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DATA 601 – Probability and Statistics
HW4

1. let $A = \{1,2,4\}$
let $B = \{2,3\}$

$$\text{Then } P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

$$P(A) * P(B) = \frac{1}{2} * \frac{1}{3} = \frac{1}{6}$$

$$P(A \cap B) = \frac{1}{6}$$

Since $P(A \cap B) = P(A) * P(B)$, A and B are independent

2.

part a

let P_1 = the ball pulled from box 1 such that $P_1 = 1,2,3,4$

let P_2 = the set of balls pulled from box 2

let $R = P_2$ pulled only red balls

$$\begin{aligned} P(R) = & P(P_2 = \{r\} | P_1 = 1) * P(P_1 = 1) + P(P_2 = \{r, r\} | P_1 = 2) * P(P_1 = 2) \\ & + P(P_2 = \{r, r, r\} | P_1 = 3) * P(P_1 = 3) \\ & + P(P_2 = \{r, r, r, r\} | P_1 = 4) * P(P_1 = 4) \end{aligned}$$

$$\text{We know that } P(P_1 = 1) = P(P_1 = 2) = P(P_1 = 3) = P(P_1 = 4) = \frac{1}{4}$$

We can solve for the rest

$$\begin{aligned} P(P_2 = \{r\} | P_1 = 1) &= \frac{5}{13} \\ P(P_2 = \{r, r\} | P_1 = 2) &= \frac{5}{13} * \frac{4}{12} \\ P(P_2 = \{r, r, r\} | P_1 = 3) &= \frac{5}{13} * \frac{4}{12} * \frac{3}{11} \\ P(P_2 = \{r, r, r, r\} | P_1 = 4) &= \frac{5}{13} * \frac{4}{12} * \frac{3}{11} * \frac{2}{10} \end{aligned}$$

$$\text{Then } P(R) = \frac{1}{4} * \frac{5}{13} * [1 + \frac{4}{12} + (\frac{4}{12} * \frac{3}{11}) + (\frac{4}{12} * \frac{3}{11} * \frac{2}{10})] = \frac{119}{858} \approx 0.14$$

part b

let P_1 = the ball pulled from box 1 such that $P_1 = 1,2,3,4$

let R = selecting only red balls

We want $P(P_1 = 2 | R)$

$$P(P_1 = 2|R) = \frac{P(P_1=2 \cap R)}{P(R)} = \frac{P(R|P_1 = 2) * P(P_1=2)}{P(R)}$$

$$P(R|P_1 = 2) = \frac{5}{13} * \frac{4}{12}$$

$$P(P_1 = 2) = \frac{1}{4}$$

$$P(R) = \frac{119}{858} \text{ from part a}$$

plugging them in we get

$$P(P_1 = 2|R) = \frac{\frac{5}{13} * \frac{4}{12} * \frac{1}{4}}{\frac{119}{858}} = \frac{55}{238} \approx 0.23$$

3.

$S = \{ (a,b), a,b \in \{1,2,3,4,5,6\} \}$ modeling rolling a dice two times
and all outcomes are equally likely so $P(\{a,b\}) = \frac{1}{36}$

have

$$X((a,b)) = a - 2$$

$$Y((a,b)) = |b - a|$$

part a

X takes 6 distinct values. $\{-1,0,1,2,3,4\}$

Y is a bit more interesting

$\downarrow b \rightarrow a$	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

So Y takes 6 values $\{0,1,2,3,4,5\}$

X+Y

$\downarrow Y \rightarrow X$	-1	0	1	2	3	4
0	-1	0	1	2	3	4
1	0	1	2	3	4	5
2	1	2	3	4	5	6
3	2	3	4	5	6	7

4	3	4	5	6	7	8
5	4	5	6	7	8	9

so $X+Y$ takes 11 values $\{-1,0,1,2,3,4,5,6,7,8,9\}$

$X * Y$

$\downarrow Y \rightarrow X$	-1	0	1	2	3	4
0	0	0	0	0	0	0
1	-1	0	1	2	3	4
2	-2	0	2	4	6	8
3	-3	0	3	6	9	12
4	-4	0	4	8	12	16
5	-5	0	5	10	15	20

so $X * Y$ takes 19 values $\{-5,-4,-3,-2,-1,0,1,2,3,4,5,6,8,9,10,12,15,16,20\}$

part b

Since X can take 6 distinct values and Y can take 6 distinct values (X,Y) should have 36 distinct vectors.

part c

$P(X+Y \leq 3) = ?$

$\downarrow Y \rightarrow X$	-1	0	1	2	3	4
0	-1	0	1	2	3	4
1	0	1	2	3	4	5
2	1	2	3	4	5	6
3	2	3	4	5	6	7
4	3	4	5	6	7	8
5	4	5	6	7	8	9

That is 15 values out of 36 values so

$$P(X+Y \leq 3) = \frac{15}{36}$$

4.

part a

we have $p = 0.6$, $q = 0.4$, $A = 4$

plugging into the formula for $f(z)$ we have

$$f(2) = \frac{1 - \left(\frac{2}{3}\right)^2}{1 - \left(\frac{2}{3}\right)^4} = \frac{9}{13}$$

this is the case of when they are tied at 0-0 so $P[A \text{ winning}] = \frac{9}{13}$

part b

let X = games played when A wins

$$P[X=2] = (0.6)^2 = 0.36 \text{ (AA)}$$

$$P[X=4] = 2 * (0.6)^3(0.4) = 0.0864 \text{ (ABAA)(BAAA)}$$

$$P[X=6] = 4 * (0.6)^4(0.4)^2 = 0.020736 \text{ (ABABAA)(ABBAAA)(BABAAA)(BAABAA)}$$

$$P[X=8] = 8 * (0.6)(0.4) * P[X=6]$$

...

$$P[X=2n] = 2^{n-1}(0.6)^{n+1}(0.4)^{n-1}$$

$$\begin{aligned} \text{so we have } P[\text{A winning}] &= \sum_{i=1}^{\infty} 2^{i-1}(0.6)^{i+1}(0.4)^{i-1} = \frac{3}{4} \sum_{i=1}^{\infty} 2^i(0.6)^i(0.4)^i \\ &= \frac{3}{4} \sum_{i=1}^{\infty} (0.48)^i \end{aligned}$$

this is an infinite geometric series

$$a = .48$$

$$\text{common ratio} = .48$$

$$S = \frac{a}{1-r} = \frac{.48}{1-.48} = \frac{48}{52} = \frac{12}{13}$$

$$\text{so we have } P[\text{A winning}] = \frac{3}{4} * \frac{12}{13} = \frac{9}{13}$$