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DIGITAL FILTERS OF THRESHOLD ELEMENTS

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1. Definition of the Digital Filters

A *threshold element* S_k^n $(n \geqslant k)$ is a binary digital circuit with n "inputs", one "output" and a "threshold" k; i.e. there are two signals (two different voltages or two different currents) which may be used as input signals of the threshold element, and can occur as its output signal with the application of these input signals. Denoting one of the two signals by 0 and the other by 1, signal 1 will appear at the output of S_n^k, provided that k or more of the inputs deliver the signal 1, or alternatively, that less than $n - k$ of the inputs deliver the signal 1; these two cases are transformed into each other by an exchange of the denotations 0 and 1. In the following let us always assume that the denotation has been chosen in such a way that the first of the two cases applies. (n, k) is called the *type* of the threshold element S_n^k. In diagrams this element is represented by the symbol in fig. 1. Let 0 and 1 be natural numbers, then the qualities of the threshold elements which alone interest us, are described by the function

$$s_{n,k} = \begin{cases} 1 \\ 0 \end{cases} \text{for} \sum_{i=1}^{n} x_i \begin{cases} \geqslant k \\ < k \end{cases},$$

with $x = x_1, x_2, \ldots, x_n$, where the x_i $(i = 1, 2, \ldots, n)$ may take the values 0 and 1.

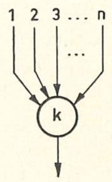

Fig. 1. Threshold element.

Let us now consider combinations of such threshold elements into more complex switching circuits, which will be called *symmetrical digital filters* and abbreviated to DFs.

One-stage DFs are the threshold elements themselves; two one-stage digital filters are called equal, if both threshold elements are of the same type. Multistage symmetrical digital filters are defined inductively. Given $r > 1$, and that the $(r - 1)$-stage DFs have already been defined:

Let $F_1, F_2, \ldots, F_n$ pairwise equal $(r - 1)$-stage DFs, and let S_k^n be any threshold element. Let the inputs of S_k^n be denoted by $x_1, x_2, \ldots, x_n$, and the inputs of F_i by $x_{i1}, x_{i2}, \ldots x_{im}$ (for $i = 1, 2, \ldots, n$). A new switching circuit is constructed from the threshold element and the $(r - 1)$-stage DFs: the inputs x_{ij} with $i + j - 1 = t$ (for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$) are combined as the input x_t' (for $t = 1, 2, \ldots, m + n$). The output of F_i is connected to the input x_i of S_k^n (for $i = 1, 2, \ldots, n$).

A digital binary switching circuit, which by this construction, can be set up of pairwise equal $(r - 1)$-stage DFs and of one threshold element, is called a *symmetrical r-stage* digital filter. Two r-stage DFs are called equal, if the two threshold elements and the $(r - 1)$-stage DFs used for their construction were respectively equal. Hence:

An r-stage DF, F is clearly defined by

$$(n_1, k_1), (n_2, k_2), \ldots, (n_r, k_r) \tag{1}$$

if (n_1, k_1) is the type of the threshold element which was assumed for the construction of F, and if (n_i, k_i) with $1 < i \leqslant r$ is the type of the threshold element which was used for the formation of F when constructing the i-th stage out of the $(i - 1)$th stage.

(1) is called the *type of F*. Fig. 2 shows a DF of type (3,2), (2,1), (3,3).

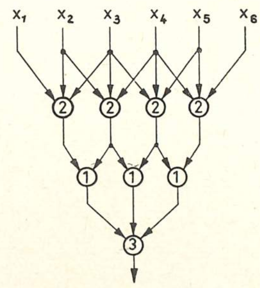

Fig. 2. 3-stage digital filter.

The number of inputs of a DF of type (1) is

$$m = n_1 + n_2 + \ldots + n_r - (r-1). \tag{2}$$

A DF of type (1) defines a function $f(x_1, .., x_m)$, with m given by (2), where the variables and the function can take the values 0 and 1. This function is obtained by replacing in the construction of the r-stage filter given above, "threshold element S_k^n" by "function $s_{n,k}(x)$", "input" by "variable", and "the connection of one output with one input" by "insertion of one function into the other". The function $f(x_1, \ldots, x_m)$ is clearly defined by the filter, but the reverse does not apply.

