ASSIGNMENT 10

MA226 : Monte Carlo Simulation
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1.0 Problem1

Generate 10 sample paths for the standard Brownian motion in the time interval [0,5] using the recursion

$$W(t_{i+1}) \ = \ W(t_i) \ + \ (\sqrt{t_{i+1} - t_i})Z_{i+1}$$

with 5000 generated values for each of the paths where $Z_{i+1} \ \sim N(0,1)$. Plot all the sample paths in a simple figure . Also estimate E[W[2]] and E[W[5]] from 10 paths that you have generated.

1.1 solution

1.2 Graph

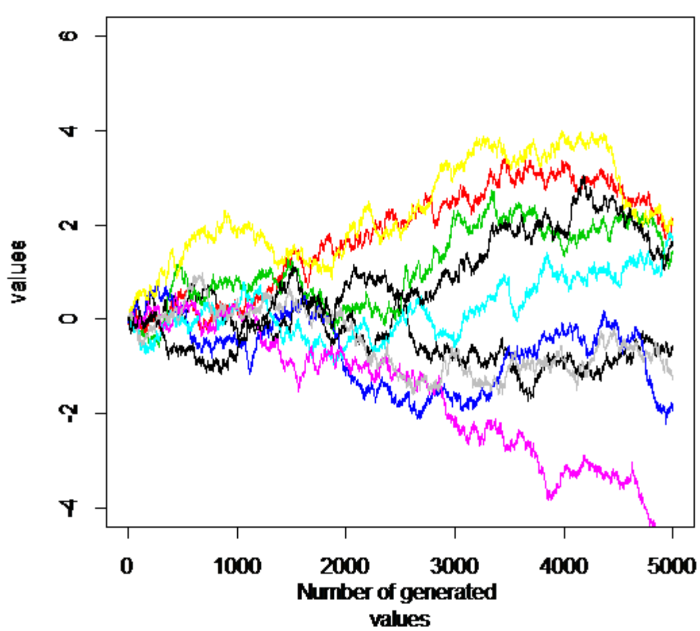

Figure 1: Brownian paths

1.3 R code

```
w<-vector()
w1<-vector()
main<-function()
{
    i=1
    count=0
    exp_w2=0
    exp_w5=0
    for(j in 0:9)
    {
        w[1+5000*j]=0
        k=1
        while(i!=5000+5000*j)
        {
            w[i+1]=w[i]+sqrt(.001)*rnorm(1)
            i=i+1
        }
        m=i-4999
        while(m!=5000+5000*j)
        {
            w1[k]=w[m]
            k=k+1;
            m=m+1;
        }
        i=i+1
        plot(w1,type="l",col=j,ylim=c(-4,6),xlab="Number of generated
        values",ylab="values")
        par(new=TRUE)
    }
    for(j in 0:9)
    {
        exp_w2=exp_w2+w[2000+5000*j]
        exp_w5=exp_w5+w[5000+5000*j]
    }
    cat("The expected value of w[2] is ",exp_w2/10,"\n")
    cat("The expected value of w[5] is ",exp_w5/10,"\n")
    par(new=FALSE)
}
main()
```

Listing 1: R Code which generates 10 Brownian paths

1.4 Results

- The expected value of W[2] is -0.07547509
- The expected value of W[5] is 0.006617122

1.5 Explanation of Results

Since $W(t) \sim N(0,t)$ for $0 < t \leq T$ .Therefore the expected theoretical value in both the cases is 0 and the result is quite nearer to it.

2.0 Problem2

Repeat the above exercise with following Brownian motion(BM(μ , σ^2)) discretization X(t_{i+1}) = X(t_i)+ $\mu(t_{i+1} - t_i)$ + $\sigma(\sqrt{t_{i+1} - t_i})Z_{i+1}$. Take X(0)=5 , μ = 0.06 and σ = 0.3

