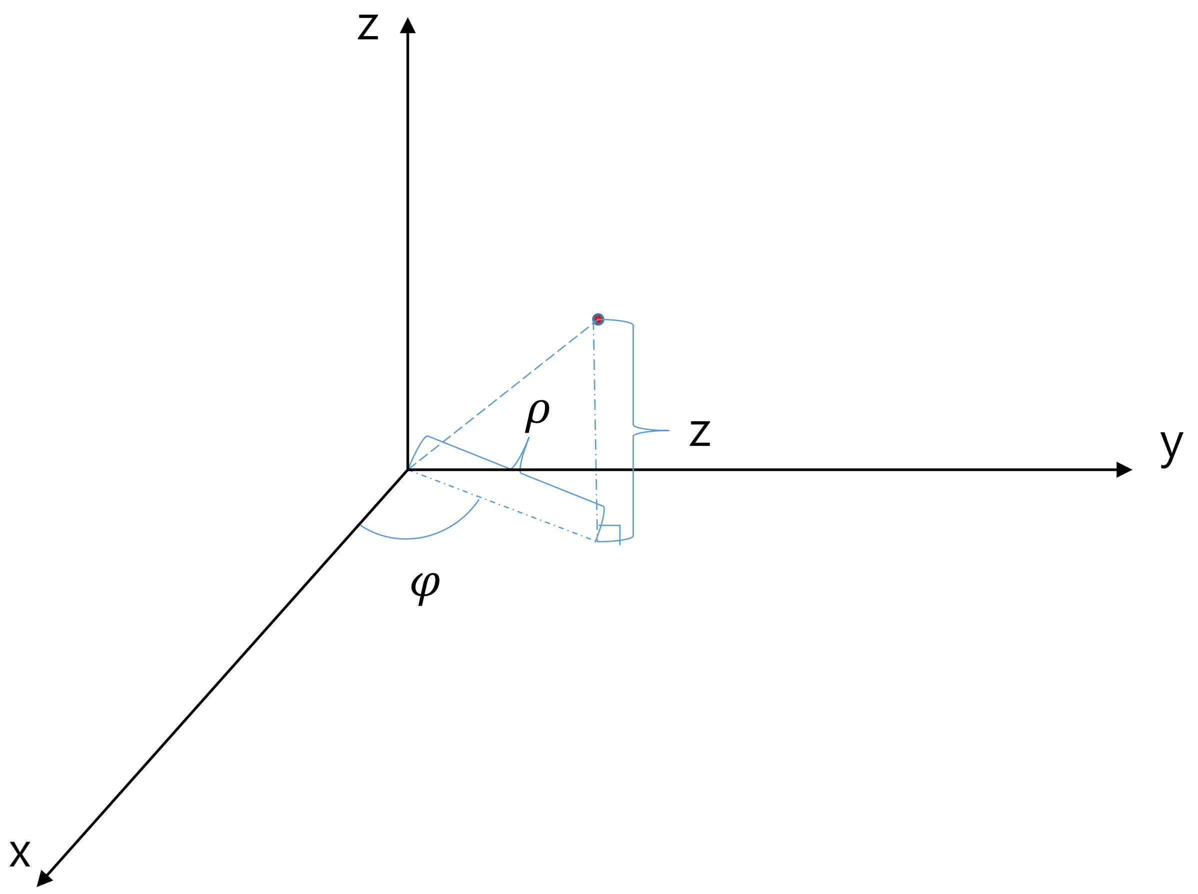
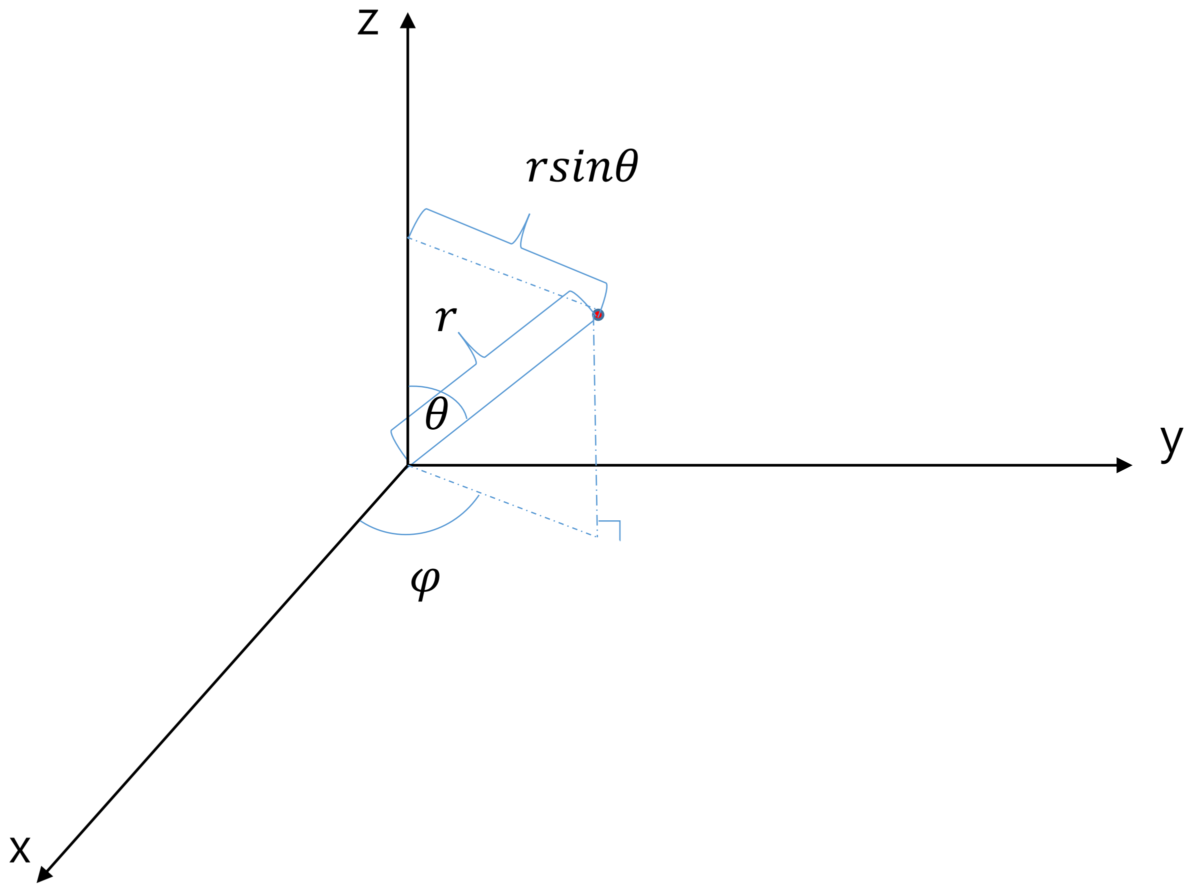
벡터 미분 연산자(델 연산자)

Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ρ, φ, z)	Spherical coordinates (r, θ, φ), where θ is the polar angle and φ is the azimuthal angle[α]
Vector field $\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\varphi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\varphi\hat{\boldsymbol{\varphi}}$
Gradient ∇f[1]	$\dfrac{\partial f}{\partial x}\hat{\mathbf{x}} + \dfrac{\partial f}{\partial y}\hat{\mathbf{y}} + \dfrac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\dfrac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \dfrac{1}{\rho}\dfrac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \dfrac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\dfrac{\partial f}{\partial r}\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \dfrac{1}{r\sin\theta}\dfrac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$
Divergence $\nabla \cdot \mathbf{A}$[1]	$\dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$	$\dfrac{1}{\rho}\dfrac{\partial(\rho A_\rho)}{\partial \rho} + \dfrac{1}{\rho}\dfrac{\partial A_\varphi}{\partial \varphi} + \dfrac{\partial A_z}{\partial z}$	$\dfrac{1}{r^2}\dfrac{\partial(r^2 A_r)}{\partial r} + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \theta}(A_\theta\sin\theta) + \dfrac{1}{r\sin\theta}\dfrac{\partial A_\varphi}{\partial \varphi}$
Curl $\nabla \times \mathbf{A}$[1]	$\left(\dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\dfrac{1}{\rho}\dfrac{\partial A_z}{\partial \varphi} - \dfrac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\dfrac{\partial A_\rho}{\partial z} - \dfrac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \dfrac{1}{\rho}\left(\dfrac{\partial(\rho A_\varphi)}{\partial \rho} - \dfrac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$	$\dfrac{1}{r\sin\theta}\left(\dfrac{\partial}{\partial \theta}(A_\varphi\sin\theta) - \dfrac{\partial A_\theta}{\partial \varphi}\right)\hat{\mathbf{r}} + \dfrac{1}{r}\left(\dfrac{1}{\sin\theta}\dfrac{\partial A_r}{\partial \varphi} - \dfrac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}} + \dfrac{1}{r}\left(\dfrac{\partial}{\partial r}(rA_\theta) - \dfrac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\varphi}}$

