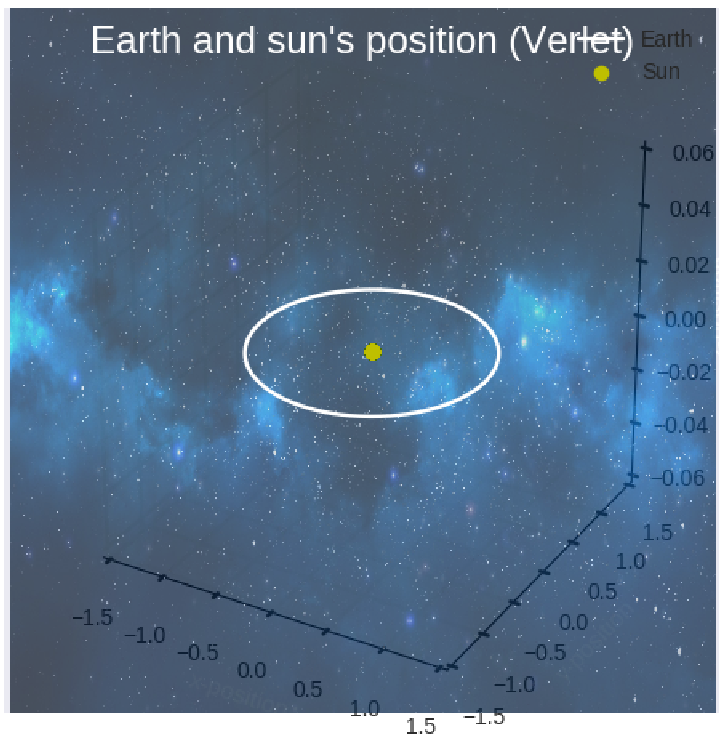
Abstract

This is an abstract



1 Introduction

We wish to solve coupled ordinary differential equations using the Verlet algorithm. From Newton's law of gravitation we get the force between the sun and the earth due to gravitation

$$F_G = \frac{GM_{sun}M_{earth}}{r^2} = \frac{M_{earth}v^2}{r} \tag{1}$$

where r is the distance between the earth and the sun. We can write this as two separate equations

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_{earth}} \tag{2}$$

$$\frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_{earth}} \tag{3}$$

which gives the differential equations

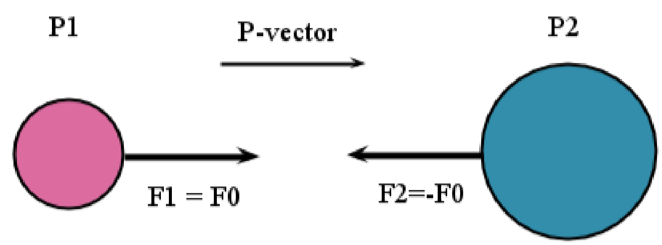
$$\frac{dv_x}{dt} = -\frac{G_{sun}M_{earth}}{r^3}x \tag{4}$$

$$\frac{dx}{dt} = v_x \tag{5}$$

$$\frac{dv_y}{dt} = -\frac{GM_{sun}}{r^3}y \tag{6}$$

$$\frac{dy}{dt} = v_y \tag{7}$$

How we define our forces:



2 The Euler method

The algorithm for the Euler method is

$$y_{n+1}^{(1)} = y_n^{(1)} + hy_{n+1}^{(2)} + \mathcal{O}(h^2) \tag{8}$$

$$y_{n+1}^{(2)} = y_n^{(2)} + ha_n + \mathcal{O}(h^2) \tag{9}$$

where $h = \frac{b-a}{N}$ and $t_{i+1} = t_i + h$. a_n is the acceleration, which also needs to be calculated. For our project this gives:

$$v_{x,i+1} = v_{x,i} - h\frac{4\pi^2}{r_i^3}x_i \tag{10}$$

$$x_{i+1} = x_i + hv_{x,i} \tag{11}$$

$$v_{y,i+1} = v_{y,i} - h\frac{4\pi^2}{r_i^3}y_i \tag{12}$$

$$y_{i+1} = y_i + hv_{y,i} \tag{13}$$

3 Changing the center of mass

We now have a system consisting of the earth, jupiter and the sun. We want to change the origin to the center of mass, in stead of the position of the sun. We then need to fulfill the conditions

$$\frac{1}{m_1 + m_2 + m_3}(m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3}) = 0 \tag{14}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = 0 \tag{15}$$