2.1 Solution

2.2 Graph

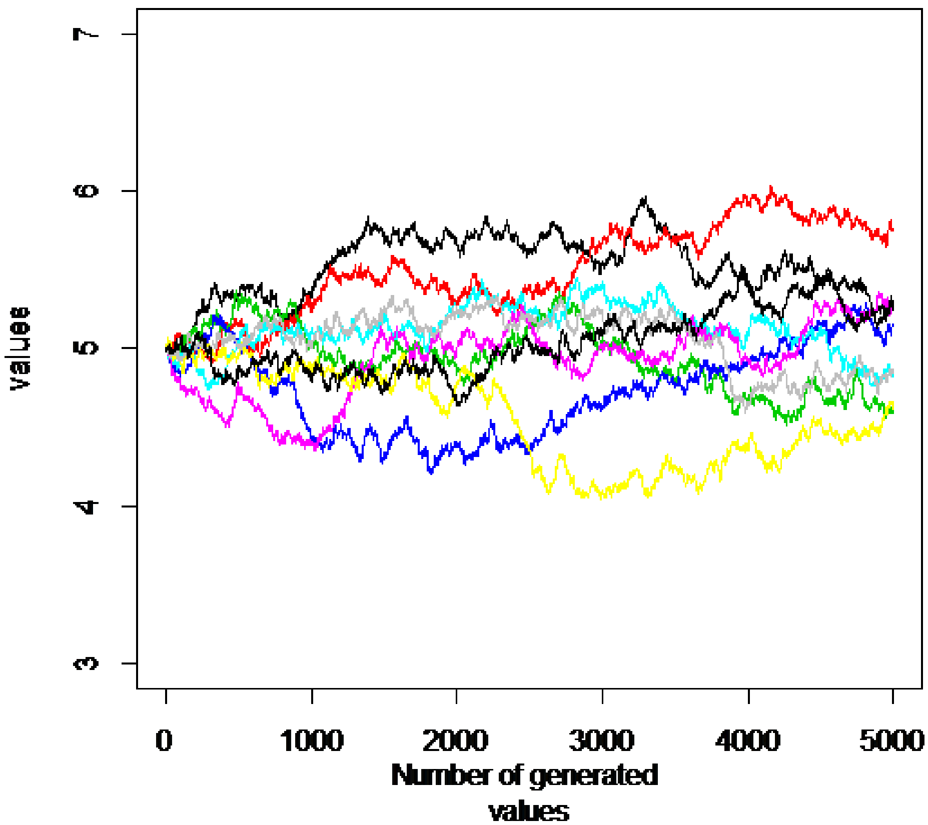

Figure 2: Brownian paths

2.2 R Code

```
w<-vector()
w1<-vector()
main<-function()
{
    i=1
    exp_w2=0
    exp_w5=0
    for(j in 0:9)
    {
        w[1+5000*j]=5
        k=1
        while(i!=5000+5000*j)
        {
            w[i+1]=w[i]+.00006+sqrt(.001)*rnorm(1)*0.3
            i=i+1
        }
        m=i-4999
        while(m!=5000+5000*j)
        {
            w1[k]=w[m]
            k=k+1;
            m=m+1;
        }
        i=i+1
        plot(w1,type="l",col=j,ylim=c(3,7),xlab="Number of generated
        values",ylab="values")
        par(new=TRUE)
    }
    for(j in 0:9)
    {
        exp_w2=exp_w2+w[2000+5000*j]
        exp_w5=exp_w5+w[5000+5000*j]
    }
    cat("The expected value of w[2] is ",exp_w2/10,"\n")
    cat("The expected value of w[5] is ",exp_w5/10,"\n")
    par(new=FALSE)
}

main()
```

Listing 2: R Code which generates 10 Brownian paths

2.3 Results

- The expected value of W[2] is 5.004518
- The expected value of W[5] is 5.32448

2.4 Explanation of Results

Since $W(t) \sim N(X(0) + \mu t, \sigma^2 t)$ for $0 < t \leq T$ .Therefore the expected theoretical value in both the cases are 5.12 and 5.30 respectively and the result is quite nearer to it.