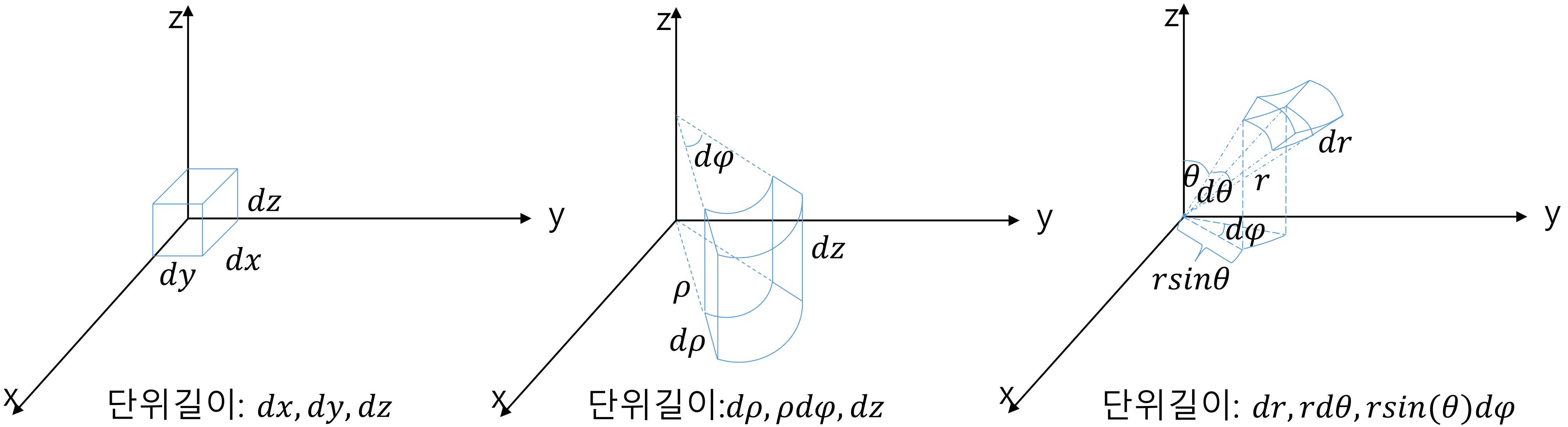
원통 좌표계



구면 좌표계



각 좌표계의 미소 성분들



단위길이: dx, dy, dz

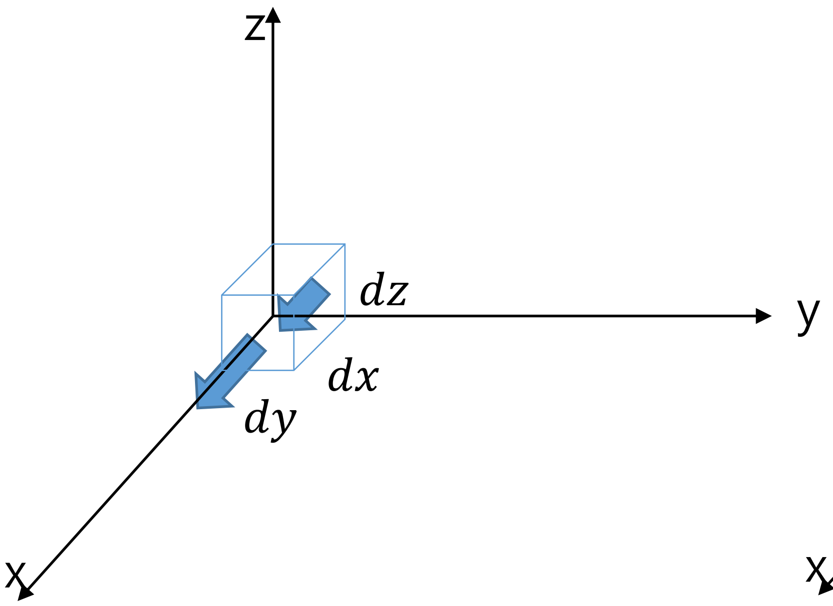
단위길이:$d\rho, \rho d\varphi, dz$

단위길이: $dr, rd\theta, rsin(\theta)d\varphi$

발산이란?

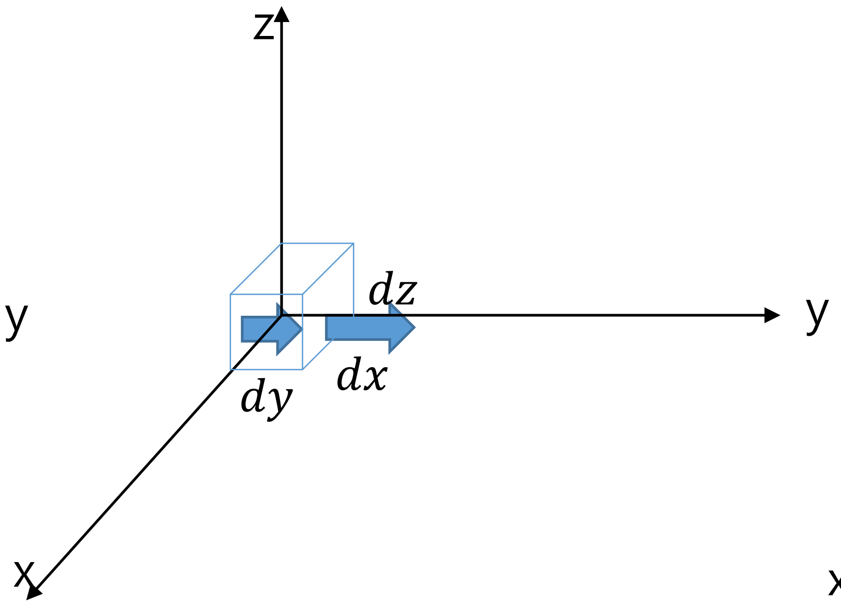
- 어떤 벡터량이 단위 미소 부피당 밖으로 빠져나가는 양
- 미소 부피의 각 미소 경계면들을 통과하는 평균 벡터량의 단위길이당 변화를 생각하면 됨
- 한 번에 한 변수만 생각-해당 단위길이는 편미분으로 바뀌어야 함

직교좌표계의 발산



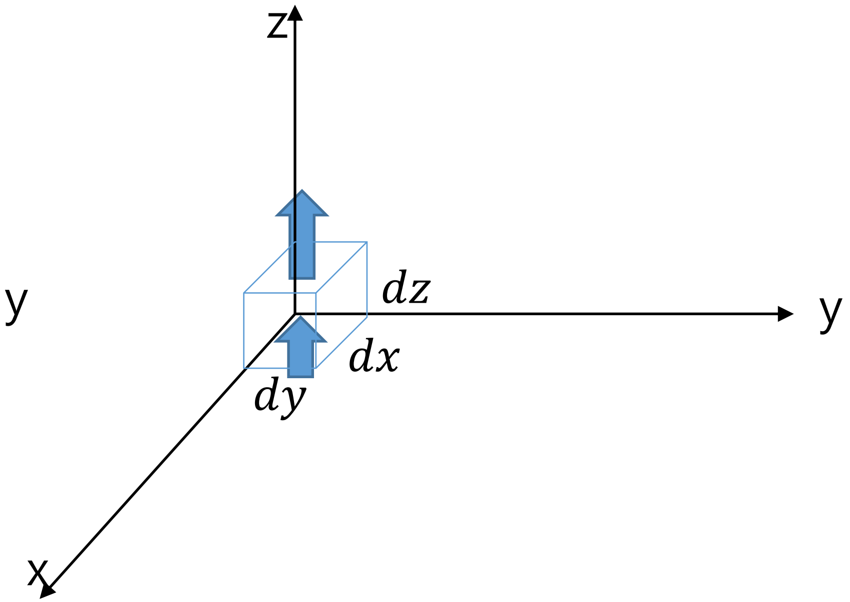
dx당 평균 통과 벡터량

$$= \frac{\partial(A_x dydz)}{dydz\partial x} = \frac{\partial(A_x)}{\partial x}$$



