

Bradley Scott  
HW1

1. **with replacement**

$\Omega = \{RR, RG, RB, GG, GR, GB, BB, BG, BR\}$   
probability of any individual outcome = 1/9

**without replacement**

$\Omega = \{RG, RB, GR, GB, BR, BG\}$   
probability of any individual outcome = 1/6

2. E = the event that the sum of the dice is odd

F = at least one of the dice lands on 1

G = the sum of the dice is 5

How many elementary outcomes are there in the events

E intersect F? 6

E union F? 23

F intersect G? 2

E \ F (E without F)? 12

E intersect F intersect G? 2

ok so the entire S is

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$   
 $(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$   
 $(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$   
 $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$   
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$   
 $(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)$   
}

The events that the sum of the dice is odd are

$E_{events} = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$   
 $(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$   
 $(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$   
 $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$   
 $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$   
 $(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)$   
}

$= \{(1,2), (1,4), (1,6),$   
 $(2,1), (2,3), (2,5),$   
 $(3,2), (3,4), (3,6),$   
 $(4,1), (4,3), (4,5),$   
 $(5,2), (5,4), (5,6),$   
 $(6,1), (6,3), (6,5)$   
}

F events = {  
    (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),  
    (2,1),(3,1),(4,1),(5,1),(6,1)  
}

G events = {  
    (1,4),(2,3),(3,2),(4,1)  
}

Then E intersect F has events

{  
    (1,2),(1,4),(1,6)  
    (2,1),(4,1),(6,1)  
}

Then E union F has events

{  
    (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),  
    (2,1),(2,3),(2,5),  
    (3,1),(3,2),(3,4),(3,6),  
    (4,1),(4,3),(4,5),  
    (5,1),(5,2),(5,4),(5,6),  
    (6,1),(6,3),(6,5)  
}

F intersect G has events

{  
    (1,4),(4,1)  
}

E without F has events

{  
    (2,3),(2,5),  
    (3,2),(3,4),(3,6),  
    (4,3),(4,5),  
    (5,2),(5,4),(5,6),  
    (6,3),(6,5)  
}

E intersect F intersect G has events

{  
    (1,4),(4,1)  
}

3. n socks. 3 are red.

if  $P(X=2) = .5$ .

That is to say that if we chose 2 socks from the  $n$  group of socks, the probability both socks chosen are red is  $.5$ .

What's the value of  $n$ ?

$$n = 3 + y$$

$y$  = the number of non red socks

We can get the number of combinations with  $n$  choose  $r = n! / (n-r)!r!$

$$= n! / (n-2)!2! = n(n-1)/2$$

we know that half of those permutations must yield 2 red socks since  $P(X=2) = .5$

How is  $P(X=2)$  determined?

$P(X=2) = \# \text{ of ways you can get two red socks} / \# \text{ of ways you can get two socks}$   
we know the denominator. The number of ways we can get two socks is  $n(n-1) / 2$

so setting up an initial equation we have  $.5 = \# \text{ of ways we can get two red socks} / n(n-1)/2$

we know there is 3 red socks. Lets call them R1, R2 and R3.

Then the distinct combinations we can make of those 3 socks are (R1,R2), (R1,R3) and (R2,R3)  
Alternatively we can say 3 choose 2 and we get  $3(2) / 2 = 3$

so now we have  $P(X=2) = .5$  and  $P(X=2) = 3 / [n(n-1)/2] = 6/(n(n-1))$

Solving  $.5 = 6/[n(n-1)]$  gives us  **$n=4$** .

4.

We have from (c)  $A_1, A_2, \dots, A_n \in \mathcal{F}$  then  $\bigcup_{i=1}^n A_i \in \mathcal{F}$

We have from (b) that if  $A_i \in \mathcal{F}$  then  $\Omega \setminus A_i \in \mathcal{F}$

Using both (c) and (b) we have that  $(\bigcup_{i=1}^n A_i)^c \in \mathcal{F}$

By De Morgan's Law we have  $(\bigcup_{i=1}^n A_i)^c = \bigcap_{i=1}^n A_i^c \in \mathcal{F}$

Once again using (b) we have that if  $\bigcap_{i=1}^n A_i^c \in \mathcal{F}$

then  $\bigcap_{i=1}^n A_i \in \mathcal{F}$