3.0 Problem3

The Euler approximated recursion with time dependent μ and σ is given by

$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)(\sqrt{t_{i+1} - t_i})Z_{i+1}$$

Repeat the above exercise by taking

$$\text{Y(0)=5 },\mu(t) = 0.0325 - 0.05t\ ,\ \sigma(t) = 0.012 + 0.0138t + 0.00125t^2$$

3.1 Solution

3.2 Graph

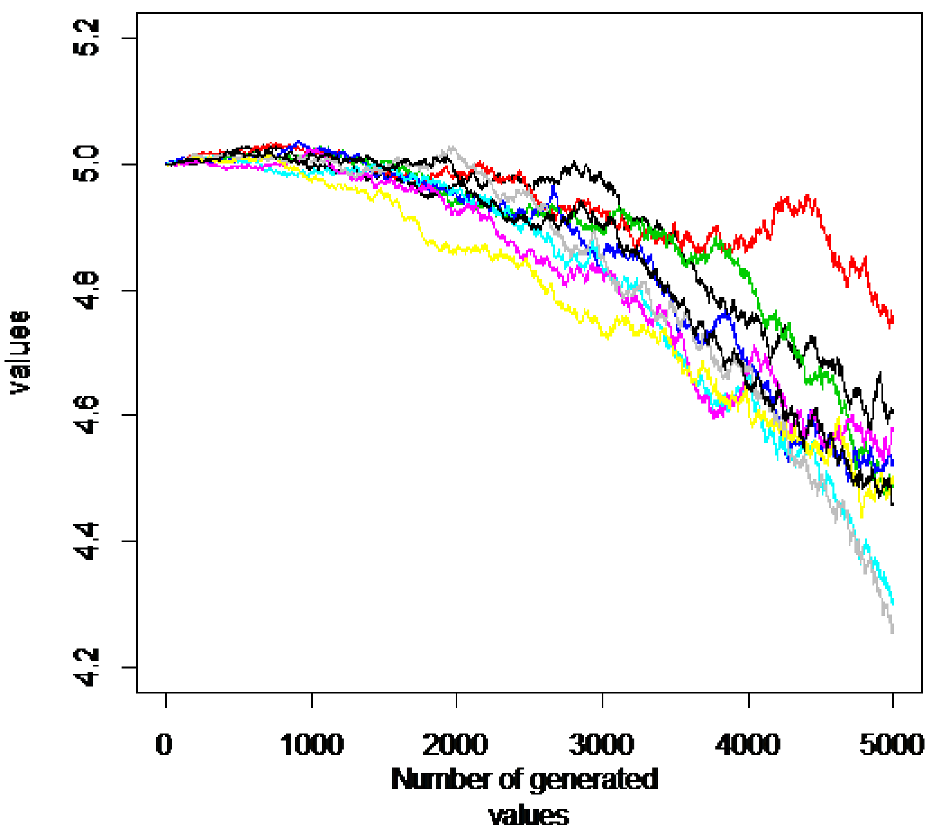

Figure 3: Brownian paths

3.2 R Code

```
w<-vector()
w1<-vector()
main<-function()
{
    i=1
    exp_w2=0
    exp_w5=0
    for(j in 0:9)
    {
        w[1+5000*j]=5
        k=1
        n=0.001
        while(i!=5000+5000*j)
        {
            w[i+1]=w[i]+(.0325-.05*n)*.001 + (.012+.0138*n+.00125*n*n)*sqrt(.001)*rnorm(1)
            i=i+1
            n=n+.001
        }
        m=i-4999
        while(m!=5000+5000*j)
        {
            w1[k]=w[m]
            k=k+1;
            m=m+1;
        }
        i=i+1
        plot(w1,type="l",col=j,ylim=c(4.2,5.2),xlab="Number of generated
        values",ylab="values")
        par(new=TRUE)
    }
    for(j in 0:9)
    {
        exp_w2=exp_w2+w[2000+5000*j]
        exp_w5=exp_w5+w[5000+5000*j]
    }
    cat("The expected value of w[2] is ",exp_w2/10,"\n")
    cat("The expected value of w[5] is ",exp_w5/10,"\n")
    par(new=FALSE)
}
main()
```

Listing 3: R Code which generates 10 Brownian paths

3.3 Results

- The expected value of W[2] is 4.954098
- The expected value of W[5] is 4.50871

3.4 Explanation of Results

Since $E(W(t)) = E(W(s)) + \int_s^t \mu(a)da$. So the theoretical expected value of W(2) and W(5) came out to be 4.965 and 4.5375 respectively and our result is quite nearer to it.