dy당 평균 통과 벡터량

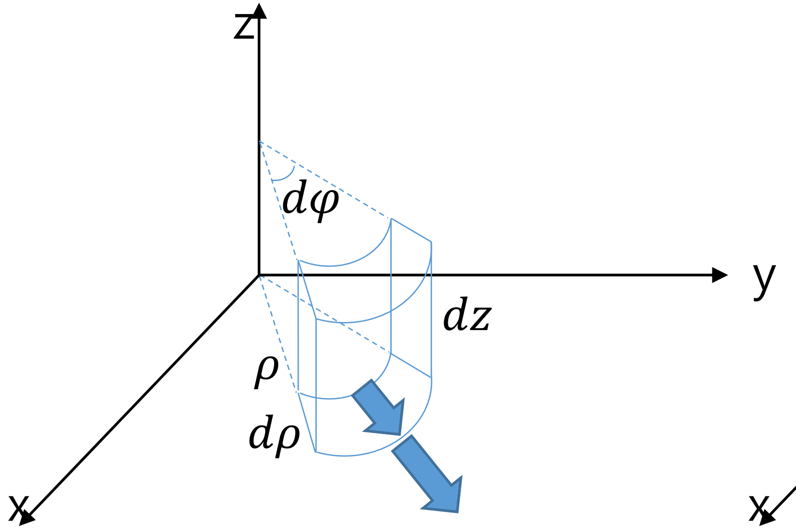
$$= \frac{\partial(A_y dxdz)}{dxdz(\partial y)} = \frac{\partial(A_y)}{\partial y}$$



dz당 평균 통과 벡터량

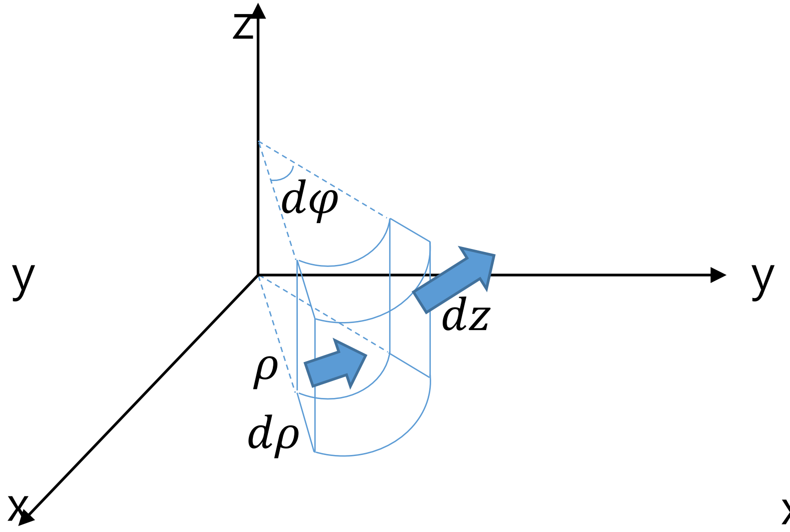
$$= \frac{\partial(A_z dxdy)}{dxdy\partial z} = \frac{\partial(A_z)}{\partial z}$$

원통좌표계의 발산



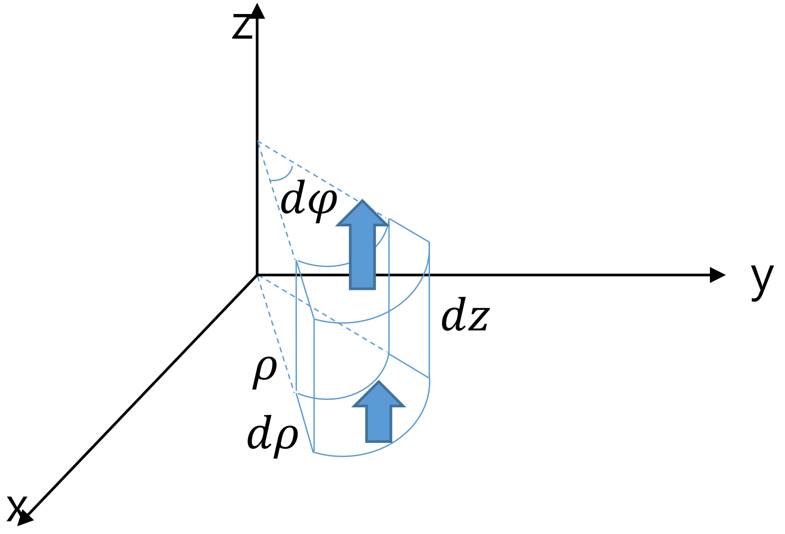
$d\rho$당 평균 통과 벡터량

$$= \frac{\partial(A_\rho \rho d\varphi dz)}{\rho d\varphi dz \partial\rho} = \frac{\partial(A_\rho \rho)}{\rho \partial\rho}$$



