Supplementary material for "Improving Maximum k-plex Solver via Second-order Reduction and Graph Color Bounding"

This is the supplementary material for manuscript "Improving Maximum k-plex Solver via Second-order Reduction and Graph Color Bounding". The remaining contents are organized as follows.

- In Section 1, we describe the heuristic search algorithm of Maplex.
- In Section 2, we complete the missing proofs.
- In Section 3, we show the complete data of the experiments.

1 Heuristic algorithm

We show the procedure of HeuristicSolution($G = (V, E), k$) in Alg. 1

Algorithm 1:

```
1  HeuristicSolution(G = (V, E), k)
2  begin
3      Sort all vertices of G by degeneracy order, denote the order as v_1, ...., v_n
4      Find the first v_i such that the degree of v_i in the subgraph induced by {v_i, ..., v_n} is at least
       n − i + 1 − k.
5      S ← {v_i, ..., v_n}                                          ▷ S is a k-plex
6      for v_j ← v_i, ..., v_1 do
7          if c(v_j) ≤ |S| − k then
8              Break
9          if S ∪ {v_j} is a k-plex then
10             S ← S ∪ {v_k}
11     return |S|
```

The heuristic algorithm relies on the *degeneracy ordering* of the vertices. A degeneracy ordering is of all vertices $G = (V, E)$ is a permutation $v_1, ..., v_n$ ($n = |V|$) such that every vertex v_i has the smallest degree in the subgraph induced by $\{v_{i+1}, ..., v_n\}$. A degeneracy ordering of graph G can be obtained in time $O(m)$ where m is the number of edges in G. [1] Given a degeneracy order $v_1, ..., v_n$, let us denote the degree of v_i in the subgraph induced by $\{v_{i+1}, ..., v_n\}$ as $c(v_i)$, a.k.a. the core number of v_i. $c(v_i)$ can be obtained along with the degeneracy ordering algorithm. So, if $c(i)$ is at lest $n - i + 1 - k$, then $v_i, ..., v_n$ is a k-plex. In lines 2-5 in Alg. 1, we find the first vertex v_i such that $v_i, ..., v_n$ is a k-plex. From line 6 to 10, we find the vertices preceding v_i that can form a larger k-plex with S. Give a k-plex S, the time complexity

of checking whether another vertex can form a k-plex with S is bounded by $O(|S|^2)$. Hence, the overall complexity of HeuristicSearch is bounded by $O(m + |S|^2 n)$ where $|S|$ is the size of the solution.

2 Proofs to the propositions

Proposition 1. Given a graph $G = (V, E)$, if V can be partitioned into c disjoint independent sets $I_1, ..., I_c$, then $\sum_{i=1}^{c} \min\{|I_i|, k\}$ is the upper bound of the size of maximum k-plex in G, a.k.a. color-bound.

Proof. Suppose that S is a maximum k-plex of G. For any $i \in \{1, ..., c\}$, let $S_i = S \cap I_i$. By hereditary property, S_i is also an k-plex. Meanwhile, the size of maximum k-plex in an independent set I_i is $min\{|I_i|, k\}$. So, $|S_i| \leq min\{|I_i|, k\}$. In sum, $|S| = \sum_{i=1}^{c} |S_i| \leq \sum_{i=1}^{c} min\{|I_i|, k\}$. □

Proposition 2. Given a subproblem with a growing k-plex P, a candidate set C, assume $\mathcal{I} = \{I_1, ..., I_c\}$ is a coloring of C. For any vertex $u \in C$, the size of k-plex S that $u \in S$ and $P \subset S$ is bounded by

$$ub_u = \sum_{i=1, u \notin I_i}^{c} \min(|I_i \cap N_G(u)|, k) + (k - |P \setminus N_G(u)|) + |P|.$$

Proof. Let us partition S into three disjoint subsets, $S_1 = S \cap (C \cap N_G(u))$, $S_2 = S \cap (C \setminus N_G(u))$ and the remaining subset P. By hereditary property, S_1, S_2 and P are k-plexes. First, because $\mathcal{I}$ is a feasible coloring, $C \cap N_G(u)$ is partitioned into independent sets $I_1 \cap N_G(u), ..., I_c \cap N_G(u)$. So, $|S_1| \leq \sum_{i=1, u \notin I_i}^{c} \min(|I_i \cap N_G(u)|, k)$ by color-bound. Second, because $u \in S_2$, by the definition of k-plex, S_2 is bounded by $k - |P \setminus N_G(u)|$. Summing up the bounds, we get $S = S_1 \cup S_2 \cup P$ bounded by the given formula. □

3 Experimental result

We report the detailed data collected in our experiments. Here is an overview of the following data.

- In Tables 1 – 3, we report the experimental results on the 139 network repository graphs at `http://networkrepository.com/`. ⋆

⋆ The graphs can be also downloaded from `http://lcs.ios.ac.cn/~caisw/graphs.html`.

- In Tables 4 – 11, we show the performance of different preprocessing strategies on the 139 network repository graphs
- In Tables 12 – 18, we show the performance of different algorithm on the 139 kernel graphs. Note that the kernel graph is pre-reduced by our preprocessing method.
- In Tables 19 and 20, we show the performance of each algorithm on 43 SNAP and partitioning graphs. SNAP graphs can be downloaded from `http://snap.stanford.edu/data/` while partitioning graphs are kept at `https://www.cc.gatech.edu/dimacs10/downloads.shtml`. The 43 graphs were selected in [2,3].
- In Table 21, we show experiments on the 6 Erdös graphs which are obtained from `https://github.com/Lweb/KPLEX`.
- In Table 22, we report the experimental results on 80 clique graphs. Clique graphs are used in the Second DIMACS Implementation Challenge and can be downloaded from `http://networkrepository.com/dimacs.php`.
- In Tables 23 and 24, we show the experimental results on 2 groups of random graphs. These random graphs are provided in the supplementary package.

Table 1
Experimental results of real world graphs for $k = 2$, 3, 4 and 5.

graph			k=2					k=3					k=4					k=5				
name	**#vtx**	**#edges**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**
bio-celegans	453	2025	10	0.00	0.00	0.00	0.06	11	0.00	0.00	0.00	4.61	13	0.00	0.00	0.00	157.73	14	0.00	0.00	0.00	N/A
bio-diseasome	516	1188	11	0.00	0.00	0.00	0.05	11	0.00	0.00	0.00	2.22	11	0.00	0.00	0.00	292.66	11	**0.00**	0.05	0.01	N/A
bio-dmela	7393	25569	8	0.01	0.01	1.58	357.70	9	0.01	0.01	2.98	N/A	10	0.01	0.01	9.50	N/A	12	0.01	0.01	24.21	N/A
bio-yeast	1458	1948	6	0.00	0.00	0.01	2.20	7	0.00	0.00	0.01	684.55	7	0.00	0.00	0.01	N/A	8	**0.00**	0.01	0.01	N/A
ca-AstroPh	17903	196972	57	0.01	0.01	0.18	N/A	57	**0.01**	0.04	0.20	N/A	57	**0.02**	0.08	0.17	N/A	57	**0.02**	0.40	0.18	N/A
ca-citeseer	227320	814134	87	**0.07**	0.11	N/A	N/A	87	**0.08**	0.11	N/A	N/A	87	0.11	**0.10**	N/A	N/A	87	0.12	**0.11**	N/A	N/A
ca-coauthors-dblp	540486	15245729	337	**0.68**	6.93	N/A	N/A	337	**0.93**	6.40	N/A	N/A	337	**1.00**	6.58	N/A	N/A	337	**0.94**	6.71	N/A	N/A
ca-CondMat	21363	91286	26	0.01	0.01	0.06	N/A	26	0.01	0.01	0.06	N/A	26	0.01	0.01	0.06	N/A	26	**0.01**	0.02	0.06	N/A
ca-CSphd	1882	1740	4	0.00	0.00	0.06	7.28	5	**0.00**	0.03	0.06	N/A	6	0.00	0.00	0.06	N/A	7	0.08	**0.00**	0.06	N/A
ca-dblp-2010	226413	716460	75	**0.06**	0.09	N/A	N/A	75	**0.06**	0.09	N/A	N/A	75	**0.07**	0.09	N/A	N/A	75	**0.08**	0.09	N/A	N/A
ca-dblp-2012	317080	1049866	114	**0.13**	0.20	N/A	N/A	114	**0.14**	0.21	N/A	N/A	114	**0.14**	0.20	N/A	N/A	114	**0.17**	0.20	N/A	N/A
ca-Erdos992	5094	7515	8	0.00	0.00	0.01	611.05	9	0.00	0.00	0.01	N/A	10	0.00	0.00	0.01	N/A	11	0.00	0.00	0.01	N/A
ca-GrQc	4158	13422	44	0.00	0.00	0.01	25.41	45	0.00	0.00	0.01	N/A	46	0.00	0.00	0.01	N/A	46	0.00	0.00	0.01	N/A
ca-HepPh	11204	117619	239	0.02	0.02	0.06	66.24	239	0.02	0.02	0.06	N/A	239	0.02	0.02	0.07	N/A	239	0.02	0.02	0.06	N/A
ca-hollywood-2009	1069126	56306653	2209	14.72	**4.80**	N/A	N/A	2209	9.14	**5.12**	N/A	N/A	2209	8.59	**4.92**	N/A	N/A	2209	8.79	**4.83**	N/A	N/A
ca-MathSciNet	332689	820644	25	0.14	0.14	N/A	N/A	25	0.19	**0.13**	N/A	N/A	25	0.18	**0.16**	N/A	N/A	25	**0.17**	0.39	N/A	N/A
ca-netscience	379	914	9	0.00	0.00	0.00	0.04	9	0.00	0.00	0.00	2.23	9	0.00	0.00	0.00	234.06	10	0.00	0.02	0.00	N/A
socfb-A-anon	3097165	23667394	28	**57.72**	292.88	N/A	N/A	32	**63.32**	461.44	N/A	N/A	35	**64.20**	626.19	N/A	N/A	37	**57.02**	426.15	N/A	N/A
socfb-B-anon	2937612	20959854	27	**113.25**	1340.44	N/A	N/A	30	**623.20**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
socfb-Berkeley13	22900	852419	47	**3.39**	42.22	N/A	N/A	51	**45.14**	474.57	N/A	N/A	52	**179.38**	306.01	N/A	N/A	53	899.31	**33.64**	N/A	N/A
socfb-CMU	6621	249959	47	**0.56**	4.29	N/A	244.06	49	**0.65**	4.00	N/A	N/A	50	**0.72**	3.13	N/A	N/A	52	**0.64**	2.54	N/A	N/A
socfb-Duke14	9885	506437	38	**72.08**	1456.46	N/A	426.18	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
socfb-Indiana	29732	1305757	51	**851.77**	907.31	N/A	N/A	55	752.78	**143.39**	N/A	N/A	57	N/A	**126.96**	N/A	N/A	59	N/A	**139.41**	N/A	N/A
socfb-MIT	6402	251230	37	**11.53**	65.11	N/A	197.73	42	94.62	**39.83**	N/A	N/A	45	321.27	**55.71**	N/A	N/A	48	745.29	**3.44**	N/A	N/A
socfb-OR	63392	816886	33	**11.97**	12.04	N/A	N/A	37	84.54	**9.40**	N/A	N/A	39	346.69	**11.46**	N/A	N/A	42	894.48	**6.06**	N/A	N/A
socfb-Penn94	41536	1362220	50	**8.30**	17.58	N/A	N/A	52	**10.05**	15.76	N/A	N/A	54	**3.49**	15.37	N/A	N/A	55	**6.71**	15.31	N/A	N/A
socfb-Stanford3	11586	568309	59	**8.85**	359.31	N/A	357.26	62	**88.28**	568.10	N/A	N/A	65	**13.28**	N/A	N/A	N/A	67	**10.95**	N/A	N/A	N/A
socfb-Texas84	36364	795325.5	68	N/A	**540.17**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	68	**540.15**	N/A	N/A	N/A
socfb-uci-uni	5879078	292208195	9	**32.97**	N/A	N/A	N/A	10	**31.73**	N/A	N/A	N/A	11	**30.94**	N/A	N/A	N/A	13	**29.44**	N/A	N/A	N/A
socfb-UCLA	20453	747604	55	**1.39**	7.20	N/A	N/A	57	**1.36**	7.37	N/A	N/A	59	**1.39**	6.56	N/A	N/A	62	**1.15**	6.30	N/A	N/A
socfb-UConn	17206	604867	53	**0.75**	5.13	326.90	1179.50	56	**0.61**	2.77	824.92	N/A	58	**0.18**	1.06	N/A	N/A	60	**0.13**	0.67	N/A	N/A
socfb-UCSB37	14917	482215	59	**20.54**	123.29	60.91	1642.37	63	5.46	**3.26**	136.49	N/A	66	5.14	**1.01**	41.70	N/A	68	**0.08**	0.49	9.28	N/A
socfb-UF	35111	732827	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	70	N/A	**847.40**	N/A	N/A	73	N/A	**202.80**	N/A	N/A
socfb-Wisconsin87	23831	835946	42	**12.19**	19.31	N/A	N/A	44	34.00	**16.76**	N/A	N/A	47	13.12	**9.58**	N/A	N/A	50	14.97	**8.22**	N/A	N/A
inf-power	4941	6594	6	0.00	0.00	0.00	506.24	6	0.00	0.00	0.11	N/A	8	0.00	0.00	0.00	N/A	9	0.03	0.00	0.00	N/A
inf-road-usa	23947347	28854312	5	N/A	**13.15**	N/A	N/A	6	**6.63**	145.52	N/A	N/A	7	**6.56**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
inf-roadNet-CA	1957027	2760388	5	**0.38**	0.64	N/A	N/A	6	0.70	**0.65**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
inf-roadNet-PA	1087562	1541514	5	**0.18**	0.30	N/A	N/A	6	**0.20**	0.69	N/A	N/A	7	**0.24**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
ia-email-EU	32430	54397	15	**0.03**	0.08	10.70	N/A	16	**0.03**	0.07	326.02	N/A	18	**0.03**	0.08	N/A	N/A	20	**0.04**	0.49	N/A	N/A
ia-email-univ	1133	5451	12	0.00	0.00	0.01	0.69	12	0.00	0.00	0.01	106.49	12	0.00	0.00	0.03	N/A	13	**0.00**	0.01	0.04	N/A
ia-enron-large	33696	180811	22	**0.42**	3.47	N/A	N/A	24	**2.86**	5.19	N/A	N/A	26	**3.20**	8.28	N/A	N/A	28	6.49	**3.06**	N/A	N/A
ia-enron-only	143	623	10	0.00	0.00	0.00	0.00	12	0.00	0.00	0.00	0.04	12	0.00	0.00	0.00	1.15	13	0.00	0.00	0.00	N/A
ia-fb-messages	1266	6451	6	0.01	0.01	4.34	3.39	8	**0.01**	0.03	30.58	740.53	9	**0.01**	0.10	61.16	N/A	10	**0.15**	0.71	145.67	29.50
ia-infect-dublin	410	2765	17	0.00	0.00	0.00	0.02	18	0.00	0.00	0.00	0.69	18	0.00	0.00	0.01	80.07	19	0.02	**0.00**	0.03	N/A
ia-infect-hyper	113	2196	19	0.01	0.01	0.26	0.02	21	0.05	**0.02**	5.10	0.20	23	0.01	0.01	31.21	2.25	25	0.01	0.01	148.39	N/A
ia-reality	6809	7680	6	0.00	0.00	0.01	1246.09	7	0.00	0.00	0.01	N/A	8	0.00	0.00	0.01	N/A	9	0.00	0.00	0.01	19.35
ia-wiki-Talk	92117	360767	18	**4.65**	28.40	N/A	N/A	21	**56.51**	121.21	N/A	N/A	23	**923.31**	1074.88	N/A	N/A	N/A	N/A	N/A	N/A	N/A
rec-amazon	91813	125704	6	**0.02**	0.04	N/A	N/A	6	**0.04**	1476.47	N/A	N/A	8	**12.74**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
rt-retweet-crawl	1112702	2278852	14	**1.26**	2.63	N/A	N/A	15	**1.32**	2.25	N/A	N/A	16	**1.35**	2.25	N/A	N/A	17	**1.16**	1.55	N/A	N/A
rt-retweet	96	117	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.03	6	0.00	0.00	0.00	0.57	7	0.00	0.00	0.00	N/A
rt-twitter-copen	761	1029	5	0.00	0.00	0.00	0.54	6	0.00	0.00	0.01	109.00	8	0.00	0.00	0.00	N/A	9	0.00	0.00	0.01	10.58
sc-nasasrb	54870	1311227	24	**40.45**	1107.81	N/A	N/A	24	**174.50**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
sc-pkustk11	87804	2565054	36	N/A	**6.01**	N/A	N/A	36	N/A	**10.16**	N/A	N/A	36	N/A	**90.65**	N/A	N/A	N/A	N/A	N/A	N/A	N/A
sc-pkustk13	94893	3260967	36	**122.57**	680.91	N/A	N/A	36	**390.21**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
sc-shipsec1	140385	1707759	24	**0.31**	1.59	N/A	N/A	24	**0.31**	3.06	N/A	N/A	24	**0.32**	N/A	N/A	N/A	24	**0.46**	N/A	N/A	N/A
sc-shipsec5	179104	2200076	24	**10.54**	47.65	N/A	N/A	24	**15.82**	N/A	N/A	N/A	24	**31.80**	N/A	N/A	N/A	26	**28.82**	N/A	N/A	N/A
soc-brightkite	56739	212945	44	**0.39**	0.71	1.42	N/A	47	1.03	**0.34**	13.48	N/A	49	0.44	**0.31**	61.42	N/A	51	**0.03**	0.11	204.15	N/A
soc-delicious	536108	1365961	23	**0.36**	1.49	N/A	N/A	27	**0.31**	1.55	N/A	N/A	29	**0.33**	1.17	N/A	N/A	30	**0.33**	1.04	N/A	N/A