Various DFs may define the same function, i.e., a DF gives a special representation of the function $f(x_1, \ldots, x_m)$ by means of the functions $s_{n,k}(x)$, but this is not uniquely defined by the function. From now on, when we refer to a DF we understand this representation of the function $f(\,)$ by means of the functions $s_{n,k}(\,)$. We now turn our attention to the qualities of these functions and their representations.

2. The Space X of the Binary Sequences Infinite in both Directions

X denotes the set of the sequences $x = \ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots, x_i = 0$ or 1 for all $i \in G$, where G is the set of the integers. Two elements $x \in X$ and $y \in X$ are called equal (written as $x = y$), when $x_i = y_i$ for all $i \in G$.

Suppose $x \in X$ and $y \in X$ and $z = \ldots, z_{-1}, z_0, z_1, \ldots$; then we define

$$\left.\begin{array}{lll} \text{I.} & z = x \vee y, & \text{when } z_i = \max(x_i, y_i) \\ \text{II.} & z = x\,y, & \text{when } z_i = \min(x_i, y_i) \\ \text{III.} & z = x' & \text{when } z_i = 0 \text{ if } x_i = 1, \\ & & \quad\text{and } z_i = 1 \text{ if } x_i = 0 \\ \text{IV.} & x \subset y & \text{when } x_i \leqslant y_i \\ \text{V.} & z = x + y & \text{when } z_i = x_i + y_i \pmod 2 \end{array}\right\} \text{ for } i \in G.$$

It can be seen that X is closed with respect to the operations I to VI, and that X forms a Boolean algebra with respect to I to III.

If $x \in X$, $x_{i-1} = 0$ and $x_i = 1$, then x_i is called a *left marginal point* of x; if $x_i = 1$ and $x_{i+1} = 0$, then x_i is called a *right marginal point* of x.

Let x_i be a left and x_k a right marginal point of $x \in X$. When $i \leqslant k$, and when there is no marginal point x_j with $i < j < k$, then $x_j = 1$ for $i \leqslant j \leqslant k$; then in this case $x_j\,(i \leqslant j \leqslant k)$ form a 1-*block* of length $k - i + 1$.

We now define three functions on X:

When x contains at least one 1-block, the length of the smallest 1-block appearing in x is denoted by $l(x)$; when $x_i = 0$ for all $i \in G$, we set $l(x) = 0$ and otherwise $l(x) = \infty$.

Let $x \in X$ and $l(x) \neq \infty$; if there is one largest 1-block in x, then the length of this 1-block is denoted by $L(x)$; when $x_i = 0$ for all $i \in G$, we set $L(x) = 0$; otherwise we define $L(x) = \infty$.

Let $x \in X$, and let x_i be a left and, x_k a right, marginal point of x with the feature $x_j = 0$ for $j < i$ and $j > k$. Then we define $N(x) = k - i + 1$. When $x_i = 0$ for all $i \in G$, we set $N(x) = 0$, and otherwise $= \infty$.

Now we write down several simple conclusions without proof.

2.1. Suppose $X_k = \{x \in X \mid l(x) \geqslant k\}$.
If $x \in X_k$ and $y \in X_k$, then $x \vee y \in X_k$.

2.2. Suppose $Y_k = \{x \in X \mid L(x) \leqslant k\}$.
If $x \in X$ and $y \in Y_k$, then $x\,y \in Y_k$.

2.3. Suppose $I_k = \{x \in X \mid N(x) \leqslant k\}$.
If $x \in I_k$ and $y \subset x$, then $y \in I_k$.

2.4. Regarding $X_k' = \{x \in X \mid l(x') \geqslant k\}$,
$Y_k' = \{x \in X \mid L(x') \leqslant k\}$,
and $I_k' = \{x \in X \mid N(x') \leqslant k\}$,
the conclusions dual to 2.1, 2.2 and 2.3 apply.

3. The Mappings of X into itself Defined by the Digital Filters

Let $f(x_1, x_2, \ldots, x_m)$ be the function pertaining to the DF, F, as explained in Section 1. We set

$$y_i = f(x_i, x_{i+1}, \ldots, x_{i+m-1}) \text{ for } i \in G,$$

where

$$x = \ldots, x_{-1}, x_0, x_1, \ldots .$$

We know that

$$y = \ldots, y_{-1}, y_0, y_1, \ldots \in X.$$

In this manner we have attached to the DF a mapping of X into itself, which we denote as

$$y = Fx;$$

i.e. in the denotation we do not distinguish between the DF and the mapping attached to it; F defines not only a certain mapping of X in itself, but also a generation of the mapping by the functions $s_{n,k}$.