4 Velocity Verlet

In the velocity Verlet method the velocity and position are calculated at the same value of the time variable. The basic algorithm is

$$x_{i+1} = x_i + v_ih + \frac{1}{2}a_ih^2 \tag{16}$$

$$v_{i+1} = v_i + \frac{a_i + a_{i+1}}{2} h \tag{17}$$

Apparently, we need some intermediate steps. We therefore divide into

$$v_{i+1/2} = v_i + \frac{1}{2} a_i h \tag{18}$$

$$x_{i+1} = x_i + v_{i+1/2} h \tag{19}$$

Calcuate the new acceleration, from $a = F/m$:

$$v_{i+1} = v_i + \frac{1}{2} a_{i+1} h \tag{20}$$

5 Energy conservation

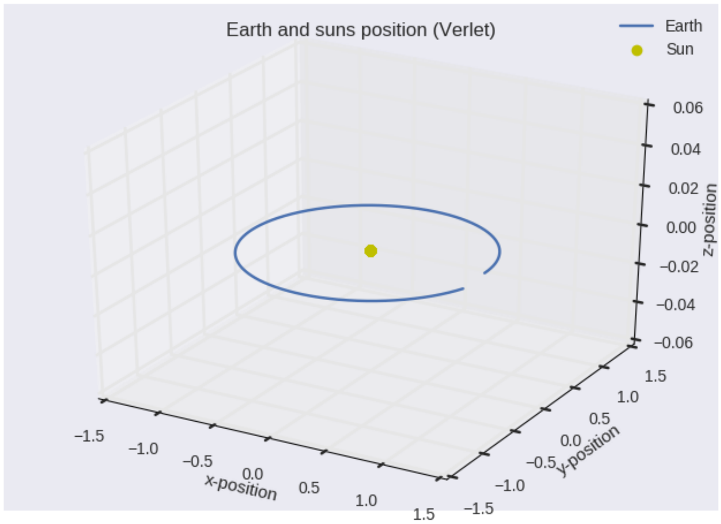
Create a small snippet of program that saves the energies for all time steps in a matrix, and finds the absolute value of the largest error

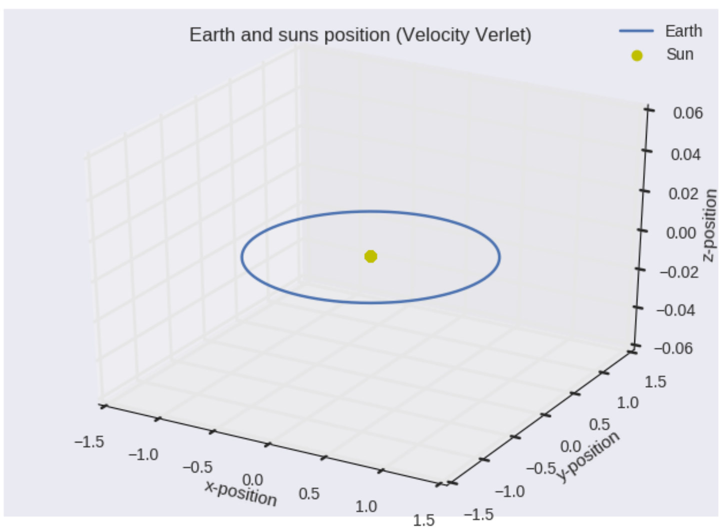
$$\epsilon_i = \frac{|E_{i,max} - E_{i,min}|}{|E_{i,max}|} \tag{21}$$

where $i = 0, 1, 2$ are the total, kinetic and potential energies.

6 Results

From including only the earth and the sun for the Euler and Verlet methods:





From checking stability of Verlet and Euler by varying the timestep dt. For Verlet we see that the movement is circular at $dt = 0.01$, so this overlaps with $dt = 0.001$. For Euler, we see that $dt = 0.001$ starts to diverge heavily, and $dt = 1$ goes almost like a tangent: