finite geometric series $1+q+q^2+\cdots+q^n = a_1\left(\frac{1-r^n}{1-r}\right)$ where a_1 = first term r = common ratio n = number of terms

infinite geometric series $1+q+q^2+q^3+\cdots = \sum_{x=0}^{\infty} q = \frac{1}{1-q}$ where q < 1

gamblers ruin

$f(z) = \frac{1-\left(\frac{q}{p}\right)^z}{1-\left(\frac{q}{p}\right)^A}$, $for\ p \neq q\ p = probability\ success,\ q = 1-p$

$f(z) = \dfrac{z}{A} for\ p = q = .5$

$A = the\ number\ of\ "jumps"\ from\ left\ to\ right\ (or\ the\ number\ of\ dollars\ you\ want\ to\ leave\ with)$

$z = the\ number\ of\ points\ to\ win\ by\ (or\ the\ number\ of\ dollars\ you're\ coming\ with)$

The Probability Mass Function (PMF) is $p_x(x \in \mathbb{R}) = P(X = x)$

the PMF graph is of the probability at each value of the random variable

$p_x(x) \geq 0$ for every $x \in \mathbb{R}$

$\sum_{x:p(X=x)\neq 0} p_x(x) = 1$

Cumulative Distribution Function (CDF)

the CDF graph is of the probability at and below each value of the random variable

if X is discrete then the CDF $F_x(x)$ is

$$F_x(x) = P(X \leq x) = \sum_{\substack{y \leq x \\ P(x=y)\neq 0}} p_x(y)$$

1. $F_x(x) \leq F_x(y)$ if x≤y (this is called monotonic - means over time the probability will increase)
2. $\lim_{x\to+\infty} F_x(x) = 1$, $\lim_{x\to-\infty} F_x(x) = 0$
3. $\lim_{y \downarrow x} F_x(y) = F_x(x)$

E[X] $= \sum_{i=1}^{\infty} x_i * P(A_i)$ if the sum converges absolutely $\sum_{i=1}^{\infty} |x_i * P(A_i)| \leq \infty$

E[c] = c , where c is a constant

$E[X_1 + X_2] = E[X_1] + E[X_2]$ and $E[aX_1 + bX_2] = aE[X_1] + bE[X_2]$ when $|E[X_1]| < \infty,\ |E[X_2]| < \infty$

if g: $\mathbb{R} \to \mathbb{R}$

then E[g(x)] $= \sum_{x:p_x(x)\neq 0} g(x)p_x(x)$

if $X_1 \leq X_2$ then $E[X_1] \leq E[X_2]$

Suppose $|E[X]| < \infty$ then var[X] $= E[\,(X - E[X])^2] = E[X^2] - (E[X])^2$

var[aX+b] $= a^2 var[X]$, $\sigma_x = \sqrt{var[X]}$

<u>Binomial Distribution with parameters (n,p)</u>

X ~ number of successes out of n experiments where probability of each success is p

each experiment is independent of the rest

N.B. the ~ in statistics means "has the distribution of"

S = $\{(a_1, \ldots, a_n),\ a_i \in (0,1)\}$

$$a_i = \begin{cases} 1, & success \\ 0, & fail \end{cases}$$

X$((a_1, \ldots, a_n)) =$ number of 1's in $(a_1, \ldots, a_n)$

P$(\{a_1, \ldots, a_n\}) = p^{\#\ of\ 1's\ in\ (a_1,\ldots,a_n)}(1-p)^{\#\ of\ 0's\ in\ (a_1,\ldots,a_n)}$

P(X=k) $= p_x(k) = \binom{n}{k} p^k (1-p)^{n-k}$, $k \in \{0,1,\ldots,n\}$

E[X] = np

var[X]=np(1-p)

<u>Geometric Distribution with parameter p</u>

X ~ the number of trials of an experiment to get the first success

P(X=k) $= p_x(k) = p(1-p)^{k-1}$

E[X] $= \frac{1}{p}$

var[X] $= \frac{1-p}{p^2}$

<u>Negative Binomial Random Variables with parameters (r,p)</u>

X ~ the number of trials of an experiment needed to get r successes

X takes values in the set {r, r+1, r+2, ...}

P(X=k) $= p_x(k) = \binom{k-1}{r-1} p^{r-1}(1-p)^{(k-1)-(r-1)} p = \binom{k-1}{r-1} p^r (1-p)^{k-r}$

E[X] $= \frac{r}{p}$

var[X] $= \frac{r(1-p)}{p^2}$

How to derive these? It's just r runs of geometric so we just multiply the geometric E[X] and var[X] by r

<u>Poisson Distribution with parameter λ</u>

X~ the number of events that occur in an infinite number of trials

P(X=k) $= p_x(k) = \frac{\lambda^k}{k!} e^{-\lambda}$

E[X] $= \lambda$

var[X] $= \lambda$

Note the poisson limit theorem which states if Y_n are binomial(n,$\frac{\lambda}{n}$) then $\lim_{n\to\infty} P[Y_n = k] = \frac{\lambda^k}{k!} e^{-\lambda}$

Continuous Random Variables

X is said to be continuous if there is a function $f: \mathbb{R} \to [0, \infty)$ such that $P(X \in [a, b]) = \int_a^b f(x)dx$
such a function is called the density of X (sometimes we'll write f_x)
if $f(x)$ serves as a density then $\int_{-\infty}^{\infty} f(x)dx = 1$

Definition: $F(x) = P(X \leq x)$ is said to be the cumulative distribution function of a random variable X.

$$F: \mathbb{R} \to [0,1]$$

N.B true for discrete or continous

$$F'(x) = f(x)$$