Contents

Chapter 1

Physics Background

In this chapter some of the physics background for this thesis is introduced. Familiarity with quentum field theory, the Standard Model of particle physics and some group theory is assumed. The Higgs mechanism and the hierarchy problem are reviewed, before some concepts in supersymmetry are outlined. Finally, the minimal supersymmetric standard model is introduced, with its corresponding field content.

1.1 The Standard Model

The Standard Model of particle physics has successfully explained almost all experimental results and predicted several phenomena. One of the most important distictions made in the Standard Model is that between *fermions* and *bosons*: particles with half-integer and integer spin values, respectively. Fermions interact via the exchange of bosons, and the Standard Model bosons are the *photon* (electromagnetic interaction), the *gluon* (strong interaction that holds atoms together), the W and Z bosons (the weak interaction) and the famously elusive *Higgs boson*. The equations of motion and allowed interactions can all be derived from the *Lagrangian* of the Standard Model. The Lagrangian, of which the time integral is the action S, is invariant to transformations under the Lorentz group — or a change of reference frame in the language of special relativity.

1.1.1 The Higgs Mechanism

The Standard Model is a gauge theory based on the symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The $SU(3)$ group is the symmetry group for strong interactions, or quantum chromodynamics, and $SU(2) \times U(1)$ is the electroweak symmetry group. In order for the particles to obtain masses the electroweak symmetry must be spontaneously broken. The symmetry is broken when the Higgs field obtains a non-zero *vacuum expectation value* (vev) — essentially stating that it has some field value when the governing potential is at its minimum. The Higgs field Φ is a self-interacting complex $SU(2)$ doublet whose Lagrangian is

$$\mathcal{L}_\Phi = \partial_\mu \Phi^\dagger \partial^\mu \Phi + V(\Phi), \tag{1.1}$$

where the first term is the kinetic term, and the scalar potential describing the Higgs $V(\Phi)$ is the famous Mexican hat potential

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2. \tag{1.2}$$

For $\mu^2 < 0$ and $\lambda > 0$ this potential aquires a non-trivial minimum given by

$$|\Phi_0| = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}, \tag{1.3}$$

where v is the vacuum expectation value. A special parameterization of the Higgs $SU(2)$ doublet, $\Phi^T(x) = \frac{1}{\sqrt{2}}(0, v + h(x))$, leads to the Lagrangian developing mass terms for fermions and the gauge bosons Z and $W^\pm$. The mass terms are proportional to v, *e.g.*

$$M_W = \frac{1}{2} v g,$$

where M_W is the mass of the W and g is the $SU(2)_L$-coupling. The Higgs' own mass provides one of the strongest arguments for introducing supersymmetry, namely the *hierarchy problem*, which is discussed below.

1.2 The Hierarchy Problem

The Higgs boson was discovered at the LHC in 2012, and its mass measured at $m_H \sim 125$ GeV [2]. The expression for the Higgs mass in the Standard Model includes loop corrections which provides a large discrepancy between theory and experiment. The Higgs mass receives fermionic or scalar loop-contributions to its mass such as those shown in Fig. (1.1). The expression for the mass can thus be written in terms of the bare parameter m_{H0} and the corrections Δm_H

$$m_H^2 = m_{H0}^2 + \Delta m_H^2.$$

Loop diagrams contain divergences, because of integrals over all possible momenta for the virtual particles in the loops. A way to get rid of these infinities is to regularize the expressions. Regularization is a neat trick that introduces a *cut-off scale*, which sets an upper limit on the momentum that is integrated over. A possible choice for the cut-off scale Λ is the Planck scale, as this is the scale where new physics is needed because gravity is not explained in the Standard

Model. The Planck scale is of the order of $\Lambda_{UV} \sim 10^{18}$ GeV. After regularization, the mass correction terms are

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2 + \frac{\lambda_s}{16\pi^2}\Lambda_{UV}^2 + ..., \tag{1.4}$$

where λ_s is the coupling of the Higgs to the scalar, and λ_f is the Higgs coupling to the fermion. The problem now becomes apparent: the correction to the mass is proportional to the Planck scale, placing it at the order of 10^{18} Gev, yet the mass has been experimentally measured around 125 GeV. There must be some colossal cancellation of terms with a tremendous tuning of the SM parameters in λ_s and λ_f. Tuning of parameters is undesirable — the model should be as natural as possible.

