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Data 601 - Probability and Statistics
HW1

1. **with replacement**

    Omega = {RR, RG, RB, GG, GR, GB, BB, BG, BR}

    probability of any individual outcome = 1/9

    without replacement

    Omega = {RG,RB,GR,GB,BR,BG}

    probability of any individual outcome = 1/6

2. E = the event that the sum of the dice is odd

    F = at least one of the dice lands on 1

    G = the sum of the dice is 5

    How many elementary outcomes are there in the events

    E interesect F? 6

    E union F? 23

    F instersect G? 2

    E\F (E without F)? 12

    E intersect F intersect G? 2

    ok so the entire S is

    S = {

    (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
    (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
    (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
    (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
    (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
    (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

    }

    The events that the sum of the dice is odd are

    E_events = {

    ~~(1,1),~~(1,2),~~(1,3)~~,(1,4),~~(1,5)~~,(1,6),
    (2,1),~~(2,2)~~,(2,3),~~(2,4)~~,(2,5)~~,(2,6)~~,
    ~~(3,1)~~,(3,2),~~(3,3)~~,(3,4),~~(3,5)~~,(3,6),
    (4,1),~~(4,2)~~,(4,3)~~,(4,4)~~,(4,5),~~(4,6),~~
    ~~(5,1)~~,(5,2),~~(5,3)~~,(5,4),~~(5,5)~~,(5,6),
    (6,1),~~(6,2)~~,(6,3),~~(6,4)~~,(6,5),~~(6,6)~~

    }

    = {

    (1,2),(1,4),(1,6),
    (2,1),(2,3),(2,5),
    (3,2),(3,4),(3,6),
    (4,1),(4,3),(4,5),
    (5,2),(5,4),(5,6),
    (6,1),(6,3),(6,5)

}

F events = {
(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(3,1),(4,1),(5,1),(6,1)
}

G events = {
(1,4),(2,3),(3,2),(4,1)
}

Then E intersect F has events
{
(1,2),(1,4),(1,6)
(2,1),(4,1),(6,1)
}

Then E union F has events
{
(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,3),(2,5),
(3,1),(3,2),(3,4),(3,6),
(4,1),(4,3),(4,5),
(5,1),(5,2),(5,4),(5,6),
(6,1),(6,3),(6,5)

}

F intersect G has events
{
(1,4),(4,1)
}

E without F has events
{

(2,3),(2,5),
(3,2),(3,4),(3,6),
(4,3),(4,5),
(5,2),(5,4),(5,6),
(6,3),(6,5)
}
E intersect F intersect G has events
{
(1,4),(4,1)
}

3. n socks. 3 are red.
   if P(X=2) = .5.
   That is to say that if we chose 2 socks from the n group of socks, the probability both socks chosen are red is .5.

What's the value of n?

n = 3 + y

y = the number of non red socks

We can get the number of combinations with n choose r = n! / (n-r)!r!

= n! / (n-2)!2! = n(n-1)/2

we know that half of those permutations must yield 2 red socks since P(X=2) = .5
How is P(X=2) determined?

P(X=2) = # of ways you can get two red socks / # of ways you can get two socks

we know the denominator. The number of ways we can get two socks is n(n-1) / 2

so setting up an initial equation we have .5 = # of ways we can get two red socks / n(n-1)/2

we know there is 3 red socks. Lets call them R1, R2 and R3.

Then the distinct combinations we can make of those 3 socks are (R1,R2),(R1,R3) and (R2,R3)
Alternatively we can say 3 choose 2 and we get 3(2) / 2 = 3

so now we have P(X=2) = .5 and P(X=2) = 3/ [n(n-1)/2] = 6/(n(n-1))

Solving .5 = 6/[n(n-1)] gives us **n=4.**

4.

