

Project report - Recommender system for movies

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This project builds recommender systems for movies using Item-Based Collaborative Filtering (IBCF) approach to compare users and items.

The idea is to compare these system using just the information if the user watched the movie or not (called binary in this project) and use the rating gave for each user (called rating). Also, compare different approaches for calculate the similarity between users and items. These measures are: - cosine distance - euclidean distance - Pearson correlation - Jaccard index

Require package and datasets used

```
library(data.table)
library(ggplot2)
library(recommenderlab)
```

```
## Loading required package: Matrix
```

```
## Loading required package: arules
```

```
##
```

```
## Attaching package: 'arules'
```

```
## The following objects are masked from 'package:base':
```

```
##
```

```
##      abbreviate, write
```

```
## Loading required package: proxy
```

```
##
```

```
## Attaching package: 'proxy'
```

```
## The following object is masked from 'package:Matrix':
```

```
##
```

```
##      as.matrix
```

```
## The following objects are masked from 'package:stats':
```

```
##
```

```
##      as.dist, dist
```

```
## The following object is masked from 'package:base':
```

```
##
```

```
##      as.matrix
```

```
## Loading required package: registry
```

```
library(countrycode)

load("~/Desktop/Ryerson/3.Data_Analytics_Capstone/MovieTweetings/latest/ratings.Rda")
load("~/Desktop/Ryerson/3.Data_Analytics_Capstone/MovieTweetings/latest/movies4.Rda")
```

Data preparation

```
# most common movie
t_m <- aggregate(cbind(count = rating) ~ movie_id,
                 data = ratings,
                 FUN = function(x){NROW(x)})

# merge
r <- merge(x=ratings, y=t_m ,by="movie_id", all.x=TRUE)

# select movies with more than 50 ratings
r <- r[r$count >= 50,]

# count movies per user
t_m_u <- aggregate(cbind(count_movie = movie_id) ~ user_id,
                  data = r,
                  FUN = function(x){NROW(x)})

# merge
r2 <- merge(x=r, y=t_m_u ,by="user_id", all.x=TRUE)

# select users with more than 20 movies
r2 <- r2[r2$count_movie >= 20,]

# delete count and timestamp
r2 <- subset(r2, , -c(rating_timestamp,count,count_movie))

# convert it into a data table
r2 <- data.table(r2)
```

I will start building recommender systems using just the information if the user watched or not the movie. So, 1 if the user rated the movie, and 0 otherwise.

Binary recommender systems

```
# delete rating
r_binary <- subset(r2, , -c(rating))

# reshape
r_binary[, value := 1]
r_wide <- reshape(data = r_binary,
                  direction = "wide",
                  idvar = "user_id",
                  timevar = "movie_id",
                  v.names = "value",
                  drop = NULL)

# keep only the columns containing ratings
# the user name will be the matrix row names, so we need to store them in the vector_users vector
```

```

vector_users <- r_wide[, user_id]
r_wide <- r_wide[, user_id := NULL]

# have the column names equal to the item names
setnames(x = r_wide,
        old = names(r_wide),
        new = substring(names(r_wide), 7))

# store the rating matrix within a recommenderlab object:
# 1) convert r_wide in a matrix
# 2) set the row names equal to the user names
matrix_wide <- as.matrix(r_wide)
rownames(matrix_wide) <- vector_users

# replace NA for zero
matrix_wide[is.na(matrix_wide)] <- 0
head(matrix_wide[, 1:6])

```

```

##      816711 2726560 3079380 1091191 2381249 1398426
## 2      1      1      1      1      1      1
## 18     0      0      0      1      0      0
## 26     1      0      0      0      0      0
## 38     0      0      0      0      0      0
## 48     0      0      0      0      0      0
## 49     0      0      1      0      0      0

```

```

# coercing matrix_wide into a binary rating matrix
ratings_matrix <- as(matrix_wide, "binaryRatingMatrix")
ratings_matrix

```

```
## 4938 x 1788 rating matrix of class 'binaryRatingMatrix' with 287862 ratings.
```

Compare recommender systems using different approaches to calculate the similarity

```

# split the data into the training and the test set
which_train <- sample(x = c(TRUE, FALSE),
                    size = nrow(ratings_matrix),
                    replace = TRUE,
                    prob = c(0.7, 0.3))
recc_data_train <- ratings_matrix[which_train, ]
recc_data_test <- ratings_matrix[!which_train, ]

percentage_training <- 0.7

# minimum number of movies watched by any user
min(rowCounts(ratings_matrix))