Two DFs F and F' are called equivalent, when

$$Fx = F'x,$$

applies for each $x \in X$. In this case we also write $F = F'$. In order to denote the equality of DF as explained in Section 1, we use the symbol "$\equiv$". Hence $F \equiv F'$ always implies $F = F'$, whereas the reverse does not apply.

Let us define a further mapping of X into itself; suppose $x \in X$, then

$$y = Tx,$$

the sequence with $y_i = x_{i+1}$ for $i \in G$.

3.1. From the definition of F and T, we may immediately conclude that

$$F(Tx) = T(Fx)$$

for each $x \in X$ and each DF, F.

Therefore we do not differentiate between X and Tx in the following theorems, classifying X modulo T.

3.1 shows how the technical realization of these mappings, by connecting a *shift register* to a digital filter, is expressed. Fig. 3 gives an example. The squares symbolize the delay elements of the shift register, which at the pulse frequency of the signals arriving at the input I accept the signal stored in the previous delay element or the newly arriving signal; the output signals appear at the output O at the same frequency.

Let F and F' be two DFs, then we define the product $F \cdot F'$ by

$$(F \cdot F')x = F(F'x).$$

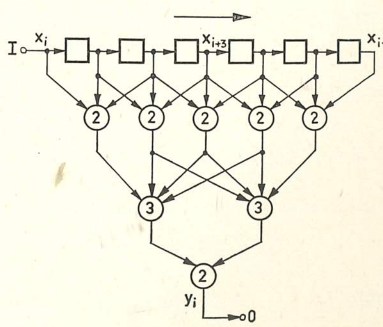

Fig. 3. Shift register connected to a digital filter.

The product thus defined is associative but, as we shall see later, not commutative.

By means of the last operation, the set of mappings under consideration generates a semi-group H, the unit element of which is represented by the 1-stage DF of type (1.1). To each element F' in H there is a DF, F with $F = F'$; this is shown by theorem 4.1 below.

When F is a DF of type

$$(n_1, k_1), (n_2, k_2), \ldots, (n_r, k_r),$$

then $F = F_{n_1,k_1} \cdot F_{n_2,k_2} \cdot \ldots \cdot F_{n_r,k_r}$, where the F_{n_i,k_i} are one-stage DFs of type (n_i, k_i) for $i = 1, 2, \ldots, r$.

This theorem will be proved in the next section. On the basis of this theorem we can restrict our discussion of the mappings of H, to mappings represented by one-stage DFs.

3.2. The mappings F of H are, for $F \neq 1$ where 1 is the unit element of H, no mappings of X onto itself, but only mappings of X into itself.

As proof it is sufficient to find a sequence $x \in X$, which for no $F \in H$ possesses a $y \in X$ such that $Fy = x$. This is true for $x = \ldots, x_{-1}, x_0, x_1 \ldots$ with $x_i = 0$ for $i = 0 \pmod 2$ and $i = 1$ and $x_i = 1$ for $i = 1 \pmod 2$ and $i \neq 1$.

It may easily be shown that the original of an alternating sequence must also be an alternating sequence. For such a sequence, however, $F_{n,k}x = 0$ for $n < 2k-1$ and $F_{n,k}x = 1$ for $n > 2k-1$, where 0 stands for the sequence $x_i = 0$, and 1 for the sequence $x_i = 1$ for all $i \in G$. The mappings $F_{n,k}$ with $n = 2k-1$, however, always lead from an alternating sequence to an alternating sequence, but for the sequence given above, no original is to be found.

In the following theorems we always assume $x \in X$ and $y \in X$.

3.3. Since $x \subset y$, $Fx \subset Fy$ for all $F \in H$.

This statement is trivial for DFs represented one-stage. For DFs represented multi-stage, it follows from theorem 4.1 by means of the product representation.

It may be specifically stated that:

If $x \subset y$ and $Fy = 0$, then $Fx = 0$.

If $x \subset y$ and $Fx = 1$, then $Fy = 1$.

If $x \subset y \subset z$ and $Fx = Fz$, then $Fy = Fz$.

