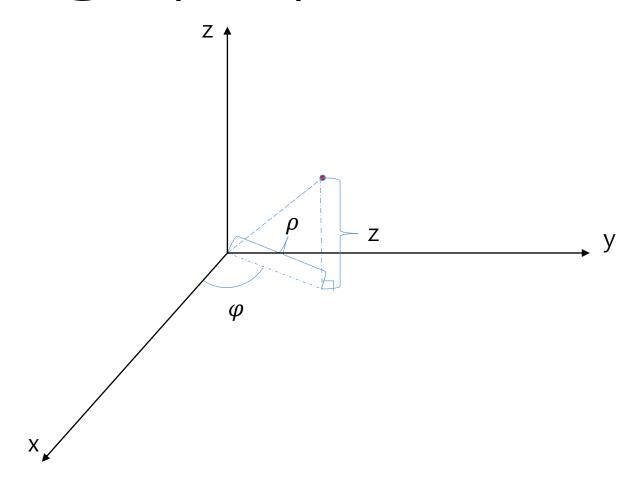
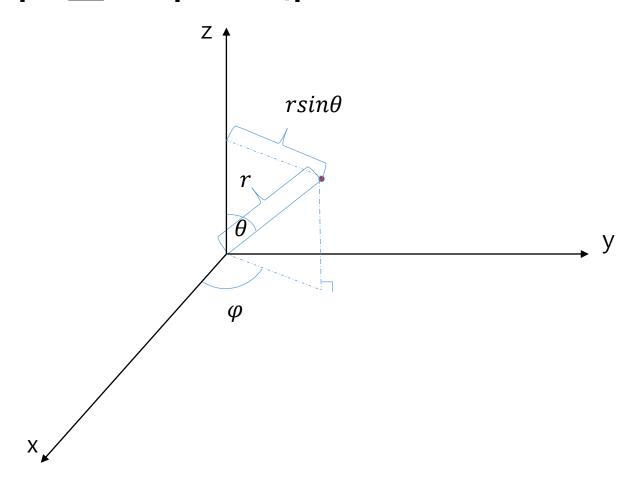
벡터 미분 연산자(델 연산자)

Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ho, ϕ, z)	Spherical coordinates (r,θ,φ) , where θ is the polar angle and φ is the azimuthal angle $^{\alpha}$
Vector field A	$A_x \hat{f x} + A_y \hat{f y} + A_z \hat{f z}$	$A_{ ho}\hat{oldsymbol{ ho}}+A_{arphi}\hat{oldsymbol{arphi}}+A_{z}\hat{oldsymbol{z}}$	$A_r \hat{f r} + A_ heta \hat{m heta} + A_arphi \hat{m \phi}$
Gradient $\nabla f^{[1]}$	$rac{\partial f}{\partial x}\hat{\mathbf{x}} + rac{\partial f}{\partial y}\hat{\mathbf{y}} + rac{\partial f}{\partial z}\hat{\mathbf{z}}$	$rac{\partial f}{\partial ho}\hat{oldsymbol{ ho}} + rac{1}{ ho}rac{\partial f}{\partial arphi}\hat{oldsymbol{arphi}} + rac{\partial f}{\partial z}\hat{f z}$	$\frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$
Divergence $\nabla \cdot \mathbf{A}^{[1]}$	$rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$	$rac{1}{ ho}rac{\partial\left(ho A_{ ho} ight)}{\partial ho}+rac{1}{ ho}rac{\partial A_{arphi}}{\partialarphi}+rac{\partial A_{z}}{\partial z}$	$rac{1}{r^2}rac{\partial \left(r^2A_r ight)}{\partial r}+rac{1}{r\sin heta}rac{\partial}{\partial heta}\left(A_ heta\sin heta ight)+rac{1}{r\sin heta}rac{\partial A_arphi}{\partial arphi}$
Curl $\nabla \times \mathbf{A}^{[1]}$	$egin{aligned} \left(rac{\partial A_z}{\partial y} - rac{\partial A_y}{\partial z} ight) \hat{\mathbf{x}} \ + \left(rac{\partial A_x}{\partial z} - rac{\partial A_z}{\partial x} ight) \hat{\mathbf{y}} \ + \left(rac{\partial A_y}{\partial x} - rac{\partial A_x}{\partial y} ight) \hat{\mathbf{z}} \end{aligned}$	$egin{aligned} igg(rac{1}{ ho}rac{\partial A_z}{\partial arphi} - rac{\partial A_{arphi}}{\partial z}igg)\hat{oldsymbol{ ho}} \ + igg(rac{\partial A_{ ho}}{\partial z} - rac{\partial A_z}{\partial ho}igg)\hat{oldsymbol{arphi}} \ + rac{1}{ ho}igg(rac{\partial \left(ho A_{arphi} ight)}{\partial ho} - rac{\partial A_{ ho}}{\partial arphi}igg)\hat{oldsymbol{z}} \end{aligned}$	$egin{aligned} &rac{1}{r\sin heta}\left(rac{\partial}{\partial heta}\left(A_{arphi}\sin heta ight)-rac{\partial A_{ heta}}{\partialarphi} ight)\hat{\mathbf{r}}\ &+rac{1}{r}\left(rac{1}{\sin heta}rac{\partial A_{r}}{\partialarphi}-rac{\partial}{\partial r}\left(rA_{arphi} ight) ight)\hat{oldsymbol{ heta}}\ &+rac{1}{r}\left(rac{\partial}{\partial r}\left(rA_{ heta} ight)-rac{\partial A_{r}}{\partial heta} ight)\hat{oldsymbol{arphi}} \end{aligned}$

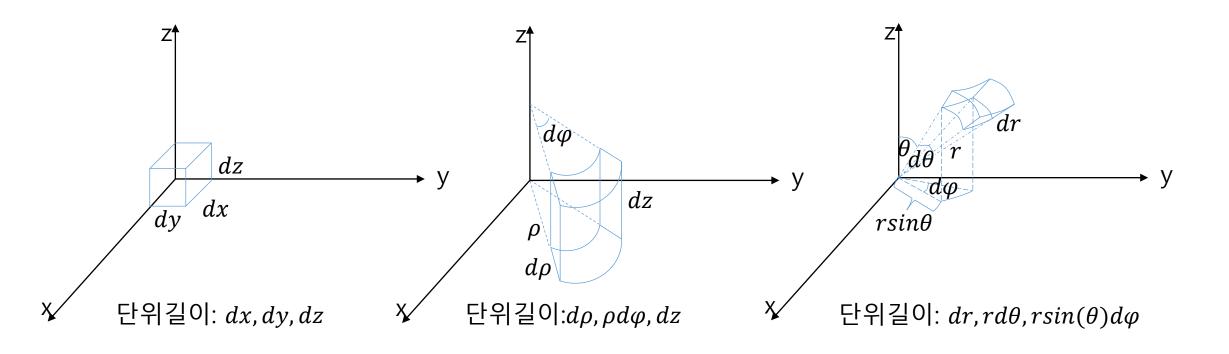
원통 좌표계



구면 좌표계



각 좌표계의 미소 성분들



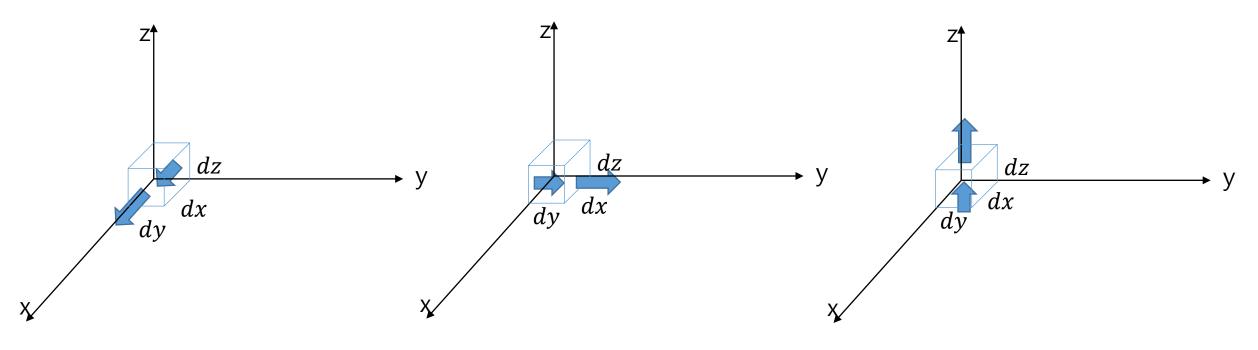
발산이란?

• 어떤 벡터량이 단위 미소 부피당 밖으로 빠져나가는 양

• 미소 부피의 각 미소 경계면들을 통과하는 평균 벡터량의 단위길이당 변화를 생각하면 됨

• 한 번에 한 변수만 생각-해당 단위길이는 편미분으로 바뀌어야 함

직교좌표계의 발산

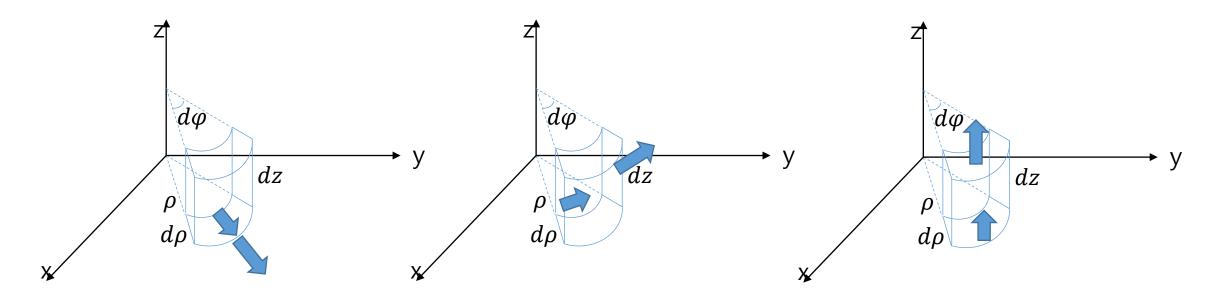


$$dx$$
당 평균 통과 벡터량
$$= \frac{\partial (A_x dy dz)}{dy dz \partial x} = \frac{\partial (A_x)}{\partial x}$$

$$dy$$
당 평균 통과 벡터량
$$= \frac{\partial (A_y dx dz)}{\partial x dz(\partial y)} = \frac{\partial (A_y)}{\partial y}$$

$$dz$$
당 평균 통과 벡터량
$$= \frac{\partial (A_z dx dy)}{\partial x dy \partial z} = \frac{\partial (A_z)}{\partial z}$$

원통좌표계의 발산

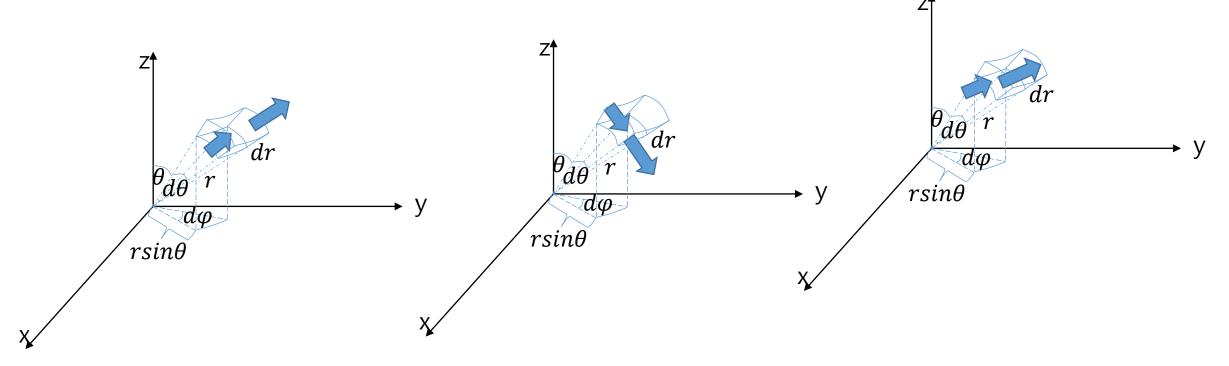


$$d\rho$$
당 평균 통과 벡터량
$$= \frac{\partial (A_{\rho}\rho d\varphi dz)}{\rho d\varphi dz \partial \rho} = \frac{\partial (A_{\rho}\rho)}{\rho \partial \rho}$$

$$\rho d\varphi$$
당 평균 통과 벡터량
$$= \frac{\partial (A_{\varphi}dzd\rho)}{dzd\rho(\rho\partial\varphi)} = \frac{\partial (A_{\varphi})}{\rho\partial\varphi}$$

$$dz$$
당 평균 통과 벡터량
$$= \frac{\partial (A_z \rho d\varphi d\rho)}{\rho d\varphi d\rho \partial z} = \frac{\partial (A_z)}{\partial z}$$

구면좌표계의 발산



$$dr$$
당 평균 통과 벡터량
$$= \frac{\partial (A_r r d\theta r s in\theta d\phi)}{r d\theta r s in\theta d\phi \partial r} = \frac{\partial (A_r r^2)}{r^2 \partial r}$$

$$rd\theta$$
당 평균 통과 벡터량
$$= \frac{\partial (A_{\theta}dr(rsin(\theta)d\varphi))}{dr(rsin(\theta)d\varphi)(r\partial\theta)} = \frac{\partial (A_{\theta}sin\theta)}{\sin(\theta)r\partial\theta}$$

