ASSIGNMENT 12

MA226 : Monte Carlo Simulation
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1 Problem 1

1.1 Statement

Generate the first 25 values of the Van der Corput sequence $x_1, x_2, \cdots, x_{25}$ using the radical inverse function $x_i := \phi_2(i)$ and list them in your report. Next, generate the first 1000 values of this sequence and plot the overlapping pairs (x_i, x_{i+1}) as a two dimensional plot. What do you observe? Now, generate the first 100 and 100000 values of this sequence and plot the sampled distributions for both cases. Compare these plots with the sampled distributions of 100 and 100000 values generated by an LCG, by plotting the sampled distributions in two graphs side by side for both the cases. Specify the LCG that you have used.
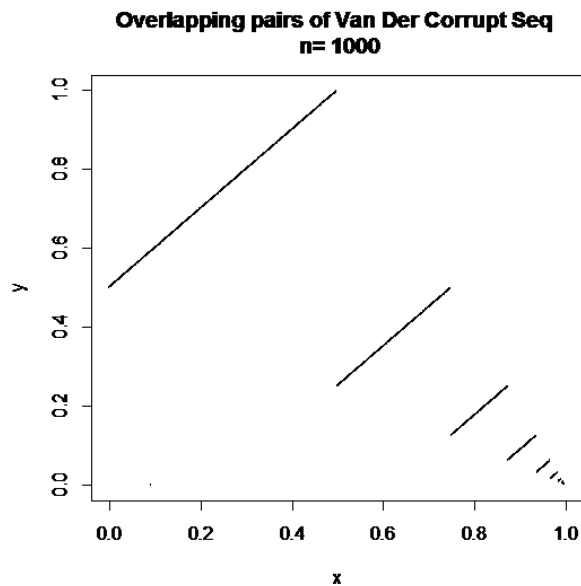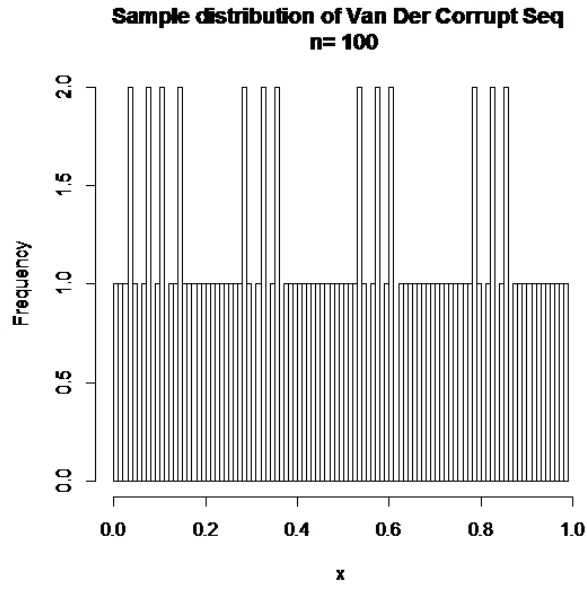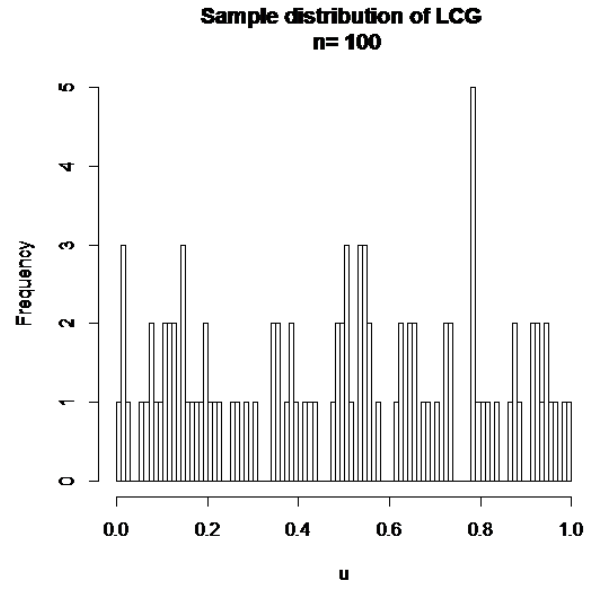
1.2 Solution

1.2.1 Graphs

Figure 1: (x_i, x_{i+1}) for N = 1000.