```

```
## [1] 20
```

```

# [1] 20

# keep 15 movies
items_to_keep <- 15

# the rating = 1 (watched the movie) will be considered good to evaluate the model
rating_threshold <- 1

# how many times we want to run the evaluation
n_eval <- 1

eval_sets <- evaluationScheme(data = ratings_matrix,
                              method = "split",
                              train = percentage_training,
                              given = items_to_keep,
                              goodRating = rating_threshold,
                              k = n_eval)

models_to_evaluate <- list(
  IBCF_jac = list(name = "IBCF", param = list(method = "jaccard")),
  IBCF_cos = list(name = "IBCF", param = list(method = "cosine")),
  IBCF_cor = list(name = "IBCF", param = list(method = "pearson")),
  IBCF_euc = list(name = "IBCF", param = list(method = "euclidean")),

  UBCF_jac = list(name = "UBCF", param = list(method = "jaccard")),
  UBCF_cos = list(name = "UBCF", param = list(method = "cosine")),
  UBCF_cor = list(name = "UBCF", param = list(method = "pearson")),
  UBCF_euc = list(name = "UBCF", param = list(method = "euclidean")),
  # ,random = list(name = "RANDOM", param=NULL)
)

# number of recommendations
n_recommendations <- c(1, 5, seq(10, 30, 5))

# We are ready to run and evaluate the models.
# The only difference from code 5.evaluate binary CF is now the input method is a list of models
list_results <- evaluate(x = eval_sets,
                        method = models_to_evaluate,
                        n = n_recommendations)

## IBCF run fold/sample [model time/prediction time]
## 1 [63.83sec/1.502sec]
## IBCF run fold/sample [model time/prediction time]
## 1 [61.962sec/1.934sec]
## IBCF run fold/sample [model time/prediction time]
## 1 [51.393sec/1.484sec]
## IBCF run fold/sample [model time/prediction time]
## 1 [109.019sec/0.888sec]
## UBCF run fold/sample [model time/prediction time]
## 1 [0.002sec/43.387sec]
## UBCF run fold/sample [model time/prediction time]
## 1 [0.001sec/42.397sec]
## UBCF run fold/sample [model time/prediction time]

```

```
## 1 Timing stopped at: 18.449 0.747 19.828
## UBCF run fold/sample [model time/prediction time]
## 1 Timing stopped at: 0.044 0.026 0.072
```

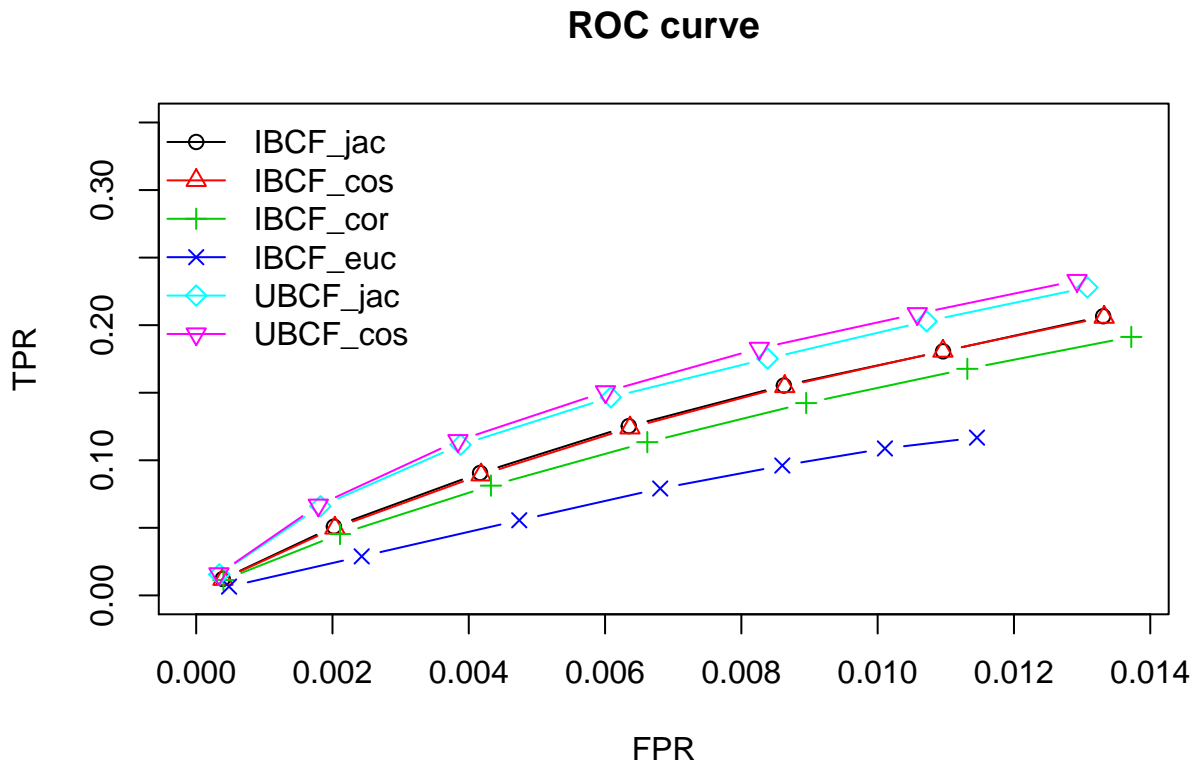
```
## Warning in .local(x, method, ...):
## Recommender 'UBCF_cor' has failed and has been removed from the results!
## Recommender 'UBCF_euc' has failed and has been removed from the results!
```

```
class(list_results)
```

```
## [1] "evaluationResultList"
## attr(,"package")
## [1] "recommenderlab"
```

```
# Calculate the ROC curve
```

