

1. **with replacement**

$\Omega = \{RR, RG, RB, GG, GR, GB, BB, BG, BR\}$
probability of any individual outcome = $1/9$

without replacement

$\Omega = \{RG, RB, GR, GB, BR, BG\}$
probability of any individual outcome = $1/6$

2. E = the event that the sum of the dice is odd

F = at least one of the dice lands on 1

G = the sum of the dice is 5

How many elementary outcomes are there in the events

E intersect F ? 6

E union F ? 23

F intersect G ? 2

$E \setminus F$ (E without F)? 12

E intersect F intersect G ? 2

ok so the entire S is

$S = \{$
 $(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
 $(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
 $(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
 $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
 $(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)$
 $\}$

The events that the sum of the dice is odd are

$E_events = \{$
 ~~$(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$~~
 ~~$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$~~
 ~~$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$~~
 ~~$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$~~
 ~~$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$~~
 ~~$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)$~~
 $\}$
 $= \{$
 $(1,2), (1,4), (1,6),$
 $(2,1), (2,3), (2,5),$
 $(3,2), (3,4), (3,6),$
 $(4,1), (4,3), (4,5),$
 $(5,2), (5,4), (5,6),$
 $(6,1), (6,3), (6,5)$
 $\}$

F events = {
 (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
 (2,1),(3,1),(4,1),(5,1),(6,1)
 }

G events = {
 (1,4),(2,3),(3,2),(4,1)
 }

Then E intersect F has events
 {
 (1,2),(1,4),(1,6)
 (2,1),(4,1),(6,1)
 }

Then E union F has events
 {
 (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
 (2,1),(2,3),(2,5),
 (3,1),(3,2),(3,4),(3,6),
 (4,1),(4,3),(4,5),
 (5,1),(5,2),(5,4),(5,6),
 (6,1),(6,3),(6,5)
 }

F intersect G has events
 {
 (1,4),(4,1)
 }

E without F has events
 {
 (2,3),(2,5),
 (3,2),(3,4),(3,6),
 (4,3),(4,5),
 (5,2),(5,4),(5,6),
 (6,3),(6,5)
 }

E intersect F intersect G has events
 {
 (1,4),(4,1)
 }

3. n socks. 3 are red.

if $P(X=2) = .5$.
 That is to say that if we chose 2 socks from the n group of socks, the probability both socks chosen are red is .5.
 What's the value of n ?

$n = 3 + y$
 y = the number of non red socks

We can get the number of combinations with n choose $r = n! / (n-r)!r!$
 $= n! / (n-2)!2! = n(n-1)/2$

we know that half of those permutations must yield 2 red socks since $P(X=2) = .5$
 How is $P(X=2)$ determined?

$P(X=2) = \# \text{ of ways you can get two red socks} / \# \text{ of ways you can get two socks}$
 we know the denominator. The number of ways we can get two socks is $n(n-1) / 2$

so setting up an initial equation we have $.5 = \# \text{ of ways we can get two red socks} / n(n-1)/2$

we know there is 3 red socks. Lets call them R_1, R_2 and R_3 .

Then the distinct combinations we can make of those 3 socks are $(R_1, R_2), (R_1, R_3)$ and (R_2, R_3)
 Alternatively we can say 3 choose 2 and we get $3(2) / 2 = 3$

so now we have $P(X=2) = .5$ and $P(X=2) = 3 / [n(n-1)/2] = 6/(n(n-1))$

Solving $.5 = 6/[n(n-1)]$ gives us **$n=4$** .

4.

We have from (c) $A_1, A_2, \dots, A_n \in \mathcal{F}$ then $\bigcup_{i=1}^n A_i \in \mathcal{F}$

We have from (b) that if $A_i \in \mathcal{F}$ then $\Omega \setminus A_i \in \mathcal{F}$

Using both (c) and (b) we have that $(\bigcup_{i=1}^n A_i)^c \in \mathcal{F}$

By De Morgan's Law we have $(\bigcup_{i=1}^n A_i)^c = \bigcap_{i=1}^n A_i^c \in \mathcal{F}$

Once again using (b) we have that if $\bigcap_{i=1}^n A_i^c \in \mathcal{F}$

then $\bigcap_{i=1}^n A_i \in \mathcal{F}$