Lab Session 8

MA-423 : Matrix Computations July-November 2014 S. Bora

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1. Write a function program $[\mathtt{iter}, \mathtt{sigma}] = \mathtt{powermethod}(\mathtt{A}, \mathtt{q}, \mathtt{k})$ for performing k iterations of Power method for finding the dominant eigenvalue of a matrix A with starting vector q which returns a matrix `iter` of dimension $n \times k$ such that $\mathtt{iter}(:, \mathtt{j})$ is the vector obtained at the end of the j^{th} iteration of the Power method and sigma is the scaling factor at the end of k iterations. If `sigma` is chosen as explained in your theory class it will approximate the dominant eigenvalue of A whenever such an eigenvalue exists.

   Run the above program with $q = [1; 1; 1]$

   $$A = \begin{bmatrix} 2.4347 & -0.8641 & 1.4661 \\ -0.8641 & 2.6630 & -3.1263 \\ 1.4661 & -3.1263 & 6.9023 \end{bmatrix}$$

   and your own choice of k.

   You can compare the observed and theoretical convergence ratios by doing the following steps:

   Use the $[\mathtt{V}, \mathtt{D}] = \mathtt{eig}(\mathtt{A})$ command to find the eigenvalues and eigenvectors of A. If v be a dominant eigenvector, then for large enough value of j examine how close the ratio $\frac{\|\mathtt{iter}(:,\mathtt{j}+1)-\mathtt{v}\|_\infty}{\|\mathtt{iter}(:,\mathtt{j})-\mathtt{v}\|_\infty}$ is to the theoretical rate of convergence which is $\frac{|\lambda_2|}{|\lambda_1|}$.

2. Write a function program $[\mathtt{iter}, \mathtt{sigma}] = \mathtt{shiftinv}(\mathtt{A}, \mathtt{q}, \mathtt{k}, \mathtt{s})$ which performs k steps of Shift and invert Method with shift `s` on a matrix A with starting vector q to produce an $n \times k$ matrix `iter` whose j^{th} column is the vector obtained at the end of the the j^{th} iteration and a scalar `sigma` which approximates an eigenvalue of $\mathtt{A} - \mathtt{s} * \mathtt{eye}(\mathtt{n})$.

   Run the above program for the same matrix A as given in the previous exercise with your own choices of shifts, starting vectors and number of iterations k (at least 5 different combinations).

   Compare the observed and theoretical rates of convergence by following the procedure outlined above.

3. Write a function program $[\mathtt{iter}, \mathtt{rho}] = \mathtt{Rayleigh}(\mathtt{A}, \mathtt{q}, \mathtt{k})$ which performs k steps of Rayleigh quotient iteration on A with initial vector q with output `iter` as in the previous exercises and a scalar `rho` which is the Rayleigh quotient obtained at the end of the k iterations.

   You should remember to convert A to upper Hessenberg form by using $[\mathtt{Q}, \mathtt{H}] = \mathtt{hess}(\mathtt{A})$ and perform the iterations with the upper Hessenberg matrix H. At the end you have to recover the eigenvector of A from the one produced by the algorithm for H.

   Run the above program on matrices and starting vectors of your choice and note whether convergence occurs and the observed convergence rate whenever it occurs. Try the program on symmetric matrices also and see if the convergence rate improves.