(a) Histogram for the van der Corput sequence with N = 100, base = 2.

(b) Histogram for the LCG with $N = 100$, $a = 16807$, $m = 2^{31} - 1$, $b = 0$ and $x_0 = 1631$.

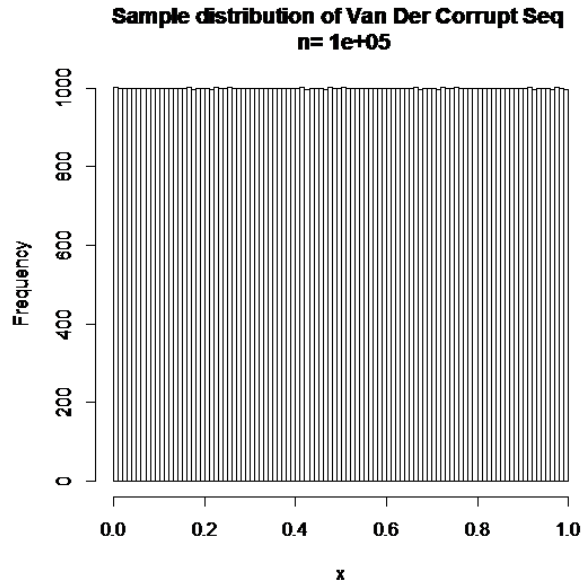Figure 2: Histograms of the van der Corput sequence and the LCG for N = 100.

(a) Histogram for the van der Corput sequence with N = 100000, base = 2.
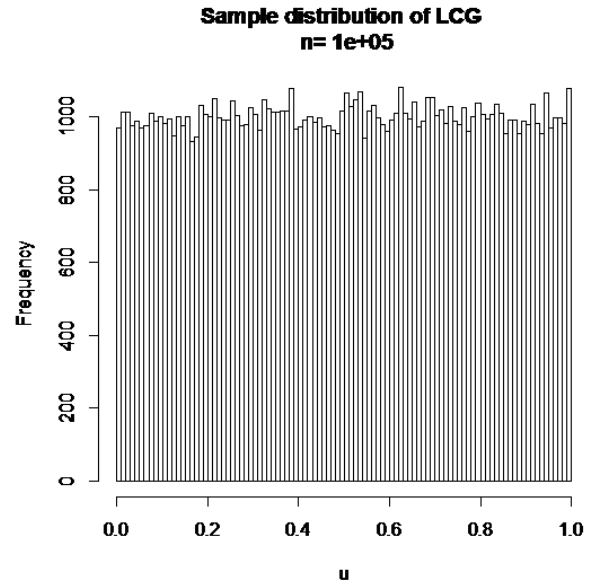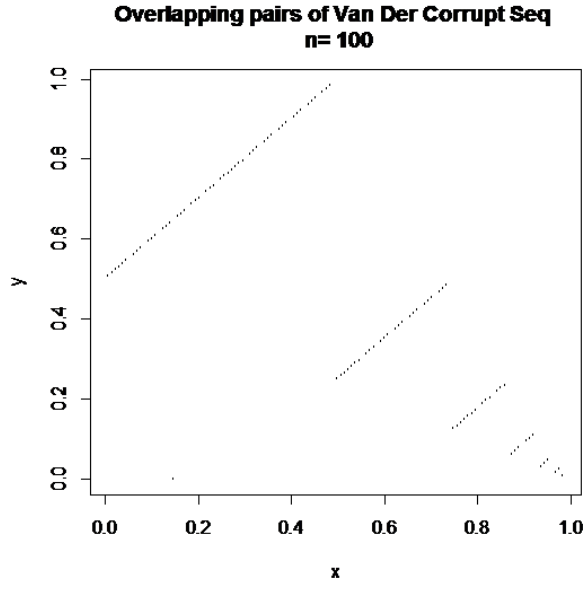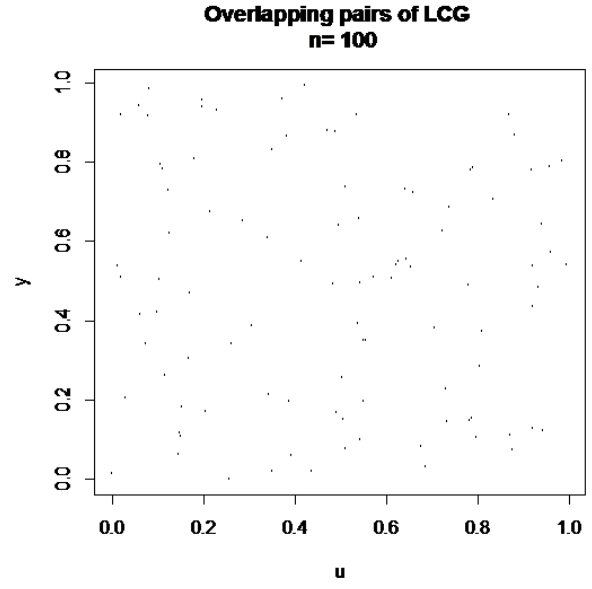
(b) Histogram for the LCG with $N = 100000$, $a = 16807$, $m = 2^{31} - 1$, $b = 0$ and $x_0 = 1631$.

