MAT1856/APM466 Assignment 1

Inho Kim, Student #: 1006751294

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Fundamental Questions - 25 points

1.
   (a) Governments issue bonds instead of printing more money as it would allow them to receive capital without causing inflation or destabilizing their currency (which printing more money every time the government needed capital would do).

   (b) An example of why the long-term part of a yield curve might flatten is when market participants anticipate a recession in the future, thus increasing demand for long-term bonds as safe assets, which in turn lowers their yield and flattens the long-term yield curve.

   (c) Quantitative easing is a monetary policy strategy that central banks use to purchase securities at a large scale (such as bonds or stocks), thus increasing the money circulating in the economy and thereby lowering longer-term interest rates - and the US (Fed) has been employing quantitative easing without a dollar or time limit in March 2020, giving the Fed the flexibility to keep purchasing assets for as long as was deemed necessary to spur economic activity. (Jackson, 2023)

2. I chose 10 bonds that had maturity dates that were evenly distributed every 6 months between March 1, 2024 and March 1, 2029. All the bonds I chose mature in March or September, and thus offer a consistent way of analyzing bonds across different maturity dates.

   The 10 bonds I chose were (in format ISIN / bond name):

   (a) CA135087J546 / CAN 2.25 Mar 1 2024
   (b) CA135087J967 / CAN 1.50 Sep 1 2024
   (c) CA135087K528 / CAN 1.25 Mar 1 2025
   (d) CA135087K940 / CAN 0.50 Sep 1 2025
   (e) CA135087L518 / CAN 0.25 Mar 1 2026
   (f) CA135087L930 / CAN 1.00 Sep 1 2026
   (g) CA135087M847 / CAN 1.25 Mar 1 2027
   (h) CA135087N837 / CAN 2.75 Sep 1 2027
   (i) CA135087P576 / CAN 3.50 Mar 1 2028
   (j) CA135087Q491 / CAN 3.25 Sep 1 2028

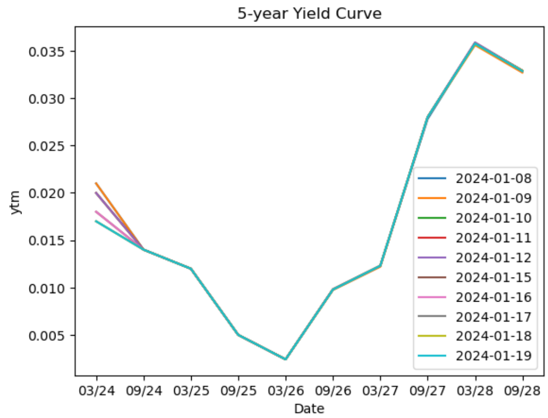
   The bonds I chose and their characteristics are also viewable on this spreadsheet here: Bond data

3. In general, the eigenvalues and eigenvectors associated with the covariance matrix of those stochastic processes would tell us the magnitude/importance and the direction of the variance in the data respectively, allowing us to analyze the importance of the data and their trends. The magnitude of the eigenvalues would determine how dominant the direction of variations are, while the eigenvectors would determine the main directions of the variation in the data.

Empirical Questions - 75 points

4.

(a) The 5-year yield curve (ytm curve) corresponding to each day is shown below:



(b) To derive the spot curve, the variables needed are: 1) dp = dirty price. 2) p = clean price 3) cr = coupon rate. 3) s = spot rate. 4) c = coupon payment. 5) Subscripts denote that the variable corresponds to a certain maturity. For example, s_t would mean that it is the 'spot rate of maturity t'.

i. Calculate the dirty price for each bond.

A. dp = p + cr × (183 - (31 - date_number) - 29) / 365

ii. Create an empty list to store 's_t' for all maturities, spot_list = [empty list]

iii. Calculate the spot rate of the bond that matures the earliest.