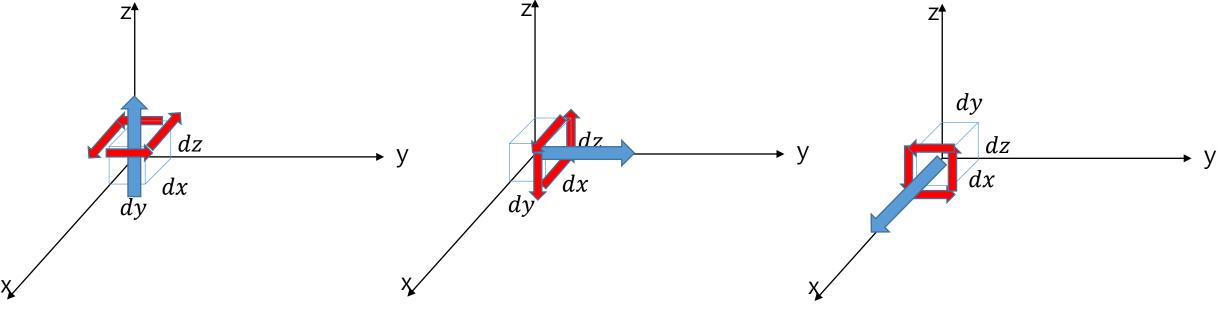
$$rsin(\theta)d\varphi$$
당 평균 통과 벡터량
$$= \frac{\partial (\overrightarrow{A_{\varphi}}rd\theta dr)}{rd\theta dr(rsin(\theta)\partial\varphi)} = \frac{\partial (\overrightarrow{A_{\varphi}})}{rsin(\theta)\partial\varphi}$$

회전이란?

• 어떤 벡터량이 단위 미소 면적당 그 가장자리를 따라 회전하는 정도

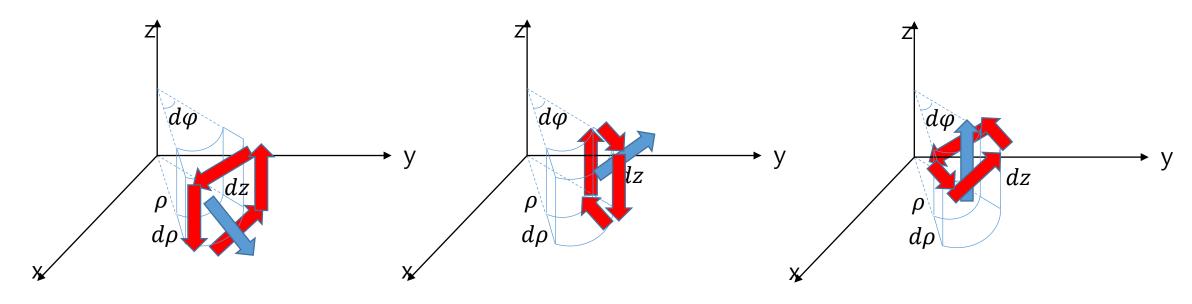
• 미소 면적의 각 미소 경계 경로선을 따라갔을 때의 단위길이당 평균 벡터량 차이

직교좌표계의 회전



 $\overrightarrow{a_z}$ 를 중심으로 하는 단위길이당 평균 회전 벡터량 = $-\frac{\partial (A_x dx)}{\partial x \partial y} + \frac{\partial (A_y dy)}{\partial y \partial x}$ = $-\frac{\partial (A_x)}{\partial x} + \frac{\partial (A_y)}{\partial x}$ $\overrightarrow{a_y}$ 를 중심으로 하는 단위길이당 평균 회전 벡터량 = $-\frac{\partial (A_z dz)}{\partial z \partial x} + \frac{\partial (A_x dx)}{\partial x \partial z}$ = $-\frac{\partial (A_z)}{\partial x} + \frac{\partial (A_x)}{\partial z}$ $\overrightarrow{a_x}$ 를 중심으로 하는 단위길이당 평균 회전 벡터량 = $-\frac{\partial(A_y dy)}{\partial y \partial z} + \frac{\partial(A_z dz)}{\partial z \partial y}$ = $-\frac{\partial(A_y)}{\partial z} + \frac{\partial(A_z)}{\partial z}$