We have from (c) $A_1, A_{2,\ldots,} A_n \in$ Ƒ then $\bigcup_{i=1}^{n} A_i \in$ Ƒ

We have from (b) that if $A_i \in$ Ƒ then $\Omega \backslash \mathrm{A}_i \in$ Ƒ

another way to write (b) is if $\mathrm{A_i} \in$ Ƒ then $\mathrm{A_i^C} \in$ Ƒ

Using both (b) and (c) we have that $\bigcup_{i=1}^{n} A_i^C \in$ Ƒ

Using (b) we then have $\left(\bigcup_{i=1}^{n} A_i^C\right)^C \in$ Ƒ

By De Morgan's Law we have $\left(\bigcup_{i=1}^{n} A_i{}^C\right)^C = (\bigcap_{i=1}^{n} (A_i^C)^C \in$ Ƒ

$$(A_i^C)^C = A_i$$

So we have $\bigcap_{\mathrm{i}=1}^{n} A_i \in$ Ƒ

N.B. I initially attempted this by showing that $A_1 \cap A_2 \cap \ldots \cap A_n \in A_1 \in \mathrm{F}$.
I had ran into some road blocks but I think I figured it out. Here's is that proof as well.

We have from (c) $A_1, A_{2,\ldots,} A_n \in \mathrm{F}$ then $\bigcup_{i=1}^{n} A_i \in \mathrm{F}$
we need to prove that
$P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i \le j \le n}^{n} P(A_i \cap A_j) + \sum_{i \le j \le l \le n}^{n} P(A_i \cap A_j \cap A_l) + \cdots \pm P(A_1 \cap A_2 \cap \ldots A_n)$

since that will show that $(A_1 \cap A_2 \cap \ldots A_n) \in \bigcup_{i=1}^{n} A_i \in \mathrm{F}$

Proof:
n=2 case

A∪B = (A\B) ∪ (A∩B) ∪ (B\A) (disjoint union)
P(A∪B) = P(A\B) + P(A∩B) + P (B\A)

add and subtract P(A∩B)
P(A∪B) = P(A\B) + P(A∩B) + P (B\A) + P(A∩B)-P(A∩B)

we also have that P(A\B) + P(A∩B) = P(A) and P(B\A) + P(A∩B) = P(B)
substituting this in we get
P(A∪B) = P(A) + P(B) - P(A∩B)
this can be rewritten to

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = \sum_{i=1}^{n=2} P(A_i) - \sum_{i \le j \le n}^{n=2} P(A_i \cap A_j)$$

n=3 case

$P(A_1 \cup A_2 \cup A_3) = \mathrm{P}(A_1 \backslash (\mathrm{A}_2 \cup A_3)) + P(A_1 \cap (A_2 \cup A_3)) + P((A_2 \cup A_3) \backslash \mathrm{A}_1)$

we have the following:
$\mathrm{P}(A_1 \backslash (\mathrm{A}_2 \cup A_3)) = P(A_1 \backslash \mathrm{A}_2) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$
$P\big(A_1 \cap (A_2 \cup A_3)\big) = P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)$
$P\big((A_2 \cup A_3) \backslash \mathrm{A}_1\big) = P(A_2 \backslash \mathrm{A}_1) + P(A_3 \backslash \mathrm{A}_1) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

$\Rightarrow P(A_1 \cup A_2 \cup A_3) = P(A_1 \backslash \mathrm{A}_2) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3) + P(A_2 \backslash \mathrm{A}_1) + P(A_3 \backslash \mathrm{A}_1) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

we also have:
$P(A_1) = P(A_1 \backslash \mathrm{A}_2) + P(A_1 \cap A_2) = P(A_1 \backslash \mathrm{A}_3) + P(A_1 \cap A_3)$
$P(A_2) = P(A_2 \backslash \mathrm{A}_1) + P(A_2 \cap A_1) = P(A_2 \backslash \mathrm{A}_3) + P(A_2 \cap A_3)$
$P(A_3) = P(A_3 \backslash \mathrm{A}_2) + P(A_3 \cap A_2) = P(A_3 \backslash \mathrm{A}_1) + P(A_3 \cap A_1)$