3.4. For each $F \in H$

$$Fx \vee Fy \subset F(x \vee y).$$

The proof follows from 3.3. Then

$$x \subset (x \vee y) \text{ and } y \subset (x \vee y);$$

and hence

$$Fx \subset F(x \vee y) \text{ and } Fy \subset F(x \vee y),$$

from which the theorem follows directly.

3.5. Of course, the dual theorem also applies:

$$F(xy) \subset (Fx)\ (Fy).$$

This follows like 3.4 from

$$xy \subset x \text{ and } xy \subset y.$$

3.6. Let $x \in X_m \cap I_m$, then

$F_{n,j}x = 0$ for $m < j$,

$F_{n,j}x \in X_{m+(n-2j+1)} \cap I_{m+(n-2j+1)}$ for $m > j$.

3.7. Let $x \in X'_m \cap I'_m$, then

$F_{n,j}x = 0$ for $m \leqslant j$,

$F_{n,j}x \in X'_{m-(n-2j+1)} \cap I'_{m-(n-2j+1)}$ for $m \geqslant n-j$.

We omit the simple proofs of 3.6 and 3.7 as well as that for the generalization of both theorems:

3.8. Let $x \in X_m \cap X'_p$ with $m \geqslant j$ and $p > n-j$, then $F_{n,j}x \in X_{m+(n-2j+1)} \cap X'_{p-(n-2j+1)}$.

Special attention should be paid to the case $n = 2j-1$, for which under the above assumptions we have

$Fx = x$; i.e. F leaves the elements of $X_j \cap X_j'$ unchanged. With a certain restriction the reverse also applies.

3.9. Let $x \in X^+$, where

$$X^+ = \{x \in X \mid N(x) < \infty\}.$$

If further, $x \neq 0$ and $x \neq 1$ and $F_{n,j}x = x$, then $n = 2j-1$ and $x \in X_j \cap X_j'$.

The proof of this theorem is given by finding the marginal point with minimum index, which exists on the basis of the assumption $x \in X^+$, and by showing that this marginal point belongs to a 1-block of length equal to or larger than j, and that $n = 2j-1$. The same condition is found for the length of the successive 0-blocks, etc., thus proving the statement.

Hence it follows in agreement with 4.1.

3.10. *Theorem*: Let $F \in H$ and be of the type $(n_1, k_1), (n_2, k_2), \ldots, (n_r, k_r)$ and $n_i \geqslant 2k_i - 1$.

Then F has a fix point $x \in X^+$ with $x \neq 0$ and $x \neq 1$, if and only if $n_i = 2k_i - 1$ for $i = 1, 2, \ldots, r$. When $k = \max(k_1, \ldots, k_r)$, then the set of the fix points is equal to $X_k \cap X_k' \cap X^+$.

3.11. *Theorem*: Let $x \in X \cup X'$ and let F be of the type (n, k) with $n = 2k-1$.

Then $Fx \in X_k \cap X_k'$,

and $F^2x = Fx$;

i.e. F is a projection of $X_k \cup X_k'$ on $X_k \cap X$

As proof it is shown that a 0-block of length less than k, between two 1-blocks of length greater than or equal to k, is converted by F into a 1-block; i.e. there will remain at best 0-blocks of length greater than or equal to k. The same applies for 1-blocks between 0-blocks. Similarly:

3.12. Let $x \in X_k' \cap Y_j$ and let F be one-stage of the type (n, k) with $n = 2k-1$, then for $j < k$

$$Fx = 0.$$

3.13. Let $x \in X^+$ and let F be one-stage of the type (n, k) with $n = 2k-1$. Then there is a natural number t with

$F^t x = X_k \cap X_k'$,

or $F^t x = 0$.

For the proof, the marginal point of x with minimum index is examined as in 3.9; if this proof pertains to a 1-block of a length k, it remains preserved with each mapping. Otherwise it becomes zero. After a maximum number of $N(x)$ iterations of the mapping, we obtain as image 0, or a sequence of which the marginal points with smallest and largest index pertain to a 1-block of a length k. To complete the proof we note that a 0-block or a 1-block of length k is enlarged by the mapping F so long as the adjacent 1-block or 0-block, respectively, has a length k, i.e., in the direction of these smaller blocks.

4. The Factorization of the Digital Filters

Let us now prove theorem 4.1.

4.1. If F is a DF of the type $(n_1, k_1), (n_2, k_2), .., (n_r, k_r)$, then $F = F_1 \cdot F_2 \cdot \ldots \cdot F_r$, where the F_i are of the type (n_i, k_i) for $i = 1, 2 \ldots, r$.