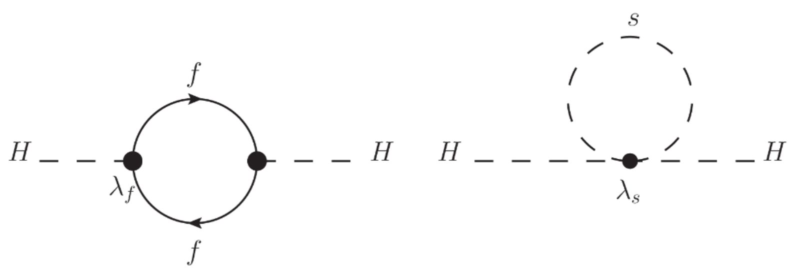

Figure 1.1: Fermion and scalar one-loop corrections to the Higgs mass. Figure from [1].

This is where supersymmetry comes in. In simple terms, supersymmetry introduces a fermionic superpartner for each boson, and vice versa. In unbroken supersymmetry these particles have identical mass, and their couplings to the Higgs are the same $\lambda_s = |\lambda_f|^2$. In addition, there are twice as many scalars as fermions, which gives a perfect cancellation of these enormous corrections. Thus, unbroken supersymmetry solves the hierarchy problem. The case of broken supersymmetry is revisited in a later section.

1.3 Supersymmetry

Supersymmetry is an extension of symmetries. Relativistic field theories are invariant under boosts, rotations and translations in spacetime. These are called Poincaré transformations, and are given by

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu, \tag{1.5}$$

where $\Lambda^\mu{}_\nu$ is a Lorentz transformation and a^μ is a translation. The assumption of supersymmetry is that Nature obeys a non-trivial extension of the related Poincaré algebra, namely the *superalgebra*.

The elements of the superalgebra and their representations can be described using *superspace*. In this space coordinates are given by $z^\pi = (x^\mu, \theta^A, \bar{\theta}_{\dot{A}})$, where x^μ are the well-known Minkowski coordinates, and $\theta^A, \bar{\theta}_{\dot{A}}$ are four Grassmann numbers. Grassmann numbers are numbers that anti-commute. Any Super-Poincaré transformation can be written in the following way

$$L(a, \alpha) = \exp[-ia^\mu P_\mu + i\alpha^A Q_A + i\bar{\alpha}^{\dot{A}} \bar{Q}_{\dot{A}}]. \tag{1.6}$$

In addition, since P_μ commutes with the generators Q one can always boost between reference frames, so in general a supersymmetry transformation is taken to mean

$$\delta_S = \alpha^A Q_A + \bar{\alpha}_{\dot{A}} \bar{Q}^{\bar{A}}. \tag{1.7}$$

The super-Poincaré algebra is given by the following commutation and anti-commutation relations [3]

$$\{Q_A, Q_B\} = \{\bar{Q}_A, \bar{Q}_B\} = 0, \tag{1.8}$$
$$\{Q_A, \bar{Q}_{\dot{A}}\} = 2(\sigma^\mu)_{A\dot{A}} P_\mu, \tag{1.9}$$
$$[Q_A, P^\mu] = [\bar{Q}_A, P^\mu] = 0, \tag{1.10}$$
$$[Q_A, M^{\mu\nu}] = \frac{1}{2}(\sigma^{\mu\nu})_A^B Q_B, \tag{1.11}$$
$$[\bar{Q}_{\dot{A}}, M^{\mu\nu}] = \frac{1}{2}(\bar{\sigma}^{\mu\nu})_{\dot{a}}^{\dot{b}} Q_{\dot{b}}^\dagger, \tag{1.12}$$

where $Q_A, A = 1, 2, 3, 4$ are the superalgebra generators, P^μ are the generators of translation, and $M^{\mu\nu}$ are the generators of the Lorentz group.

