Data 602
Midterm
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Problem 1

$$P[k\ red\ balls\ from\ a\ sample\ of\ 20\ with\ replacement] = \binom{20}{k}\left(\frac{3}{10}\right)^k\left(\frac{7}{10}\right)^{20-k}$$

We do $\binom{20}{k}$ because there are that many ways the sequence can occur
We do $\left(\frac{3}{10}\right)^k$ because $\frac{3}{10}$ is the probability of pulling a red ball and k is how many we pull

We do $\left(\frac{7}{10}\right)^{20-k}$ because $\frac{7}{10}$ is the probability of pulling a green ball and 20-k is how many green balls we pull

$$P[k\ red\ balls\ from\ a\ sample\ of\ 20\ without\ replacement] = \frac{\binom{30}{k}\binom{70}{20-k}}{\binom{100}{20}}$$

This is just using combinatorics. We choose k red balls from the 30 red balls. We choose 20-k green balls from the 70 green balls. We divide the whole thing but the total number of variations we could make of 20 balls.

Problem 2

The washing machine can eat 4 socks each from a different pair leaving us with 6 intact pairs.

The washing machine can eat 1 whole pair and 2 socks from 2 other pairs, leaving us with 7 intact pairs.

The washing machine can eat 4 socks from 2 whole pairs leaving us with 8 intact pairs.

There is no way for us to get more or less pairs since 4 socks are always eaten.

We can use combinatorics to figure out the probabilites of each scenario.

$$P[6\ intact\ pairs\ are\ left] = \frac{\binom{10}{4}\binom{2}{1}^4}{\binom{20}{4}}$$

from left to right and then denominator explanation:
since we're choosing 4 pairs from the 10 pairs we have
we're choosing 1 sock from each pair
and we divide by the number of ways to select 4 socks from the 20 socks we have

$$P[7\ intact\ pairs\ are\ left] = \frac{\binom{10}{2}\binom{2}{1}^2\binom{8}{1}\binom{2}{2}}{\binom{20}{4}}$$

from left to right and then denominator explanation:

since we're choosing 2 pairs from the 10 pairs we have to have 1 sock eaten each
so we have 2 choose 1 sock from each of those 2 pairs
then we choose 1 pair from the remaining 8 to have both socks eaten
and we choose 2 socks from the 2 in the pair
then we divide by the number of ways to select 4 socks from the 20 socks we have

$$P[8\ intact\ pairs\ are\ left] = \frac{\binom{10}{2}\binom{2}{2}}{\binom{20}{4}}$$

from left to right and then denominator explanation:

since we're choosing 2 pairs from the 10 pairs to have both socks eaten
and we choose 2 socks from the 2 in the pair
then we divide by the number of ways to select 4 socks from the 20 socks we have

$$E[X] = x * P[X] = 6 * \left(\frac{\binom{10}{4}\binom{2}{1}^4}{\binom{20}{4}}\right) + 7 * \left(\frac{\binom{10}{2}\binom{2}{1}^2\binom{8}{1}\binom{2}{2}}{\binom{20}{4}}\right) + 8 * \left(\frac{\binom{10}{2}\binom{2}{2}}{\binom{20}{4}}\right)$$

$$= \frac{6\left(\frac{10!}{4!\,6!} * 16\right) + 7\left(\frac{10!}{2!\,8!} * 4 * 8\right) + 8\left(\frac{10!}{2!\,8!}\right)}{\frac{20!}{4!\,16!}}$$

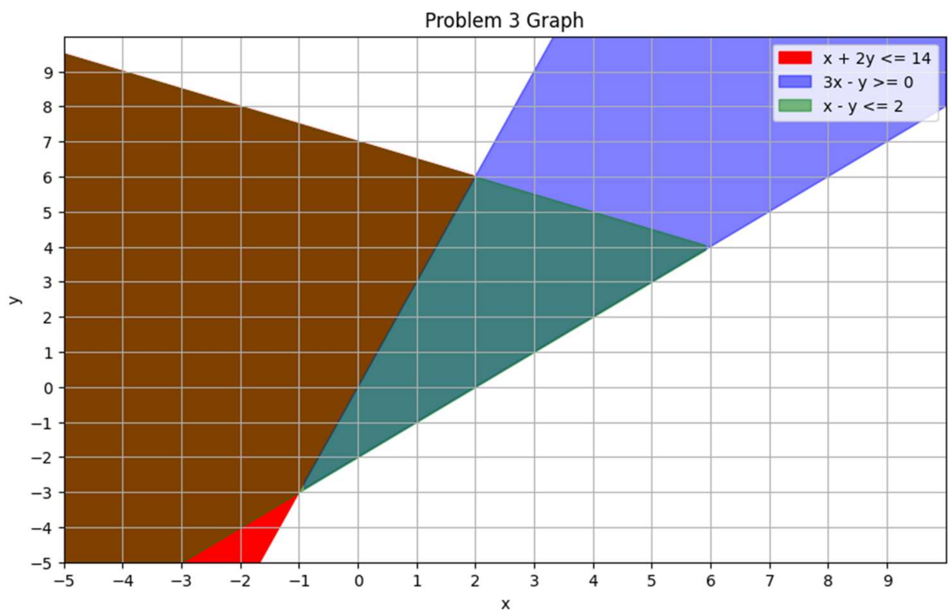
$$= \frac{96\left(\frac{10 * 9 * 8 * 7}{4 * 3 * 2}\right) + 224\left(\frac{10 * 9}{2}\right) + 8\left(\frac{10 * 9}{2}\right)}{\frac{20 * 19 * 18 * 17}{4 * 3 * 2}}$$

$$= \frac{96(10 * 3 * 7) + 224(45) + 8(45)}{5 * 19 * 3 * 17}$$

$$= \frac{20{,}160 + 10{,}080 + 360}{4845} = \frac{30{,}600}{4845} = \frac{6120}{969} = \frac{2040}{323} = \frac{120}{19}$$

Problem 3

Using the below python code, I made the following graph so I could find the region where the inequalities overlap.