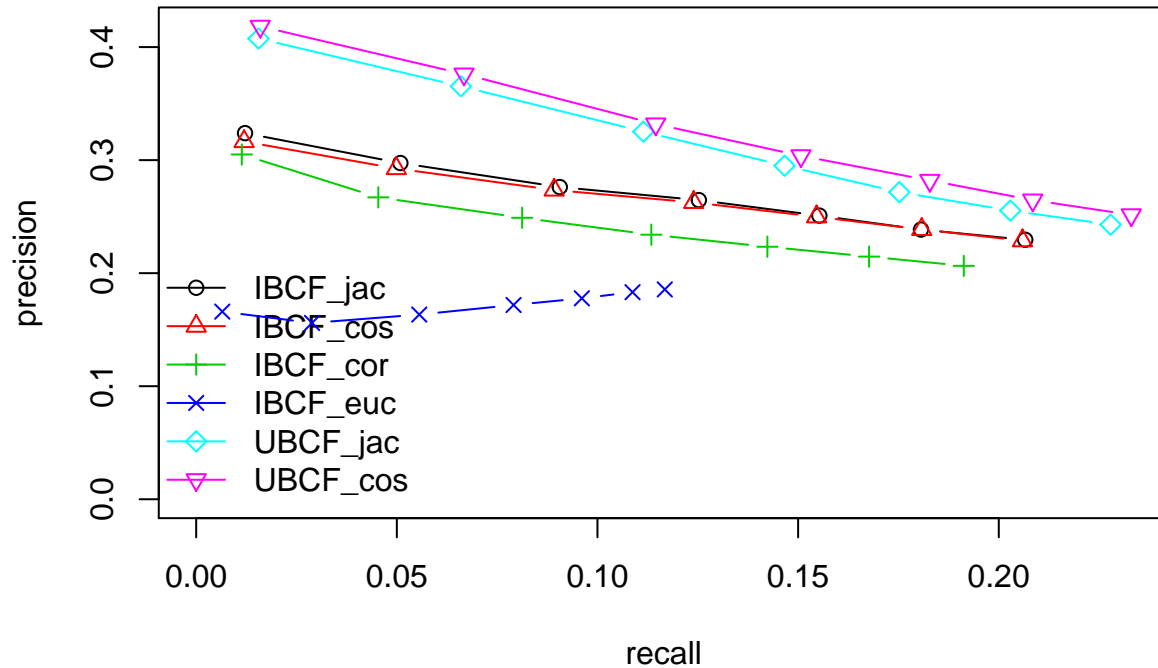
```
plot(list_results, legend = "topleft", ylim = c(0, 0.35))
title("ROC curve")
```



```
# Calculate the precision and recall
```

```
plot(list_results, "prec/rec", legend = "bottomleft")
title("Precision-recall binary models")
```

