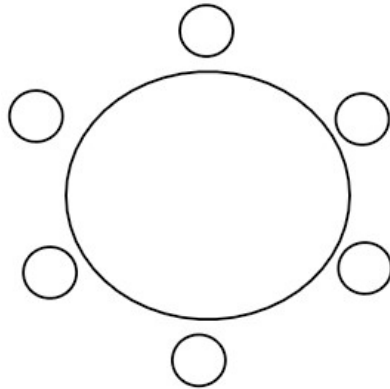


Problem 1



$$P(\text{all couples seated next to each other}) = \frac{\# \text{ of ways all couples can be seated together}}{\# \text{ of ways to arrange all 6 people}}$$

of ways to arrange all 6 people = 6!

of ways all couples can be seated together = ?

Pick any couple and pick one person from that couple

The person has 6 options to sit at

Their partner has 2 options to sit at (left or right of their partner)

There are 12 total options

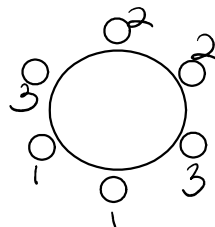
Pick a second couple from the two remaining couples and pick one person from that couple

The person has 4 options to sit at

Their partner has only 1 option because certain choices mean we can't sit our third couple together

such as the below drawing

There are 4 options



Pick the third couple and pick a person from that couple

The person has 2 options

Their partner has 1 option

There are 2 options

So we have $12 * 4 * 2 = 96$ different ways to sit all the couples together

$$P(\text{all couples seated next to each other}) = \frac{\# \text{ of ways all couples can be seated together}}{\# \text{ of ways to arrange all 6 people}} = \frac{96}{6!} = \frac{6 * 2 * 4 * 2}{6 * 5 * 4 * 3 * 2} = \frac{2}{15}$$

Problem 2

part a

Let A = at least one die shows a 5

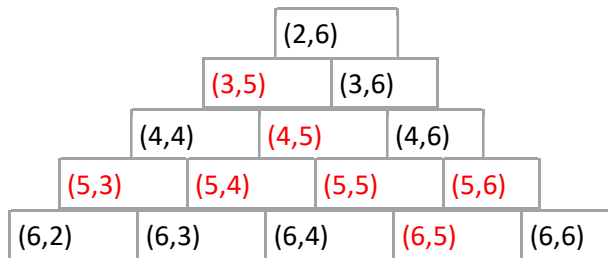
let B = sum of the two die is larger than 7

What is $P(A|B)$?

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(B) = ?$

There are the following two die rolls that are greater than 7



So there are 15 ways out of 36 ways

$$\text{so } P(B) = \frac{15}{36}$$

$$P(A \cap B) = \text{the red ones above divided by 36} = \frac{7}{36}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{7}{15}$$

part b

Let A = at least one die shows a 5

let B = sum of the two die is larger than 7

What is $P(B|A)$?

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = 1 - P(\text{no die shows a 5}) = 1 - \frac{\binom{5}{1}\binom{5}{1}}{\binom{6}{1}\binom{6}{1}} = 1 - \frac{25}{36} = \frac{11}{36}$$

$$P(B \cap A) = P(A \cap B) = \frac{7}{36} \text{ from the last problem}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{7}{11}$$

Problem 3

Part a)

Let A = first ace is the 20th card

let B = next card is the ace of spades

What is $P(B|A)$?

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$P(B)$ is the same as asking what is the probability that the first card is the ace of spades out of a deck of 32 with only 3 aces (1 of which must be the ace of spades for this to work)

So $P(B) = \frac{1}{32}$ since we have 1 card out of 32 we want

$$P(B \cap A) = \binom{48}{19} \binom{3}{1} \frac{1}{32}$$

the first term is pulling the 19 first cards out of the 48 that are non aces

the second term is pulling 1 ace from the 3 aces that are not the ace of spades

the last term is pulling the ace of spades from the remaining 32 cards

$P(A) = \binom{48}{19} \binom{4}{1}$ using the same logic as above but noting that the 20th card Ace could have been the ace of spades for just event A

$$\text{then } P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\binom{48}{19} \binom{3}{1} \frac{1}{32}}{\binom{48}{19} \binom{4}{1}} = \frac{3}{128}$$

Part b)

Let A = first ace is the 20th card

let B = next card is the 2 of clubs

What is $P(B|A)$?

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = \binom{48}{19} \binom{4}{1}$$

$$P(B \cap A) = \binom{47}{19} \binom{4}{1} \frac{1}{32}$$

above uses similar logic to part a

$$\text{then } P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\binom{47}{19} \binom{4}{1} \frac{1}{32}}{\binom{48}{19} \binom{4}{1}} = \frac{\frac{47!}{19! \cdot 28!} \cdot \frac{1}{32}}{\frac{48!}{19! \cdot 29!}} = \frac{29}{48 \cdot 32} = \frac{29}{1536}$$

Problem 4

Coin 1 is a fair coin

Coin 2 is tails up 60%

Coin 3 is tails up 100%

Let A = a random coin from our 3 coins landed tails up

Let B = the random coin from our 3 coins is coin 1

Want P(B|A)

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = \frac{1}{3}(.5) + \frac{1}{3}(.6) + \frac{1}{3}(1) = \frac{21}{30}$$

$$P(A \cap B) = \frac{1}{3}(.5) = \frac{5}{30}$$

$$\text{then } P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{5}{21}$$