Programming Paradigms 2022
Session 12: Lazy evaluation

Problems for solving and discussing

Hans Hüttel

29 November 2022

Problems that we will definitely talk about

1. ***(Everyone at the table – 15 minutes)***

   In Haskell, the value undefined has type a. One can put it anywhere and it will compile. But one tries to evaluate it, it throws the exception "undefined".

   Here is a function called indflet.

   ```
   indflet _ []       = []
   indflet _ [x]      = [x]
   indflet e (x:y:ys) = x : e : indflet e (y:ys)
   ```

   First try to figure out *without asking the Haskell interpreter* what the type of indflet is and what the function does. Next try to figure out *without asking the Haskell interpreter* why an exception is throwh when you evaluate

   ```
   head (indflet 1 (2:undefined))
   ```

2. ***(Work in pairs – 15 minutes)***

   Now define a version of the function from the previous problem that is called fletind and does not throw an exception when you evaluate

   ```
   head (fletind 1 (2:undefined))
   ```

3. ***(Everyone at the table – 20 minutes)***

   Define a function allBinaries :: [String] that wil give us the infinite ordered list of all binary numbers, with the least significant bit first, no trailing zeros, i.e.

   ```
   allBinaries = ["0","1","01","11","001",...].
   ```

4. ***(Work in pairs – 25 minutes)***

   Trees can be defined by

   ```
   data Tree = Node Tree Tree | Leaf
   data Direction = L | R -- left and right
   type Path = [Direction]
   ```

   Define a function allFinitePaths :: Tree -> [Path] that takes a binary tree t :: Tree (which may be an infinite tree!) and gives us a list of all finite paths from the root to any leaf of t.

More problems to solve at your own pace

a) A problem, due to the mathematician W. R. Hamming, is to write a program that produces an infinite list of natural numbers with the following properties:

   i The list is in ascending order, without duplicates.

   ii The list begins with the number 1.

   iii If the list contains the number x, then it also contains the numbers $2x$, $3x$, and $5x$.

   iv iv The list contains no other numbers.

   Define a function hamming that will give us such a list.