Lab Session 6

MA-423 : Matrix Computations July - November 2013 S. Bora

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1. Write a MATLAB function program $[\mathtt{Q},\mathtt{R}] = \mathtt{cgs}(\mathtt{V})$ to orthonormalize the columns of an $n \times m$ matrix V by the Classical Gram Schmidt procedure so that Q is the isometry satisfying

$$\begin{aligned} \mathrm{span}\{Q(:,1)\} &= \mathrm{span}\{V(:,1)\} \\ \mathrm{span}\{Q(:,1), Q(:,2)\} &= \mathrm{span}\{V(:,1), V(:,2)\}, \\ &\vdots \\ \mathrm{span}\{Q(:,1), Q(:,2), \ldots, Q(:,m)\} &= \mathrm{span}\{V(:,1), V(:,2), \ldots, V(:,m)\} \end{aligned}$$

   and R is an upper triangular matrix such that $R(i,j) = \langle V(:,j), Q(:,i)\rangle$.

2. A slight modification of the above program leads to the Modified Gram Schmidt procedure for orthonormalizing the columns of V. Perform this modification to obtain another function program $[\mathtt{Q},\mathtt{R}] = \mathtt{mgs}(\mathtt{V})$.

3. Write a function program $[\mathtt{Q},\mathtt{R}] = \mathtt{cgssrep}(\mathtt{V})$ that performs Classified Gram Schmidt with reorthogonalization by making appropriate changes to your function program `cgs`.

4. Write a function program $[\mathtt{Q},\mathtt{R}] = \mathtt{mgssrep}(\mathtt{V})$ that performs Classified Gram Schmidt with reorthogonalization by making appropriate changes to your function program `mgs`.

   Note: Please look up the pseudocode in *Fundamentals of Matrix Computations* for efficient execution of the reorthogonalization process.

   Take care to replace *for loops* by matrix-vector multiplications as far as possible in each of the above programs.

4. Orthonormalise the columns of the Hilbert matrices of size 7 and 12 respectively via `cgs`, `mgscgsrep`, `mgsrep` and `QR` (using the program you made in your last class) respectively and calculate the deviation from orthonormality $\|I - Q^*Q\|_2$ in each case.

   Prepare a report of your experiments in tabular form.