From here we need to evaluate at the vertex's of the darkest green area where all three inequalities are satisified. The vertex's are at (-1,-3), (2,6) and (6,4). Plugging them into 5x + 3y we get -14, 28 and 42. So our maximum of 5x + 3y that satisfies the 3 equations is when x = 6 and y=4 and gives us a value of 42.

Using pulp validated our answer. The code and output is below the graph code.

```
#Note: Use ChatGPT to help with the plotting
#import the needed modules
import numpy as np
import matplotlib.pyplot as plt
#define the range of values
x = np.linspace(-5,10,100)
# the 3 equations
y1 = 7 - (x / 2)
y2 = 3 * x
y3 = x - 2
#plot and shade the regions
plt.figure(figsize=(10, 6))
plt.fill_between(x, y1, y2, where=(y1 > y2), color='red', label='x + 2y <=
14')
plt.fill_between(x, y2, y3, where=(y2 > y3), color='blue', label='3x - y >=
0', alpha=0.5)
plt.fill_between(x, y1, y3, where=(y1 > y3), color='green', label='x - y <=
2', alpha=0.5)
# Set x and y axis ticks
plt.xticks(np.arange(-5, 10, 1))
plt.yticks(np.arange(-5, 10, 1))
# Add legends, labels, and display the plot
plt.legend()
plt.xlabel('x')
```

```
plt.ylabel('y')
plt.title('Problem 3 Graph')
plt.grid(True)
plt.xlim([-5, 10])
plt.ylim([-5, 10])
plt.show()
```

```
import pulp as p
Lp_prob = p.LpProblem('Problem', p.LpMaximize)
# Create problem Variables
x = p.LpVariable("x", lowBound = -10, cat = 'Integer')   # Create a variable
x >= -10
y = p.LpVariable("y", lowBound = -10, cat = 'Integer')   # Create a variable
y >= -10
z = p.LpVariable("z", lowBound = -10, cat = 'Integer')   # Create a variable
z >= -10
# Objective Function
Lp_prob += 5 * x + 3 * y
# Constraints:
Lp_prob += x + 2*y <= 14
Lp_prob += 3*x - y >= 0
Lp_prob += x-y <= 2
status = Lp_prob.solve()   # Solver
print(p.LpStatus[status])   # The solution status
# Printing the final solution
print(p.value(x), p.value(y), p.value(Lp_prob.objective))
```

Optimal
6.0 4.0 42.0

Problem 4

Let x_i = the fraction of the company stock of s_i we decide to buy

constraint $0 \leq x_i \leq 1 \; \forall \, i \in$ n

We want to maximize $\sum_{i=1}^{n} w_i x_i$

N.B. I'm assuming w_i is the expected profit of buying every available stock of company s_i based on the problem statement.

We have constraint $\sum_{(i=1)}^{n} c_i x_i \leq B$
that is to say we can't spend more than our budget.

Problem 5

See attached midterm_problem5 file

Problem 6

see attached midterm_problem6 file

Problem 7

A two way marketplace is one with a buyer and a seller but the marketplace creates revenue from both buyers and sellers. An example would be ebay that connects buyers with sellers and profits off the transaction between them. Uber would be another example since it connects riders with drivers and profits of fthe transaction between them.

One of the challenges with a two way marketplace is you need a lot of volume in order to create revenue. This is because you need to keep your charge on the transaction low otherwise why wouldn't the buyer and seller go through a different platform or not use the marketplace at all. Or if you can't keep the charge low you need to make the experience of using the platform worth the price.

A three sided marketplace is one with a buyer, a seller and a facilitator. An example would be food delivery since you have the restauraunt selling the food, the delivery driver taking the food to the buyer and the buyer who buys the food. Another example could be appliance delivery. The appliance store sells the appliance, the logistics company delivers the appliance and the buyer buys the appliance.

A challenge with three sided marketplaces is that there is more pieces to the puzzle to try to fine tune and optimize. There may be less oversight or influence as well. When things are done "in house" they can be tailored and optimized specifically to ones needs. As we outsource things like delivery to an external company, we don't have as much say as to how things are done.

Some of the sites used when researching this are:
https://www.youtube.com/@hajiaghayi/search?query=marketplace
https://zapier.com/blog/two-sided-marketplace/
https://blog.muttdata.ai/2021/01/14/three-sided-intro.html

Problem 8

SSP is a supply side platform. It allows a publisher to make their ads available to buyers of ads. The idea is to maximize the price for what the ad is sold for.

DSP is a demand side platform. It is what advertisers use to buy ad spaces on various sites, but it's targeted to specific people based on information about them (like their location or browsing history).

Display ad networks facilitate the connection between business wanting to run ads and advertisers(websites) wanting to host them. They will buy up ad space that goes unsold (or buy up premium ad space) and sell it to companies wishing to run ads.

Some of the sites used when researching this are:
Part 5: Potentials of E-commerce for Retails (Dr. Mohammad Hajiaghayi and Nishan Subedi): Display Ad


MONETIZATION
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