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Data 601 – Probability and Statistics
HW4

1.
let A = {1,2,4}
let B = {2,3}

Then P(A) = $\frac{3}{6} = \frac{1}{2}$
P(B) = $\frac{2}{6} = \frac{1}{3}$

$\mathrm{P(A)} * \mathrm{P(B)} = \frac{1}{2} * \frac{1}{3} = \frac{1}{6}$

$\mathrm{P(A \cap B)} = \frac{1}{6}$

Since $\mathrm{P(A \cap B)} = \mathrm{P(A)} * \mathrm{P(B)}$, A and B are independent

2.
part a
let $P_1 =$ the ball pulled from box 1 such that $P_1 = 1{,}2{,}3{,}4$
let $P_2 =$ the set of balls pulled from box 2
let $\mathrm{R} = P_2$ pulled only red balls

$$\begin{aligned}\mathrm{P(R)} = P(P_2 = \{r\}\,|P_1 = 1) * P(P_1 = 1) + &\ P(P_2 = \{r,r\}\,|P_1 = 2) * P(P_1 = 2) \\ &+ P(P_2 = \{r,r,r\}\,|P_1 = 3) * P(P_1 = 3) \\ &+ P(P_4 = \{r,r,r,r\}\,|P_1 = 4) * P(P_1 = 4)\end{aligned}$$

We know that $P(P_1 = 1) = P(P_1 = 2) = P(P_1 = 3) = P(P_1 = 4) = \frac{1}{4}$

We can solve for the rest

$$P(P_2 = \{r\}\,|P_1 = 1) = \frac{5}{13}$$
$$P(P_2 = \{r,r\}\,|P_1 = 2) = \frac{5}{13} * \frac{4}{12}$$
$$P(P_2 = \{r,r,r\}\,|P_1 = 3) = \frac{5}{13} * \frac{4}{12} * \frac{3}{11}$$
$$P(P_4 = \{r,r,r,r\}\,|P_1 = 4) = \frac{5}{13} * \frac{4}{12} * \frac{3}{11} * \frac{2}{10}$$

Then $\mathrm{P(R)} = \frac{1}{4} * \frac{5}{13} \ * [1 + \frac{4}{12} + \left(\frac{4}{12} * \frac{3}{11}\right) + \left(\frac{4}{12} * \frac{3}{11} * \frac{2}{10}\right) = \frac{119}{858} \approx 0.14$

part b
let $P_1 =$ the ball pulled from box 1 such that $P_1 = 1{,}2{,}3{,}4$
let $\mathrm{R} =$ selecting only red balls

We want P$(P_1 = 2|R)$

P$(P_1 = 2|R) = \frac{P(P_1=2 \cap R)}{P(R)} = \frac{P(R|P_1 = 2) * P(P_1=2)}{P(R)}$

$$P(R|P_1 = 2) = \frac{5}{13} * \frac{4}{12}$$

P$(P_1 = 2) = \frac{1}{4}$

P(R) $= \frac{119}{858}$ from part a

plugging them in we get

P$(P_1 = 2|R) = \frac{\frac{5}{13}*\frac{4}{12}*\frac{1}{4}}{\frac{119}{858}} = \frac{55}{238} \approx 0.23$

3.

S= { (a,b), a,b ∈ {1,2,3,4,5,6}} modeling rolling a dice two times and all outcomes are equally likely so P({a,b}) $= \frac{1}{36}$

have

X((a,b)) = a - 2
Y ((a,b))= |b - a|

part a

X takes 6 distinct values. {-1,0,1,2,3,4}

Y is a bit more interesting

$\downarrow \boldsymbol{b} \rightarrow \boldsymbol{a}$	1	2	3	4	5	6
1	*0*	*1*	*2*	*3*	*4*	*5*
2	*1*	*0*	*1*	*2*	*3*	*4*
3	*2*	*1*	*0*	*1*	*2*	*3*
4	*3*	*2*	*1*	*0*	*1*	*2*
5	*4*	*3*	*2*	*1*	*0*	*1*
6	*5*	*4*	*3*	*2*	*1*	*0*

