Lab Session 6

MA-423 : Matrix Computations July - November 2014 S. Bora

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1. Write a MATLAB function program $[\mathtt{Q},\mathtt{R}] = \mathtt{cgs(V)}$ to orthonormalize the columns of an $n \times m$ matrix V, $(n \geq m)$ by the Classical Gram Schmidt procedure so that Q is the isometry satisfying

$$\begin{aligned}
\text{span}\{Q(:,1)\} &= \text{span}\{V(:,1)\} \\
\text{span}\{Q(:,1), Q(:,2)\} &= \text{span}\{V(:,1), V(:,2)\}, \\
&\vdots \\
\text{span}\{Q(:,1), Q(:,2), \ldots, Q(:,m)\} &= \text{span}\{V(:,1), V(:,2), \ldots, V(:,m)\}
\end{aligned}$$

   and R is an upper triangular matrix such that $R(i,j) = \langle V(:,j), Q(:,i) \rangle$.

2. A slight modification of the above program leads to the Modified Gram Schmidt procedure for orthonormalizing the columns of V. Perform this modification to obtain another function program $[\mathtt{Q},\mathtt{R}] = \mathtt{mgs(V)}$.

3. Write a function program $[\mathtt{Q},\mathtt{R}] = \mathtt{cgsrep(V)}$ that performs Classified Gram Schmidt with reorthogonalization by making appropriate changes to your function program `cgs`. (Find out how to do this efficiently from your textbook.)

   Take care to replace *for loops* by matrix-vector multiplications as far as possible in each of the above programs.

4. Orthonormalise the columns of the Hilbert matrices H of size $3, 4, \ldots, 10$ respectively via `cgs`, `mgs`, `cgsrep` and `QR` (using the program you made in your last class) respectively and calculate the deviation from orthonormality $\|I - Q^*Q\|_2$ as well as $\|H - QR\|_2$ in each case.

   Prepare a report of your experiments in tabular form which also contains the condition numbers of H in one of the columns. Make conclusions about the efficacy of each method in orthonormalising the columns of H.