Precision-recall binary models



```
# average confusion matrix
avg_matrices <- lapply(list_results, avg)
```

```
# IBCF cosine distance
head(avg_matrices$IBCF_cos[,1:8])
```

```
##          TP          FP          FN          TN precision    recall      TPR
## 1  0.3164642  0.6835358  45.05128  1726.949  0.3164642  0.01194274  0.01194274
## 5  1.4628880  3.5371120  43.90486  1724.095  0.2925776  0.04987898  0.04987898
## 10 2.7354926  7.2645074  42.63225  1720.368  0.2735493  0.08921586  0.08921586
## 15 3.9392713  11.0607287  41.42848  1716.572  0.2626181  0.12397890  0.12397890
## 20 4.9973009  15.0026991  40.37045  1712.630  0.2498650  0.15459799  0.15459799
## 25 5.9676113  19.0323887  39.40013  1708.600  0.2387045  0.18079897  0.18079897
##          FPR
## 1  0.0003934291
## 5  0.0020353745
## 10 0.0041810570
## 15 0.0063657448
## 20 0.0086363172
## 25 0.0109575799
```

```
# IBCF Pearson correlation
head(avg_matrices$IBCF_cor[, 1:8])
```

```
##          TP          FP          FN          TN precision    recall      TPR
## 1  0.3049933  0.6950067  45.06275  1726.937  0.3049933  0.01137293  0.01137293
## 5  1.3353576  3.6646424  44.03239  1723.968  0.2670715  0.04538827  0.04538827
## 10 2.4898785  7.5101215  42.87787  1720.122  0.2489879  0.08123494  0.08123494
```

```
## 15 3.5107962 11.4892038 41.85695 1716.143 0.2340531 0.11341921 0.11341921
## 20 4.4676113 15.5323887 40.90013 1712.100 0.2233806 0.14234366 0.14234366
## 25 5.3663968 19.6336032 40.00135 1707.999 0.2146559 0.16768605 0.16768605
##
##      FPR
## 1  0.0004000936
## 5  0.0021102720
## 10 0.0043253723
## 15 0.0066196578
## 20 0.0089512772
## 25 0.0113157768
```

```
# IBCF jaccard
head(avg_matrices$IBCF_jac[, 1:8])
```

```
##      TP      FP      FN      TN precision  recall      TPR
## 1  0.3238866 0.6761134 45.04386 1726.956 0.3238866 0.01217523 0.01217523
## 5  1.4871795 3.5128205 43.88057 1724.119 0.2974359 0.05088415 0.05088415
## 10 2.7618084 7.2381916 42.60594 1720.394 0.2761808 0.09060615 0.09060615
## 15 3.9696356 11.0303644 41.39811 1716.602 0.2646424 0.12526513 0.12526513
## 20 5.0182186 14.9817814 40.34953 1712.650 0.2509109 0.15529337 0.15529337
## 25 5.9642375 19.0357625 39.40351 1708.596 0.2385695 0.18061832 0.18061832
##
##      FPR
## 1  0.0003891262
## 5  0.0020213464
## 10 0.0041659378
## 15 0.0063482765
## 20 0.0086240180
## 25 0.0109594116
```

```
# IBCF Eucliden
head(avg_matrices$IBCF_euc[, 1:8])
```

```
##      TP      FP      FN      TN precision  recall      TPR
## 1  0.1659919 0.8340081 45.20175 1726.798 0.1659919 0.00650464 0.00650464
## 5  0.7746289 4.2024291 44.59312 1723.430 0.1559829 0.02887252 0.02887252
## 10 1.5742240 8.2004049 43.79352 1719.432 0.1633943 0.05559240 0.05559240
## 15 2.3481781 11.7746289 43.01957 1715.858 0.1718435 0.07908990 0.07908990
## 20 2.9770580 14.8663968 42.39069 1712.766 0.1777548 0.09610670 0.09610670
## 25 3.4635628 17.4628880 41.90418 1710.169 0.1831799 0.10873960 0.10873960
##
##      FPR
## 1  0.0004820242
## 5  0.0024290700
## 10 0.0047406859
## 15 0.0068092014
## 20 0.0086007651
## 25 0.0101067197
```

```
# UBCF cosine distance
head(avg_matrices$UBCF_cos[, 1:8])
```

```
##      TP      FP      FN      TN precision  recall      TPR
## 1  0.4183536 0.5816464 44.94939 1727.051 0.4183536 0.01596944 0.01596944
```

```
## 5  1.8805668  3.1194332 43.48718 1724.513 0.3761134 0.06675333 0.06675333
## 10 3.3191633  6.6808367 42.04858 1720.951 0.3319163 0.11451138 0.11451138
## 15 4.5573549 10.4426451 40.81039 1717.190 0.3038237 0.15070815 0.15070815
## 20 5.6403509 14.3596491 39.72740 1713.273 0.2820175 0.18282154 0.18282154
## 25 6.6160594 18.3839406 38.75169 1709.248 0.2646424 0.20846502 0.20846502
##
##          FPR
## 1  0.0003339639
## 5  0.0017923477
## 10 0.0038414728
## 15 0.0060063893
## 20 0.0082629222
## 25 0.0105814364
```

```
# UBCF jaccard
head(avg_matrices$UBCF_jac[, 1:8])
```

```
##          TP          FP          FN          TN precision      recall      TPR
## 1  0.4075574  0.5924426 44.96019 1727.040 0.4075574 0.01555400 0.01555400
## 5  1.8265857  3.1734143 43.54116 1724.459 0.3653171 0.06597394 0.06597394
## 10 3.2523617  6.7476383 42.11538 1720.885 0.3252362 0.11144961 0.11144961
## 15 4.4257760 10.5742240 40.94197 1717.058 0.2950517 0.14664062 0.14664062
## 20 5.4338731 14.5661269 39.93387 1713.066 0.2716937 0.17523697 0.17523697
## 25 6.3839406 18.6160594 38.98381 1709.016 0.2553576 0.20288348 0.20288348
##
##          FPR
## 1  0.0003401687
## 5  0.0018238739
## 10 0.0038807414
## 15 0.0060850176
## 20 0.0083846592
## 25 0.0107185442
```