Let F_1 be of the type $(n_1, k_1), \ldots, (n_{r-1}, k_{r-1})$, and F_2 of the type (n_r, k_r).
Further let $y = Fx, z = F_1 x$ and $y' = F_2 z$.

As proof it must be shown that $y = y'$ for all $x \in X$.

To do this we return to the construction principle for one-stage filters. When $y_i = f(x_i, \ldots, x_{i+j})$ and $z_i = f_1(x_i, \ldots, x_{i+m})$ for $i \in G$ are the functions defining F and F_1, then $f(x_i, \ldots, x_{i+j})$ is obtained by inserting the function $f_1(x_{i+k}, \ldots, x_{i+m+k})$ for z_{i+k} in s_{n_r,k_r} $(z_i, \ldots, z_{n_r+i-1})$, for $k = 0, 1, \ldots, n_r - 1$. This results in $y' = F_2 z$, and thus the statement is proved.

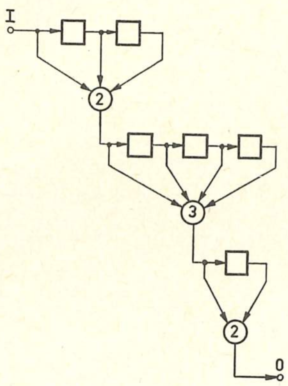

Fig. 4. Factorization of the filter of fig. 3.

In fig. 4 the factorization of the DF represented in fig. 3, is carried out, namely a factorization into a product of one-stage DFs. On the basis of theorem 4.1, each element of H may be represented by a product of one-stage DFs; i.e. H is generated by elements corresponding to these. The question arises whether this kind of representation is unique; i.e. whether this generating set of H is a free one, and whether there exists such a set at all. The generating set is unfree, since the following theorem applies:

4.2. Let the one-stage filter of the type (n, k) be denoted by $F_{n,k}$.

In this notation we have

$$F_{n,1} \cdot F_{m,1} = F_{n+m,1}$$
$$F_{n,n} \cdot F_{m,m} = F_{n+m,n+m}.$$

This results in the exchangeability of these two special classes of DFs. However, these are the only non-trivial cases in which generating sets are exchangeable, since it is a rule that:

4.3. If $(n, j) \neq (r, k)$ and $n > 1$ and $r > 1$, then

$$F_{n,j} \cdot F_{r,k} = F_{r,k} \cdot F_{n,j}$$

except for the cases considered under 4.2.

One might assume that a unique normal form might still be found by using the formulae of 4.2 in order to combine exchangeable factors into one factor. This is not so, however, since the exchangeability of $F_{2k-1,k}$ and $F_{2r-1,r}$ in $F_{n,1} \cdot F_{2k-1,k} \cdot F_{2r-1,r}$ for $k \leqslant n$ and $r \leqslant n$ follows from $F_{n,1} x \in x_n$ for each $x \in x$, plus theorem 3.8.

Similarly it may be shown that there exists no free generating set for H, i.e. H is unfree.

ABSTRACTS

Pyramid-like combinations of binary threshold elements to form more complex switching circuits with several inputs and one output, are considered. The inputs are connected to the outputs of the delay elements of a shift register, so that a sequential network is obtained with one binary input and one binary output; these electrical networks are our digital filters.

We investigate the mappings into itself of the set of binary sequences, infinite in both directions, defined by these filters. The filter-mapping relation is not one-to-one, and this leads to a classification of the filters, i.e. filters, which result in the same mapping, come within the same class. For these classes, normal forms are found which are not, however, unique.

Мы рассматриваем пирамидальные соединения элементов в комплексных коммутируемых схемах с несколькими входами и одним выхожом. Входы соединяются с выходами замедляющих звеньев цепи передвижения, таким образом мы получаем коммутатор с двойным входом и двойным выходом, эта электрическая схема представляет собой наш цифровой фильтр.

Мы исследуем определяемое этим фильтром изображение большого количества двойных соединений (бесконечных в обе стороны). Расположение изображения, определяемого фильтром, не однозначно, что приводит к классификации фильтров, т.е. фильтры, дающие одинаковые изображения, относятся к одному классу. Для этих классов мы определили нормальные формы, которые все же невсегда однозначны.