$\Rightarrow P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_3) + P(A_2 \backslash \mathrm{A}_1) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

add and subtract $P(A_2 \cap A_1)$ to the right hand side
$\Rightarrow P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_2 \cap A_1) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) = \sum_{i=1}^{n=3} P(A_i) - \sum_{i \le j \le n}^{n=3} P\left(A_i \cap A_j\right) + \sum_{i \le j \le l \le n}^{n=3} P(A_i \cap A_j \cap A_l)$

n=n case
let

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i \le j \le n}^{n} P\left(A_i \cap A_j\right) + \sum_{i \le j \le l \le n}^{n} P\left(A_i \cap A_j \cap A_l\right) + \cdots \pm P(A_1 \cap A_2 \cap \dots A_n)$$

be true

then prove that

$$P(A_1 \cup A_2 \cup \dots \cup A_{n+1}) = \sum_{i=1}^{n+1} P(A_i) - \sum_{i \le j \le n+1}^{n+1} P\left(A_i \cap A_j\right) + \sum_{i \le j \le l \le n}^{n+1} P\left(A_i \cap A_j \cap A_l\right) + \cdots \pm P(A_1 \cap A_2 \cap \dots A_{n+1})$$

$$P(A_1 \cup A_2 \cup \dots \cup A_{n+1}) = P\{(A_1 \cup A_2 \cup \dots \cup A_n) \cup A_{n+1}\}$$

we can use the n=2 case on this second part

$P\{(A_1 \cup A_2 \cup \dots \cup A_n) \cup A_{n+1}\} =$
$P(A_1 \cup A_2 \cup \dots \cup A_n) + P(A_{n+1}) - P\{(A_1 \cup A_2 \cup \dots \cup A_n) \cap$ *A_(n+1)*}

$P\{(A_1 \cup A_2 \cup \dots \cup A_n) \cap A_{n+1}\} = P\{(A_1 \cap A_{n+1}) \cup (A_2 \cap A_{n+1}) \cup \dots$
U(A_nnA_(n+1))}

so we have

$$P(A_1 \cup A_2 \cup \dots \cup A_{n+1}) = P(A_1 \cup A_2 \cup \dots \cup A_n) + P(A_{n+1}) - P\{(A_1 \cap A_{n+1}) \cup (A_2 \cap A_{n+1}) \cup \dots \cup (A_n \cap A_{n+1})\}$$

The first and third term we can use the formula on so we get.

$= \sum_{i=1}^{n} P(A_i) - \sum_{i \le j \le n}^{n} P\left(A_i \cap A_j\right) + \sum_{i \le j \le l \le n}^{n} P\left(A_i \cap A_j \cap A_l\right) + \cdots \pm P(A_1 \cap A_2 \cap \dots A_n)$
$+ P(A_{n+1})$
-
$\sum_{i=1}^{n} P(A_i \cap A_{n+1}) + \sum_{i \le j \le n}^{n} P\left(A_i \cap A_{n+1} \cap A_j\right) - \sum_{i \le j \le l \le n}^{n} P\left(A_i \cap A_{n+1} \cap A_j \cap A_l\right) + \cdots \pm P(A_1 \cap A_2 \cap \dots A_n \cap A_{n+1})$

Notice how $\sum_{i=1}^{n} P(A_i) + P(A_{n+1}) = \sum_{i=1}^{n+1} P(A_i)$

and how $- \sum_{i \le j \le n}^{n} P\left(A_i \cap A_j\right) - \sum_{i=1}^{n} P(A_i \cap A_{n+1}) = -\sum_{i \le j \le n}^{n+1} P(A_i \cap A_j)$

and the rest of the terms combine as well
which gives us

$$P(A_1 \cup A_2 \cup \dots \cup A_{n+1}) = \sum_{i=1}^{n+1} P(A_i) - \sum_{i \le j \le n+1}^{n+1} P\left(A_i \cap A_j\right) + \sum_{i \le j \le l \le n}^{n+1} P\left(A_i \cap A_j \cap A_l\right) + \cdots \pm P(A_1 \cap A_2 \cap \dots A_{n+1})$$