Now, I will use the rating information to build the recommender systems.

```
# Data preparation
# reshape
r_wide <- reshape(data = r2,
                  direction = "wide",
                  idvar = "user_id",
                  timevar = "movie_id",
                  drop = NULL)

head(r_wide[, 1:5, with = FALSE])
```

```
##      user_id rating.816711 rating.2726560 rating.3079380 rating.1091191
## 1:         2             8             9             8             7
## 2:        18            NA            NA            NA            8
## 3:        26             5            NA            NA            NA
## 4:        38            NA            NA            NA            NA
## 5:        48            NA            NA            NA            NA
## 6:        49            NA            NA            10            NA
```



```

vector_users <- r_wide[, user_id]
r_wide <- r_wide[, user_id := NULL]

# have the column names equal to the item names
setnames(x = r_wide,
        old = names(r_wide),
        new = substring(names(r_wide), 7))

matrix_wide <- as.matrix(r_wide)
rownames(matrix_wide) <- vector_users
head(matrix_wide[, 1:6])

##      .816711 .2726560 .3079380 .1091191 .2381249 .1398426
## 2         8         9         8         7         5         8
## 18        NA        NA        NA         8        NA        NA
## 26         5        NA        NA        NA        NA        NA
## 38        NA        NA        NA        NA        NA        NA
## 48        NA        NA        NA        NA        NA        NA
## 49        NA        NA        10        NA        NA        NA

```

```

# coercing matrix_wide into a binary rating matrix
ratings_matrix <- as(matrix_wide, "realRatingMatrix")
ratings_matrix

```

4938 x 1788 rating matrix of class 'realRatingMatrix' with 287862 ratings.

```

# Train and test
which_train <- sample(x = c(TRUE, FALSE),
                    size = nrow(ratings_matrix),
                    replace = TRUE,
                    prob = c(0.7, 0.3))

recc_data_train <- ratings_matrix[which_train, ]
recc_data_test <- ratings_matrix[!which_train, ]

eval_sets <- evaluationScheme(data = ratings_matrix,
                             method = "split",
                             train = percentage_training,
                             given = items_to_keep,
                             goodRating = rating_threshold,
                             k = n_eval)

models_to_evaluate <- list(
  IBCF_jac = list(name = "IBCF", param = list(method = "jaccard")),
  IBCF_cos = list(name = "IBCF", param = list(method = "cosine")),
  IBCF_cor = list(name = "IBCF", param = list(method = "pearson")),
  IBCF_euc = list(name = "IBCF", param = list(method = "euclidean")),

  UBCF_jac = list(name = "UBCF", param = list(method = "jaccard")),
  UBCF_cos = list(name = "UBCF", param = list(method = "cosine")),
  UBCF_cor = list(name = "UBCF", param = list(method = "pearson")),
  UBCF_euc = list(name = "UBCF", param = list(method = "euclidean")),
  # , random = list(name = "RANDOM", param=NULL)