$\rho d\varphi$당 평균 통과 벡터량

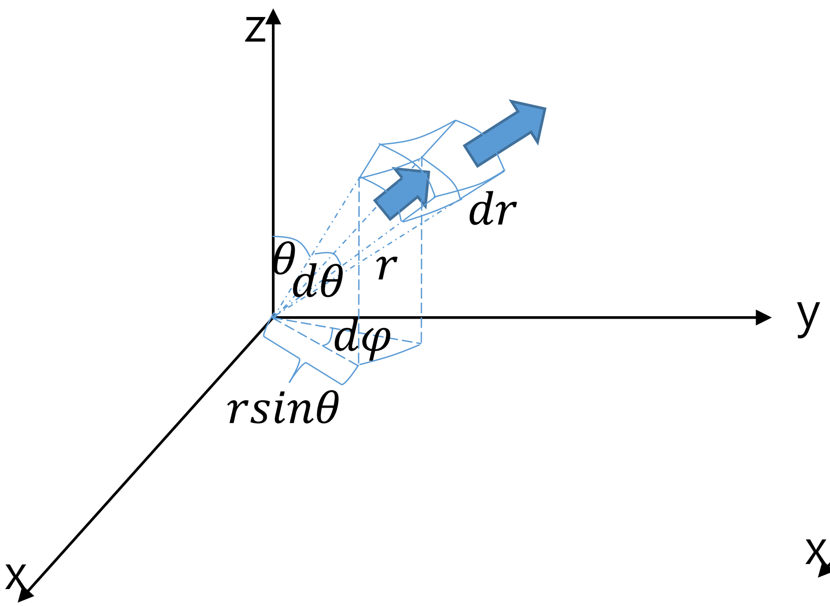
$$= \frac{\partial(A_\varphi dz d\rho)}{dz d\rho(\rho \partial\varphi)} = \frac{\partial(A_\varphi)}{\rho \partial\varphi}$$

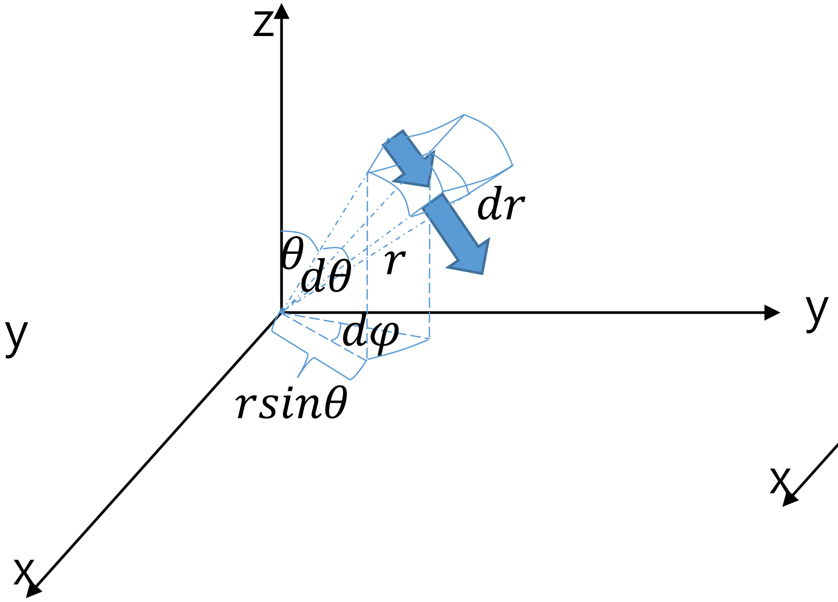


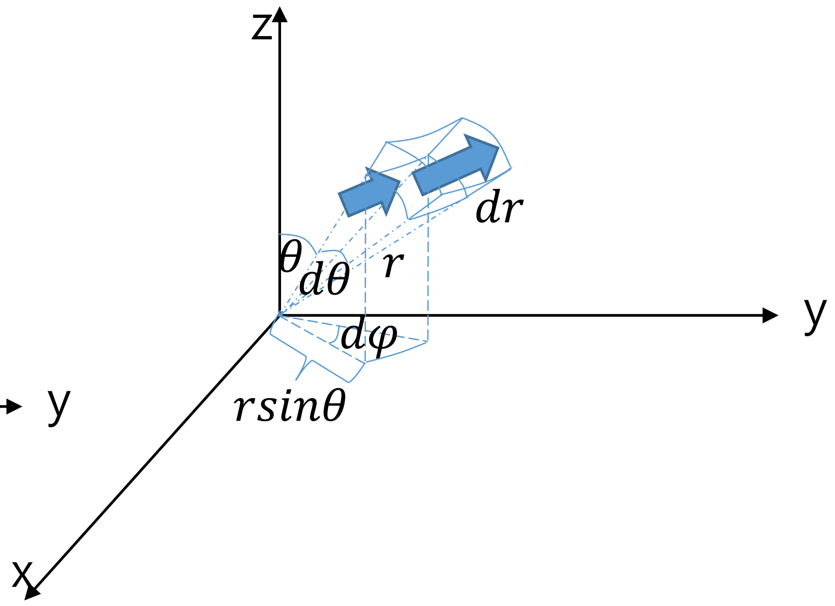
dz당 평균 통과 벡터량

$$= \frac{\partial(A_z \rho d\varphi d\rho)}{\rho d\varphi d\rho \partial z} = \frac{\partial(A_z)}{\partial z}$$

구면좌표계의 발산







dr당 평균 통과 벡터량

$$= \frac{\partial(A_r r d\theta r sin\theta d\varphi)}{r d\theta r sin\theta d\varphi \partial r} = \frac{\partial(A_r r^2)}{r^2 \partial r}$$

$rd\theta$당 평균 통과 벡터량

$$= \frac{\partial(A_\theta dr(rsin(\theta)d\varphi))}{dr(rsin(\theta)d\varphi)(r\partial\theta)} = \frac{\partial(A_\theta sin\theta)}{\sin(\theta) r\partial\theta}$$