The supersymmetry generators turn fermions into bosons and vice versa. More specifically, these operators have the following commutation relations with the rotation generator J^3

$$[Q_A, J^3] = \frac{1}{2}(\sigma^3)_A^B Q_B, \tag{1.13}$$

which for the Q_1 generator becomes

$$[Q_1, J^3] = \frac{1}{2} Q_1. \tag{1.14}$$

Using this operator on a state in an irreducible representation of the Poincaré algebra with mass m and spin j_3 gives

$$J^3 Q_1 |m, j_3\rangle = (j_3 - \frac{1}{2}) Q_1 |m, j_3\rangle, \tag{1.15}$$

thus lowering the spin of the state by 1/2. Similarly, Q_2 would increase the spin. They do not, however, change the mass. This can be seen from Eq. (1.10)

$$P^\mu P_\mu Q_A |m, j_3\rangle = Q_A P^\mu P_\mu |m, j_3\rangle = m^2 Q_A |m, j_3\rangle. \tag{1.16}$$

States that transform into eachother via Q_A and $\bar{Q}_{\dot{A}}$ are called *superpartners*. In unbroken supersymmetry, therefore, the partnering fermions and bosons have the same mass. If this were the case supersymmetric particles would already have been discovered, so supersymmetry must be a broken symmetry.

1.3.1 Superfields

A supersymmetric Lagrangian will need a derivative that is invariant under supersymmetry transformations. The general covariant derivatives are defined as

$$D_A \equiv \partial_A + i(\sigma^\mu \bar{\theta})_A \partial_\mu, \tag{1.17}$$

$$\bar{D}^{\dot{A}} \equiv -\partial^{\dot{A}} - i(\sigma^\mu \theta)^{\dot{A}} \partial_\mu. \tag{1.18}$$

The covariant derivatives work on the *superfields* Φ, which are functions on superspace $\Phi(x, \theta, \bar{\theta})$. These are affected by the covariant derivatives in the following way

$$\bar{D}_{\dot{A}} \Phi(x, \theta, \bar{\theta}) = 0 \qquad \text{(left-handed scalar superfield)}, \tag{1.19}$$

$$D^A \Phi^\dagger(x, \theta, \bar{\theta}) = 0 \qquad \text{(right-handed scalar superfield)} \tag{1.20}$$

$$\Phi(x, \theta, \bar{\theta}) = \Phi^\dagger(x, \theta, \bar{\theta}) \qquad \text{(vector superfield)}. \tag{1.21}$$

The fields Φ are required to be Lorentz scalars or pseudoscalars, which restricts the properties of their component fields. It can be shown that the left- and right-handed scalar fields can be written in terms of their component fields as [1]

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) =& A(x) + i(\theta \sigma^\mu \bar{\theta}) \partial_\mu A(x) - \frac{1}{4} \theta\theta\bar{\theta}\bar{\theta} \Box A(x) + \sqrt{2} \theta \psi(x) \\ & - \frac{i}{\sqrt{2}} \theta\theta \partial_\mu \psi(x) \sigma^\mu \bar{\theta} + \theta\theta F(x), \end{aligned} \tag{1.22}$$

$$\begin{aligned} \Phi^\dagger(x, \theta, \bar{\theta}) =& A^*(x) - i(\theta \sigma^\mu \bar{\theta}) \partial_\mu A^*(x) - \frac{1}{4} \theta\theta\bar{\theta}\bar{\theta} \Box A^*(x) + \sqrt{2} \bar{\theta} \bar{\psi}(x) \\ & - \frac{i}{\sqrt{2}} \bar{\theta}\bar{\theta} \theta \sigma^\mu \partial_\mu \bar{\psi}(x) + \bar{\theta}\bar{\theta} F^*(x), \end{aligned} \tag{1.23}$$

where $A(x)$ and $F(x)$ are complex scalars and $\psi_A(x)$ and $\bar{\psi}^{\dot{A}}(x)$ are left-handed and right-handed Weyl spinors, respectively. From Eq. (1.21) the structure of a general vector field should be [1]

