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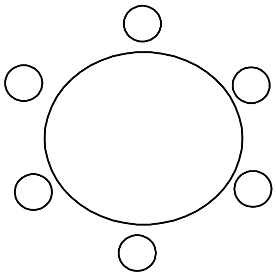
Data 601 - Probability and Statistics
HW3

Problem 1

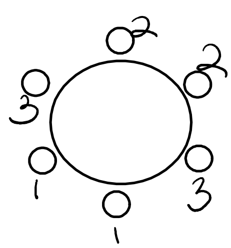


P(all couples seated next to each other) = $\frac{\#\ of\ ways\ all\ couples\ can\ be\ seated\ together}{\#\ of\ ways\ to\ arrange\ all\ 6\ people}$

# of ways to arrange all 6 people = 6!

# of ways all couples can be seated together = ?
    Pick any couple and pick one person from that couple
        The person has 6 options to sit at
        Their partner has 2 options to sit at (left or right of their partner)
        There are 12 total options

    Pick a second couple from the two remaining couples and pick one person from that couple
        The person has 4 options to sit at
        Their partner has only 1 option because certain choices mean we can't sit our third couple together
            such as the below drawing
        There are 4 options



Pick the third couple and pick a person from that couple

The person has 2 options

Their partner has 1 option

There are 2 options

So we have 12 * 4 * 2 = 96 different ways to sit all the couples together

P(all couples seated next to each other) = $\frac{\boldsymbol{\#\ of\ ways\ all\ couples\ can\ be\ seated\ together}}{\boldsymbol{\#\ of\ ways\ to\ arrange\ all\ 6\ people}} = \frac{\mathbf{96}}{\mathbf{6!}} = \frac{\mathbf{2*4*6}}{\mathbf{6*5*4*3*2}} = \frac{\mathbf{1}}{\mathbf{15}}$

Problem 2

part a

Let A = at least one die shows a 5

let B = sum of the two die is larger than 7

What is P(A|B)?

P(A|B) = $\frac{P(A \cap B)}{P(B)}$

P(B) = ?

There are the following two die rolls that are greater than 7

(2,6)				
(3,5)	(3,6)			
(4,4)	(4,5)	(4,6)		
(5,3)	(5,4)	(5,5)	(5,6)	
(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

So there are 15 ways out of 36 ways

so P(B) $= \frac{15}{36}$

$P(A \cap B)$= the red ones above divided by 36 $= \frac{7}{36}$

P(A|B) = $\frac{\boldsymbol{P(A \cap B)}}{\boldsymbol{P(B)}} = \frac{\mathbf{7}}{\mathbf{15}}$

part b

Let A = at least one die shows a 5

let B = sum of the two die is larger than 7

What is P(B|A)?

P(B|A) = $\frac{P(B \cap A)}{P(A)}$

P(A) = 1 - P(no die shows a 5) = 1 - $\frac{\binom{5}{1}\binom{5}{1}}{\binom{6}{1}\binom{6}{1}} = 1 - \frac{25}{36} = \frac{11}{36}$

P$(B \cap A) = P(A \cap B)$ = $\frac{7}{36}$ from the last problem

P(B|A) = $\boldsymbol{\frac{P(B \cap A)}{P(A)} = \frac{7}{11}}$

Problem 3

Part a)

Let A = first ace is the 20th card

let B = next card is the ace of spades

What is P(B|A)?

P(B|A) = $\frac{P(B \cap A)}{P(A)}$

P(B) is the same as asking what is the probability that the first card is the ace of spades out of a deck of 32 with only 3 aces(1 of which must be the ace of spades for this to work)

So P(B) $= \frac{1}{32}$ since we have 1 card out of 32 we want

$P(B \cap A) = \binom{48}{19}\binom{3}{1}$ *1/32*

the first term is pulling the 19 first cards out of the 48 that are non aces

the second term is pulling 1 ace from the 3 aces that are not the ace of spades

the last term is pulling the ace of spades from the remaining 32 cards

P(A) = $\binom{48}{19}\binom{4}{1}$ using the same logic as above but noting that the 20th card Ace could have been the ace of spades for just event A

then **P(B|A) =** $\boldsymbol{\frac{P(B \cap A)}{P(A)} = \frac{\binom{48}{19}\binom{3}{1}\frac{1}{32}}{\binom{48}{19}\binom{4}{1}} = \frac{3}{128}}$

Part b)

Let A = first ace is the 20th card

let B = next card is the 2 of clubs

What is P(B|A)?

P(B|A) = $\frac{P(B \cap A)}{P(A)}$

P(A) = $\binom{48}{19}\binom{4}{1}$

P($B \cap A$) = $\binom{47}{19}\binom{4}{1}\frac{1}{32}$

above uses similar logic to part a

then P(B|A) = $\boldsymbol{\frac{P(B \cap A)}{P(A)} = \frac{\binom{47}{19}\binom{4}{1}\frac{1}{32}}{\binom{48}{19}\binom{4}{1}} = \frac{\frac{47!}{19!*28!*32}}{\frac{48!}{19!*29!}} = \frac{29}{48*32} = \frac{29}{1536}}$

Problem 4

Coin 1 is a fair coin
Coin 2 is tails up 60%
Coin 3 is tails up 100%

Let A = a random coin from our 3 coins landed tails up
Let B = the random coin from our 3 coins is coin 1

Want P(B|A)

P(B|A) = $\frac{P(B \cap A)}{P(A)}$

$\mathrm{P(A)} = \frac{1}{3}(.5) + \frac{1}{3}(.6) + \frac{1}{3}(1) = \frac{21}{30}$

P($A \cap B$) = $\frac{1}{3}(.5) = \frac{5}{30}$

then P(B|A) = $\boldsymbol{\frac{P(B \cap A)}{P(A)} = \frac{5}{21}}$