```

```

)

list_results <- evaluate(x = eval_sets,
                        method = models_to_evaluate,
                        n = n_recommendations)

## IBCF run fold/sample [model time/prediction time]
## 1 [81.655sec/2.628sec]
## IBCF run fold/sample [model time/prediction time]
## 1 [116.702sec/2.005sec]
## IBCF run fold/sample [model time/prediction time]
## 1 [66.172sec/1.344sec]
## IBCF run fold/sample [model time/prediction time]
## 1 [86.755sec/1.342sec]
## UBCF run fold/sample [model time/prediction time]
## 1 [0.042sec/98.061sec]
## UBCF run fold/sample [model time/prediction time]
## 1 [0.042sec/143.112sec]
## UBCF run fold/sample [model time/prediction time]
## 1 [0.042sec/38.178sec]
## UBCF run fold/sample [model time/prediction time]
## 1 [0.04sec/90.231sec]

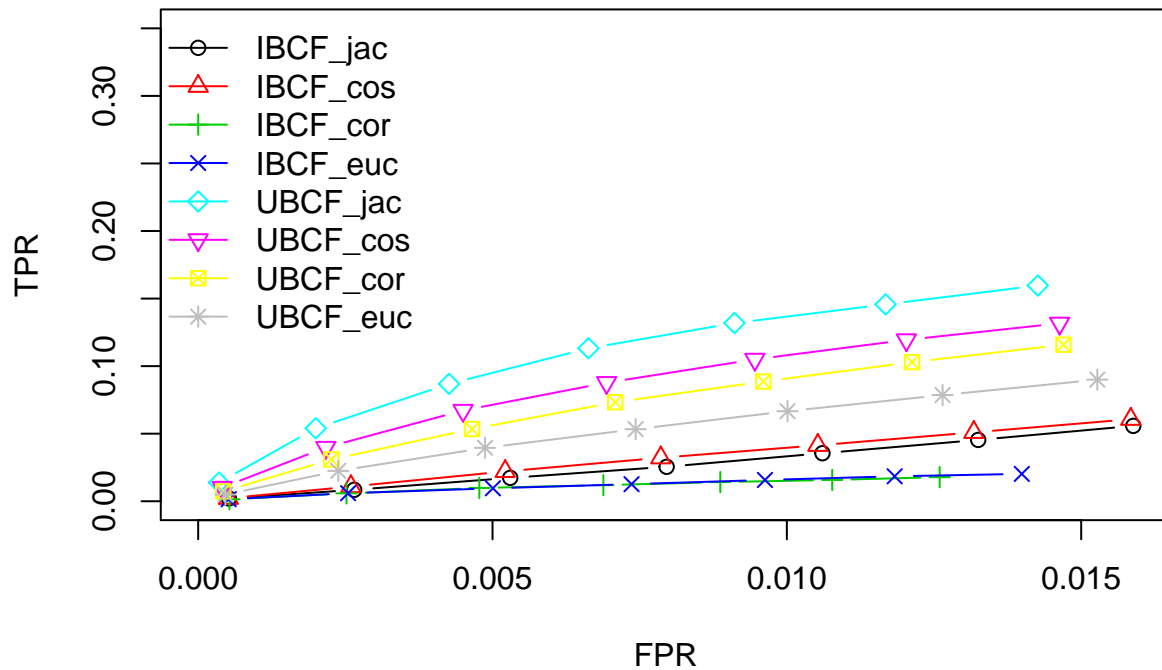
class(list_results)

## [1] "evaluationResultList"
## attr(,"package")
## [1] "recommenderlab"

# Calculate the ROC curve
plot(list_results, legend = "topleft", ylim = c(0, 0.35))
title("ROC curve - rating models")

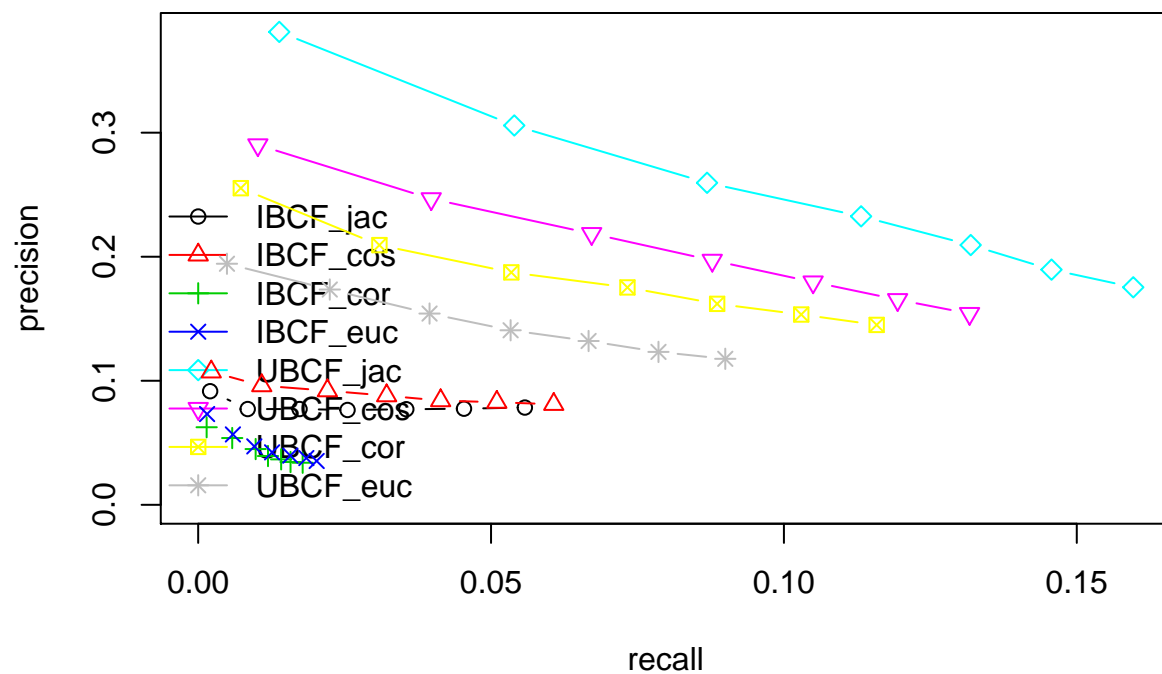
```