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) =& f(x) + \theta \varphi(x) + \bar{\theta} \bar{\varphi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta} m^*(x) \\ & + \theta \sigma^\mu \bar{\theta} V_\mu(x) + \theta\theta\bar{\theta} \bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta \lambda(x) + \theta\theta\bar{\theta}\bar{\theta} d(x), \end{aligned} \tag{1.24}$$

where $f(x)$, $d(x)$ are real scalar fields, $\varphi_A(x)$, $\lambda_A(x)$ are Weyl spinors, $m(x)$ is a complex scalar field and $V_\mu(x)$ is a real Lorentz four-vector. An example of a vector field is the product $V = \Phi^\dagger \Phi$. In the $j = \frac{1}{2}$ representation of the superalgebra, this field does not correspond to the promised number of degrees of freedom. This problem is fixed by the super-gauge.

1.3.2 Supersymmetric Lagrangian

Symmetry transformations of the Lagrangian should leave the action

$$S \equiv \int d^4x \mathcal{L}, \tag{1.25}$$

invariant. This is automatically fulfilled if the Lagrangian only changes by a total derivative. It can be shown that the highest order component fields in θ and $\bar{\theta}$ of the scalar and vector superfields have this property. To ensure that the action is invariant under supersymmetry transformations, the Lagrangian is redefined such that

$$S = \int d^4x \int d^4\theta \mathcal{L}, \tag{1.26}$$

where the integral over Grassmann four numbers is defined as $d^4\theta = d^2\theta d^2\bar{\theta}$, and $\int d^4\theta(\theta\theta)(\bar{\theta}\bar{\theta}) = 1$.

Restrictions on the supersymmetric Lagrangian, such as invariance under supersymmetric transformations and renormalizability, mean that the most general Lagrangian as a function of the scalar superfields Φ_i is

$$\mathcal{L} = \Phi_i^\dagger \Phi_i + \bar{\theta}\bar{\theta} W[\Phi] + \theta\theta W[\Phi^\dagger], \tag{1.27}$$

where $\Phi_i^\dagger \Phi_i$ is the kinetic term, and $W[\Phi]$ is the *superpotential*

$$W[\Phi] = g_i \Phi_i + m_{ij} \Phi_i \Phi_j + \lambda_{ijk} \Phi_i \Phi_j \Phi_k, \tag{1.28}$$

where m_{ij} and λ_{ijk} are symmetric. So to specify a supersymmetric Lagrangian all that is needed is to specify the superpotential.

Abelian Supergauge

A natural further step is to require that the Lagrangian be gauge invariant. An Abelian, or $U(1)$, supergauge transformation (global or local) on left-handed scalar superfields Φ_i is defined as [1]

$$\Phi_i \to \Phi'_i = e^{-i\Lambda q_i} \Phi_i, \tag{1.29}$$

where q_i is the $U(1)$ charge of the superfield Φ_i and Λ is the parameter of the transformation. For a left-handed superfield Φ_i the parameter Λ must also be a left-handed superfield, and correspondingly a right-handed superfield $\Phi_i^\dagger$ must have a right-handed superfield $\Lambda^\dagger$.