Table 2
Experimental results of real world graphs for $k = 2$, 3, 4 and 5.

graph			k=2					k=3					k=4					k=5				
name	**#vtx**	**#edges**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**
soc-dolphins	62	159	6	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.07	9	0.00	0.00	0.00	N/A
soc-douban	154908	327162	12	**0.23**	0.88	N/A	N/A	14	**0.24**	0.81	N/A	N/A	16	**0.21**	0.45	N/A	N/A	17	**0.19**	0.58	N/A	0.37
soc-epinions	26588	100120	18	**0.13**	0.73	240.77	N/A	21	**0.13**	0.82	N/A	N/A	23	**0.46**	0.62	N/A	N/A	25	0.99	**0.37**	N/A	N/A
soc-flixster	2523386	7918801	38	**125.47**	1086.01	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
soc-FourSquare	639014	3214986	35	**850.21**	N/A	N/A	N/A	39	1363.04	**1057.65**	N/A	N/A	42	N/A	**896.75**	N/A	N/A	44	N/A	**866.91**	N/A	N/A
soc-gowalla	196591	950327	30	**1.14**	45.92	N/A	N/A	31	**1.24**	43.60	N/A	N/A	32	**3.61**	91.76	N/A	N/A	32	**72.82**	235.07	N/A	N/A
soc-karate	34	78	6	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	8	0.00	0.00	0.00	0.01	9	0.00	0.00	0.00	N/A
soc-lastfm	1191805	4519330	18	**10.75**	283.87	N/A	N/A	21	**66.48**	879.72	N/A	N/A	24	**1293.93**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	**0.03**
soc-livejournal	403313727933062		214	**6.27**	6.60	N/A	N/A	214	**5.89**	6.65	N/A	N/A	214	**5.72**	6.49	N/A	N/A	214	**5.89**	6.71	N/A	N/A
soc-LiveMocha	104103	2193083	19	**49.58**	484.29	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
soc-pokec	1632803223019	64	31	**42.74**	179.11	N/A	N/A	32	**34.74**	218.87	N/A	N/A	32	**35.97**	136.84	N/A	N/A	34	**42.17**	162.47	N/A	N/A
soc-slashdot	70068	358647	31	**14.37**	47.26	120.30	N/A	34	590.20	**330.97**	N/A	N/A	37	N/A	**519.60**	N/A	N/A	40	N/A	**202.84**	N/A	N/A
soc-twitter-follows	404719	713319	8	**0.84**	11.36	N/A	N/A	9	**0.71**	23.98	N/A	N/A	11	**0.59**	32.74	N/A	N/A	13	**0.49**	35.90	N/A	N/A
soc-wiki-Vote	889	2914	8	0.00	0.00	0.01	0.67	9	0.00	0.00	0.03	197.22	11	0.00	0.00	0.01	N/A	12	0.00	0.00	0.01	N/A
soc-youtube-snap	1134890	2987624	20	**4.94**	38.50	N/A	N/A	21	**53.42**	82.56	N/A	N/A	24	213.74	**81.84**	N/A	N/A	26	1712.64	**107.34**	N/A	N/A
soc-youtube	495957	1936748	20	**3.27**	19.62	N/A	N/A	21	**16.56**	30.07	N/A	N/A	24	55.62	**22.67**	N/A	N/A	26	1245.28	**52.65**	N/A	N/A
tech-as-caida2007	26475	53381	17	0.01	0.01	0.18	N/A	18	**0.02**	0.03	3.87	N/A	21	**0.01**	0.02	9.10	N/A	23	**0.01**	0.02	19.90	N/A
tech-as-skitter	169461611094209		69	**284.76**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
tech-internet-as	40164	85123	18	**0.02**	0.05	0.31	N/A	20	**0.02**	0.03	2.57	N/A	22	**0.01**	0.03	12.26	N/A	22	**0.02**	0.06	143.82	N/A
tech-p2p-gnutella	62561	147878	5	**0.12**	0.13	377.07	N/A	6	0.11	0.11	426.65	N/A	8	0.09	0.09	158.53	N/A	10	**0.02**	0.03	12.89	N/A
tech-RL-caida	190914	607610	20	**0.82**	3.39	N/A	N/A	23	5.22	**3.89**	N/A	N/A	24	**21.10**	N/A	N/A	N/A	26	**1.39**	2.22	N/A	N/A
tech-routers-rf	2113	6632	17	0.00	0.00	0.01	2.98	18	0.00	0.00	0.01	927.15	19	0.00	0.00	0.01	N/A	20	0.00	0.00	0.01	N/A
tech-WHOIS	7476	56943	64	N/A	N/A	**113.91**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
scc_enron-only	146	9828	121	0.00	0.00	0.00	0.03	121	0.00	0.00	0.00	0.04	122	0.16	0.12	**0.00**	0.62	123	**0.00**	0.12	0.01	N/A
scc_fb-forum	488	71011	266	N/A	N/A	**0.69**	N/A	268	N/A	N/A	**7.85**	N/A	270	N/A	N/A	**19.96**	N/A	272	N/A	N/A	26.33	**2.10**
scc_fb-messages	1303	531893	708	**0.02**	0.16	0.27	1094.35	709	**0.02**	0.17	0.28	N/A	709	0.38	**0.17**	0.29	N/A	709	0.37	**0.16**	0.27	N/A
scc_infect-dublin	10972	175573	84	**0.01**	0.05	0.11	262.06	84	**0.01**	0.05	0.11	N/A	84	**0.01**	0.05	0.12	N/A	84	**0.01**	0.05	0.11	N/A
scc_infect-hyper	113	6222	106	0.00	0.00	0.00	0.01	107	0.00	0.00	0.00	0.21	107	0.00	0.00	0.00	1.42	107	0.00	0.00	0.00	N/A
scc_reality	6809	4714485	1236	2.05	**0.80**	3.12	N/A	1236	2.04	**0.79**	3.17	N/A	1236	2.03	**0.80**	3.12	N/A	1237	1356.34	N/A	**3.14**	5.39
scc_retweet-crawl	17151	24015	21	0.00	0.00	0.01	N/A	21	0.00	0.00	0.01	N/A	22	0.00	0.00	0.01	N/A	22	0.00	0.00	0.01	N/A
scc_retweet	1206	65990	166	**0.06**	1.99	1.66	1049.06	167	**5.93**	25.48	70.69	N/A	169	**0.06**	1.31	528.18	N/A	170	8.68	**2.29**	N/A	N/A
scc_rt_alwefaq	72	355	17	0.00	0.00	0.00	0.00	18	0.00	0.00	0.00	0.00	18	0.00	0.00	0.00	0.01	19	0.00	0.00	0.00	N/A
scc_rt_assad	34	96	9	0.00	0.00	0.00	0.00	9	0.00	0.00	0.00	0.00	11	0.00	0.00	0.00	0.00	12	0.00	0.00	0.00	0.12
scc_rt_bahrain	72	129	9	0.00	0.00	0.00	0.00	9	0.00	0.00	0.00	0.00	10	0.00	0.00	0.00	0.02	11	0.00	0.00	0.00	0.00
scc_rt_barackobama	80	226	11	0.00	0.00	0.00	0.00	12	0.00	0.00	0.00	0.00	13	0.00	0.00	0.00	0.03	14	0.00	0.00	0.00	0.21
scc_rt_damascus	34	41	6	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.00	8	0.00	0.00	0.00	0.19
scc_rt_dash	31	39	6	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.00	8	0.00	0.00	0.00	0.02
scc_rt_gmanews	135	1078	22	0.00	0.00	0.00	0.00	23	0.00	0.00	0.00	0.01	24	0.01	0.00	0.00	0.11	25	0.01	0.00	0.00	0.01
scc_rt_gop	13	7	3	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	1.06
scc_rt_http	5	6	4	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00
scc_rt_israel	22	12	3	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00
scc_rt_justinbieber	62	442	18	0.00	0.00	0.00	0.00	19	0.00	0.00	0.00	0.00	19	0.00	0.00	0.00	0.01	20	0.00	0.00	0.00	0.00
scc_rt_ksa	21	23	6	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.05
scc_rt_lebanon	10	5	2	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00
scc_rt_libya	27	26	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.00
scc_rt_lolgop	273	4510	43	0.00	0.00	0.00	0.01	43	0.00	0.00	0.00	0.14	43	0.00	0.00	0.00	1.78	44	0.00	0.04	0.00	0.01
scc_rt_mittromney	102	108	6	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.02	8	0.00	0.00	0.00	0.37	9	0.00	0.00	0.00	45.73
scc_rt_obama	8	4	2	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	10.03
scc_rt_occupy	55	60	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	8	0.00	0.00	0.00	0.01	8	0.00	0.00	0.00	0.00
scc_rt_occupywallstnyc	127	931	19	0.00	0.00	0.00	0.00	19	0.00	0.00	0.00	0.05	20	0.00	0.00	0.00	0.45	22	0.00	0.01	0.00	0.27
scc_rt_oman	16	13	4	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	2.10
scc_rt_onedirection	35	368	27	0.00	0.00	0.00	0.00	27	0.00	0.00	0.00	0.00	27	0.00	0.00	0.00	0.00	27	0.00	0.00	0.00	0.00
scc_rt_p2	26	15	3	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00
scc_rt_qatif	14	11	3	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.01
scc_rt_saudi	28	91	9	0.00	0.00	0.00	0.00	10	0.00	0.00	0.00	0.00	11	0.00	0.00	0.00	0.00	12	0.00	0.00	0.00	0.00
scc_rt_tcot	26	18	4	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00
scc_rt_tlot	13	8	3	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.01
scc_rt_uae	18	12	3	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00
scc_rt_voteonedirection	7	5	3	0.00	0.00	0.00	0.00	4	0.00	0.00	0.00	0.00	5	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00
scc_twitter-copen	2623	473614	581	8.85	N/A	**0.75**	1491.85	581	1654.25	N/A	**5.47**	N/A	581	N/A	N/A	**229.63**	N/A	582	N/A	N/A	897.87	**0.00**
web-arabic-2005	163598	1747269	102	**0.09**	0.15	N/A	N/A	102	**0.09**	0.14	N/A	N/A	102	**0.09**	0.14	N/A	N/A	102	**0.09**	0.15	N/A	N/A

Table 3
Experimental results of real world graphs for $k = 2$, 3, 4 and 5.

graph			k=2					k=3					k=4					k=5				
name	**#vtx**	**#edges**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**
web-BerkStan	12305	19500	29	0.00	0.00	0.01	1074.75	29	0.00	0.00	0.01	N/A	29	0.00	0.00	0.01	N/A	29	0.00	0.00	0.01	N/A
web-edu	3031	6474	30	0.00	0.00	0.00	66.71	30	0.00	0.00	0.00	N/A	30	0.00	0.00	0.00	N/A	30	0.00	0.00	0.00	N/A
web-google	1299	2773	19	0.00	0.00	0.00	0.68	19	0.00	0.00	0.00	105.02	19	0.00	0.00	0.00	N/A	19	0.00	0.00	0.00	N/A
web-indochina-2004	11358	47606	50	**0.00**	0.01	0.02	N/A	50	**0.00**	0.01	0.02	N/A	50	**0.00**	0.01	0.02	N/A	50	**0.00**	0.01	0.02	N/A
web-it-2004	509338	7178413	432	**0.40**	15.13	N/A	N/A	432	**0.50**	74.90	N/A	N/A	432	**0.51**	719.89	N/A	N/A	432	**0.51**	N/A	N/A	N/A
web-polblogs	643	2280	12	0.00	0.00	0.00	0.20	14	0.00	0.00	0.00	9.39	15	0.00	0.00	0.00	1775.84	17	0.00	0.00	0.00	N/A
web-sk-2005	121422	334419	82	**0.04**	0.15	N/A	N/A	83	**0.04**	0.34	N/A	N/A	83	**0.04**	3.87	N/A	N/A	83	**0.04**	37.39	N/A	N/A
web-spam	4767	37375	21	**0.14**	0.94	14.20	64.16	24	6.58	**1.62**	164.03	N/A	27	2.96	**0.86**	1552.16	N/A	30	0.36	**0.22**	N/A	N/A
web-uk-2005	129632	11744049	500	**0.56**	27.76	N/A	N/A	500	**0.56**	135.99	N/A	N/A	500	**0.71**	1493.97	N/A	N/A	500	**0.71**	N/A	N/A	N/A
web-webbase-2001	16062	25593	33	0.00	0.00	0.01	N/A	33	0.00	0.00	0.01	N/A	33	0.00	0.00	0.01	N/A	33	0.00	0.00	0.01	N/A
web-wikipedia2009	1864433	4507315	32	**4.85**	419.15	N/A	N/A	32	**4.36**	N/A	N/A	N/A	32	**2.93**	N/A	N/A	N/A	32	**3.21**	N/A	N/A	N/A

