Bradley Scott
HW1

1. **with replacement**

    Omega = {RR, RG, RB, GG, GR, GB, BB, BG, BR}
    probability of any individual outcome = 1/9

   without replacement

    Omega = {RG,RB,GR,GB,BR,BG}
    probability of any individual outcome = 1/6

2. E = the event that the sum of the dice is odd
   F = at least one of the dice lands on 1
   G = the sum of the dice is 5
   How many elementary outcomes are there in the events
   E interesect F? 6
   E union F? 23
   F instersect G? 2
   E\F (E without F)? 12
   E intersect F intersect G? 2

   ok so the entire S is

   S = {
   (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
   (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
   (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
   (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
   (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
   (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)
   }

   The events that the sum of the dice is odd are

   E_events = {
   ~~(1,1)~~,(1,2),~~(1,3)~~,(1,4),~~(1,5)~~,(1,6),
   (2,1),~~(2,2)~~,(2,3),~~(2,4)~~,(2,5),~~(2,6)~~,
   ~~(3,1)~~,(3,2),~~(3,3)~~,(3,4),~~(3,5)~~,(3,6),
   (4,1),~~(4,2)~~,(4,3),~~(4,4)~~,(4,5),~~(4,6),~~
   ~~(5,1)~~,(5,2),~~(5,3)~~,(5,4),~~(5,5)~~,(5,6),
   (6,1),~~(6,2)~~,(6,3),~~(6,4)~~,(6,5),~~(6,6)~~
   }
   = {
   (1,2),(1,4),(1,6),
   (2,1),(2,3),(2,5),
   (3,2),(3,4),(3,6),
   (4,1),(4,3),(4,5),
   (5,2),(5,4),(5,6),
   (6,1),(6,3),(6,5)
   }

F events = {
(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(3,1),(4,1),(5,1),(6,1)
}

G events = {
(1,4),(2,3),(3,2),(4,1)
}

Then E intersect F has events
{
(1,2),(1,4),(1,6)
(2,1),(4,1),(6,1)
}

Then E union F has events
{
(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,3),(2,5),
(3,1),(3,2),(3,4),(3,6),
(4,1),(4,3),(4,5),
(5,1),(5,2),(5,4),(5,6),
(6,1),(6,3),(6,5)

}

F intersect G has events
{
(1,4),(4,1)
}

E without F has events
{

(2,3),(2,5),
(3,2),(3,4),(3,6),
(4,3),(4,5),
(5,2),(5,4),(5,6),
(6,3),(6,5)
}
E intersect F intersect G has events
{
(1,4),(4,1)
}

3. n socks. 3 are red.

if P(X=2) = .5.
That is to say that if we chose 2 socks from the n group of socks, the probability both socks chosen are red is .5.

What's the value of n?

n = 3 + y

y = the number of non red socks

We can get the number of combinations with n choose r = n! / (n-r)!r!

= n! / (n-2)!2! = n(n-1)/2

we know that half of those permutations must yield 2 red socks since P(X=2) = .5

How is P(X=2) determined?

P(X=2) = # of ways you can get two red socks / # of ways you can get two socks

we know the denominator. The number of ways we can get two socks is n(n-1) / 2

so setting up an initial equation we have .5 = # of ways we can get two red socks / n(n-1)/2

we know there is 3 red socks. Lets call them R1, R2 and R3.

Then the distinct combinations we can make of those 3 socks are (R1,R2),(R1,R3) and (R2,R3)

Alternatively we can say 3 choose 2 and we get 3(2) / 2 = 3

so now we have P(X=2) = .5 and P(X=2) = 3/ [n(n-1)/2] = 6/(n(n-1))

Solving .5 = 6/[n(n-1)] gives us **n=4.**

4.

From (3) we have that if $A_1, A_{2,\ldots,} A_n \in \mathfrak{F}$ then $\bigcup_{i=1}^{n} A_i \in \mathfrak{F}$

n = 2 case

From (3) we know that $A_1 \cup A_2 \in \mathfrak{F}$ since $A_1 \cup A_2 \in \quad \cup_{i=1}^{n} A_i \in \mathfrak{F}$

We know that $A_1 \cup A_2$ is the same as $(A_1 \backslash A_2) \cup (A_1 \cap A_2) \cup (A_2 \backslash A_1)$

that is to say A_1union A_2 is composed of A_1 without A_2, the intersection of A_1 and A_2, and A_2 without A_1

so we have $(A_1 \backslash A_2) \cup (A_1 \cap A_2) \cup (A_2 \backslash A_1) = A_1 \cup A_2 \in \mathfrak{F}$

naturally it follows that

$$(A_1 \backslash A_2) \cup (A_1 \cap A_2) \cup (A_2 \backslash A_1) \in \mathfrak{F}$$

we know that $(A_1 \backslash A_2) \cup (A_1 \cap A_2) = A_1$

which implies that $(A_1 \cap A_2) \in A_1$

so we have $A_1 \cap A_2 \in A_1 \in \mathfrak{F}$

which implies that $A_1 \cap A_2 \in \mathfrak{F}$

extending to n = n case

let $(((A_1\backslash \mathrm{A}_2)\backslash \mathrm{A}_3)..\backslash \mathrm{A}_n)$ mean A_1 without any other A_n for every n
similarly, let $(((A_2\backslash \mathrm{A}_1)\backslash \mathrm{A}_3)..\backslash \mathrm{A}_n)$ mean A_2 without any other A_n for every n

then $(((A_1\backslash \mathrm{A}_2)\backslash \mathrm{A}_3)..\backslash \mathrm{A}_n) \cup (A_1 \cap A_2 \cap A_3 \dots \cap A_n) \cup (((A_2\backslash \mathrm{A}_1)\backslash \mathrm{A}_3)..\backslash \mathrm{A}_n) = \cup_{i=1}^{n} A_i$

we know from the n=2 case logic that
$(A_1 \cap A_2 \cap A_3 \dots \cap A_n) \in \quad A_1$

we know that $A_1 \in \mathcal{F}$

so we have $(A_1 \cap A_2 \cap A_3 \dots \cap A_n) \in \quad A_1 \in \mathcal{F}$
which implies that $(A_1 \cap A_2 \cap A_3 \dots \cap A_n) \in \mathcal{F}$