For the Lagrangian to be gauge invariant, W must also be. From the requirement that $W[\Phi] = W[\Phi']$, the following restrictions on the superpotential follow

$$g_i = 0 \text{ if } q_i \neq 0 \tag{1.30}$$
$$m_{ij} = 0 \text{ if } q_i + q_j \neq 0 \tag{1.31}$$
$$\lambda_{ijk} = 0 \text{ if } q_i + q_j + q_k \neq 0. \tag{1.32}$$

The kinetic term must also be invariant under gauge transformations

$$\Phi_i^\dagger \Phi_i \to \Phi_i^\dagger e^{i\Lambda^\dagger q_i} e^{-i\Lambda q_i} \Phi_i' = e^{i(\Lambda^\dagger - \Lambda)q_i} \Phi_i^\dagger \Phi_i. \tag{1.33}$$

For this term to be invariant, a gauge compensating vector superfield V with the appropriate gauge transformation is introduced. The kinetic term can then be written as $\Phi_i^\dagger e^{q_i V} \Phi_i$, and the kinetic term transforms as

$$\Phi_i^\dagger e^{q_i V} \Phi_i \to \Phi_i^\dagger e^{i\Lambda^\dagger q_i} e^{q_i(V_i + i\Lambda - i\Lambda^\dagger)} e^{-i\Lambda q_i} \Phi_i = \Phi_i^\dagger e^{q_i V} \Phi_i, \tag{1.34}$$

and so the kinetic term is invariant to Abelian supergauge transformations.

Non-Abelian Supergauge

The above discussion can be extended to non-Abelian gauge theories. Consider a group G with the Lie algebra of group generators t^a that fullfil

$$[t_a, t_b] = i f_{ab}^c t_c, \tag{1.35}$$

where f_{ab} are the structure constants. Using the exponential map, a unitary representation of an element g in the group G, can be written as $U(g) = e^{i\lambda^a t_a}$. Superfields then transform as

$$\Phi \to \Phi' = e^{-iq\Lambda_a t^a} \Phi, \tag{1.36}$$

where q is the charge of Φ under G.

The superpotential is invariant under non-Abelian gauge transformations if

$$g_i = 0 \text{ if } g_i U_{ir} \neq g_r, \tag{1.37}$$
$$m_{ij} = 0 \text{ if } m_{ij} U_{ir} U_{js} \neq m_{rs}, \tag{1.38}$$
$$\lambda_{ijk} = 0 \text{ if } \lambda_{ijk} U_{ir} U_{js} U_{kt} \neq \lambda_{rst}, \tag{1.39}$$

where the indices on U are matrix indices.

A similar kinetic term for non-abelian gauge theories as abelian gauge theories is then $\Phi^\dagger e^{qV^a T_a} \Phi$, where a specific representation T_a of the generators t_a has been chosen. Under a non-Abelian gauge transformation this term transforms as

$$\Phi^\dagger e^{qV^a T_a} \Phi \to \Phi'^\dagger e^{qV'^a T_a} \Phi' = \Phi^\dagger e^{iq\Lambda^{a\dagger} T_a} e^{qV'^a T_a} e^{-iq\Lambda^a T_a} \Phi, \tag{1.40}$$

meaning that the vector superfield V^a is required to transform as

$$e^{qV'^a T_a} = e^{-iq\Lambda^{a\dagger} T_a} e^{qV^a T_a} e^{iq\Lambda^a T_a}. \tag{1.41}$$

A more efficient way of writing the supergauge transformations of vector superfields is independent of the representation

$$e^{V'} = e^{-i\Lambda^\dagger} e^V e^{i\Lambda}, \tag{1.42}$$

where $\Lambda \equiv q\Lambda^a T_a$ and $V \equiv qV^a T_a$, such that $e^V e^{-V} = e^{V'} e^{-V'} = 1$.