Table 4
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
bio-celegans (453,2025)	2	51	383	0.00	19	0.00	20	123	0.00
	3	31	228	0.00	0	0.00	0	0	0.00
	4	31	228	0.00	27	0.00	27	191	0.00
	5	25	176	0.00	20	0.00	20	129	0.00
bio-diseasome (516,1188)	2	0	0	0.00	0	0.00	0	0	0.00
	3	21	100	0.00	21	0.00	21	100	0.00
	4	38	183	0.00	38	0.00	38	177	0.00
	5	59	277	0.00	59	0.00	59	275	0.00
bio-dmela (7393,25569)	2	1718	12760	0.00	0	0.01	15	46	0.01
	3	1718	12760	0.00	21	0.01	21	80	0.01
	4	1379	10792	0.00	0	0.01	0	0	0.01
	5	1379	10792	0.00	27	0.01	27	152	0.01
bio-yeast (1458,1948)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	12	29	0.00	12	0.00	12	29	0.00
	5	142	318	0.00	142	0.00	142	318	0.00
ca-AstroPh (17903,196972)	2	0	0	0.01	0	0.01	0	0	0.01
	3	113	3166	0.01	113	0.04	113	3136	0.01
	4	113	3166	0.01	113	0.04	113	3136	0.01
	5	165	4704	0.01	165	0.05	165	4566	0.01
ca-citeseer (227320,814134)	2	0	0	0.03	0	0.11	0	0	0.03
	3	0	0	0.04	0	0.11	0	0	0.04
	4	0	0	0.05	0	0.10	0	0	0.05
	5	0	0	0.06	0	0.11	0	0	0.06
ca-coauthors-dblp (540486,15245729)	2	0	0	0.33	0	6.90	0	0	0.33
	3	0	0	0.47	0	6.37	0	0	0.47
	4	0	0	0.45	0	6.54	0	0	0.45
	5	0	0	0.44	0	6.68	0	0	0.44
ca-CondMat (21363,91286)	2	0	0	0.00	0	0.01	0	0	0.00
	3	0	0	0.00	0	0.01	0	0	0.00
	4	0	0	0.00	0	0.01	0	0	0.00
	5	49	578	0.00	49	0.01	49	578	0.00
ca-CSphd (1882,1740)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	1882	0.00	0	0	0.00
	4	0	0	0.00	113	0.00	0	0	0.00
	5	113	133	0.00	0	0.00	113	133	0.00
ca-dblp-2010 (226413,716460)	2	0	0	0.03	0	0.09	0	0	0.03
	3	0	0	0.03	0	0.09	0	0	0.03
	4	0	0	0.03	0	0.09	0	0	0.03
	5	0	0	0.03	0	0.09	0	0	0.03
ca-dblp-2012 (317080,1049866)	2	0	0	0.07	0	0.20	0	0	0.07
	3	0	0	0.08	0	0.21	0	0	0.08
	4	0	0	0.07	0	0.19	0	0	0.07
	5	0	0	0.08	0	0.20	0	0	0.08
ca-Erdos992 (5094,7515)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
ca-GrQc (4158,13422)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
ca-HepPh (11204,117619)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
ca-hollywood-2009 (1069126,56306653)	2	0	0	2.19	0	2.76	0	0	2.19
	3	0	0	1.63	0	3.00	0	0	1.63
	4	0	0	1.42	0	2.87	0	0	1.42
	5	0	0	1.49	0	2.80	0	0	1.49
ca-MathSciNet (332689,820644)	2	0	0	0.08	0	0.14	0	0	0.08
	3	49	576	0.10	49	0.13	49	576	0.10
	4	72	829	0.08	72	0.14	72	829	0.08
	5	72	829	0.09	72	0.14	72	829	0.09
ca-netscience (379,914)	2	0	0	0.00	0	0.00	0	0	0.00
	3	25	92	0.00	25	0.00	25	92	0.00
	4	46	157	0.00	46	0.00	46	155	0.00
	5	69	231	0.00	69	0.00	69	231	0.00
ia-email-EU (32430,54397)	2	719	10772	0.00	128	0.06	127	1920	0.02
	3	605	9581	0.00	91	0.05	88	1173	0.02
	4	526	8650	0.00	90	0.04	92	1249	0.01
	5	526	8650	0.00	122	0.05	122	1877	0.01

Table 5
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
ia-email-univ (1133,5451)	2	0	0	0.00	0	0.00	0	0	0.00
	3	121	845	0.00	0	0.00	12	66	0.00
	4	238	1837	0.00	0	0.00	12	66	0.00
	5	238	1837	0.00	77	0.00	0	0	0.00
ia-enron-large (33696,180811)	2	2561	73666	0.01	455	1.65	432	11550	0.30
	3	2276	68430	0.01	348	2.38	318	8345	0.24
	4	2055	63929	0.01	286	1.64	202	4904	0.23
	5	1944	61511	0.01	175	0.82	202	4934	0.23
ia-enron-only (143,623)	2	17	100	0.00	15	0.00	15	83	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	16	92	0.00	16	0.00	16	92	0.00
ia-fb-messages (1266,6451)	2	753	5557	0.00	155	0.01	144	708	0.01
	3	666	5219	0.00	407	0.01	107	540	0.00
	4	666	5219	0.00	753	0.01	328	2243	0.00
	5	666	5219	0.00	666	0.01	666	5219	0.00
ia-infect-dublin (410,2765)	2	32	351	0.00	0	0.00	18	149	0.00
	3	32	351	0.00	0	0.00	0	0	0.00
	4	32	351	0.00	0	0.00	0	0	0.00
	5	32	351	0.00	32	0.00	32	349	0.00
ia-infect-hyper (113,2196)	2	101	2051	0.00	45	0.01	48	751	0.01
	3	98	2000	0.00	45	0.01	48	751	0.00
	4	96	1964	0.00	45	0.01	44	672	0.00
	5	94	1926	0.00	44	0.01	44	673	0.00
ia-reality (6809,7680)	2	71	301	0.00	26	0.00	0	0	0.00
	3	71	301	0.00	0	0.00	0	0	0.00
	4	132	533	0.00	132	0.00	132	533	0.00
	5	71	301	0.00	71	0.00	71	301	0.00
ia-wiki-Talk (92117,360767)	2	6549	180546	0.02	461	3.74	828	34563	0.98
	3	5222	162193	0.03	543	4.73	618	25477	0.86
	4	4296	146593	0.02	715	5.05	417	16595	0.76
	5	4001	141040	0.03	715	6.40	413	16478	1.09
inf-power (4941,6594)	2	12	36	0.00	12	0.00	12	30	0.00
	3	36	106	0.00	32	0.00	32	92	0.00
	4	36	106	0.00	36	0.00	36	106	0.00
	5	231	479	0.00	36	0.00	231	479	0.00
inf-road-usa (23947347,28854312)	2	16919524	21826489	14.82	0	13.15	16919524	21826489	41.68
	3	3712	5916	3.28	3712	16.43	3712	5916	3.28
	4	0	0	3.35	1.7E+07	62.14	0	0	3.35
	5	3712	5916	3.82	2.4E+07	64.39	3712	5916	3.82
inf-roadNet-CA (1957027,2760388)	2	0	0	0.19	0	0.64	0	0	0.19
	3	4454	7393	0.19	0	0.65	4454	7393	0.19
	4	1589938	2393299	0.87	4454	2.73	1589938	2393299	2.77
	5	4454	7393	0.26	1589938	5.32	4454	7393	0.26
inf-roadNet-PA (1087562,1541514)	2	916	1491	0.09	0	0.30	355	562	0.09
	3	916	1491	0.09	916	0.37	916	1491	0.09
	4	916	1491	0.12	873219	2.73	916	1491	0.12
	5	916	1491	0.12	1087562	2.58	916	1491	0.12
rec-amazon (91813,125704)	2	0	0	0.01	1496	0.02	0	0	0.01
	3	1504	3065	0.01	15029	0.05	1496	3047	0.01
	4	15029	26042	0.01	15029	0.25	15029	26042	0.02
	5	61351	95242	0.04	15029	0.26	61351	95242	0.10
rt-retweet-crawl (1112702,2278852)	2	23022	312247	0.39	0	2.63	38	378	0.89
	3	23022	312247	0.41	0	2.25	38	378	0.91
	4	23022	312247	0.41	0	2.25	38	378	0.92
	5	16450	236099	0.41	0	1.55	0	0	0.77
rt-retweet (96,117)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	34	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
rt-twitter-copen (761,1029)	2	21	62	0.00	0	0.00	0	0	0.00
	3	21	62	0.00	87	0.00	12	32	0.00
	4	21	62	0.00	0	0.00	21	62	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
sc-ldoor (909537,20770807)	2	909447	20769277	1.33	882715	360.19	882399	20128569	1480.93
	3	909537	20770807	1.41	908841	143.65	908841	20748670	58.47
	4	909537	20770807	1.40	909395	138.84	909395	20760756	23.16
	5	909537	20770807	1.40	909537	136.52	909537	20763402	8.10
sc-msdoor (404785,9378650)	2	404705	9377290	0.56	380695	80.79	379472	8771089	153.00
	3	404785	9378650	0.62	403825	61.05	403802	9350810	45.85
	4	404785	9378650	0.58	404705	58.69	404705	9370298	8.86
	5	404785	9378650	0.61	404785	58.07	404785	9371658	3.71

Table 6
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
sc-nasasrb (54870,1311227)	2	53675	1293082	0.05	12960	3.04	12972	322518	0.96
	3	53945	1298844	0.06	21575	7.72	21586	470123	1.96
	4	54012	1300215	0.06	51075	10.08	51069	1193313	1.47
	5	54012	1300215	0.06	51154	8.67	51153	1205309	1.40
sc-pkustk11 (87804,2565054)	2	87084	2558070	0.14	588	5.89	87084	2546406	1.07
	3	87084	2558070	0.11	588	5.94	87084	2546406	1.05
	4	87084	2558070	0.11	588	5.95	87084	2546406	1.06
	5	87084	2558070	0.12	1692	10.18	87084	2556918	1.08
sc-pkustk13 (94893,3260967)	2	92533	3194949	0.11	27558	113.48	27120	644421	3.40
	3	92539	3195147	0.12	27789	143.68	27120	644448	2.50
	4	92708	3200612	0.15	27615	138.00	27120	644448	3.61
	5	92730	3201296	0.15	94795	61.05	60849	1982268	45.75
sc-pwtk (217891,5653221)	2	216897	5641349	0.25	204275	36.80	204263	5302197	5.10
	3	217112	5645965	0.26	204924	37.41	204924	5323510	15.75
	4	217115	5646028	0.27	208567	39.59	208547	5426854	27.70
	5	217116	5646048	0.26	212461	35.30	212326	5507216	12.12
sc-shipsec1 (140385,1707759)	2	11085	227028	0.07	240	1.58	240	2760	0.24
	3	11594	236570	0.07	240	2.88	240	2760	0.24
	4	11665	238009	0.07	9357	5.70	280	3672	0.24
	5	11804	240748	0.07	10501	5.73	300	4186	0.37
sc-shipsec5 (179104,2200076)	2	24250	448770	0.07	10602	2.57	10602	162297	0.32
	3	26224	487252	0.08	10602	4.04	10602	162322	0.37
	4	26842	499123	0.08	11162	4.57	11162	175015	0.41
	5	27490	510743	0.08	11313	5.15	11297	178650	0.54
scc_enron-only (146,9828)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	131	8435	0.00	131	0.08	131	8435	0.00
	5	0	0	0.00	131	0.08	0	0	0.00
scc_fb-forum (488,71011)	2	315	48325	0.00	288	8.87	288	41113	0.10
	3	316	48589	0.00	288	7.73	292	42184	0.10
	4	312	47529	0.00	282	6.04	282	39523	0.16
	5	313	47795	0.00	285	12.00	286	40594	0.10
scc_fb-messages (1303,531893)	2	0	0	0.01	0	0.01	0	0	0.01
	3	0	0	0.01	0	0.01	0	0	0.01
	4	0	0	0.02	0	0.01	0	0	0.02
	5	0	0	0.02	0	0.01	0	0	0.02
scc_infect-dublin (10972,175573)	2	0	0	0.01	0	0.05	0	0	0.01
	3	0	0	0.00	0	0.05	0	0	0.00
	4	0	0	0.00	0	0.05	0	0	0.00
	5	0	0	0.00	0	0.05	0	0	0.00
scc_infect-hyper (113,6222)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_reality (6809,4714485)	2	0	0	0.11	0	0.10	0	0	0.11
	3	0	0	0.11	0	0.10	0	0	0.11
	4	0	0	0.11	0	0.10	0	0	0.11
	5	1267	801295	0.11	N/A	N/A	1239	766923	3.96
scc_retweet-crawl (17151,24015)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_retweet (1206,65990)	2	255	28652	0.00	0	1.98	172	14674	0.04
	3	255	28652	0.00	174	2.73	174	15010	0.04
	4	255	28652	0.00	0	1.30	0	0	0.05
	5	255	28652	0.00	174	2.04	174	15010	0.05
scc_rt_alwefaq (72,355)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	20	177	0.00	20	0.00	20	177	0.00
scc_rt_assad (34,96)	2	12	53	0.00	0	0.00	12	53	0.00
	3	14	65	0.00	12	0.00	14	65	0.00
	4	0	0	0.00	14	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_bahrain (72,129)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_barackobama (80,226)	2	14	80	0.00	0	0.00	14	80	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	14	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00

Table 7
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
scc_rt_damascus (34,41)	2	0	0	0.00	0	0.00	0	0	0.00
	3	6	14	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	8	0.00	8	19	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_dash (31,39)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	8	20	0.00	8	0.00	8	20	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_gmanews (135,1078)	2	34	451	0.00	33	0.00	33	407	0.00
	3	34	451	0.00	37	0.00	34	446	0.00
	4	34	451	0.00	37	0.00	34	451	0.00
	5	37	511	0.00	37	0.00	37	511	0.00
scc_rt_gop (13,7)	2	13	7	0.00	13	0.00	13	7	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	13	7	0.00	13	0.00	13	7	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_http (5,6)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_israel (22,12)	2	22	12	0.00	22	0.00	22	12	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	22	0.00	0	0	0.00
scc_rt_justinbieber (62,442)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	25	0.00	0	0	0.00
scc_rt_ksa (21,23)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_lebanon (10,5)	2	10	5	0.00	10	0.00	10	5	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	10	5	0.00	10	0.00	10	5	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_libya (27,26)	2	0	0	0.00	9	0.00	0	0	0.00
	3	9	13	0.00	0	0.00	9	13	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	9	13	0.00	0	0.00	9	13	0.00
scc_rt_lolgop (273,4510)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	52	0.00	0	0	0.00
scc_rt_mittromney (102,108)	2	0	0	0.00	11	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_obama (8,4)	2	8	4	0.00	8	0.00	8	4	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	8	4	0.00	8	0.00	8	4	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_occupy (55,60)	2	8	20	0.00	0	0.00	0	0	0.00
	3	8	20	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_occupywallstnyc (127,931)	2	0	0	0.00	33	0.00	0	0	0.00
	3	33	372	0.00	36	0.00	33	362	0.00
	4	33	372	0.00	42	0.00	33	365	0.00
	5	36	420	0.00	43	0.00	36	420	0.00
scc_rt_oman (16,13)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_onedirection (35,368)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_p2 (26,15)	2	26	15	0.00	26	0.00	26	15	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	26	0.00	0	0	0.00