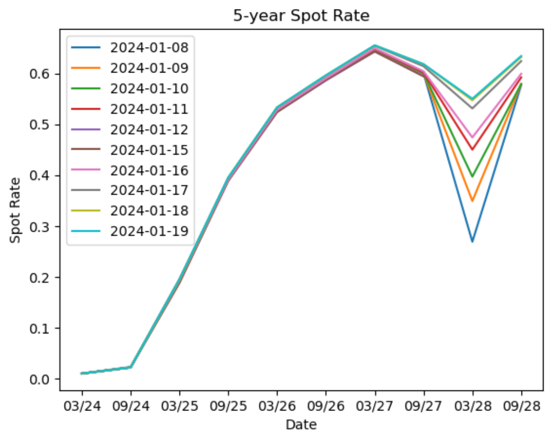
A. $s_1 = (\frac{100 + c_1}{dp_1}) - 1$

iv. Calculate the spot rates of all following bonds by solving for 's_i' in the following equation, then append it to the spot_list. This value will then be called in the subsequent iterations of the bootstrap method.

A. $dp_n = \sum_{t=1}^{n-1}[\frac{c_n}{(1+s_t)^t} + \frac{c_n + 100}{(1+s_n)^n}]$

v. Repeat Steps i-vi for each date in Jan.8-22.

The 5-year spot curve corresponding to each day is shown below:



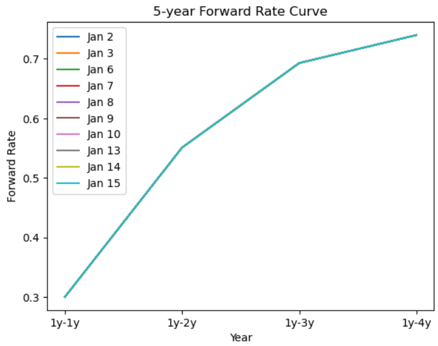
(c) To derive the one-year forward curve, the variables needed are: 1) $F_{t,\ t+n}$ is the forward rate for the period between time t and t+n. 2) S_t is the spot rate of time t.

i. Calculate $F_{t,\ t+n}$ by setting March 2024 as n = 1, and iterating through $F_{1,\ 2}$, $F_{1,\ 3}$, $F_{1,\ 4}$, and $F_{1,\ 5}$.

A. $F_{t,t+n} = [\frac{(1+S_{t+n})^{2(t+n)}}{(1+S_t)^{2t}}]^{1/(2n)} - 1$

ii. Repeat for each date in Jan.8-22.

The one-year forward curve corresponding to each day is shown below:



5. The covariance matrix for the time series of daily log-returns of yield is:

$$\begin{matrix} 7.31e^{-3} & 0 & 0 & 0 & -6.16e^{-6} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -6.16e^{-6} & 0 & 0 & 0 & 6.33e^{-6} \end{matrix}$$

The covariance matrix for the time series of daily log-returns of forward rates is:

$$\begin{matrix} 0.07738248 & 0.07738248 & 0.07738248 & 0.07738248 \\ 0.07738248 & 0.07738248 & 0.07738248 & 0.07738248 \\ 0.07738248 & 0.07738248 & 0.07738248 & 0.07738248 \\ 0.07738248 & 0.07738248 & 0.07738248 & 0.07738248 \end{matrix}$$

6. The first eigenvalue and its associated eigenvector implies the largest variation in the data (magnitude shown by the eigenvalue) in the direction of the eigenvector.

The eigenvalues of the yield covariance matrix is: [7.30e-03 6.33e-06 0 0 0]. The eigenvectors of the yield covariance matrix is:

$$\begin{matrix} 1 & 8.44e^{-4} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -8.44e^{-4} & 1 & 0 & 0 & 0 \end{matrix}$$

The eigenvalues of the forward covariance matrix is: [2.78e-17 3.10e-01 -2.04e-35 0]. The eigenvectors of the forward covariance matrix is:

$$\begin{matrix} -8.66e^{-1} & -5e^{-1} & -3.11e^{-18} & 8.15e^{-18} \\ 2.89e^{-1} & -5e^{-1} & 8.16e^{-1} & -6.07e^{-16} \\ 2.89e^{-1} & -5e^{-1} & -4.08e^{-1} & -7.07e^{-1} \\ 2.89e^{-1} & -5e^{-1} & -4.08e^{-1} & 7.07e^{-1} \end{matrix}$$

References and GitHub Link to Code

GitHub Link
Bond data

Works Cited:

1. Jackson, A.-L. (2023, March 18). Quantitative easing explained. Forbes. https://www.forbes.com/advisor/investing/easing-qe/