Supersymmetric Field Strength

The supersymmetric field strengths are defined by the spinor scalar superfields

$$W_A \equiv -\frac{1}{4}\bar{D}\bar{D}e^{-V} D_A e^V, \tag{1.43}$$

$$\bar{W}_{\dot{A}} \equiv -\frac{1}{4}DDe^{-V}\bar{D}_{\dot{A}} e^V, \tag{1.44}$$

where $V = V^a T_a$. W_A is a left-handed superfield, and $\mathrm{Tr}[W_A W^A]$ is supergauge invariant [1]. The Lagrangian for a supersymmetric theory with (possibly) non-Abelian gauge groups is then

$$\mathcal{L} = \Phi^\dagger e^V \Phi + \delta^2(\bar{\theta})W[\Phi] + \delta^2(\theta)W[\Phi^\dagger] + \frac{1}{2T(R)}\delta^2(\bar{\theta})\mathrm{Tr}[W_A W^A], \tag{1.45}$$

where $T(R)$ is the Dynkin index for normalization, $\delta^2(\bar{\theta}) = \bar{\theta}\bar{\theta}$ and $\delta^2(\theta) = \theta\theta$. The Dynkin index of the representation R in terms of matrices T_a is given by $\mathrm{Tr}[T_a, T_b] = T(R)\delta_{ab}$.

1.3.3 Soft Supersymmetry Breaking

As previously mentioned, an unbroken supersymmetry would mean that the particle-sparticle pairs should have the same mass. Since these particles have not been observed, supersymmetry must be a broken symmetry. In this section soft supersymmetry breaking is considered as a way of giving mass to particles, without comprimising the solution to the hierarchy problem.

As it turns out, the best way of breaking SUSY is through *soft breaking*. This entails adding terms to the Lagrangian which cause spontaneous symmetry breaking, while preserving the cancellations of divergences that fixes the hierarchy problem.

Supersymmetry is assumed to be broken at some unknown scale $\sqrt{\langle F \rangle}$, where the supertrace relation (which states that the sum of scalar particle masses squared equals the sum of fermion masses squared) is fulfilled. Initially, this introduces 104 new parameters (in addition to the existing SM ones). Luckily, various bounds, like the ones from gauge symmetry, reduce the number of parameters significantly. The allowed soft terms may be written in terms of their component fields

$$\mathcal{L}_{soft} = -\frac{1}{2}\lambda^A\lambda_A - \left(\frac{1}{6}a_{ijk}A_iA_jA_k + \frac{1}{2}b_{ij}A_iA_j + t_iA_i + \frac{1}{2}c_{ijk}A_i^*A_jA_k + \text{ c.c.}\right) - m_{ij}^2A_i^*A_j \tag{1.46}$$

At this point we can put restrictions on the newly introduced parameters so as not to reintroduce the hierarchy problem. If the parameters are not too large, the correcting mass terms are at most

$$\Delta m_h^2 = -\frac{\lambda_s}{16\pi^2}m_s^2 \ln\frac{\Lambda_{UV}}{m_s^2} + ...,$$

at leading order in the breaking scale Λ_{UV}, and where m_s is the soft breaking scale. In this scheme m_s is restricted to $m_s \sim \mathcal{O}(1 \text{ TeV})$.

1.4 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is 'minimal' in the sense that it requires the least amount of new fields introduced. In the MSSM each fermion-boson pair requires a left-handed and a right-handed supersymmetric Weyl spinor, along with complex scalar fields, which in turn leaves 4 fermionic and 4 bosonic degrees of freedom. These degrees of freedom become two fermions (a particle and an antiparticle), and four scalars (two particle-antiparticle pairs).

1.4.1 Field Content

For leptons we use l_i and $\bar{E}_i$, and ν_i for neutrinos, giving us the $SU(2)$ doublets $L_i = (\nu_i, l_i)$. For quarks we get u_i, $\bar{U}_i$ (up-type) and d_i, $\bar{D}_i$, which form the $SU(2)$ doublets $Q_i = (u_i, d_i)$.