Table 8
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
scc_rt_qatif (14,11)	2	0	0	0.00	0	0.00	0	0	0.00
	3	14	11	0.00	14	0.00	14	11	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	14	11	0.00	14	0.00	14	11	0.00
scc_rt_saudi (28,91)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	12	55	0.00	0	0.00	12	55	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_tcot (26,18)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	26	18	0.00	26	0.00	26	18	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_tlot (13,8)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	13	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
scc_rt_uae (18,12)	2	0	0	0.00	0	0.00	0	0	0.00
	3	18	12	0.00	18	0.00	18	12	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	18	12	0.00	18	0.00	18	12	0.00
scc_rt_voteonedirection (7,5)	2	0	0	0.00	0	0.00	0	0	0.00
	3	7	5	0.00	7	0.00	7	5	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	7	5	0.00	7	0.00	7	5	0.00
scc_twitter-copen (2623,473614)	2	651	208822	0.01	636	92.62	636	200020	0.60
	3	653	209969	0.01	636	90.21	636	200106	0.48
	4	651	208822	0.02	636	88.61	636	200100	0.88
	5	651	208822	0.02	636	90.04	636	200106	0.86
soc-BlogCatalog (88784,2093195)	2	12670	1457074	0.12	3190	389.37	3296	544518	83.62
	3	11943	1419626	0.10	3071	399.02	2957	496595	66.72
	4	11237	1380473	0.09	2748	357.64	2654	451418	59.61
	5	10443	1332843	0.11	2305	338.58	2302	395755	52.53
soc-brightkite (56739,212945)	2	204	8264	0.01	63	0.16	63	1799	0.03
	3	197	7964	0.01	63	0.17	63	1799	0.02
	4	195	7876	0.01	63	0.16	63	1799	0.02
	5	182	7288	0.01	0	0.11	0	0	0.02
soc-buzznet (101163,2763066)	2	27285	2006350	0.19	2613	705.43	2598	299136	53.68
	3	25166	1932364	0.17	2225	637.51	2208	257731	46.14
	4	22156	1815250	0.16	2062	560.77	1642	193764	35.58
	5	21101	1770452	0.17	1442	477.29	1538	180962	35.69
soc-delicious (536108,1365961)	2	6794	123606	0.11	181	1.33	183	3724	0.25
	3	5201	99775	0.10	202	1.40	183	3724	0.21
	4	4133	82481	0.11	179	1.06	181	3688	0.21
	5	2827	58718	0.12	142	0.95	137	2677	0.19
soc-digg (770799,5907132)	2	46088	3687178	0.61	11885	1304.75	10665	1564270	243.33
	3	44423	3649023	0.53	10289	1422.43	10665	1564270	236.43
	4	37555	3472632	0.52	9838	1495.63	9021	1366306	212.25
	5	36496	3442021	0.59	9416	1221.10	9020	1366277	200.65
soc-dolphins (62,159)	2	0	0	0.00	0	0.00	0	0	0.00
	3	36	109	0.00	20	0.00	20	52	0.00
	4	36	109	0.00	45	0.00	36	109	0.00
	5	45	135	0.00	45	0.00	45	135	0.00
soc-douban (154908,327162)	2	15406	133240	0.03	0	0.88	341	1824	0.20
	3	15406	133240	0.03	0	0.81	957	5293	0.22
	4	12351	118179	0.03	0	0.45	570	3493	0.19
	5	10124	105100	0.03	19	0.58	315	2123	0.16
soc-epinions (26588,100120)	2	2142	39858	0.01	198	0.40	263	5040	0.08
	3	1312	28485	0.00	198	0.37	140	2287	0.04
	4	1217	26891	0.00	146	0.38	141	2313	0.04
	5	1078	24560	0.01	82	0.28	140	2290	0.06
soc-flickr (513969,3190452)	2	9829	1236014	0.27	2251	342.27	2221	516467	69.73
	3	8282	1131146	0.21	1995	295.89	1999	471860	61.34
	4	7364	1062030	0.23	1880	283.89	1854	442332	70.85
	5	6530	994583	0.25	1753	296.48	1756	420999	56.83
soc-flixster (2523386,7918801)	2	21774	856061	0.70	282	14.31	275	12479	2.94
	3	15651	631892	0.71	252	9.99	263	11618	2.40
	4	7545	312522	0.68	237	4.42	239	10187	1.48
	5	844	38648	0.54	237	3.48	226	9407	0.65
soc-FourSquare (639014,3214986)	2	23248	860065	0.23	N\A	N\A	12743	481491	122.41
	3	18709	724599	0.19	100	1048.56	9692	374193	92.11
	4	17257	678326	0.20	84	889.88	9612	371635	80.77
	5	17257	678326	0.22	84	863.22	10645	410671	76.10

Table 9
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
soc-gowalla (196591,950327)	2	10054	261440	0.08	922	7.79	913	26555	1.04
	3	7487	214737	0.08	577	6.25	532	16431	0.83
	4	5590	174851	0.08	463	5.76	375	10909	0.61
	5	5062	162536	0.08	460	5.14	370	10738	0.60
soc-karate (34,78)	2	0	0	0.00	0	0.00	0	0	0.00
	3	10	25	0.00	0	0.00	0	0	0.00
	4	22	55	0.00	22	0.00	22	55	0.00
	5	22	55	0.00	0	0.00	22	55	0.00
soc-lastfm (1191805,4519330)	2	79932	1696417	0.50	1254	35.22	2020	77629	8.03
	3	51344	1291703	0.39	805	25.46	960	42582	5.02
	4	46162	1204599	0.33	976	23.72	960	42582	4.48
	5	N/A	N/A	N/A	677	24.93	N/A	N/A	N/A
soc-livejournal (4033137,27933062)	2	0	0	2.85	0	6.59	0	0	2.85
	3	0	0	2.91	0	6.63	0	0	2.91
	4	0	0	2.82	0	6.47	0	0	2.82
	5	0	0	2.77	0	6.69	0	0	2.77
soc-LiveMocha (104103,2193083)	2	66980	2059265	0.15	4210	99.71	4064	182930	16.23
	3	63437	2031014	0.19	6031	130.53	4057	182887	15.82
	4	60099	2001067	0.19	4182	90.17	4036	182688	19.12
	5	57177	1971921	0.18	5884	112.14	4005	182378	18.41
soc-orkut (2997166,106349209)	2	1954140	94256779	12.12	N/A	N/A	158470	3882134	1454.54
	3	1698831	87924565	16.99	N/A	N/A	92224	4702365	1391.10
	4	1596517	85033746	18.52	N/A	N/A	81267	4178156	1434.97
	5	1496616	82012999	15.20	N/A	N/A	71325	3727013	1223.57
soc-pokec (1632803,22301964)	2	605699	15871320	2.74	838	174.10	2359	50231	40.03
	3	510227	14174602	1.70	298	214.70	1194	25050	33.29
	4	480453	13586210	1.55	298	124.54	1192	25010	33.12
	5	422488	12355265	1.40	298	123.04	825	17403	28.84
soc-slashdot (70068,358647)	2	2032	65142	0.02	143	1.57	136	5197	0.15
	3	1794	58313	0.02	122	0.84	136	5197	0.17
	4	1449	47851	0.02	122	0.59	134	5069	0.14
	5	258	10547	0.02	113	0.37	102	3521	0.07
soc-twitter-follows (404719,713319)	2	46454	304801	0.09	5700	8.04	5700	27620	0.54
	3	29081	252715	0.08	3994	6.69	3994	22407	0.44
	4	20721	219304	0.08	2522	5.65	2522	16438	0.38
	5	16070	196074	0.08	1815	4.85	1815	12751	0.35
soc-wiki-Vote (889,2914)	2	127	808	0.00	0	0.00	0	0	0.00
	3	127	808	0.00	0	0.00	0	0	0.00
	4	127	808	0.00	0	0.00	35	210	0.00
	5	44	288	0.00	23	0.00	23	131	0.00
soc-youtube-snap (1134890,2987624)	2	29658	748678	0.41	1034	23.32	1004	33811	3.56
	3	26901	707751	0.33	1015	22.26	985	33649	3.35
	4	22388	634365	0.28	752	21.56	700	22903	2.87
	5	20593	602346	0.21	1015	21.93	702	23009	2.59
soc-youtube (495957,1936748)	2	25613	634540	0.22	703	12.92	665	22210	2.62
	3	23212	599031	0.23	703	12.63	667	22260	2.47
	4	19251	534632	0.22	415	9.76	499	16060	2.04
	5	19251	534632	0.21	700	11.56	663	22187	2.23
socfb-A-anon (3097165,23667394)	2	390144	14512286	2.94	2239	234.83	2155	64767	50.16
	3	357908	13760467	1.85	1657	216.17	1578	46887	48.21
	4	327525	12991403	1.84	1182	197.86	1101	32196	42.58
	5	298641	12203325	1.71	819	186.89	777	21635	39.81
socfb-B-anon (2937612,20959854)	2	469342	15355191	2.85	37564	440.91	25847	645480	78.17
	3	450382	15072871	1.80	26887	513.00	25555	641827	72.27
	4	432416	14787481	2.85	36219	597.15	24916	633217	79.60
	5	399864	14222285	2.89	34767	593.33	17124	452253	70.70
socfb-Berkeley13 (22900,852419)	2	12322	663509	0.03	693	12.96	777	24575	1.69
	3	11343	626123	0.06	383	10.09	404	13904	1.86
	4	10322	582878	0.04	383	9.74	292	9421	1.68
	5	10136	574544	0.04	292	10.23	292	9421	1.69
socfb-CMU (6621,249959)	2	3616	197658	0.01	289	3.97	291	8045	0.52
	3	3487	192707	0.01	0	4.00	169	4578	0.62
	4	3487	192707	0.01	56	3.13	246	7055	0.60
	5	3310	185587	0.01	54	2.53	169	4578	0.57
socfb-Duke14 (9885,506437)	2	7013	465047	0.02	685	14.46	630	28590	2.11
	3	6745	455562	0.03	553	12.89	396	20459	2.11
	4	6745	455562	0.03	553	12.98	487	23704	2.07
	5	6536	447455	0.04	399	11.87	330	18225	2.01
socfb-Indiana (29732,1305757)	2	19088	1104303	0.07	2316	31.04	2317	84021	3.95
	3	17038	1021599	0.07	1601	27.85	1541	54605	3.55
	4	16238	986201	0.07	1031	22.28	1224	43309	3.18
	5	15503	952191	0.08	976	24.37	985	35164	3.26

Table 10
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
socfb-MIT (6402,251230)	2	3815	214184	0.01	280	3.06	283	8754	0.63
	3	3815	214184	0.01	294	3.21	291	9108	0.80
	4	3696	209981	0.01	279	3.27	268	7678	0.76
	5	3503	202661	0.01	60	2.71	151	4183	0.64
socfb-OR (63392,816886)	2	10506	383449	0.04	294	5.33	404	11600	0.95
	3	9271	350579	0.05	279	5.34	381	10749	0.83
	4	7599	300020	0.03	263	3.46	288	8062	0.51
	5	6937	278716	0.03	180	3.98	285	7946	0.47
socfb-Penn94 (41536,1362220)	2	20353	978684	0.08	0	17.58	609	17020	2.91
	3	18403	905141	0.08	0	15.76	420	11761	2.62
	4	16521	828657	0.08	0	15.37	241	6722	2.42
	5	16521	828657	0.08	59	15.13	350	9590	2.32
socfb-Stanford3 (11586,568309)	2	7692	529815	0.02	3021	42.49	2669	99399	4.45
	3	7523	525207	0.03	2235	35.17	2369	88458	4.08
	4	7160	513875	0.03	1455	24.75	1458	54763	2.49
	5	6993	508016	0.03	1388	25.57	1276	48664	2.42
socfb-Texas84 (36364,1590651)	2	16086	1046859	0.10	452	30.80	377	18140	4.39
	3	13721	922322	0.09	317	25.11	233	11103	3.47
	4	12211	836795	0.10	128	21.23	133	6820	3.14
	5	10402	727916	0.09	111	17.12	118	5686	2.71
socfb-uci-uni (58790782,92208195)	2	165396	1179070	15.49	N\A	N\A	24	138	17.43
	3	67366	543298	15.51	N\A	N\A	0	0	16.29
	4	67366	543298	15.22	N\A	N\A	0	0	15.99
	5	26159	234484	14.73	N\A	N\A	0	0	15.02
socfb-UCLA (20453,747604)	2	7631	452461	0.03	66	6.94	66	2088	1.22
	3	7318	435788	0.04	68	7.09	66	2086	1.19
	4	7318	435788	0.04	65	6.29	66	2088	1.20
	5	6877	411479	0.04	65	6.29	65	2031	1.11
socfb-UConn (17206,604867)	2	4574	234632	0.02	63	2.85	66	2072	0.56
	3	3910	200603	0.03	63	2.54	66	2072	0.50
	4	1088	57155	0.02	63	0.84	63	1912	0.13
	5	899	46932	0.02	63	0.67	63	1912	0.11
socfb-UCSB37 (14917,482215)	2	1050	57497	0.02	148	0.86	77	2818	0.13
	3	874	47721	0.01	76	0.64	76	2754	0.07
	4	874	47721	0.01	76	0.56	76	2754	0.07
	5	814	44014	0.01	0	0.49	0	0	0.07
socfb-UF (35111,1465654)	2	13673	882549	0.09	1527	35.26	1420	65851	3.64
	3	12496	820553	0.08	1263	32.80	1188	55226	3.20
	4	10672	718151	0.08	1000	21.74	1043	47775	2.51
	5	8526	589551	0.08	714	17.97	715	32691	1.99
socfb-UIllinois (30795,1264421)	2	24242	1196254	0.11	6629	84.52	6025	223569	8.46
	3	23234	1173182	0.08	5268	76.02	5217	198327	7.06
	4	22180	1145986	0.10	4277	59.58	4464	173942	7.98
	5	21421	1124495	0.08	3938	60.55	4125	161738	7.10
socfb-Wisconsin87 (23831,835946)	2	12709	615478	0.05	266	9.25	262	7663	1.83
	3	11617	572039	0.05	137	7.40	137	4147	1.57
	4	10839	539211	0.05	132	7.48	132	3900	1.49
	5	10431	521379	0.05	132	7.14	132	3899	1.44
tech-as-caida2007 (26475,53381)	2	115	1987	0.00	27	0.01	27	304	0.00
	3	121	2077	0.00	35	0.02	37	482	0.00
	4	115	1987	0.00	35	0.02	35	452	0.00
	5	90	1578	0.00	29	0.01	33	413	0.00
tech-as-skitter (1694616,11094209)	2	5608	389663	0.87	422	27.53	423	29468	2.53
	3	5041	357594	0.80	404	26.82	404	27713	2.34
	4	4196	303506	0.85	403	25.00	334	21022	2.11
	5	3740	272755	0.85	334	21.47	204	9598	1.92
tech-internet-as (40164,85123)	2	175	2990	0.01	20	0.05	34	416	0.01
	3	175	2990	0.01	0	0.03	46	656	0.01
	4	129	2289	0.01	0	0.03	22	219	0.01
	5	129	2289	0.01	58	0.03	46	656	0.01
tech-p2p-gnutella (62561,147878)	2	24222	100020	0.01	0	0.13	108	185	0.11
	3	19765	86829	0.01	0	0.11	0	0	0.09
	4	16174	72853	0.01	0	0.09	0	0	0.08
	5	1004	4554	0.01	0	0.03	0	0	0.01
tech-RL-caida (190914,607610)	2	29626	260808	0.05	36	3.38	4570	41977	0.57
	3	41170	313510	0.09	0	3.89	16720	128624	0.84
	4	41170	313510	0.07	28437	9.75	28437	213348	0.60
	5	15936	180843	0.05	40	2.21	4409	49296	0.48
tech-routers-rf (2113,6632)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	24	238	0.00	0	0.00	24	238	0.00

