Lab Session 5

MA-423 : Matrix Computations July-November 2013 S. Bora

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1 Given a vector $x \in \mathbb{R}^n$, write a MATLAB function program $[\mathtt{u}, \gamma, \tau] = \mathtt{reflect(x)}$ to compute $u \in \mathbb{R}^n$, and $\gamma \in \mathbb{R}$ so that $Qx = [-\tau, 0, \ldots, 0]^T$ where $\tau = \pm\|x\|_2$ and the sign is chosen so that it is the same as that of the first entry of x.

2 Write another function program $\mathtt{B} = \mathtt{applreflect(u, gamma, A)}$ to *efficiently* perform the multiplication QA where $Q = I - \gamma uu^T$.

3. Use the programs written above to write another function program $[\mathtt{Q}, \mathtt{R}] = \mathtt{reflectqr(A)}$ that computes the QR decomposition of $A \in \mathbb{R}^{n\times m}$, $n \geq m$ via reflectors. The program should be such that the lower triangular part of A contains the vectors u (apart from the leading 1 entry) required to construct the reflectors used at each stage and the values of γ corresponding to each reflector are stored in a separate vector.

   Also (as explained in theory class) if $Q_1, Q_2, \ldots Q_m$ are the reflectors such that $Q = Q_1 Q_2 \cdots Q_m$, then use the fact that

$$Q(:, k) = Qe_k = Q_1 Q_2 Q_3 \cdots Q_k e_k$$

   to compute Q.

   Your program should use the `reflect` program every time you create a reflector and the `applreflect` program whenever you have to apply a reflector to a matrix or a vector. Verify your answer by computing $\|A - QR\|_2$ which will be $\mathcal{O}(u)$ if your program is correct. Compare your Q and R with that obtained from the built in MATLAB command $[\mathtt{Q}, \mathtt{R}] = \mathtt{qr(A)}$ which also uses the same method to compute Q and R.

Please look up relevant pages (pp. 199-200) of your textbook for writing the programs for [1] and [2].