

a.  $6! = 720$

b.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| E | F |   |   |   |   |
|   | E | F |   |   |   |
|   |   | E | F |   |   |
|   |   |   | E | F |   |
|   |   |   |   | E | F |

5 ways to put E and F together \* 2 because EF and FE \* 4! ways to put A,B,C and D and divided by 720 for the number of permutations from part a and we get

$$\frac{2 * 4! * 5}{6!} = \frac{1}{3}$$

c)

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| C | D |   |   |   |   |
|   | C | D |   |   |   |
|   |   | C | D |   |   |
|   |   |   | C | D |   |
|   |   |   |   | C | D |

each of the above rows has 2 ways to arrange CD (CD and DC)

The first row has 3 ways to put AB

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| C | D | A | B |   |   |
| C | D |   | A | B |   |
| C | D |   |   | A | B |

The second row has 2 ways to put AB

|  |   |   |   |   |   |
|--|---|---|---|---|---|
|  | C | D | A | B |   |
|  | C | D |   | A | B |

The third row was 2 ways to put AB

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| A | B | C | D |   |   |
|   |   | C | D | A | B |

The fourth row is symmetric with the second row and has 2 ways

The fifth row is symmetric with the first row and has 3 ways

So calculating for each row

| CD & DC | AB & BA | ways to place AB | total count for the row |
|---------|---------|------------------|-------------------------|
| 2       | 2       | 3                | 12                      |
| 2       | 2       | 2                | 8                       |

|   |   |   |    |
|---|---|---|----|
| 2 | 2 | 2 | 8  |
| 2 | 2 | 2 | 8  |
| 2 | 2 | 3 | 12 |

so there is 48 total ways

$$\frac{48}{720} = \frac{1}{15}$$

d.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| A | B | C | D |   |   |
|   | A | B | C | D |   |
|   |   | A | B | C | D |
| A | B |   | C | D |   |
| A | B |   |   | C | D |
|   | A | B |   | C | D |

Row 1 has 2 ways (EF and FE)

Row 2 has 0 ways

Row 3 has 2 ways

Row 4 has 0 ways

Row 5 has 2 ways

Row 6 has 0 ways

That's out of 2 (EF and FE) \* 6 rows = 12

so we have  $\frac{6}{12} = \frac{1}{2}$