Table 11
The expermental results of three preprocessing techniques, peel-reduct, subg-reduct and second-reduct.

graph	k	peel-reduct			subgraph-reduct		second-reduct		
		#vtx	#edges	time	#vtx	time	#vtx	#edges	time
tech-WHOIS (7476,56943)	2	266	16523	0.00	174	0.79	174	9778	0.07
	3	256	15871	0.00	129	0.39	128	7003	0.04
	4	248	15327	0.00	118	0.26	121	6386	0.04
	5	248	15327	0.00	121	0.31	128	6976	0.03
web-arabic-2005 (163598,1747269)	2	0	0	0.04	0	0.15	0	0	0.04
	3	0	0	0.04	0	0.14	0	0	0.04
	4	0	0	0.04	0	0.14	0	0	0.04
	5	0	0	0.04	0	0.15	0	0	0.04
web-BerkStan (12305,19500)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
web-edu (3031,6474)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
web-google (1299,2773)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	51	414	0.00	51	0.00	51	410	0.00
web-indochina-2004 (11358,47606)	2	0	0	0.00	0	0.01	0	0	0.00
	3	0	0	0.00	0	0.01	0	0	0.00
	4	0	0	0.00	0	0.01	0	0	0.00
	5	0	0	0.00	0	0.01	0	0	0.00
web-it-2004 (509338,7178413)	2	0	0	0.16	0	15.05	0	0	0.16
	3	864	186191	0.16	864	31.69	864	186191	0.32
	4	864	186191	0.16	864	31.01	864	186191	0.32
	5	864	186191	0.16	864	32.13	864	186191	0.33
web-polblogs (643,2280)	2	34	305	0.00	16	0.00	0	0	0.00
	3	34	305	0.00	0	0.00	14	83	0.00
	4	27	229	0.00	20	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
web-sk-2005 (121422,334419)	2	246	9963	0.02	246	0.15	246	9963	0.02
	3	248	10123	0.02	248	0.14	248	10123	0.02
	4	248	10123	0.02	248	0.13	248	10123	0.02
	5	248	10123	0.02	250	0.15	248	10123	0.02
web-spam (4767,37375)	2	580	13364	0.00	109	0.14	71	1595	0.03
	3	580	13364	0.00	88	0.13	89	2127	0.03
	4	238	6285	0.00	67	0.08	58	1213	0.01
	5	184	5002	0.00	57	0.05	55	1112	0.01
web-uk-2005 (129632,11744049)	2	1000	249500	0.14	1000	27.67	1000	249500	0.40
	3	1000	249500	0.14	1000	27.53	1000	249500	0.40
	4	1498	373253	0.14	1498	48.15	1498	373253	0.53
	5	1498	373253	0.14	1498	48.12	1498	373253	0.53
web-webbase-2001 (16062,25593)	2	0	0	0.00	0	0.00	0	0	0.00
	3	0	0	0.00	0	0.00	0	0	0.00
	4	0	0	0.00	0	0.00	0	0	0.00
	5	0	0	0.00	0	0.00	0	0	0.00
web-wikipedia2009 (1864433,4507315)	2	61293	595014	1.07	5824	10.18	5724	78542	3.77
	3	39199	431535	0.98	3513	13.42	4740	73941	3.36
	4	15235	223991	0.83	4127	9.81	3083	57027	2.01
	5	15235	223991	0.98	4867	8.72	3586	67144	2.18

Table 12
The experimental results of Maplex, Maplex-NoCol, BnBk, BS and RDS. The input graphs for each algorithm are pruned by our preprocessing method.

graph	k	opt	Maplex		Maplex-NoCol		BnBk	BS	RDS
			branches	time	branches	time			
bio-celegans	2	10	0	0.00	0	0.00	0.00	0.00	0.00
	3	11	0	0.00	0	0.00	0.00	0.00	0.00
	4	13	84	0.00	162	0.00	0.00	0.00	0.00
	5	14	26	0.00	42	0.00	0.00	0.00	0.00
bio-diseasome	2	11	0	0.00	0	0.00	0.00	0.00	0.00
	3	11	4	0.00	11	0.00	0.00	0.00	0.00
	4	11	12	0.00	36	0.00	0.00	0.00	0.01
	5	11	55	0.00	104	0.00	0.04	0.00	0.74
bio-dmela	2	8	10	0.00	15	0.00	0.00	0.00	0.00
	3	9	16	0.00	47	0.00	0.00	0.00	0.00
	4	10	0	0.00	0	0.00	0.00	0.00	0.00
	5	12	0	0.00	0	0.00	0.00	0.00	0.01
bio-yeast	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	7	0	0.00	0	0.00	0.00	0.00	0.00
	4	7	5	0.00	6	0.00	0.00	0.00	0.00
	5	8	1156	0.01	6422	0.01	0.03	**0.00**	135.27
ca-AstroPh	2	57	0	0.00	0	0.00	0.00	0.00	0.00
	3	57	4	0.00	57	0.00	0.02	0.00	0.03
	4	57	6	0.00	57	0.00	0.16	0.00	0.65
	5	57	13	0.00	161	0.00	2.40	0.01	175.43
ca-citeseer	2	87	0	0.00	0	0.00	0.00	0.00	0.00
	3	87	0	0.00	0	0.00	0.00	0.00	0.00
	4	87	0	0.00	0	0.00	0.00	0.00	0.00
	5	87	0	0.00	0	0.00	0.00	0.00	0.00
ca-coauthors-dblp	2	337	0	0.00	0	0.00	0.00	0.00	0.00
	3	337	0	0.00	0	0.00	0.00	0.00	0.00
	4	337	0	0.00	0	0.00	0.00	0.00	0.00
	5	337	0	0.00	0	0.00	0.00	0.00	0.00
ca-CondMat	2	26	0	0.00	0	0.00	0.00	0.00	0.00
	3	26	0	0.00	0	0.00	0.00	0.00	0.00
	4	26	0	0.00	0	0.00	0.00	0.00	0.00
	5	26	6	0.00	24	0.00	0.01	0.00	0.24
ca-CSphd	2	4	0	0.00	0	0.00	0.00	0.00	0.00
	3	5	0	0.00	0	0.00	0.00	0.00	0.00
	4	6	0	0.00	0	0.00	0.00	0.00	0.00
	5	7	588044	2.89	128653650	173.75	0.00	0.00	31.58
ca-dblp-2010	2	75	0	0.00	0	0.00	0.00	0.00	0.00
	3	75	0	0.00	0	0.00	0.00	0.00	0.00
	4	75	0	0.00	0	0.00	0.00	0.00	0.00
	5	75	0	0.00	0	0.00	0.00	0.00	0.00
ca-dblp-2012	2	114	0	0.00	0	0.00	0.00	0.00	0.01
	3	114	0	0.00	0	0.00	0.00	0.00	0.01
	4	114	0	0.00	0	0.00	0.00	0.00	0.00
	5	114	0	0.00	0	0.00	0.00	0.00	0.00
ca-Erdos992	2	8	0	0.00	0	0.00	0.00	0.00	0.00
	3	9	0	0.00	0	0.00	0.00	0.00	0.00
	4	10	0	0.00	0	0.00	0.00	0.00	0.00
	5	11	0	0.00	0	0.00	0.00	0.00	0.00
ca-GrQc	2	44	0	0.00	0	0.00	0.00	0.00	0.00
	3	45	0	0.00	0	0.00	0.00	0.00	0.00
	4	46	0	0.00	0	0.00	0.00	0.00	0.00
	5	46	0	0.00	0	0.00	0.00	0.00	0.00
ca-HepPh	2	239	0	0.00	0	0.00	0.00	0.00	0.00
	3	239	0	0.00	0	0.00	0.00	0.00	0.00
	4	239	0	0.00	0	0.00	0.00	0.00	0.00
	5	239	0	0.00	0	0.00	0.00	0.00	0.00
ca-hollywood-2009	2	2209	0	0.00	0	0.00	0.00	0.00	0.00
	3	2209	0	0.00	0	0.00	0.00	0.00	0.00
	4	2209	0	0.00	0	0.00	0.00	0.00	0.00
	5	2209	0	0.00	0	0.00	0.00	0.00	0.00
ca-MathSciNet	2	25	0	0.00	0	0.00	0.00	0.00	0.00
	3	25	4	0.00	25	0.00	0.00	0.00	0.00
	4	25	7	0.00	48	0.00	0.01	0.00	0.12
	5	25	10	0.00	48	0.00	0.17	0.00	1.66
ca-netscience	2	9	0	0.00	0	0.00	0.00	0.00	0.00
	3	9	5	0.00	17	0.00	0.00	0.00	0.00
	4	9	11	0.00	38	0.00	0.00	0.00	0.02
	5	10	36	0.00	136	0.00	0.01	0.00	1.22
ia-email-EU	2	15	855	**0.00**	1787	0.01	0.02	0.31	0.02
	3	16	1375	0.00	1968	0.00	0.01	0.70	0.10
	4	18	1326	0.00	1938	0.00	0.05	5.17	1.84
	5	20	2099	0.01	3806	0.01	0.29	330.15	138.01

Table 13
The experimental results of Maplex, Maplex-NoCol, BnBk, BS and RDS. The input graphs for each algorithm are pruned by our preprocessing method.

graph	k	opt	Maplex		Maplex-NoCol		BnBk	BS	RDS
			branches	time	branches	time			
ia-email-univ	2	12	0	0.00	0	0.00	0.00	0.00	0.00
	3	12	0	0.00	0	0.00	0.00	0.00	0.00
	4	12	0	0.00	0	0.00	0.00	0.00	0.00
	5	13	0	0.00	0	0.00	0.00	0.00	0.00
ia-enron-large	2	22	71543	**0.19**	194634	0.44	0.67	14.89	0.34
	3	24	461995	1.42	830013	1.73	**1.39**	195.97	15.36
	4	26	724521	2.07	1130082	2.09	**1.87**	616.34	124.74
	5	28	1481417	4.83	2486109	**4.61**	11.48	N/A	N/A
ia-enron-only	2	10	0	0.00	0	0.00	0.00	0.00	0.00
	3	12	0	0.00	0	0.00	0.00	0.00	0.00
	4	12	0	0.00	0	0.00	0.00	0.00	0.00
	5	13	13	0.00	35	0.00	0.00	0.00	0.00
ia-fb-messages	2	6	0	0.00	0	0.00	0.00	0.02	0.01
	3	8	103	0.00	261	0.00	0.00	0.03	0.09
	4	9	2085	0.01	6049	0.01	0.02	7.03	476.81
	5	10	10948	**0.06**	41275	0.08	0.75	151.49	N/A
ia-infect-dublin	2	17	0	0.00	0	0.00	0.00	0.00	0.00
	3	18	0	0.00	0	0.00	0.00	0.00	0.00
	4	18	0	0.00	0	0.00	0.00	0.00	0.00
	5	19	847	0.00	1244	0.00	0.00	0.02	0.04
ia-infect-hyper	2	19	2863	0.01	5823	0.01	0.01	0.01	**0.00**
	3	21	9105	0.01	12527	0.02	0.01	0.08	0.01
	4	23	5245	0.01	6500	0.01	**0.00**	0.06	0.06
	5	25	3089	0.01	3778	0.01	**0.00**	0.03	0.18
ia-reality	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	7	0	0.00	0	0.00	0.00	0.00	0.00
	4	8	74	0.00	523	0.00	0.00	0.00	4.90
	5	9	340	0.00	10938	0.01	0.00	0.00	3.41
ia-wiki-Talk	2	18	344721	**1.40**	1012352	2.76	16.32	535.50	5.02
	3	21	11950526	**36.56**	23440699	51.31	101.71	N/A	343.29
	4	23	511839842	1638.08	881316482	1790.78	**449.27**	N/A	N/A
inf-power	2	6	3	0.00	7	0.00	0.00	0.00	0.00
	3	6	33	0.00	60	0.00	0.00	0.00	0.00
	4	8	27	0.00	58	0.00	0.00	0.00	0.01
	5	9	13055	0.07	77437	0.13	0.00	0.00	N/A
inf-road-usa	3	6	395	0.21	3725	0.94	54.75	**0.07**	N/A
	4	7	0	0.00	0	0.00	0.00	0.00	0.00
	5	8	N/A	N/A	N/A	N/A	N/A	**0.07**	N/A
inf-roadNet-CA	2	5	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	660	0.36	4479	1.71	144.15	**0.26**	N/A
	5	8	N/A	N/A	N/A	N/A	N/A	**0.25**	N/A
inf-roadNet-PA	2	5	0	0.00	0	0.00	0.00	0.01	0.07
	3	6	131	0.01	922	0.03	0.48	0.01	269.37
	4	7	17292	0.14	32849	0.17	19.31	**0.01**	N/A
	5	8	4087016	27.41	12490978	34.46	720.67	**0.01**	N/A
rec-amazon	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	334	**0.04**	1496	0.10	92.45	0.14	N/A
	4	8	19699648	1718.32	39442451	1432.30	N/A	**1.11**	N/A
	5	8	N/A	N/A	N/A	N/A	N/A	**1.27**	N/A
rt-retweet-crawl	2	14	0	0.00	0	0.00	0.00	0.00	0.00
	3	15	0	0.00	0	0.00	0.00	0.00	0.00
	4	16	0	0.00	0	0.00	0.00	0.00	0.01
	5	17	0	0.00	0	0.00	0.00	0.00	0.00
rt-retweet	2	4	0	0.00	0	0.00	0.00	0.00	0.00
	3	5	0	0.00	0	0.00	0.00	0.00	0.00
	4	6	0	0.00	0	0.00	0.00	0.00	0.00
	5	7	0	0.00	0	0.00	0.00	0.00	0.00
rt-twitter-copen	2	5	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	17	0.00	24	0.00	0.00	0.00	0.00
	4	8	63	0.00	112	0.00	0.00	0.00	0.00
	5	9	0	0.00	0	0.00	0.00	0.00	0.00
sc-nasasrb	2	24	114134	**31.38**	415682	39.27	1041.54	204.67	N\A
	3	24	567036	**153.13**	2485953	207.67	N\A	558.78	N\A
	4	24	1676545	**1781.37**	7995967	N\A	N\A	N\A	N\A
sc-pkustk11	2	36	8648	**1.96**	22614	2.22	N/A	N/A	N/A
	3	36	162847	**2.63**	503983	3.53	N/A	N/A	N/A
	4	36	1349879	**9.73**	4471409	14.31	N/A	N/A	N/A
sc-pkustk13	2	36	505380	144.66	1136684	326.77	279.54	**60.19**	N\A
	3	36	11113284	762.31	28308270	1338.09	N\A	**168.43**	N\A
	4	36	N\A	N\A	N\A	N\A	N\A	**1517.46**	N\A
sc-shipsec1	2	24	11	0.00	217	0.00	0.00	0.01	0.02
	3	24	21	0.00	217	0.00	0.14	0.01	1.20
	4	24	2493	0.01	3457	0.01	7.05	0.01	137.26
	5	24	4673	0.02	7876	0.02	376.41	0.09	N/A