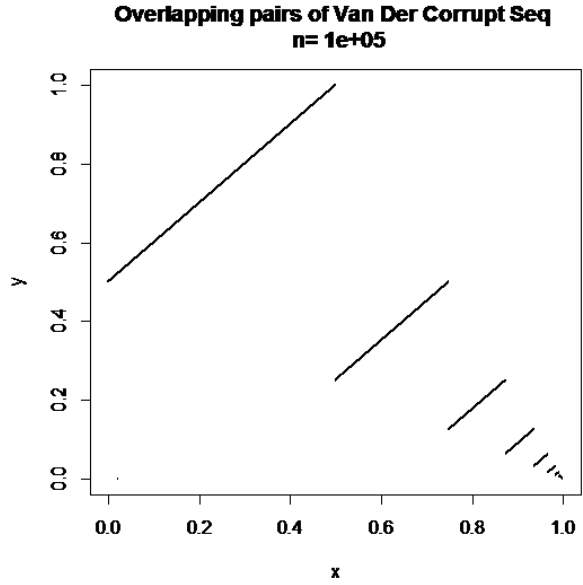Figure 3: Histograms of the van der Corput sequence and the LCG for N = 100000.

**Overlapping pairs of Van Der Corrupt Seq
n= 100**

**Overlapping pairs of LCG
n= 100**

(a) (x_i, x_{i+1}) for the van der Corput sequence with N = 100, base = 2.

(b) (x_i, x_{i+1}) for the LCG with $N = 100$, $a = 16807$, $m = 2^{31} - 1$, $b = 0$ and $x_0 = 1631$.

Figure 4: (x_i, x_{i+1}) for the van der Corput sequence and the LCG for N = 100.

**Overlapping pairs of Van Der Corrupt Seq
n= 1e+05**

**Overlapping pairs of LCG
n= 1e+05**

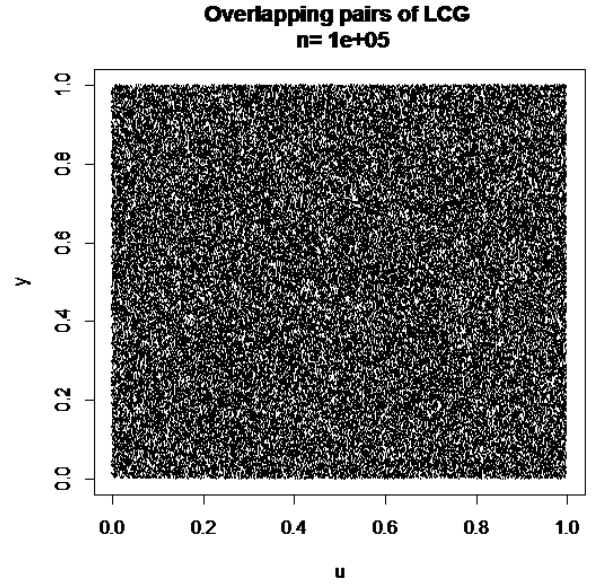(a) (x_i, x_{i+1}) for the van der Corput sequence with N = 100000, base = 2.