ROC curve – rating models



```
# Calculate the precision and recall
plot(list_results, "prec/rec", legend = "bottomleft")
title("Precision-recall rating models")
```

Precision–recall rating models



```
# average confusion matrix
avg_matrices <- lapply(list_results, avg)
```

```
# IBCF cosine distance
head(avg_matrices$IBCF_cos[,1:8])
```

```
##           TP           FP           FN           TN precision recall
## 1  0.1066127  0.8879892  44.86707 1727.138 0.10719132 0.002231592
## 5  0.4777328  4.4952767  44.49595 1723.531 0.09606513 0.010857502
## 10 0.9149798  9.0310391  44.05870 1718.995 0.09199457 0.022062247
## 15 1.3124157 13.6066127  43.66127 1714.420 0.08796924 0.032177022
## 20 1.6707152 18.2213225  43.30297 1709.805 0.08398915 0.041397161
## 25 2.0566802 22.8083671  42.91700 1705.218 0.08271370 0.050984408
##           TPR           FPR
## 1  0.002231592 0.0005122931
## 5  0.010857502 0.0025950535
## 10 0.022062247 0.0052153509
## 15 0.032177022 0.0078584410
## 20 0.041397161 0.0105257930
## 25 0.050984408 0.0131759576
```

```
# IBCF Pearson correlation
head(avg_matrices$IBCF_cor[, 1:8])
```

```
##           TP           FP           FN           TN precision recall
## 1  0.06140351  0.9203779  44.91228 1727.106 0.06254296 0.001446846
## 5  0.26045884  4.3569501  44.71323 1723.669 0.05382589 0.005808662
## 10 0.42240216  8.2516869  44.55128 1719.775 0.04495173 0.009813845
## 15 0.53171390 11.8846154  44.44197 1716.142 0.03931299 0.011922175
## 20 0.63292848 15.3070175  44.34076 1712.719 0.03663704 0.014163713
## 25 0.71524966 18.5829960  44.25843 1709.443 0.03448819 0.015765982
##           TPR           FPR
## 1  0.001446846 0.0005321392
## 5  0.005808662 0.0025192432
## 10 0.009813845 0.0047761147
## 15 0.011922175 0.0068831885
## 20 0.014163713 0.0088692688
## 25 0.015765982 0.0107704038
```

```
# IBCF jaccard
head(avg_matrices$IBCF_jac[, 1:8])
```

```
##           TP           FP           FN           TN precision recall
## 1  0.09109312  0.9035088  44.88259 1727.123 0.09158752 0.002038986
## 5  0.38394062  4.5890688  44.58974 1723.437 0.07720488 0.008455769
## 10 0.76788124  9.1781377  44.20580 1718.848 0.07720488 0.017305251
## 15 1.14035088 13.7786775  43.83333 1714.248 0.07643600 0.025502802
## 20 1.53238866 18.3596491  43.44130 1709.667 0.07703528 0.035529894
## 25 1.92645074 22.9385965  43.04723 1705.088 0.07747626 0.045382171
##           TPR           FPR
## 1  0.002038986 0.0005213819
## 5  0.008455769 0.0026501149
```

```
## 10 0.017305251 0.0053011412
## 15 0.025502802 0.0079579261
## 20 0.035529894 0.0106047162
## 25 0.045382171 0.0132502283
```

```
# IBCF Eucliden
```

```
head(avg_matrices$IBCF_euc[, 1:8])
```

```
##          TP          FP          FN          TN precision      recall
## 1  0.0708502  0.8974359  44.90283  1727.129 0.07317073 0.001521440
## 5  0.2719298  4.4021592  44.70175  1723.624 0.05672474 0.005929194
## 10 0.4433198  8.6491228  44.53036  1719.377 0.04708534 0.009585519
## 15 0.5843455  12.7125506  44.38934  1715.314 0.04227051 0.012623287
## 20 0.7179487  16.6214575  44.25574  1711.405 0.03963719 0.015736500
## 25 0.8380567  20.4203779  44.13563  1707.606 0.03766949 0.018465357
##          TPR          FPR
## 1  0.001521440 0.0005185225
## 5  0.005929194 0.0025466127
## 10 0.009585519 0.0050064645
## 15 0.012623287 0.0073618511
## 20 0.015736500 0.0096279057
## 25 0.018465357 0.0118314209
```