Table 14
The experimental results of Maplex, Maplex-NoCol, BnBk, BS and RDS. The input graphs for each algorithm are pruned by our preprocessing method.

graph	k	opt	Maplex		Maplex-NoCol		BnBk	BS	RDS
			branches	time	branches	time			
sc-shipsec5	2	24	76256	11.39	252786	23.84	41.22	**5.07**	N/A
	3	24	477106	22.25	2011240	46.30	N/A	**12.65**	N/A
	4	24	1116747	56.88	5096605	86.50	N/A	**55.31**	N/A
	5	26	856366	**48.85**	3629490	82.18	N/A	93.40	N/A
scc_enron-only	2	121	0	0.00	0	0.00	0.00	0.00	0.00
	3	121	0	0.00	0	0.00	0.00	0.00	0.01
	4	122	49071	0.93	55635	0.86	0.10	**0.00**	0.06
	5	123	0	0.00	0	0.00	0.00	0.00	0.00
scc_fb-forum	2	266	N/A	N/A	N/A	N/A	N/A	0.32	**0.10**
	3	268	N/A	N/A	N/A	N/A	N/A	**4.49**	49.23
	4	270	N/A	N/A	N/A	N/A	N/A	**7.25**	141.17
	5	272	N/A	N/A	N/A	N/A	N/A	**18.34**	N/A
scc_fb-messages	2	708	0	0.00	0	0.00	0.00	0.00	0.00
	3	709	0	0.00	0	0.00	0.00	0.00	0.00
	4	709	0	0.00	0	0.00	0.00	0.00	0.00
	5	709	0	0.00	0	0.00	0.00	0.00	0.00
scc_infect-dublin	2	84	0	0.00	0	0.00	0.00	0.00	0.00
	3	84	0	0.00	0	0.00	0.00	0.00	0.00
	4	84	0	0.00	0	0.00	0.00	0.00	0.00
	5	84	0	0.00	0	0.00	0.00	0.00	0.00
scc_infect-hyper	2	106	0	0.00	0	0.00	0.00	0.00	0.00
	3	107	0	0.00	0	0.00	0.00	0.00	0.00
	4	107	0	0.00	0	0.00	0.00	0.00	0.00
	5	107	0	0.00	0	0.00	0.00	0.00	0.00
scc_reality	2	1236	0	0.00	0	0.00	0.00	0.00	0.00
	3	1236	0	0.00	0	0.00	0.00	0.00	0.00
	4	1236	0	0.00	0	0.00	0.00	0.00	0.00
	5	1237	N/A	N/A	N/A	N/A	N/A	**0.28**	10.86
scc_retweet-crawl	2	21	0	0.00	0	0.00	0.00	0.00	0.00
	3	21	0	0.00	0	0.00	0.00	0.00	0.00
	4	22	0	0.00	0	0.00	0.00	0.00	0.00
	5	22	0	0.00	0	0.00	0.00	0.00	0.00
scc_retweet	2	166	0	**0.00**	0	0.01	0.00	0.01	0.02
	3	167	279969	4.52	286613	4.40	0.15	**0.02**	0.04
	4	169	0	0.00	0	0.00	0.00	0.00	0.00
	5	170	166822	6.23	167300	5.73	0.24	**0.01**	0.02
scc_rt_alwefaq	2	17	0	0.00	0	0.00	0.00	0.00	0.00
	3	18	N/A	0.00	0	0.00	0.00	0.00	0.00
	4	18	0	0.00	0	0.00	0.00	0.00	0.00
	5	19	20	0.00	20	0.00	0.00	0.00	0.00
scc_rt_assad	2	9	0	0.00	0	0.00	0.00	0.00	0.00
	3	9	30	0.00	47	0.00	0.00	0.00	0.00
	4	11	0	0.00	0	0.00	0.00	0.00	0.00
	5	12	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_bahrain	2	9	0	0.00	0	0.00	0.00	0.00	0.00
	3	9	0	0.00	0	0.00	0.00	0.00	0.00
	4	10	0	0.00	0	0.00	0.00	0.00	0.00
	5	11	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_barackobama	2	11	0	0.00	0	0.00	0.00	0.00	0.00
	3	12	0	0.00	0	0.00	0.00	0.00	0.00
	4	13	0	0.00	0	0.00	0.00	0.00	0.00
	5	14	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_damascus	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	0	0.00	0	0.00	0.00	0.00	0.00
	4	7	8	0.00	8	0.00	0.00	0.00	0.00
	5	8	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_dash	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	0	0.00	0	0.00	0.00	0.00	0.00
	4	7	8	0.00	8	0.00	0.00	0.00	0.00
	5	8	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_gmanews	2	22	234	0.00	683	0.00	0.00	0.00	0.00
	3	23	2461	0.01	5730	0.01	0.00	0.00	0.00
	4	24	4581	0.01	8685	0.01	0.01	0.00	0.00
	5	25	4553	0.01	8365	0.01	0.00	0.00	0.01
scc_rt_gop	2	3	5	0.00	13	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_http	2	4	0	0.00	0	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	5	0	0.00	0	0.00	0.00	0.00	0.00

Table 15
The experimental results of Maplex, Maplex-NoCol, BnBk, BS and RDS. The input graphs for each algorithm are pruned by our preprocessing method.

graph	k	opt	Maplex		Maplex-NoCol		BnBk	BS	RDS
			branches	time	branches	time			
scc_rt_israel	2	3	13	0.00	22	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_justinbieber	2	18	0	0.00	0	0.00	0.00	0.00	0.00
	3	19	0	0.00	0	0.00	0.00	0.00	0.00
	4	19	0	0.00	0	0.00	0.00	0.00	0.00
	5	20	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_ksa	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	0	0.00	0	0.00	0.00	0.00	0.00
	4	6	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_lebanon	2	2	6	0.00	9	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	4	75	0.00	210	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_libya	2	4	0	0.00	0	0.00	0.00	0.00	0.00
	3	5	8	0.00	18	0.00	0.00	0.00	0.00
	4	6	0	0.00	0	0.00	0.00	0.00	0.00
	5	7	10	0.00	29	0.00	0.00	0.00	0.00
scc_rt_lolgop	2	43	0	0.00	0	0.00	0.00	0.00	0.00
	3	43	0	0.00	0	0.00	0.00	0.00	0.00
	4	43	0	0.00	0	0.00	0.00	0.00	0.00
	5	44	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_mittromney	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	7	0	0.00	0	0.00	0.00	0.00	0.00
	4	8	0	0.00	0	0.00	0.00	0.00	0.00
	5	9	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_obama	2	2	5	0.00	7	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	4	35	0.00	70	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_occupy	2	5	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	0	0.00	0	0.00	0.00	0.00	0.00
	4	8	0	0.00	0	0.00	0.00	0.00	0.00
	5	8	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_occupywallstnyc	2	19	0	0.00	0	0.00	0.00	0.00	0.00
	3	19	185	0.00	990	0.00	0.00	0.00	0.00
	4	20	249	0.00	1426	0.00	0.00	0.00	0.00
	5	22	107	0.00	153	0.00	0.00	0.00	0.02
scc_rt_oman	2	4	0	0.00	0	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_onedirection	2	27	0	0.00	0	0.00	0.00	0.00	0.00
	3	27	0	0.00	0	0.00	0.00	0.00	0.00
	4	27	0	0.00	0	0.00	0.00	0.00	0.00
	5	27	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_p2	2	3	7	0.00	27	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_qatif	2	3	0	0.00	0	0.00	0.00	0.00	0.00
	3	4	11	0.00	44	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_saudi	2	9	0	0.00	0	0.00	0.00	0.00	0.00
	3	10	0	0.00	0	0.00	0.00	0.00	0.00
	4	11	0	0.00	0	0.00	0.00	0.00	0.00
	5	12	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_tcot	2	4	0	0.00	0	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	5	95	0.00	12709	0.01	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_tlot	2	3	0	0.00	0	0.00	0.00	0.00	0.00
	3	4	0	0.00	0	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	0	0.00	0	0.00	0.00	0.00	0.00
scc_rt_uae	2	3	0	0.00	0	0.00	0.00	0.00	0.00
	3	4	15	0.00	45	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	53	0.00	6219	0.01	0.00	0.00	0.00

Table 16
The experimental results of Maplex, Maplex-NoCol, BnBk, BS and RDS. The input graphs for each algorithm are pruned by our preprocessing method.

graph	k	opt	Maplex		Maplex-NoCol		BnBk	BS	RDS
			branches	time	branches	time			
scc_rt_voteonedirection	2	3	0	0.00	0	0.00	0.00	0.00	0.00
	3	4	7	0.00	11	0.00	0.00	0.00	0.00
	4	5	0	0.00	0	0.00	0.00	0.00	0.00
	5	6	7	0.00	7	0.00	0.00	0.00	0.00
scc_twitter-copen	2	581	0	0.27	0	0.49	23.44	**0.16**	0.88
	3	581	N/A	N/A	N/A	N/A	N/A	**11.48**	N/A
	4	581	N/A	N/A	N/A	N/A	N/A	**180.88**	N/A
	5	582	N/A	N/A	N/A	N/A	N/A	**619.67**	N/A
soc-brightkite	2	44	4299	0.02	7045	0.02	0.18	0.02	**0.01**
	3	47	36054	0.12	40276	0.14	0.06	**0.02**	0.03
	4	49	222787	0.70	242976	0.75	0.06	**0.01**	0.05
	5	51	0	0.00	0	0.00	0.00	0.00	0.00
soc-delicious	2	23	3115	**0.02**	44764	0.09	0.07	0.37	0.04
	3	27	2134	**0.01**	13992	0.03	0.04	2.04	1.06
	4	29	3621	0.02	7622	0.02	0.05	31.56	12.18
	5	30	6383	**0.02**	16649	0.03	0.04	4.47	38.41
soc-dolphins	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	7	0	0.00	0	0.00	0.00	0.00	0.00
	4	7	54	0.00	131	0.00	0.00	0.00	0.01
	5	9	4840	0.02	27712	0.03	0.00	0.00	0.18
soc-douban	2	12	64	0.00	112	0.00	0.00	0.00	0.06
	3	14	0	0.00	0	0.00	0.00	0.01	153.36
	4	16	29	0.00	30	0.00	0.00	0.00	1242.40
	5	17	34	0.00	45	0.00	0.00	0.00	N/A
soc-epinions	2	18	32121	**0.08**	100463	0.20	0.27	2.00	0.10
	3	21	88160	**0.16**	142080	0.25	0.16	2.85	0.38
	4	23	532397	1.31	796053	1.41	**0.13**	15.92	10.43
	5	25	1767034	4.57	2667841	4.79	**0.16**	51.27	235.27
soc-flixster	2	38	51893366	289.48	259887884	693.63	644.25	120.17	**4.07**
	3	42	N/A	N/A	N/A	N/A	N/A	N/A	**464.16**
soc-FourSquare	2	35	9118949	**1158.08**	N/A	N/A	N/A	N/A	N/A
	3	39	8452295	**87.69**	16477192	97.59	591.75	N/A	N/A
	4	42	44859662	**151.64**	69717963	168.36	169.47	N/A	N/A
	5	44	7279628	**31.05**	11031373	31.67	65.57	N/A	N/A
soc-gowalla	2	30	2751	**0.06**	20874	0.13	12.87	37.69	2.36
	3	31	44729	**0.16**	99122	0.29	14.39	N/A	89.47
	4	32	886207	**2.83**	1426044	3.14	38.22	N/A	1600.17
	5	32	43239897	133.02	68187459	132.96	**100.57**	N/A	N/A
soc-karate	2	6	0	0.00	0	0.00	0.00	0.00	0.00
	3	6	0	0.00	0	0.00	0.00	0.00	0.00
	4	8	44	0.00	86	0.00	0.00	0.00	0.00
	5	9	60	0.00	220	0.00	0.00	0.00	0.00
soc-lastfm	2	18	134475	**2.47**	4338550	14.74	75.08	N/A	35.09
	3	21	7254034	**54.55**	94051342	257.55	325.20	N/A	N/A
	4	24	160552910	**1096.57**	N/A	N/A	1158.60	N/A	N/A
soc-livejournal	2	214	0	0.00	0	0.00	0.00	0.00	0.00
	3	214	0	0.00	0	0.00	0.00	0.00	0.00
	4	214	0	0.00	0	0.00	0.00	0.00	0.00
	5	214	0	0.00	0	0.00	0.00	0.00	0.00
soc-LiveMocha	2	19	2211000	**10.03**	8538232	21.56	110.07	N\A	230.58
	3	22	231628480	**822.13**	496640471	1041.74	1557.97	N\A	N\A
soc-pokec	2	31	229	**0.26**	1549	0.28	3.01	5.55	19.58
	3	32	11145	**0.10**	27377	0.12	2.23	36.18	272.39
	4	32	1510477	**4.92**	3436847	7.66	6.82	485.93	N\A
	5	34	7399842	27.09	15851749	33.75	**25.18**	1612.22	N\A
soc-slashdot	2	31	301614	1.26	826010	2.12	19.50	8.46	**0.42**
	3	34	17430542	58.13	27223570	66.15	143.27	384.71	**31.08**
	4	37	261312873	704.21	344296432	798.13	**262.50**	N/A	782.52
	5	40	N/A	N/A	N/A	N/A	**96.83**	N/A	N/A
soc-twitter-follows	2	8	19	**0.02**	865	0.03	0.03	6.01	257.47
	3	9	35	0.02	114	**0.01**	0.02	27.10	N/A
	4	11	35	0.01	91	0.01	0.01	135.98	N/A
	5	13	23	0.01	39	0.01	0.01	236.87	N/A
soc-wiki-Vote	2	8	0	0.00	0	0.00	0.00	0.00	0.00
	3	9	0	0.00	0	0.00	0.00	0.00	0.00
	4	11	75	0.00	137	0.00	0.00	0.00	0.01
	5	12	213	0.00	379	0.00	0.00	0.00	0.00
soc-youtube-snap	2	20	160436	**0.65**	563820	1.52	3.86	286.05	3.58
	3	21	10548963	37.09	23608355	49.00	**18.04**	N/A	N/A
	4	24	78784092	263.67	147503341	312.65	**37.88**	N/A	N/A
	5	26	N/A	N/A	N/A	N/A	**44.55**	N/A	N/A

Table 17
The experimental results of Maplex, Maplex-NoCol, BnBk, BS and RDS. The input graphs for each algorithm are pruned by our preprocessing method.

