Task 1 Coordinate Frames and Transformation Matrix

Ingrid Castillo Buitrago*, Juan Jesús Quesada†, Maykol David Echeverry‡
Universidad Pontificia Bolivariana, Medellín, 050031, Colombia

September 2025

1 Introduction

This section presents a Python program designed to transform velocity vectors between different reference frames. The program allows the user to select reference frames and flight scenarios, with the objective of performing these transformations and computing the angle of attack (α), sideslip angle (β), and climb/descent angle (γ). The following subsection explains the program's workflow and key calculations.

2 Backend: Matrices, Equations, and Computational Modules

For the calculation of aerodynamic angles, the program uses Python functions to transform velocity vectors between the vehicle, body, and wing reference frames. The transformations are based on standard rotation matrices defined by the Euler angles (roll ϕ, pitch θ, yaw ψ), as well as additional rotations defined by the angle of attack α and sideslip angle β to account for the aerodynamic orientation of the aircraft.

2.1 Rotation Matrices

To transform vectors between reference frames, rotation matrices are defined based on the aircraft's Euler angles: roll (ϕ), pitch (θ), and yaw (ψ). The rotation matrices about each axis are defined as:

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}, \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, \quad R_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The complete rotation matrix to transform is given by the following:

$$Rot_{matriz} = R_z(\psi)R_y(\theta)R_x(\phi) \tag{2}$$

The complete rotation matrix used to transform vectors from the body frame to the vehicle frame, and vice versa, is given by:

$$V_B = \left(R_z(\psi)R_y(\theta)R_x(\phi)\right) V_V \qquad V_V = \left(R_z(\psi)R_y(\theta)R_x(\phi)\right)^T V_B \tag{3}$$

For the wing-to-body transformation, the rotation matrices corresponding to the angle of attack (α) and sideslip (β) are:

$$R_\alpha = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}, \quad R_\beta = \begin{bmatrix} \cos(-\beta) & \sin(-\beta) & 0 \\ -\sin(-\beta) & \cos(-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The combined wing-to-body rotation matrix is then:

$$R_{WB} = R_\alpha R_\beta \tag{5}$$

*Student, Aeronautical Engineering, 500403, ingrid.castillo@upb.edu.co
†Student, Aeronautical Engineering, 489185, juan.quesadam@upb.edu.co
‡Student, Aeronautical Engineering, 487042, maykol.echeverry@upb.edu.co

2.2 Program Workflow

After defining the rotation matrices and computing the aerodynamic angles, the program guides the user through an interactive menu to select the desired reference frame and flight scenario. The main features of this workflow are:

- **Reference Frame Selection:** The user can choose between the Body Fixed Frame, Vehicle Carried Frame, or Wing/Air Trajectory Frame. This selection determines how the input velocity vector will be interpreted and transformed.
- **Flight Case Selection:** Once the reference frame is chosen, the program presents three flight cases: straight-and-level flight (Case A), crosswind flight (Case B), and climb or descent with wind (Case C). Each case affects the calculation of the aerodynamic angles, including angle of attack (α), sideslip angle (β), and climb/descent angle (γ).
- **Input of Velocity and Angular Rates:** The user is prompted to enter the components of the velocity vector and the aircraft's angular rates (roll, pitch, yaw). Depending on the selected flight case, some components may be set to zero to reflect the physical scenario.
- **Vector Transformation:** Using the previously defined rotation matrices, the program transforms the input velocity vector from the selected reference frame to the target frame. This allows the calculation of aerodynamic angles in the correct frame.
- **Output:** Finally, the program displays the transformed velocity vector and the computed aerodynamic angles.

This structured workflow ensures that users can easily explore different flight scenarios and understand the effect of reference frames on the aircraft's aerodynamic behavior.

3 Computation of Aerodynamic Angles

The aerodynamic angles are defined as follows:

- **Angle of Attack** (α): the angle between the body x-axis and the relative wind in the x-z plane of the body frame,
$$\alpha = \arctan 2(w, u) \tag{6}$$
- **Sideslip Angle** (β): the angle between the aircraft velocity projection on the x-y plane and the x-axis of the body frame,
$$\beta = \arcsin(v/V) \tag{7}$$
- **Climb/Descent Angle** (γ): the difference between the aircraft pitch angle and the angle of attack,
$$\gamma = \theta - \alpha \tag{8}$$

These definitions ensure α and β reflect the relative orientation of the aircraft to the airflow, while γ indicates the inclination of the flight path.

4 Key Observations from Test Cases

The following observations summarize the expected aircraft behavior in each flight case:

- **Case A – Straight and Level Flight:** The absence of wind leads to a symmetric condition where $\beta = 0°$ and $\gamma = 0°$. The aircraft maintains a steady trajectory aligned with its attitude, and the angle of attack α coincides with the pitch angle.
- **Case B – Crosswind Flight:** Although the code assumes $\gamma = 0°$, the presence of lateral wind introduces sideslip effects. In reality, the aircraft would require a bank or yaw correction to maintain course. The simplification in the model keeps $\alpha = \theta$, which limits fidelity but illustrates the influence of wind on the trajectory.
- **Case C – Climb or Descent with Wind:** Both vertical velocity and wind components are considered, producing more representative values of α and γ. The angle of attack depends directly on the velocity components and relative airflow, allowing the model to capture realistic variations during climb or descent.