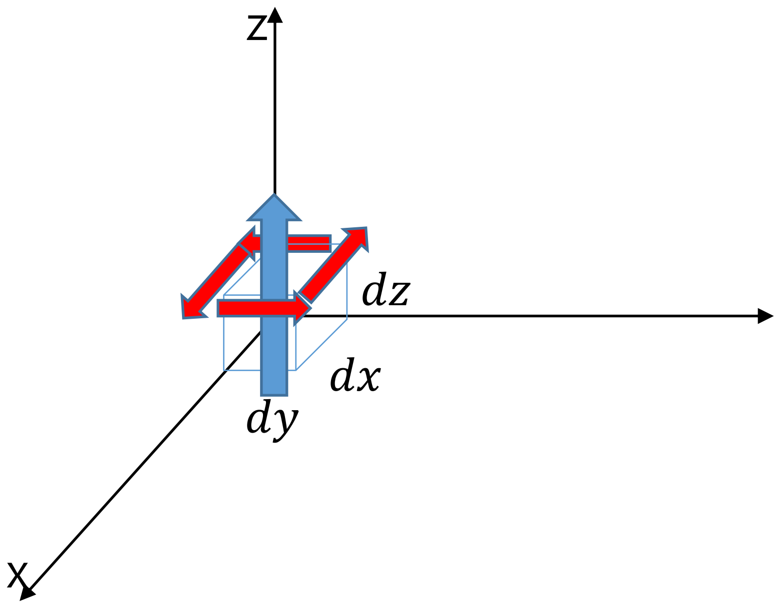
$rsin(\theta)d\varphi$당 평균 통과 벡터량

$$= \frac{\partial(\overrightarrow{A_\varphi} r d\theta dr)}{r d\theta dr(rsin(\theta)\partial\varphi)} = \frac{\partial(\overrightarrow{A_\varphi})}{rsin(\theta)\partial\varphi}$$

회전이란?

- 어떤 벡터량이 단위 미소 면적당 그 가장자리를 따라 회전하는 정도
- 미소 면적의 각 미소 경계 경로선을 따라갔을 때의 단위길이당 평균 벡터량 차이

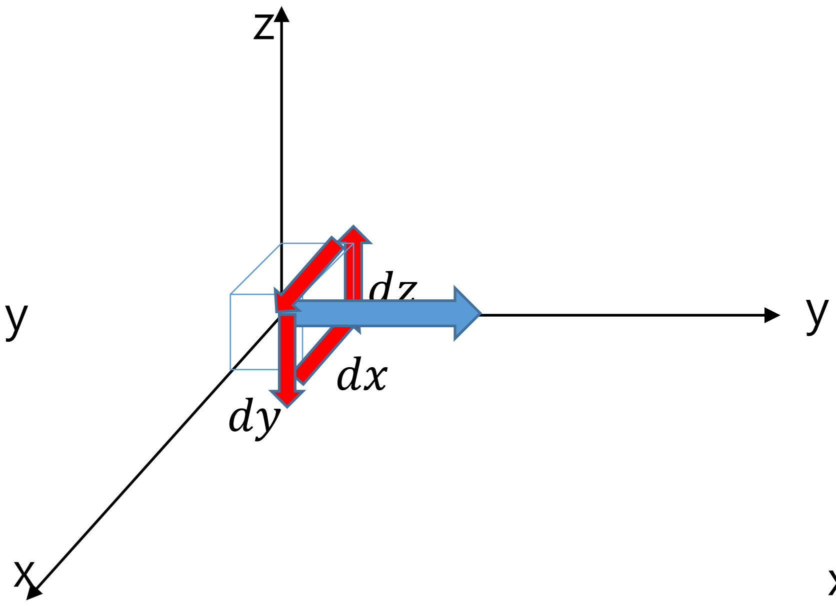
직교좌표계의 회전



$\overrightarrow{a_z}$를 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial(A_x dx)}{dx\partial y} + \frac{\partial(A_y dy)}{dy\partial x}$$

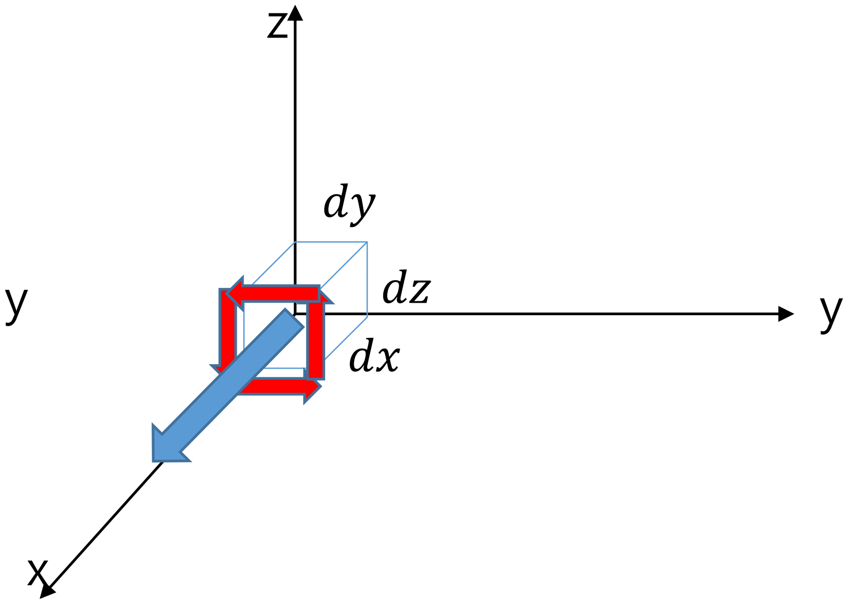
$$= -\frac{\partial(A_x)}{\partial y} + \frac{\partial(A_y)}{\partial x}$$