graph	k	opt	Maplex		Maplex-NoCol		BnBk	BS	RDS
			branches	time	branches	time			
soc-youtube	2	20	99453	**0.48**	397699	1.03	1.75	104.04	1.42
	3	21	5907729	19.17	12141147	26.56	**5.37**	N/A	308.51
	4	24	13874326	41.67	23356764	49.14	**7.31**	N/A	N/A
	5	26	194675260	609.16	326957763	630.57	**15.14**	N/A	N/A
socfb-A-anon	2	28	222243	**3.08**	7799968	28.41	26.65	267.68	24.21
	3	32	758323	**8.22**	9580924	33.51	76.81	N\A	1776.45
	4	35	1032676	**8.37**	8210105	25.63	264.98	N\A	N\A
	5	37	1962993	**12.26**	15228233	42.22	251.80	N\A	N\A
socfb-B-anon	2	27	185317	**3.13**	11423410	31.93	62.72	N\A	N\A
	3	30	3906023	**40.58**	80012762	259.72	84.49	N\A	N\A
	4	33	14744144	**90.46**	233789118	534.07	142.73	N\A	N\A
	5	35	136913172	708.45	N\A	N\A	**354.37**	N\A	N\A
socfb-Berkeley13	2	47	191875	**1.68**	624373	2.45	57.68	4.37	1.83
	3	51	101977	**0.52**	136787	0.55	329.43	46.48	24.40
	4	52	306809	1.54	405861	**1.53**	44.33	91.14	114.83
	5	53	1916622	**9.86**	2565827	10.01	11.26	225.54	N/A
socfb-CMU	2	47	0	0.00	0	0.00	0.01	0.01	0.04
	3	49	0	0.00	0	0.00	0.00	0.00	0.17
	4	50	1250	0.01	1361	0.01	0.02	0.01	12.81
	5	52	437	0.00	464	0.00	0.01	0.00	76.03
socfb-Duke14	2	38	31094914	174.87	88884311	292.12	897.85	643.76	**33.13**
	3	43	N\A	N\A	N\A	N\A	N\A	N\A	**1704.69**
socfb-Indiana	2	51	2943995	69.70	13596023	110.39	284.06	31.54	**30.79**
	3	55	65689191	**646.09**	106278245	671.05	N\A	857.00	N\A
socfb-MIT	2	37	4001670	16.20	6674088	17.79	46.32	**2.33**	8.02
	3	42	8489388	24.02	10402987	26.17	13.21	**2.43**	29.36
	4	45	6067013	19.21	7786512	22.75	4.11	**1.74**	165.52
	5	48	1744697	6.33	2144562	5.98	**0.54**	0.59	204.77
socfb-OR	2	33	2097662	6.61	3861834	9.59	10.67	2.79	**0.72**
	3	37	768721	**2.26**	1007295	2.63	17.14	20.79	15.17
	4	39	36718346	121.23	50869290	128.37	**14.44**	124.19	264.66
	5	42	3161129	11.87	4301970	11.11	**7.21**	250.21	N/A
socfb-Penn94	2	50	6962	0.04	7426	0.04	0.06	**0.01**	0.34
	3	52	0	0.00	0	0.00	0.01	0.01	5.23
	4	54	0	0.00	0	0.00	0.01	0.01	60.02
	5	55	6877	0.03	8522	0.03	0.10	**0.01**	N/A
socfb-Stanford3	2	59	377788	**13.54**	961581	15.67	211.15	42.58	57.69
	3	62	3038795	**63.71**	4615936	64.51	N/A	N/A	N/A
	4	65	667581	12.35	847730	**11.41**	N/A	N/A	N/A
	5	67	489728	7.83	614469	**7.77**	N/A	N/A	N/A
socfb-Texas84	2	55	N/A	N/A	N/A	N/A	N/A	90.01	**21.28**
	3	60	N/A	N/A	N/A	N/A	N/A	471.72	**206.19**
	5	68	N/A	N/A	N/A	N/A	**243.59**	1439.47	N/A
socfb-uci-uni	2	9	0	0.00	0	0.00	0.00	0.00	0.00
	3	10	0	0.00	0	0.00	0.00	0.00	0.00
	4	11	0	0.00	0	0.00	0.00	0.00	0.00
	5	13	0	0.00	0	0.00	0.00	0.00	0.00
socfb-UCLA	2	55	45831	0.18	62740	0.24	0.09	0.01	**0.00**
	3	57	180503	0.80	205791	0.81	0.09	**0.01**	0.02
	4	59	18415	0.12	21561	0.11	0.09	**0.01**	0.12
	5	62	493	0.00	540	0.00	0.01	0.00	0.00
socfb-UConn	2	53	84573	0.23	91757	0.34	0.91	0.01	0.01
	3	56	38930	0.20	45049	0.19	0.09	**0.01**	0.02
	4	58	8928	0.05	10123	0.05	0.08	**0.00**	0.01
	5	60	571	0.00	633	0.00	0.01	0.00	0.00
socfb-UCSB37	2	59	15705	0.10	17928	0.10	53.27	0.10	**0.03**
	3	63	12174	0.08	13862	0.07	1.03	**0.04**	0.06
	4	66	1181788	5.64	1354989	5.14	0.18	**0.03**	0.06
	5	68	0	0.00	0	0.00	0.00	0.00	0.00
socfb-UF	2	60	N/A	N/A	N/A	N/A	N/A	261.98	**116.38**
socfb-UIllinois	2	63	N/A	N/A	N/A	N/A	N/A	**278.61**	756.44
socfb-Wisconsin87	2	42	192049	0.89	402799	1.15	9.07	0.91	**0.29**
	3	44	1417225	5.41	1895090	5.67	4.75	6.07	**3.05**
	4	47	2130295	9.05	2535243	8.29	**0.92**	9.85	23.22
	5	50	593951	2.58	716658	2.62	**0.31**	39.62	236.07
tech-as-caida2007	2	17	438	0.00	576	0.00	0.00	0.00	0.00
	3	18	7185	0.02	9724	0.02	0.01	0.03	0.01
	4	21	2704	0.01	3566	0.01	**0.00**	0.01	0.02
	5	23	648	0.00	822	0.00	0.00	0.00	0.01

Table 18
The experimental results of Maplex, Maplex-NoCol, BnBk, BS and RDS. The input graphs for each algorithm are pruned by our preprocessing method.

graph	k	opt	Maplex		Maplex-NoCol		BnBk	BS	RDS
			branches	time	branches	time			
tech-as-skitter	2	69	4446674	**39.72**	17593205	72.18	N/A	375.31	107.10
	3	71	212149171	1376.42	394548503	1563.76	**1316.18**	N/A	N/A
	4	74	N/A	N/A	N/A	N/A	**1202.27**	N/A	N/A
	5	75	N/A	N/A	N/A	N/A	**223.57**	N/A	N/A
tech-internet-as	2	18	86	0.00	89	0.00	0.00	0.00	0.00
	3	20	171	0.00	236	0.00	0.00	0.07	0.02
	4	22	0	0.00	0	0.00	0.00	0.00	0.00
	5	22	1170	0.00	1977	0.00	0.01	0.25	0.54
tech-p2p-gnutella	2	5	21	0.00	121	0.00	0.00	0.00	0.00
	3	6	0	0.00	0	0.00	0.00	0.00	0.00
	4	8	0	0.00	0	0.00	0.00	0.00	0.00
	5	10	0	0.00	0	0.00	0.00	0.00	0.00
tech-RL-caida	2	20	523	0.01	1030	0.01	0.02	0.56	287.33
	3	23	0	0.02	0	0.02	0.02	3.68	N/A
	4	24	1533	0.04	1802	**0.03**	0.05	62.64	N/A
	5	26	18861	0.06	24885	0.06	**0.02**	151.19	N/A
tech-routers-rf	2	17	0	0.00	0	0.00	0.00	0.00	0.00
	3	18	0	0.00	0	0.00	0.00	0.00	0.00
	4	19	0	0.00	0	0.00	0.00	0.00	0.00
	5	20	0	0.00	0	0.00	0.00	0.00	0.00
tech-WHOIS	2	64	N/A	N/A	N/A	N/A	N/A	62.52	**8.74**
	3	71	N/A	N/A	N/A	N/A	1188.27	328.16	**124.39**
	4	74	N/A	N/A	N/A	N/A	**506.18**	N/A	N/A
	5	76	N/A	N/A	N/A	N/A	**984.36**	N/A	N/A
web-arabic-2005	2	102	0	0.00	0	0.00	0.00	0.00	0.00
	3	102	0	0.00	0	0.00	0.00	0.00	0.00
	4	102	0	0.00	0	0.00	0.00	0.00	0.00
	5	102	0	0.00	0	0.00	0.00	0.00	0.00
web-BerkStan	2	29	0	0.00	0	0.00	0.00	0.00	0.00
	3	29	0	0.00	0	0.00	0.00	0.00	0.00
	4	29	0	0.00	0	0.00	0.00	0.00	0.00
	5	29	0	0.00	0	0.00	0.00	0.00	0.00
web-edu	2	30	0	0.00	0	0.00	0.00	0.00	0.00
	3	30	0	0.00	0	0.00	0.00	0.00	0.00
	4	30	0	0.00	0	0.00	0.00	0.00	0.00
	5	30	0	0.00	0	0.00	0.00	0.00	0.00
web-google	2	19	0	0.00	0	0.00	0.00	0.00	0.00
	3	19	0	0.00	0	0.00	0.00	0.00	0.00
	4	19	0	0.00	0	0.00	0.00	0.00	0.00
	5	19	7	0.00	33	0.00	0.00	0.00	0.20
web-indochina-2004	2	50	0	0.00	0	0.00	0.00	0.00	0.00
	3	50	0	0.00	0	0.00	0.00	0.00	0.00
	4	50	0	0.00	0	0.00	0.00	0.00	0.00
	5	50	0	0.00	0	0.00	0.00	0.00	0.00
web-it-2004	2	432	0	0.00	0	0.00	0.00	0.00	0.00
	3	432	5	0.19	433	0.20	17.92	**0.13**	78.81
	4	432	7	0.20	433	0.21	302.50	**0.15**	N/A
	5	432	9	0.19	433	0.22	N/A	**0.18**	N/A
web-polblogs	2	12	0	0.00	0	0.00	0.00	0.00	0.00
	3	14	0	0.00	0	0.00	0.00	0.00	0.00
	4	15	0	0.00	0	0.00	0.00	0.00	0.00
	5	17	0	0.00	0	0.00	0.00	0.00	0.00
web-sk-2005	2	82	4	0.00	165	0.00	0.06	0.01	0.02
	3	83	86	0.01	248	0.01	0.19	0.01	0.94
	4	83	90	0.01	248	0.01	2.49	0.01	43.01
	5	83	93	0.01	248	0.01	24.54	0.01	1290.60
web-spam	2	21	45620	0.14	193185	0.37	0.26	0.47	**0.03**
	3	24	1526350	4.06	3114362	5.63	**0.79**	11.76	1.58
	4	27	5003748	12.98	7976019	13.92	**0.43**	15.91	5.49
	5	30	1898726	4.89	2859783	5.31	**0.06**	7.07	11.46
web-uk-2005	2	500	3	0.29	501	0.33	22.82	**0.16**	1.32
	3	500	5	0.28	501	0.30	78.23	**0.18**	130.71
	4	500	8	0.48	999	0.50	1422.58	**0.27**	N/A
	5	500	11	0.44	999	0.54	N/A	**0.27**	N/A
web-webbase-2001	2	33	0	0.00	0	0.00	0.00	0.00	0.00
	3	33	0	0.00	0	0.00	0.00	0.00	0.00
	4	33	0	0.00	0	0.00	0.00	0.00	0.00
	5	33	0	0.00	0	0.00	0.00	0.00	0.00
web-wikipedia2009	2	32	74	**0.12**	8261	3.41	159.17	548.59	601.17
	3	32	74	**0.14**	9855	2.46	N/A	N/A	N/A
	4	32	117	**0.12**	15111	0.83	N/A	N/A	N/A
	5	32	1430	**0.51**	161203	3.20	N/A	N/A	N/A

Table 19
Experimental results of SNAP graphs for $k = 2$, 3, 4 and 5.

graph			k=2					k=3					k=4					k=5				
name	**#vtx**	**#edges**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**
cit-HepPh	34546	420877	24	**0.35**	1.34	97.88	N/A	27	**0.41**	0.89	56.72	N/A	30	**0.28**	0.44	17.34	N/A	32	**0.15**	0.46	7.47	N/A
cit-HepTh	27769	352285	28	**1.10**	3.16	504.56	N/A	31	**1.69**	4.00	925.27	N/A	34	7.59	**1.72**	91.37	N/A	37	3.01	**0.83**	13.67	N/A
email-EuAll	265009	364481	19	**0.38**	1.62	N/A	N/A	22	**1.23**	1.29	N/A	N/A	25	1.89	**1.23**	N/A	N/A	27	1.56	**1.31**	N/A	N/A
p2p-Gnutella04	10876	39994	5	0.06	**0.03**	39.42	N/A	7	0.06	**0.02**	22.97	N/A	9	0.05	**0.02**	13.13	N/A	10	0.04	**0.02**	15.49	N/A
p2p-Gnutella24	26518	65369	5	0.11	**0.04**	49.32	N/A	6	0.09	**0.03**	58.40	N/A	8	57.14	**0.03**	31.15	N/A	9	59.23	**0.00**	34.90	N/A
p2p-Gnutella25	22687	54705	5	0.09	**0.03**	44.76	N/A	6	0.07	**0.03**	52.72	N/A	8	0.06	**0.02**	19.01	N/A	10	0.02	**0.00**	0.46	N/A
soc-Epinions1	75879	405740	28	**11.73**	34.38	N/A	N/A	32	432.33	**65.78**	N/A	N/A	37	137.46	**9.61**	N/A	N/A	39	194.41	**8.62**	N/A	N/A
soc-Slashdot0811	77360	469180	31	**21.51**	53.63	N/A	N/A	34	573.83	**390.53**	N/A	N/A	38	N/A	**133.45**	N/A	N/A	40	N/A	**286.44**	N/A	N/A
soc-Slashdot0902	82168	504230	32	**23.99**	70.11	N/A	N/A	35	808.83	**605.86**	N/A	N/A	40	1638.30	**41.51**	N/A	N/A	42	1685.54	**75.52**	N/A	N/A
web-BerkStan	685230	6649470	202	**0.42**	3.08	N/A	N/A	202	**0.85**	35.59	N/A	N/A	202	**2.11**	382.31	N/A	N/A	202	3.44	**0.00**	N/A	N/A
web-Google	875713	4322051	46	**0.66**	0.93	N/A	N/A	47	**0.69**	0.98	N/A	N/A	48	**0.69**	1.00	N/A	N/A	48	**0.69**	0.98	N/A	N/A
web-NotreDame	325729	1090108	155	**0.16**	4.56	N/A	N/A	155	**0.20**	6.28	N/A	N/A	155	**0.20**	115.36	N/A	N/A	155	**0.20**	N/A	N/A	N/A
web-Stanford	281903	1992636	64	**47.35**	323.39	N/A	N/A	64	486.66	**330.50**	N/A	N/A	65	N/A	**309.71**	N/A	N/A	66	N/A	**301.27**	N/A	N/A
wiki-Vote	7115	100762	21	**0.82**	6.34	N/A	453.53	24	26.96	**8.90**	N/A	N/A	27	5.15	**2.24**	N/A	N/A	28	1260.83	**23.10**	N/A	N/A

Table 20
Experimental results of 10th DIMACS graphs for $k = 2$, 3, 4 and 5.