(b) (x_i, x_{i+1}) for the LCG with $N = 100000$, $a = 16807$, $m = 2^{31} - 1$, $b = 0$ and $x_0 = 1631$.

Figure 5: (x_i, x_{i+1}) for the van der Corput sequence and the LCG for N = 100000.

1.2.2 Result

- First 25 values are:-
0.50000, 0.25000, 0.75000, 0.12500, 0.62500, 0.37500, 0.87500, 0.06250, 0.56250, 0.31250, 0.81250, 0.18750, 0.68750, 0.43750, 0.93750, 0.03125, 0.53125, 0.28125, 0.78125, 0.15625, 0.65625, 0.40625, 0.90625, 0.09375, 0.59375.

- From Figure 1, we see that all the numbers generated by the van der Corput sequence lie on parallel lines in the interval [0, 1).

- From Figure 2 and Figure 3, we see that the van der Corput sequence and the LCG are both uniformly distributed on the inverval [0, 1). For $n = 100000$ almost all the points have equal frequency.

- From Figure 4 and Figure 5, we see that the van der Corput sequence lies on parallel lines in the interval [0, 1) while the sequence generated by the LCG is uniformly distributed in [0, 1).

1.3 R Code

```r
a<-vector()
b<-vector()
x<-vector()
y<-vector()
z<-vector()
u<-vector()
convert<-function(n){
    i=1
    while(n!=0){
        a[i]=n%%2
        n=floor(n/2)
        i=i+1
    }
    return(a)
}
radical<-function(b){
    t=length(b)
    sum1=0
    for(i in 1:t){
        sum1=sum1+b[i]*((1/2)^i)
    }
    return(sum1)
}
lcg<-function(a1,b1,m1,seed,m){
    z[1]=seed
    u[1]=z[1]/m1
    for(i in 2:m){
        z[i]=(a1*z[i-1])%%m1
        u[i]=z[i]/m1
    }
    return(u)
}
generate<-function(x,m){
    for(i in 1:m-1){
```

```r
        y[i]=x[i+1]
    }
    y[m]=0
    return(y)
}
main<-function(m){
    for(i in 1:m){
        b=convert(i)
        x[i]=radical(b)
    }
    y=generate(x,m)
    print(x)
    plot(x,y,cex=0.1,main=paste("Overlapping pairs of Van Der Corrupt Seq",
        "\n","n=",paste(m)))
    x11()
    hist(x,breaks=99,main=paste("Sample distribution of Van Der Corrupt Seq",
        "\n","n=",paste(m)))
    x11()
    u=lcg(16807,0,2^31-1,1631,m)
    hist(u,breaks=99,main=paste("Sample distribution of LCG","\n",
        "n=",paste(m)))
    y=generate(u,m)
    x11()
    plot(u,y,cex=0.1,main=paste("Overlapping pairs of LCG","\n",
    "n=",paste(m)))
}
main(1000)
x11()
main(100)
x11()
main(100000)
```

Listing 1: R code which generates van der Corput sequence and the LCG

2 Problem 2

2.1 Statement

Generate the Halton sequence $x_i = (\phi_2(i), \phi_3(i))$ (as points in $\mathbb{R}^2$) and plot the first 100 and 100000 values. What are your observations?

