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Data 601 - Probability and Statistics
HW2

1)

a. P(random student chosen, not in any class)
 = 1 - P(student is in at least one class)

 P(student is in at least one class) = $\frac{\#students\ in\ at\ least\ one\ class}{100\ students\ total}$

\# students in at least one class = S + F + G - S⋂F - S⋂G - F⋂G + S⋂F⋂G =
 28 + 26 + 16 - 12 - 4 - 6 + 2 = 50

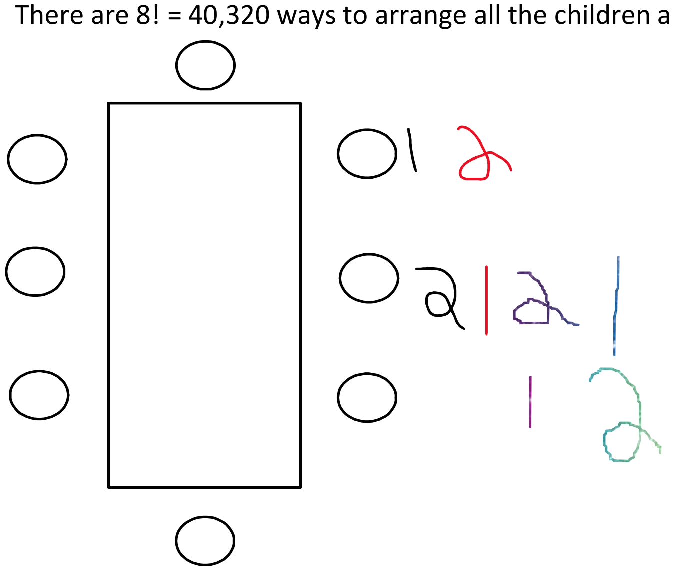
 P(random student chosen, not in any class) = .5

b. P(random student chosen, taking exactly one class) = $\frac{\#\ students\ in\ exactly\ one\ class}{100}$
 \# students in exactly one class = 50 - S⋂F - S⋂G - F⋂G + 2S⋂F⋂G =
 50 - 12 - 4 - 6 + 2(2) = 50-18=32

 P(random student chosen, taking exactly one language class) = $\mathbf{\frac{32}{100} = .32}$

c. P(two students chosen randomly, at least 1 is taking a language class) = 1- P(none of the 2 students is taking any language class)
 $=1 - \frac{C_2^{50}}{C_2^{100}} = 1 - \frac{1225}{4950} = \frac{3725}{4950} = \mathbf{\frac{149}{198}}$

2.
 There are 8! = 40,320 ways to arrange all the children and adults.



There are 4 configurations on the right side of the table that have the children next to each other. Due to symmetry, there is also 4 configurations on the left side of the table that have the children next to each other.

child 1
child 2
adult

child 2
child 1
adult

adult
child 2
child 1

adult
child 1
child 2

For each configuration where the children are next to each other we have 6! number of ways the adults can be arranged
So the total number of ways that the children can be arranged together is 2*4*6! = 5,760

P(the two children are seated next to each other) =
$$\frac{\#of\ ways\ that\ the\ children\ end\ up\ next\ to\ each\ other}{\#\ of\ ways\ to\ arrange\ everyone} = \frac{5760}{40320} = \frac{2*4*6!}{8*7*6*5*4*3*2} = \mathbf{\frac{1}{7}}$$

3.

The probability (X = pulling exactly one black ball) = $\frac{C_1^m C_2^{8-m}}{C_3^8}$ where m is the number of black balls out of 8
we want to maximize the above probability.
To maximize it, we notice that only the $m * C_2^{8-m}$ can change since really we have $\frac{m*C_2^{8-m}}{56}$

$$m * C_2^{8-m}$$

Let's first simplify it.
$$\frac{C_1^m C_2^{8-m}}{C_3^8} = m\ * \frac{(8-m)!}{2!(8-m-2)!} * \frac{1}{56} = \frac{m}{112} * (8-m) * (8-m-1) = \frac{m}{112} * (m^2 - 15m + 56)$$

to maximize we can take the derivative, set it to 0, and solve for m. This gives us the places where the derivative is zero and so the slope of the function at these points is 0 like it would be where the function is maximized. We then use the solutions in the formula to see which gives us a high probability.

$\frac{d}{dm}[\frac{m}{112} * (m^2 - 15m + 56)] = \frac{1}{112} * (m^2 - 15m + 56) + \frac{m}{112}(2m - 15)$ (using the product rule)

$$= \frac{1}{112}(m^2 - 15m + 56 + 2m^2 - 15m) = \frac{1}{112}(3m^2 - 30m + 56)$$

setting it equal to 0 and solving

$$\frac{1}{112}(3m^2 - 30m + 56) = 0$$
$$(3m^2 - 30m + 56) = 0$$

this doesn't factor nicely so we'll need to use the quadratic formula

$$m = \frac{30 \pm \sqrt{(-30)^2 - 4(3)(56)}}{2(3)}$$
$$m = \frac{30 \pm \sqrt{228}}{6}$$

$$m \approx 7.5 \; and \; m \approx 2.5$$

We need to use integers so we would want to test 2,3,7 and 8. However, right off the bat we can tell that 7 or 8 is not going to be a reasonable solution for the number of black balls since then there is not 2 white balls to grab to maximize the scenario we want - 2 white balls and 1 black ball.

Testing 2 and 3 we get

$m * C_2^{8-m}$= 30 when m is 2
$m * C_2^{8-m}$= 30 when m is 3

To be certain we can test the other scenarios
$m * C_2^{8-m}$= 20 when m is 1
$m * C_2^{8-m}$= 24 when m is 4
$m * C_2^{8-m}$= 15 when m is 5
$m * C_2^{8-m}$= 6 when m is 5

Therefore, to maximize the scenario that we pull exactly 1 black ball and 2 white balls out of the 8, we should put 2 black balls and 6 white balls or 3 black balls and 5 white balls in the box.