On considère des circuits de commutation complexes à plusieurs entrées et à une seule sortie obtenus par un branchement en pyramide d'éléments à seuil binaires. On relie les entrées de ces circuits à la sortie de retardateurs d'un registre de décalage de façon à obtenir un montage de commutation à une entrée et une sortie binaires; ce sont ces réseaux binaires qui constituent nos filtres digitaux.

Nous examinons les représentations que donnent ces filtres de l'ensemble des séquences infinies dans les deux sens en elles-mêmes. La relation filtre-représentation n'est pas bi-univoque, ce qui conduit à une classification de ces filtres, c'est à dire que des filtres qui fournissent des représentations identiques appartiennent à la même classe. On trouve pour ces classes des formes réduites qui ne sont cependant pas univoques.

Wir betrachten pyramidenförmige Zusammenschaltungen von binären Schwellenelementen zu komplexeren Schaltkreisen mit mehreren Eingängen und einem Ausgang. Die Eingänge werden mit den Ausgängen der Verzögerungsglieder einer Schiebekette verbunden, so dass wir ein Schaltwerk mit einem binären Eingang und einem binären Ausgang erhalten; diese elektrischen Netzwerke sind unsere Digitalfilter.

Wir untersuchen die durch diese Filter definierten Abbildungen der Menge der nach beiden Seiten unendlichen binären Folgen in sich. Die Zuordnung Filter — Abbildung ist nicht eineindeutig, was zu einer Klassifikation der Filter führt; das heisst, Filter, die die gleiche Abbildung liefern, liegen in der gleichen Klasse. Für diese Klassen finden wir Normalformen, die allerdings nicht eindeutig sind.

Se investigan los mapas en sí mismos del sistema de sucesiode corte binarios para circuitos más complejos de comutación con varias entradas y una salida. Las entradas se hayan conectadas con la salida de los elementos de retardo de un registro de desplazamientos, de tal forma que se obtiene una red secuencial con una entrada y una salida binarias. Estas redes eléctricas son nuestros filtros digitales.

Se investigan los mapas en sí mismos del sistema de sucesiones binarias, infinitas por ambos lados, definidas por estos filtros. La relación filtro-mapa, no es del tipo uni-univoca, que conduciría a una clasificación de los filtros, por ejemplo filtros que resultan tener el mismo mapa, se clasifican bajo la misma clase. Para estas clases se encontraron formas normales, que no obstante no son uni-univocas.

DISCUSSION

A. Wilhelmy *(Germany)*. The definition of the set X of all (admissible) sequences x should be modified to take account of sequences $x, y, \in X$, for which it is impossible to decide whether they are equal or not. From the point of view of symbolic logic, Dr. Hotz's paper contains an interesting extension to parts of Leśniewski's protothetics (cf., A. Church, Introduction to Mathematical Logic, I, Princeton, 1956).

G. Hotz *(Germany)*. In this connection it does not matter whether or not a finite algorithm can decide the equality of two sequences.

COMPUTER PROGRAMMING AND FORMAL SYSTEMS

EDITORS

P. BRAFFORT AND D. HIRSCHBERG

6 × 9″ 170 pp. **Gld. 20.—;** **40s.; $ 5.60**

Computer programmers and logicians are becoming increasingly interested in one another's techniques. This is due to the growing diversity of computer usage, a situation which justifies a more fundamental approach to the symbol manipulation involved and also to the fact that one of the applications happens to be automatic theorem proving.

This book presents a collection of articles dealing with various aspects of the relationship between formal system theory and computer programming and with a number of borderline problems. The papers are based on contributions made at two seminars which were held at the IBM WTEC Education Centre of Blaricum (Holland), in April and October 1961.

CONTENTS — INTRODUCTION; 1. Mechanical Mathematics and Inferential Analysis (HAO WANG); 2. Observations concerning Computing Deduction and Heuristics (E. W. BETH); 3. A Basis for a Mathematical Theory of Computation (JOHN McCARTHY); 4. An Abstract Computer with a Lisp-like Machine Language without a Label Operator (P. C. GILMORE); 5. A Simplified Proof Method for Elementary Logic (STIG KANGER); 6. A Basis for the Mechanization of the Theory of Equations (A. ROBINSON); 7. Programming and the Theory of Automata (ARTHUR W. BURKS); 8. The Algebraic Theory of Context-Free Languages (N. CHOMSKY and M. P. SCHUTZENBERGER).