In addition, vector superfields are needed, which come from the vector field $V = t_aV^a$. Since we need as many vector fields as generators, these are the usual $SU(3) \times SU(2) \times U(1)$ vector bosons, namely C^a, W^a and B^0. Their superpartners, constructed from their respective Weyl spinors, are $\tilde{g}$ (the gluino), $\tilde{W}^0$ (the wino) and $\tilde{B}^0$ (the bino).

$SU(2)$ doublets and the mixing of left- and right-handed particles require that we introduce at least two Higgs $SU(2)_L$ doublets. These are called H_u and H_d, where the index tells us which quark they give mass to. These must have weak hypercharge $y = \pm 1$, so we get the doublets

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}. \tag{1.47}$$

The entire field content of the MSSM can be found in Table (1.1).

Supermultiplet	Scalars	Fermions	Vectors	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_i	$(\tilde{u}_{iL}, \tilde{d}_{iL})$	(u_{iL}, d_{iL})		3	2	$\frac{1}{6}$
$\bar{u}_i$	$\tilde{u}^*_{iR}$	$u^\dagger_{iR}$		$\bar{3}$	1	$-\frac{2}{3}$
$\bar{d}_i$	$\tilde{d}^*_{iR}$	$d^\dagger_{iR}$		$\bar{3}$	1	$\frac{1}{3}$
L_i	$(\tilde{\nu}_{iL}, \tilde{e}_{iL})$	(ν_{iL}, e_{iL})		1	2	$-\frac{1}{2}$
$\bar{e}_i$	$\tilde{e}^*_{iR}$	$e^\dagger_{iR}$		1	1	1
H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$		1	2	$\frac{1}{2}$
H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$		1	2	$-\frac{1}{2}$
g		$\tilde{g}$	g	8	1	0
W		$\tilde{W}^{1,2,3}$	$W^{1,2,3}$	1	3	0
B		$\tilde{B}$	B	1	1	0

Table 1.1: Gauge and chiral supermultiplets in the Minimal Supersymmetric Standard Model. The index $i = 1, 2, 3$ runs over the three generations of quarks and lepton. Table from [3].

1.4.2 R-parity

The supersymmetric Lagrangian results in couplings that violate both lepton and baryon numbers. However, such violations are under strict restrictions from experiment, such as the proton decay $p \to e^+\pi^0$. Therefore, we introduce a new, multiplicative conserved quantity, named R-parity [4]

$$P_R = (-1)^{3(B-L)+2s}, \tag{1.48}$$

where s is spin, B is baryon number and L is lepton number. This quantity is $+1$ for SM particles, and -1 for the sparticles. If R-parity is to be conserved sparticles must therefore always be produced and annihilated in pairs. A further consequence is that there must exist a stable, *lightest supersymmetric particle* (LSP), to which all other supersymmetric particles decay eventually. For this particle to be stable it should have zero eletric and color charge. These properties make the LSP a good candidate for dark matter [5].

1.4.3 Soft Breaking terms

In order to give masses to the particles in the MSSM we need soft symmetry breaking terms. All the allowed terms are as follows

$$\mathcal{L}_{MSSM,soft} = -\frac{1}{2}M_1\tilde{B}\tilde{B} + M_2\tilde{W}^a\tilde{W}^a + M_3\tilde{g}^a\tilde{g}^a + c.c. \quad (1.49)$$

$$- a^u_{ij}\tilde{Q}_iH_u\tilde{u}^*_{jR} - a^d_{ij}\tilde{Q}_iH_d\tilde{d}^*_{iR} - a^e_{ij}\tilde{L}_iH_d\tilde{e}^*_{jR} + c.c. \quad (1.50)$$

$$- (m^2_u)_{ij}\tilde{u}^*_{iR}\tilde{u}_{jR} - (m^2_d)_{ij}\tilde{d}^*_{iR}\tilde{d}_{jR} - (m^2_e)_{ij}\tilde{e}^*_{iR}\tilde{e}_{jR} \quad (1.51)$$

$$- (m^2_Q)_{ij}\tilde{Q}^\dagger_i\tilde{Q}_j - (m^2_L)_{ij}\tilde{L}^\dagger_i\tilde{L}_j \quad (1.52)$$

$$- m^2_{H_u}H^*_uH_u - m^2_{H_d}H^*_dH_d - (bH_uH_d + c.c.). \quad (1.53)$$

So the new MSSM Lagrangian has introduced a total of 105 new parameters, where 104 come from the soft terms.