$\overrightarrow{a_y}$를 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial(A_z dz)}{dz\partial x} + \frac{\partial(A_x dx)}{dx\partial z}$$

$$= -\frac{\partial(A_z)}{\partial x} + \frac{\partial(A_x)}{\partial z}$$

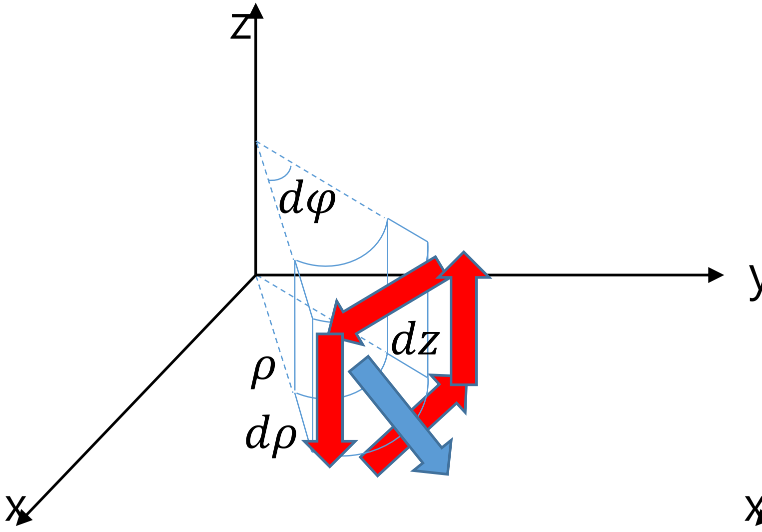


$\overrightarrow{a_x}$를 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial(A_y dy)}{dy\partial z} + \frac{\partial(A_z dz)}{dz\partial y}$$

$$= -\frac{\partial(A_y)}{\partial z} + \frac{\partial(A_z)}{\partial y}$$

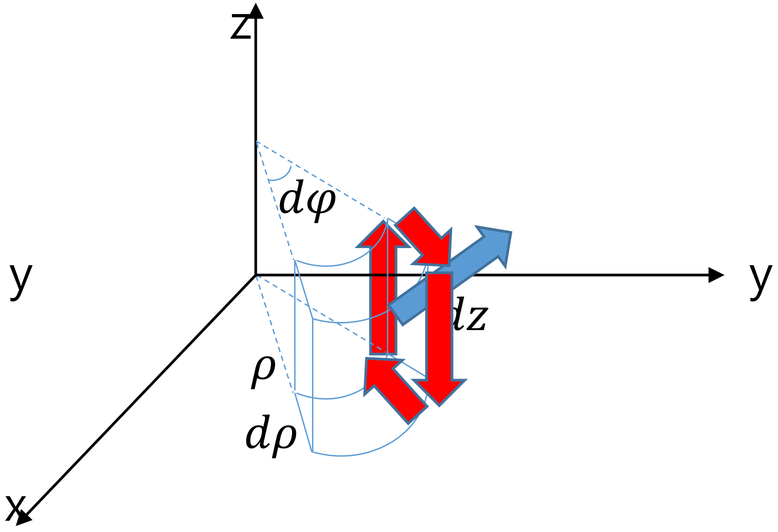
원통좌표계의 회전



$\overrightarrow{a_\rho}$를 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial(A_\varphi \rho d\varphi)}{\rho d\varphi \partial z} + \frac{\partial(A_z dz)}{dz \rho \partial \varphi}$$

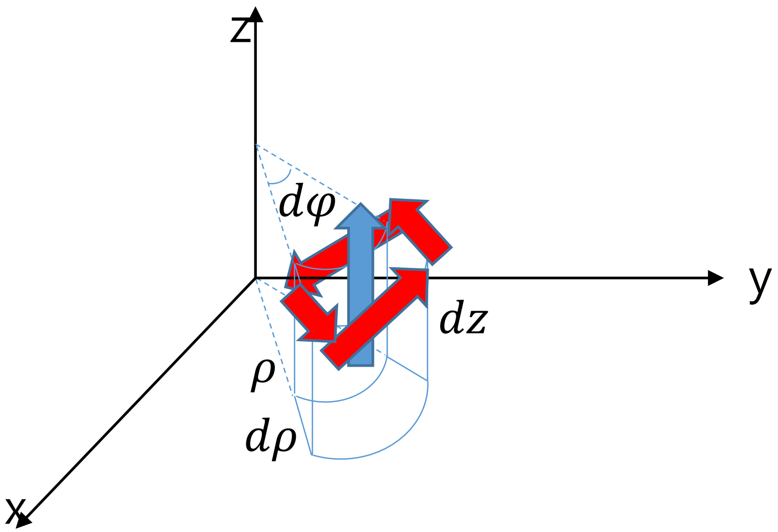
$$= -\frac{\partial(A_\varphi)}{\partial z} + \frac{\partial(A_z)}{\rho \partial \varphi}$$



$\overrightarrow{a_\varphi}$를 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial(A_z dz)}{dz \partial \rho} + \frac{\partial(A_\rho d\rho)}{d\rho \partial z}$$

