Bradley Scott
Data601
HW6

Problem 1

biased coin lands on heads with probability p
it's flipped 10 times
6 heads is the result
find the conditional probability that the first 3 outcomes are H,T,T

let X = the number of heads flipped out of 10 trials

P(first three flips are H,T,T | X = 6 for 10 flips) = ?

X is then binomial with (n,p) = (10,p)

P(X=6) = $\binom{10}{6} p^6(1-p)^4$

The probability that the first three flips is H,T,T is $p(1-p)^2$

P(H,T,T | X = 6 for 10 flips) = $\frac{P(HTT \cap X=6)}{P(X=6)}$

$P(HTT \cap X = 6)$ is the number of ways to get X=6 when our first 3 outcomes are HTT and we do 10 tosses total

so we have things like H,T,T,H,H,H,H,H,T,T and H,T,T,H,H,H,H,T,H,T

this is like deciding the first 3 rolls and then out of 7 choosing 5 to be success

so $P(HTT \cap X = 6) = p(1-p)^2 \binom{7}{5} p^5(1-p)^2 = \binom{7}{5} p^6(1-p)^4$

Then P(H,T,T | X = 6 for 10 flips) = $\frac{P(HTT \cap X=6)}{P(X=6)} = \frac{\binom{7}{5} p^6(1-p)^4}{\binom{10}{6} p^6(1-p)^4} = \frac{\binom{7}{5}}{\binom{10}{6}} = \frac{\frac{7!}{5!2!}}{\frac{10!}{6!4!}} = \frac{1}{10}$

Problem 2

let X = the number of boxes needed to get a prize

let X_1 = the number of boxes needed to get the first unique coupon

let X_2 = the number of boxes needed to get the second unique coupon

...
up to letting X_5 = the number of boxes needed to get the fifth and final unique coupon

then

$\text{E[X]} = E[X_1] + E[X_2] + E[X_3] + E[X_4] + E[X_5]$

$E[X_1] = 1$ since you'll always need exactly 1 box to get the first unique coupon

X_2 is a geometric distribution since we want the first success with probably $\frac{4}{5}$ since there are still 4 unique to us coupons we can obtain out of 5 total unique coupons

the expectation value for a geometric distrbution is $\frac{1}{p}$

hence $E[X_2] = \frac{5}{4}$

X_3 is also geometric but now $\text{p} = \frac{3}{5}$ and so $\text{E}[X_3] = \frac{5}{3}$

it seems clear enough from here that $E[X_4] = \frac{5}{2}$ and $\text{E}[X_5] = 5$

then we have

$\text{E[X]} = E[X_1] + E[X_2] + E[X_3] + E[X_4] + E[X_5] = 1 + \frac{5}{4} + \frac{5}{3} + \frac{5}{2} + 5 = \frac{137}{12} \approx 12$

We rounded up to 12 since we can't buy a fraction of a box

Problem 3

let X = the number of rainy days in a given year

we're given that P[X≥1] = 1/2

we know that P[X=0] + P[X≥1] = 1

$$\Rightarrow P[X = 0] = \frac{1}{2}$$

We can model X as a poisson distribution and set up an equation from P[X=0] to calculate λ

$\frac{1}{2} = \text{P[X=0]} = \frac{\lambda^0}{0!} e^{-\lambda} = e^{-\lambda}$

taking the ln of both sides

$$\ln\left(\frac{1}{2}\right) = -\lambda \ln(e)$$
$$\ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = -\ln(2)$$
$$\ln(e) = 1$$

so we have

$$-\ln(2) = -\lambda$$
$$\Rightarrow \lambda = \ln(2)$$

let Y = the number of rainy days in 265 days

since poisson is over a period of time and we have changed the time interval, our λ changes as well

let λ_y be the λ for the Y distribution

then $\lambda_y = \frac{250}{365}\lambda = \frac{50}{73}\ln(2)$

we want P[Y=3] $= \frac{(\lambda_y)^3}{3!}e^{-\lambda_y} = \frac{\left(\frac{50}{73}\ln(2)\right)^3}{3!}e^{-\frac{50}{73}\ln(2)}$

this can be simplified to

$$= \frac{\frac{50^3}{70^3} * (\ln(2))^3}{6} * \frac{1}{2^{\frac{50}{73}}}$$

from here we can approximate $\left(\frac{50}{73}\right) \approx \left(\frac{5}{7}\right) \ln(2) \approx \frac{7}{10}$ to get

$$\frac{1}{6} * \left(\frac{5}{7}\right)^3 \left(\frac{7}{10}\right)^3 \left(\frac{1}{2^{\frac{5}{7}}}\right) = \frac{5^3 * 7^3}{6 * 7^3 * 10^3 * 2^{\frac{5}{7}}} = \frac{1}{48 * 2^{\frac{5}{7}}} \approx 0.0127$$

so the P[Y=3]$\approx$ 0.0127 or about a $\frac{1}{100}$ chance of it raining 3 times in 25 days

This seems reasonable since there's a 50% chance it doesn't rain for a year.