```
# UBCF cosine distance
```

```
head(avg_matrices$UBCF_cos[, 1:8])
```

```
##          TP          FP          FN          TN precision      recall      TPR
## 1  0.2901484  0.7098516  44.68354  1727.316 0.2901484 0.01018963 0.01018963
## 5  1.2334683  3.7665317  43.74022  1724.260 0.2466937 0.03979122 0.03979122
## 10 2.1862348  7.8137652  42.78745  1720.213 0.2186235 0.06718063 0.06718063
## 15 2.9561404  12.0438596  42.01754  1715.982 0.1970760 0.08776730 0.08776730
## 20 3.5917679  16.4082321  41.38192  1711.618 0.1795884 0.10498868 0.10498868
## 25 4.1342780  20.8657220  40.83941  1707.161 0.1653711 0.11941242 0.11941242
##          FPR
## 1  0.0004087764
## 5  0.0021682408
## 10 0.0044995445
## 15 0.0069378490
## 20 0.0094563526
## 25 0.0120302592
```

```
# UBCF jaccard
```

```
head(avg_matrices$UBCF_jac[, 1:8])
```

```
##          TP          FP          FN          TN precision      recall      TPR
## 1  0.3812416  0.6187584  44.59244  1727.408 0.3812416 0.01384676 0.01384676
## 5  1.5290148  3.4709852  43.44467  1724.555 0.3058030 0.05396808 0.05396808
## 10 2.5958165  7.4041835  42.37787  1720.622 0.2595816 0.08688196 0.08688196
## 15 3.4885290  11.5114710  41.48516  1716.515 0.2325686 0.11318172 0.11318172
## 20 4.1882591  15.8117409  40.78543  1712.215 0.2094130 0.13189186 0.13189186
## 25 4.7395412  20.2604588  40.23414  1707.766 0.1895816 0.14568772 0.14568772
##          FPR
```

```
## 1 0.0003555711
## 5 0.0019977789
## 10 0.0042627475
## 15 0.0066302096
## 20 0.0091104678
## 25 0.0116780869
```

```
# UBCF euclidean
head(avg_matrices$UBCF_euc[, 1:8])
```

```
##          TP          FP          FN          TN precision      recall      TPR
## 1 0.1943320 0.805668 44.77935 1727.221 0.1943320 0.00491603 0.00491603
## 5 0.8677463 4.132254 44.10594 1723.894 0.1735493 0.02247098 0.02247098
## 10 1.5418354 8.458165 43.43185 1719.568 0.1541835 0.03950176 0.03950176
## 15 2.1106613 12.889339 42.86302 1715.137 0.1407108 0.05333939 0.05333939
## 20 2.6410256 17.358974 42.33266 1710.667 0.1320513 0.06665376 0.06665376
## 25 3.0816464 21.918354 41.89204 1706.108 0.1232659 0.07858338 0.07858338
##          FPR
## 1 0.0004636386
## 5 0.0023796893
## 10 0.0048732083
## 15 0.0074307641
## 20 0.0100116683
## 25 0.0126453276
```

```
# UBCF correlation
head(avg_matrices$UBCF_cor[, 1:8])
```

```
##          TP          FP          FN          TN precision      recall
## 1 0.2537112 0.7402159 44.71997 1727.286 0.2552614 0.007281998
## 5 1.0404858 3.9291498 43.93320 1724.097 0.2093686 0.030941939
## 10 1.8623482 8.0769231 43.11134 1719.949 0.1873727 0.053461079
## 15 2.6126856 12.2962213 42.36100 1715.730 0.1752433 0.073286201
## 20 3.2206478 16.6578947 41.75304 1711.368 0.1620163 0.088599897
## 25 3.8117409 21.0364372 41.16194 1706.990 0.1534012 0.102957772
##          TPR          FPR
## 1 0.007281998 0.000425691
## 5 0.030941939 0.002262984
## 10 0.053461079 0.004652751
## 15 0.073286201 0.007084640
## 20 0.088599897 0.009600603
## 25 0.102957772 0.012126494
```