1.4.4 Neutralinos and Charginos

Because the Electroweak symmetry is broken, the gauge fields are now free to mix. The only requirement is that fields of the same $U(1)_{em}$ charge mix. This gives us fields like the photino and zino, which are supersymmetric partners to the photon and Z boson. These are mixes of the neutral $\tilde{B}^0$ and $\tilde{W}^0$. However, the gauge fields are also free to mix with the Higgsinos, giving particles known as *neutralinos*. There are four neutralinos,

$$\tilde{\chi}^0_i = N_{i1}\tilde{B}^0 + N_{i2}\tilde{W}^0 + N_{i3}\tilde{H}^0_d + N_{i4}\tilde{H}^0_u, \quad (1.54)$$

where N_{ij} indicates how much of each component field is mixed in the neutralino. We can also make charged particles, known as *charginos*, which are similar to the neutralinos but mixes of $\tilde{W}^+$, $\tilde{H}^+_u$, $\tilde{W}^-$ and $\tilde{H}^-_d$.

1.4.5 Parameters of the MSSM

The supersymmetry breaking sector of the MSSM contains the following parameters: three complex gaugino Majorana mass parameters, M_1, M_2 and M_3; five diagonal sfermion squared-mass parameters $M^2_{\tilde{Q}}$, $M^2_{\tilde{U}}$, $M^2_{\tilde{D}}$, $M^2_{\tilde{L}}$ and $M^2_{\tilde{E}}$; three Higgs-slepton-slepton and Higgs-squark-squark trilinear interaction terms, with complex coefficients $T_U \equiv \lambda_u A_U$, $T_D \equiv \lambda_d A_D$ and $T_E \equiv \lambda_e A_E$; two real m^2_1, m^2_2 and one complex squared-mass parameter m^2_{12} from the MSSM scalar Higgs potential [4]

$$V(H_u, H_d) =(|\mu|^2 + m^2_1)H^\dagger_dH_d + (|\mu|^2 + m^2_2)H^\dagger_uH_u + (m^2_{12}H_uH_d + h.c.) \quad (1.55)$$

$$+ \frac{1}{8}(g^2 + g^{'2})(H^\dagger_dH_d - H^\dagger_uH_u)^2 + \frac{1}{2}|H^\dagger_dH_u|^2. \quad (1.56)$$

These mass parameters can be expressed in terms of the Higgs vevs $v_u \equiv \frac{1}{\sqrt{2}} \langle H_u \rangle$ and $v_d \equiv \frac{1}{\sqrt{2}} \langle H_d \rangle$. The Fermi constant G_F gives bounds on v_d and v_u, which leaves us with one free parameter

$$\tan\beta \equiv \frac{v_u}{v_d}. \tag{1.57}$$

Assuming a unification of masses at some high scale M_X we can set

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}, \tag{1.58}$$

where M_i are the gaugino masses. In the minimal supergravity scheme (mSUGRA) we may put further constraints on the model using the Kähler potential [4]. This allows us to define a mass m_0 to which all sfermion and Higgs masses coverge at the mass scale M_X: $M^2_{\tilde{f},H}(M_X) = m_0^2$. We can also set $A_i(M_X) = A_0$ for $i = U, D, E$. Using bounds set by m_Z the parameters necessary to define this constrained minimal supersymmetric standard model (cMSSM) are

$$m_{1/2}\ ,\ A_0\ ,\ m_0\ ,\ \tan\beta \text{ and } \mathrm{sgn}(\mu). \tag{1.59}$$

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