원통좌표계의 회전



 $\overrightarrow{a_{\rho}}$ 를 중심으로 하는 단위길이당 평균 회전 벡터량

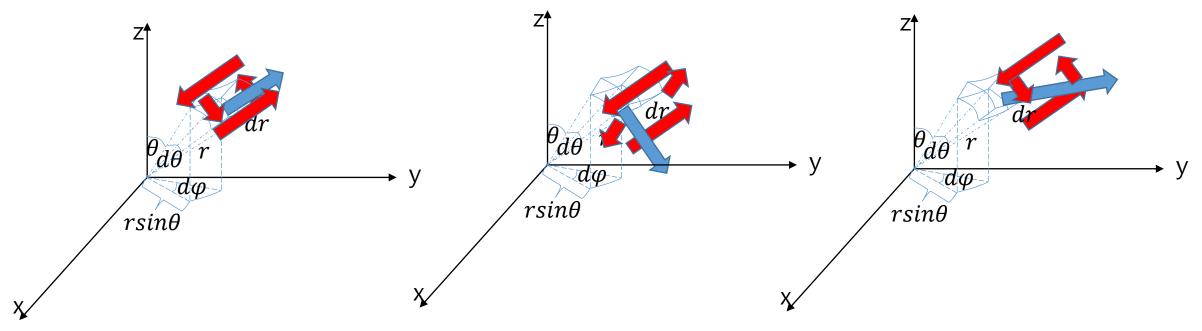
$$= -\frac{\partial (A_{\varphi}\rho d\varphi)}{\rho d\varphi \partial z} + \frac{\partial (A_{z}dz)}{\partial z \rho \partial \varphi}$$
$$= -\frac{\partial (A_{\varphi})}{\partial z} + \frac{\partial (A_{z})}{\rho \partial \varphi}$$

 $\overrightarrow{a_{\varphi}}$ 를 중심으로 하는 단위길이당 평균 회전 벡터량 = $-\frac{\partial (A_z dz)}{\partial z} + \frac{\partial (A_\rho d\rho)}{\partial z}$

$$= -\frac{\partial (A_z dz)}{\partial z \partial \rho} + \frac{\partial (A_\rho d\rho)}{\partial \rho \partial z}$$
$$= -\frac{\partial (A_z)}{\partial \rho} + \frac{\partial (A_\rho)}{\partial z}$$

 $\overrightarrow{a_z}$ 를 중심으로 하는 단위길이당 평균 회전 벡터량 = $-\frac{\partial(A_\rho d\rho)}{\partial \rho \partial \varphi} + \frac{\partial(A_\varphi \rho d\varphi)}{\rho \partial \varphi \partial \rho}$ = $-\frac{\partial(A_\rho)}{\partial \varphi} + \frac{\partial(A_\varphi \rho)}{\rho \partial \varphi}$

구면좌표계의 회전



 $\overrightarrow{a_r}$ 을 중심으로 하는 단위길이당 평균 회전 벡터량

$$\begin{split} &= -\frac{\partial (A_{\theta} r d\theta)}{r d\theta r sin(\theta) \partial \varphi} + \frac{\partial \left(A_{\varphi} r sin(\theta) d\varphi\right)}{r sin(\theta) d\varphi r \partial \theta} \\ &= -\frac{\partial (A_{\theta})}{r sin(\theta) \partial \varphi} + \frac{\partial \left(A_{\varphi} sin(\theta)\right)}{r sin(\theta) \partial \theta} \end{split}$$

 $\overrightarrow{a_{\theta}}$ 을 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial \left(A_{\varphi} r sin(\theta) d\varphi\right)}{r sin(\theta) d\varphi r \partial r} + \frac{\partial (A_r dr)}{r dr sin(\theta) \partial \varphi}$$

$$= -\frac{\partial \left(A_{\varphi} r\right)}{r r \partial r} + \frac{\partial (A_r)}{r sin(\theta) \partial \varphi}$$

 $\overrightarrow{a_{\varphi}}$ 을 중심으로 하는 단위길이당 평균 회전 벡터량 = $-\frac{\partial(A_r dr)}{rdr\partial\theta} + \frac{\partial(A_{\theta} r d\theta)}{rd\theta\partial r}$

$$= -\frac{\partial (A_r dr)}{r dr \partial \theta} + \frac{\partial (A_\theta r d\theta)}{r d\theta \partial r}$$
$$= -\frac{\partial (A_r)}{r \partial \theta} + \frac{\partial (A_\theta r)}{r \partial r}$$