So Y takes 6 values {0,1,2,3,4,5}

X+Y

$\downarrow \boldsymbol{Y} \rightarrow \boldsymbol{X}$	-1	0	1	2	3	4
0	*-1*	*0*	*1*	*2*	*3*	*4*
1	*0*	*1*	*2*	*3*	*4*	*5*

2	*1*	*2*	*3*	*4*	*5*	*6*
3	*2*	*3*	*4*	*5*	*6*	*7*
4	*3*	*4*	*5*	*6*	*7*	*8*
5	*4*	*5*	*6*	*7*	*8*	*9*

so X+Y takes 11 values {-1,0,1,2,3,4,5,6,7,8,9}

X * Y

$\downarrow \boldsymbol{Y} \rightarrow \boldsymbol{X}$	-1	0	1	2	3	4
0	*0*	*0*	*0*	*0*	*0*	*0*
1	*-1*	*0*	*1*	*2*	*3*	*4*
2	*-2*	*0*	*2*	*4*	*6*	*8*
3	*-3*	*0*	*3*	*6*	*9*	*12*
4	*-4*	*0*	*4*	*8*	*12*	*16*
5	*-5*	*0*	*5*	*10*	*15*	*20*

so X * Y takes 19 values {-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,10,12,15,16,20}

part b

Since X can take 6 distinct values and Y can take 6 distinct values (X,Y) should have 36 distinct vectors.

part c

P(X+Y ≤ 3) = ?

$\downarrow \boldsymbol{Y} \rightarrow \boldsymbol{X}$	-1	0	1	2	3	4
0	*-1*	*0*	*1*	*2*	*3*	*4*
1	*0*	*1*	*2*	*3*	*4*	*5*
2	*1*	*2*	*3*	*4*	*5*	*6*
3	*2*	*3*	*4*	*5*	*6*	*7*
4	*3*	*4*	*5*	*6*	*7*	*8*
5	*4*	*5*	*6*	*7*	*8*	*9*

That is 15 values out of 36 values so

$P(X+Y \leq 3) = \frac{15}{36} = \frac{5}{12}$

4.

part a

we have p = 0.6, q = 0.4, A = 4

plugging into the formula for f(z) we have

$f(2) = \frac{1-\left(\frac{2}{3}\right)^2}{1-\left(\frac{2}{3}\right)^4} = \frac{9}{13}$

this is the case of when they are tied at 0-0 so P[A winning] $= \frac{9}{13}$

part b

let X = games played when A wins

P[X=2] $= (0.6)^2 = \ 0.36\ (AA)$

P[X=4] $= 2 * (0.6)^3(0.4) = \ 0.0864\ (ABAA)(BAAA)$

P[X=6] $= 4 * (0.6)^4(0.4)^2 = 0.020736\ (ABABAA)(ABBAAA)(BABAAA)(BAABAA)$

P[X=8] = 8 * (0.6)(0.4) * P[X=6]

...

P[X=2n]=$2^{n-1}(0.6)^{n+1}(0.4)^{n-1}$

so we have P[A winning] $= \sum_{i=1}^{\infty} 2^{i-1}(0.6)^{i+1}(0.4)^{i-1} = \frac{3}{4}\sum_{i=1}^{\infty} 2^{i}(0.6)^{i}(0.4)^{i}$

$= \ \frac{3}{4}\sum_{i=1}^{n}(0.48)^{i}$

this is an infinite geometric series

a = .48

common ratio = .48

$S = \frac{a}{1-r} = \frac{.48}{1-.48} = \frac{48}{52} = \frac{12}{13}$

so we have P[A winning] $= \frac{3}{4} * \frac{12}{13} = \frac{9}{13}$