$$= -\frac{\partial(A_z)}{\partial \rho} + \frac{\partial(A_\rho)}{\partial z}$$

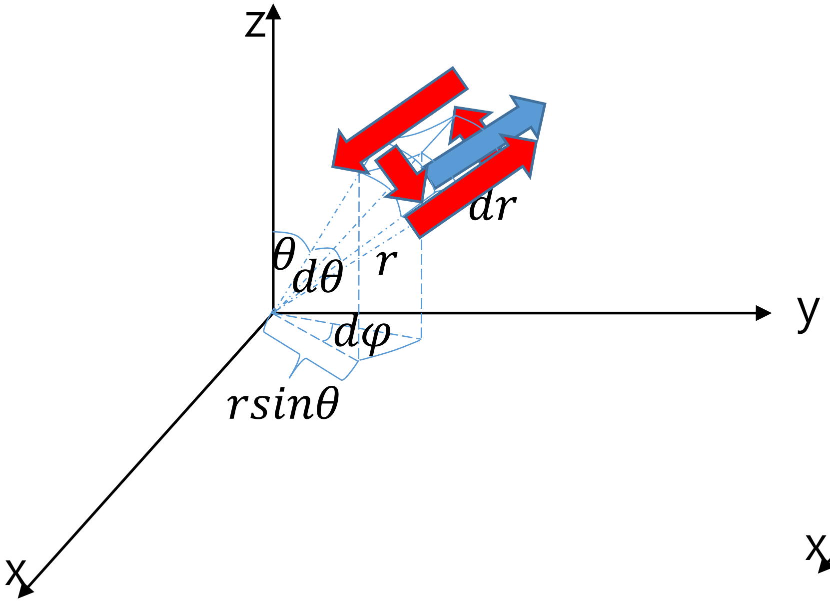


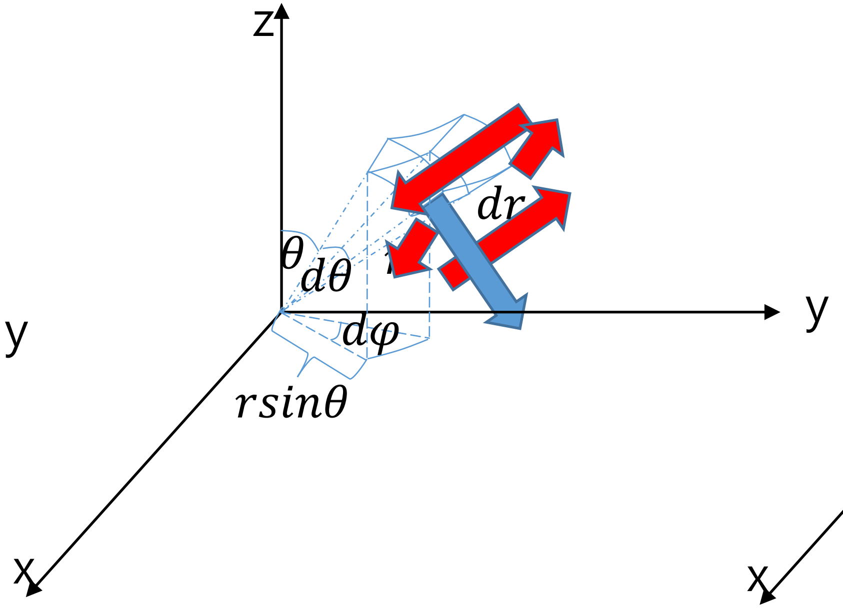
$\overrightarrow{a_z}$를 중심으로 하는 단위길이당 평균 회전 벡터량

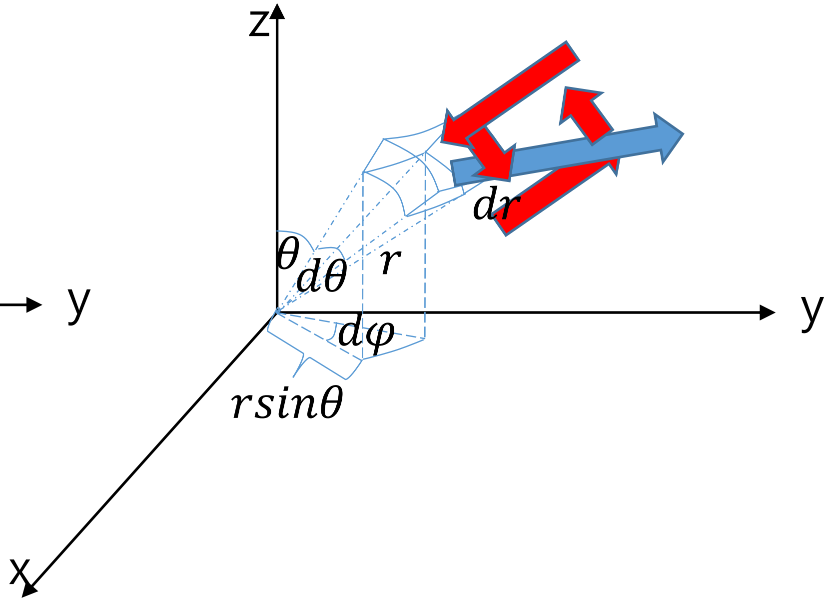
$$= -\frac{\partial(A_\rho d\rho)}{d\rho \partial \varphi} + \frac{\partial(A_\varphi \rho d\varphi)}{\rho d\varphi \partial \rho}$$

$$= -\frac{\partial(A_\rho)}{\partial \varphi} + \frac{\partial(A_\varphi \rho)}{\rho \partial \rho}$$

구면좌표계의 회전







$\overrightarrow{a_r}$을 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial(A_\theta r d\theta)}{r d\theta r\sin(\theta)\partial\varphi} + \frac{\partial\left(A_\varphi r\sin(\theta)d\varphi\right)}{r\sin(\theta)d\varphi r\partial\theta}$$

$$= -\frac{\partial(A_\theta)}{r\sin(\theta)\partial\varphi} + \frac{\partial\left(A_\varphi \sin(\theta)\right)}{r\sin(\theta)\partial\theta}$$

$\overrightarrow{a_\theta}$을 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial\left(A_\varphi r\sin(\theta)d\varphi\right)}{r\sin(\theta)d\varphi r\partial r} + \frac{\partial(A_r dr)}{r dr\sin(\theta)\partial\varphi}$$

$$= -\frac{\partial\left(A_\varphi r\right)}{r r\partial r} + \frac{\partial(A_r)}{r\sin(\theta)\partial\varphi}$$

$\overrightarrow{a_\varphi}$을 중심으로 하는 단위길이당 평균 회전 벡터량

$$= -\frac{\partial(A_r dr)}{r dr\partial\theta} + \frac{\partial(A_\theta r d\theta)}{r d\theta\partial r}$$

$$= -\frac{\partial(A_r)}{r\partial\theta} + \frac{\partial(A_\theta r)}{r\partial r}$$