graph			k=2					k=3					k=4					k=5				
name	**#vtx**	**#edges**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**	**opt**	**Maplex**	**BnBk**	**BS**	**RDS**
adjnoun	112	425	6	0.00	0.00	0.01	0.00	8	0.00	0.00	0.01	0.03	8	0.00	0.00	0.05	0.84	10	0.01	0.00	0.00	15.04
as-22july06	22963	48436	19	**0.01**	0.03	0.34	N/A	21	**0.02**	0.05	4.19	N/A	22	**0.02**	0.04	51.78	N/A	24	0.13	**0.08**	194.76	N/A
astro-ph	16706	121251	57	0.01	**0.00**	0.08	N/A	57	**0.01**	0.02	0.08	N/A	57	**0.01**	0.12	0.08	N/A	57	**0.01**	1.67	0.08	N/A
caidaRouterLevel	192244	609066	20	**0.73**	2.84	N/A	N/A	23	3.31	**3.27**	N/A	N/A	24	**18.40**	1757.20	N/A	N/A	26	**1.45**	2.24	N/A	N/A
celegansneural	297	2148	10	0.00	0.00	0.03	0.02	11	0.00	0.00	0.05	0.83	12	0.00	0.00	0.09	40.90	13	0.00	0.00	0.15	N/A
celegans_metabolic	453	2025	10	0.00	0.00	0.00	0.06	11	0.00	0.00	0.00	4.65	13	0.00	0.00	0.00	162.39	14	0.00	0.00	0.00	N/A
chesapeake	39	170	7	0.00	0.00	0.00	0.00	8	0.00	0.00	0.00	0.00	9	0.00	0.00	0.00	0.01	11	0.00	0.00	0.00	0.05
cnr-2000	325557	2738969	85	**0.09**	0.12	N/A	N/A	86	**0.09**	0.12	N/A	N/A	86	**0.10**	0.16	N/A	N/A	86	**0.10**	0.16	N/A	N/A
coAuthorsCiteseer	227320	814134	87	**0.10**	0.12	N/A	N/A	87	**0.10**	0.11	N/A	N/A	87	**0.09**	0.12	N/A	N/A	87	**0.08**	0.11	N/A	N/A
coAuthorsDBLP	299067	977676	115	**0.10**	0.18	N/A	N/A	115	**0.11**	0.18	N/A	N/A	115	**0.10**	0.19	N/A	N/A	115	**0.14**	0.19	N/A	N/A
cond-mat-2003	31163	120029	25	0.01	0.01	0.09	N/A	26	0.01	0.01	0.07	N/A	27	0.01	0.01	0.07	N/A	27	0.01	0.01	0.07	N/A
cond-mat-2005	40421	175691	30	0.01	0.01	0.16	N/A	30	0.02	**0.01**	0.14	N/A	30	0.02	**0.01**	0.14	N/A	30	**0.02**	0.03	0.15	N/A
cond-mat	16726	47594	18	0.00	0.00	0.03	N/A	18	0.00	0.00	0.05	N/A	19	**0.00**	0.01	0.05	N/A	20	**0.01**	0.04	0.05	N/A
dolphins	62	159	6	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.00	7	0.00	0.00	0.00	0.09	9	0.00	0.00	0.00	0.49
email	1133	5451	12	0.00	0.00	0.01	0.76	12	0.00	0.00	0.01	117.12	12	0.00	0.00	0.03	N/A	13	**0.00**	0.01	0.03	N/A
football	115	613	10	0.00	0.00	0.00	0.00	11	0.00	0.00	0.00	0.03	12	0.00	0.02	0.00	0.28	12	0.00	1.06	0.00	9.15
hep-th	8361	15751	24	0.00	0.00	0.01	N/A	24	0.00	0.00	0.01	N/A	24	0.00	0.00	0.01	N/A	24	0.00	0.00	0.01	N/A
jazz	198	2742	30	0.00	0.00	0.00	0.01	30	0.00	0.00	0.00	0.06	30	0.00	0.00	0.00	0.56	30	0.00	0.00	0.00	6.76
karate	34	78	6	0.00	0.00	0.00	0.00	6	0.00	0.00	0.00	0.00	8	0.00	0.00	0.00	0.01	9	0.00	0.00	0.00	0.03
lesmis	77	254	10	0.00	0.00	0.00	0.00	12	0.00	0.00	0.00	0.02	12	0.00	0.00	0.00	0.21	12	0.00	0.00	0.00	2.47
memplus	17758	54196	97	0.00	0.00	0.02	N/A	97	0.00	0.00	0.02	N/A	97	0.00	0.00	0.02	N/A	97	0.00	0.00	0.02	N/A
netscience	1589	2742	20	0.00	0.00	0.00	4.96	20	0.00	0.00	0.00	1794.27	20	0.00	0.00	0.00	N/A	20	0.00	0.00	0.00	N/A
PGPgiantcompo	10680	24316	29	**0.00**	0.02	0.01	588.15	31	0.00	0.00	0.02	N/A	33	0.00	0.00	0.01	N/A	35	**0.00**	0.01	0.01	N/A
polblogs	1490	16715	23	**0.71**	1.15	11.65	2.64	27	2.94	**0.77**	113.63	297.77	29	16.01	**1.48**	1231.39	N/A	32	1.43	**0.38**	N/A	N/A
polbooks	105	441	7	0.00	0.00	0.00	0.00	9	0.00	0.00	0.00	0.04	10	0.00	0.00	0.00	0.68	11	0.00	0.00	0.00	14.10
power	4941	6594	6	0.00	0.00	0.00	336.15	6	0.00	0.00	0.10	N/A	8	0.00	0.00	0.00	N/A	9	0.04	0.00	0.00	N/A
rgg_n_2_17_s0	131072	728753	16	0.08	**0.06**	N/A	N/A	17	0.08	**0.07**	N/A	N/A	18	0.08	**0.06**	N/A	N/A	18	0.09	**0.06**	N/A	N/A
rgg_n_2_19_s0	524288	3269766	19	0.40	**0.30**	N/A	N/A	19	0.45	**0.30**	N/A	N/A	20	0.46	**0.29**	N/A	N/A	21	0.46	**0.30**	N/A	N/A
rgg_n_2_20_s0	1048576	6891620	18	0.95	**0.68**	N/A	N/A	19	1.04	**0.72**	N/A	N/A	20	1.03	**0.77**	N/A	N/A	20	1.03	**0.73**	N/A	N/A

Table 21
Experimental results of Erdös graphs for $k = 2$, 3, 4 and 5.

graph			k=2					k=3					k=4					k=5				
name	#vtx	#edges	opt	Maplex	BnBk	BS	RDS	opt	Maplex	BnBk	BS	RDS	opt	Maplex	BnBk	BS	RDS	opt	Maplex	BnBk	BS	RDS
Erdos971	433	1314	8	0.00	0.00	0.01	0.05	9	0.00	0.00	0.01	4.18	11	0.00	0.00	0.00	307.02	12	0.00	0.00	0.01	N/A
Erdos972	5482	8972	8	0.00	0.00	0.02	543.15	9	0.00	0.00	0.03	N/A	11	0.00	0.00	0.01	N/A	12	0.00	0.00	0.01	N/A
Erdos981	445	1381	8	0.00	0.00	0.01	0.06	9	0.00	0.00	0.01	4.00	11	0.00	0.00	0.00	259.49	12	0.00	0.00	0.01	N/A
Erdos982	5816	9505	8	0.00	0.00	0.02	475.71	9	0.00	0.00	0.04	N/A	11	0.00	0.00	0.01	N/A	12	0.00	0.00	0.02	N/A
Erdos991	454	1417	8	0.00	0.00	0.01	0.06	9	0.00	0.00	0.02	4.42	11	0.00	0.00	0.00	295.67	12	0.00	0.00	0.01	N/A
Erdos992	6094	9939	8	0.00	0.00	0.02	859.10	9	0.00	0.00	0.05	N/A	11	0.00	0.00	0.02	N/A	12	0.00	0.00	0.02	N/A

Table 22
Experimental results of clique graphs for $k = 2$, 3, 4 and 5.

graph			k=2					k=3					k=4					k=5				
name	#vtx	#edges	opt	Maplex	BnBk	BS	RDS	opt	Maplex	BnBk	BS	RDS	opt	Maplex	BnBk	BS	RDS	opt	Maplex	BnBk	BS	RDS
brock200_2	200	9876	13	12.26	115.23	660.48	**10.28**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
brock200_3	200	12048	17	268.24	1774.15	N/A	**200.62**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
brock200_4	200	13089	20	**1371.68**	N/A	N/A	1434.81	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
c-fat200-1	200	1534	12	0.00	0.00	0.01	0.00	12	**0.00**	0.06	0.01	0.10	12	**0.00**	4.13	0.02	4.43	14	**0.00**	2.81	0.01	58.38
c-fat200-2	200	3235	24	0.00	0.00	0.01	0.00	24	**0.00**	0.03	0.01	0.02	24	**0.00**	0.83	0.01	0.50	24	**0.00**	32.48	0.06	17.68
c-fat200-5	200	8473	58	0.03	0.06	0.03	**0.00**	58	1.55	0.96	0.05	**0.01**	58	32.59	15.23	**0.07**	0.09	58	336.99	219.68	**0.49**	1.46
c-fat500-1	500	4459	14	**0.00**	0.01	0.02	0.05	14	**0.02**	0.30	0.03	1.98	14	**0.02**	74.99	0.07	306.05	15	**0.02**	1186.09	0.12	N/A
c-fat500-10	500	46627	126	0.48	0.84	0.52	**0.02**	126	145.82	22.25	0.24	**0.06**	N/A	N/A	473.37	0.68	**0.58**	126	N/A	N/A	**3.17**	7.77
c-fat500-2	500	9139	26	**0.02**	0.07	0.03	0.03	26	0.17	1.80	**0.05**	0.43	26	0.59	111.74	**0.09**	22.42	26	1.04	N/A	**0.84**	1421.49
c-fat500-5	500	23191	64	0.05	0.10	0.13	**0.02**	64	3.18	1.71	**0.07**	0.10	64	88.11	55.53	**0.15**	1.65	64	1234.07	1735.45	**0.58**	29.78
hamming10-2	1024	518656	512	N/A	N/A	N/A	**2.61**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
hamming6-2	64	1824	32	24.05	27.89	42.93	**0.00**	32	N/A	N/A	N/A	**0.13**	40	N/A	N/A	N/A	**127.05**	48	663.01	N/A	N/A	**13.38**
hamming6-4	64	704	6	0.00	0.01	0.07	0.00	8	0.02	0.03	1.64	0.02	10	**0.05**	0.09	10.88	0.16	12	0.08	**0.04**	143.53	2.00
hamming8-2	256	31616	128	N/A	N/A	N/A	**0.03**	128	N/A	N/A	N/A	**271.75**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
hamming8-4	256	20864	16	N/A	N/A	N/A	**8.67**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
johnson8-2-4	28	210	5	0.00	0.00	0.01	0.00	8	0.01	0.01	0.05	**0.00**	9	0.09	0.10	0.55	**0.05**	12	0.09	**0.04**	0.54	0.05
johnson8-4-4	70	1855	14	8.50	16.92	364.95	**0.02**	18	768.50	707.84	N/A	**5.72**	22	N/A	N/A	N/A	**839.88**	28	N/A	N/A	N/A	**1129.68**
keller4	171	9435	15	490.92	N/A	N/A	**118.22**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
MANN_a9	45	918	26	0.16	26.73	6.84	**0.02**	36	0.75	0.24	**0.06**	0.15	36	143.01	90.27	**8.85**	12.77	45	0.00	0.00	0.00	0.00
p_hat1000-1	1000	122253	13	**723.49**	N/A	N/A	766.69	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
p_hat300-1	300	10933	10	**0.84**	19.99	46.47	0.94	12	**63.93**	343.32	N/A	99.35	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
p_hat500-1	500	31569	12	**16.13**	472.19	N/A	17.33	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
p_hat700-1	700	60999	13	87.90	N/A	N/A	**59.98**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
san200_0.7_2	200	13930	26	**4.69**	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 23
Expermental results for random graphs with edge probability ranging form 0.05 to 0.3 and n = 100.

graph	k	heuristic	reduction			opt	branches	totime
			#vtx	#edges	time			
random100_0.05 (100,268)	2	4	0	0	0.00	4	0	0.00
	3	5	86	244	0.00	6	49	0.00
	4	6	86	244	0.00	7	443	0.00
	5	7	86	244	0.00	8	1240443	3.73
	6	8	86	244	0.00	9	2529589	9.50
	7	9	86	244	0.00	10	15099323	57.62
	8	11	64	181	0.00	11	8128391	38.33
random100_0.1 (100,521)	2	5	0	0	0.00	5	0	0.00
	3	6	79	312	0.00	6	127	0.00
	4	7	99	518	0.00	8	636	0.00
	5	8	99	518	0.00	9	12259	0.03
	6	10	96	506	0.00	10	178473	0.56
	7	11	96	506	0.00	11	4.95E+08	1190.59
random100_0.15 (100,761)	2	5	76	375	0.00	6	49	0.00
	3	7	41	180	0.00	7	65	0.00
	4	8	97	669	0.00	9	1065	0.00
	5	9	100	761	0.00	10	80833	0.20
	6	10	100	761	0.00	11	2733276	7.25
	7	12	100	761	0.00	12	52756140	156.12
random100_0.2 (100,1003)	2	7	0	0	0.00	7	0	0.00
	3	8	23	113	0.00	8	26	0.00
	4	9	100	912	0.00	10	3481	0.01
	5	11	99	890	0.00	11	4356	0.01
	6	12	100	985	0.00	12	797450	1.95
	7	13	100	1003	0.00	14	1.57E+08	409.82
	8	15	100	1003	0.00	15	4.47E+08	1422.63
random100_0.25 (100,1255)	2	6	100	1184	0.00	7	1100	0.00
	3	8	100	1184	0.00	8	17552	0.04
	4	10	100	1184	0.00	10	38548	0.08
	5	11	100	1236	0.00	12	1081578	2.31
	6	13	100	1236	0.00	14	10275050	24.18
	7	14	100	1251	0.00	15	6.15E+08	1600.52
	8	16	100	1251	0.00	N/A	N/A	N/A
random100_0.3 (100,1493)	2	7	100	1461	0.00	8	3779	0.01
	3	8	100	1481	0.00	9	76711	0.14
	4	10	100	1481	0.00	11	980333	1.93
	5	11	100	1491	0.00	12	19648261	39.31
	6	13	100	1491	0.00	14	1.88E+08	404.97

Table 24
Expermental results for random graphs with edge probability ranging form 0.05 to 0.3 and n = 200.

graph	k	heuristic	reduction			opt	branches	totime
			#vtx	#edges	time			
random200_0.05 (199,955)	2	4	43	99	0.00	5	17	0.00
	3	5	199	955	0.00	6	214	0.00
	4	6	199	955	0.00	7	2141	0.01
	5	7	199	955	0.00	8	11311488	29.66
random200_0.1 (200,1933)	2	5	24	69	0.00	6	12	0.00
	3	7	0	0	0.00	7	0	0.00
	4	7	200	1933	0.00	8	2594	0.01
	5	10	197	1627	0.00	10	782	0.01
random200_0.15 (200,2883)	2	5	200	2653	0.00	6	1146	0.00
	3	6	200	2838	0.00	8	4478	0.01
	4	8	200	2838	0.00	9	252639	0.57
	5	9	200	2883	0.00	10	4521860	11.71
random200_0.2 (200,3842)	2	6	200	3751	0.00	7	9173	0.02
	3	7	200	3817	0.00	9	87284	0.19
	4	8	200	3839	0.00	10	2882727	6.29
	5	11	200	3817	0.00	12	1.96E+08	445.47
random200_0.25 (200,4841)	2	6	200	4837	0.00	8	34414	0.08
	3	8	200	4837	0.00	9	1938117	4.29
	4	11	200	4830	0.00	11	43836525	100.08
random200_0.3 (200,5842)	2	8	200	5841	0.01	9	70977	0.15
	3	9	200	5842	0.01	10	5676220	13.02
	4	11	200	5842	0.00	12	2.23E+08	524.13

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