Recall that the radical inverse function is defined by $\phi_b(i) = \sum_{k=0}^{j} d_k b^{-k-1}$, where $i = \sum_{k=0}^{j} d_k b^k$.
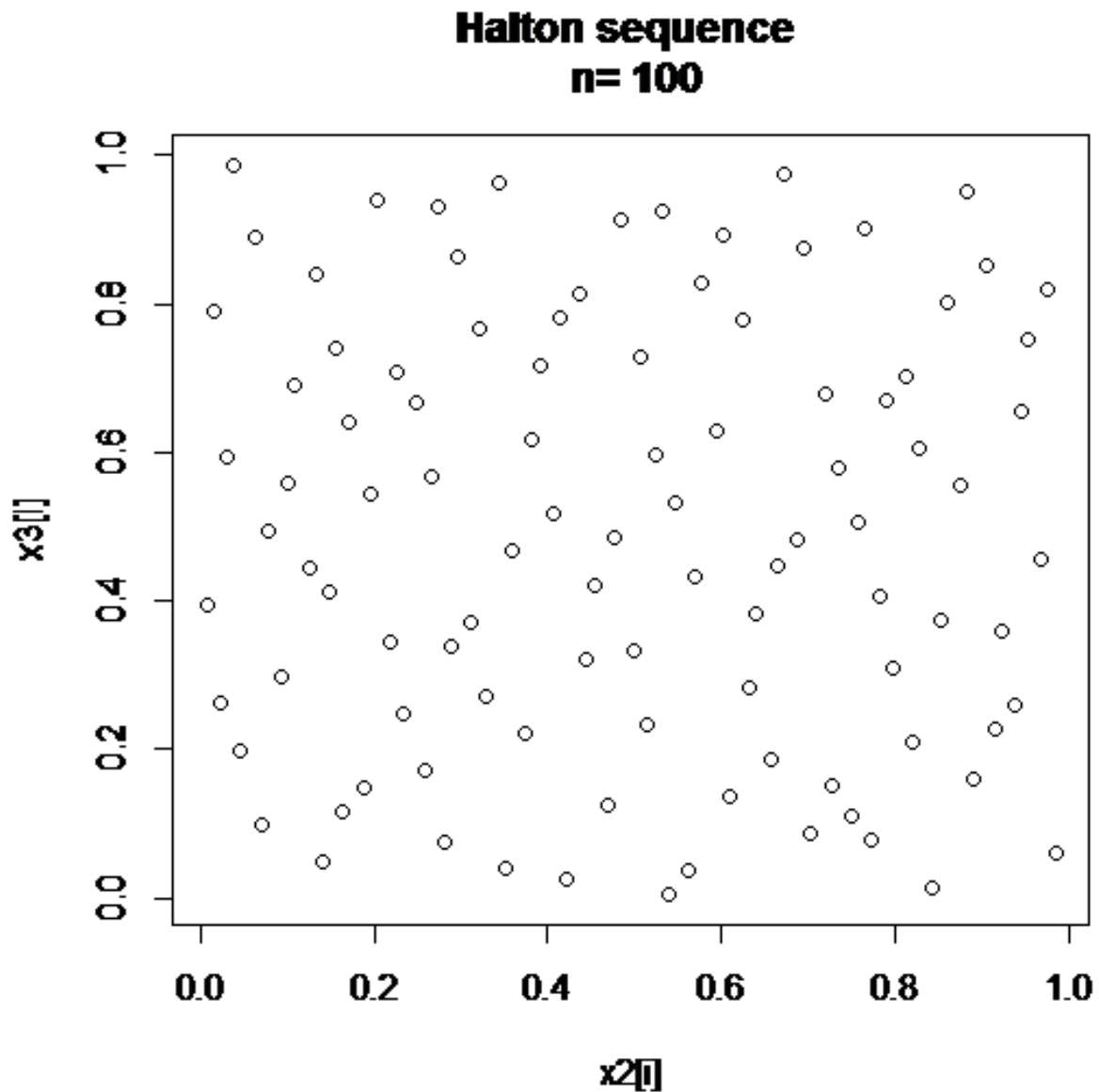
2.2 Solution
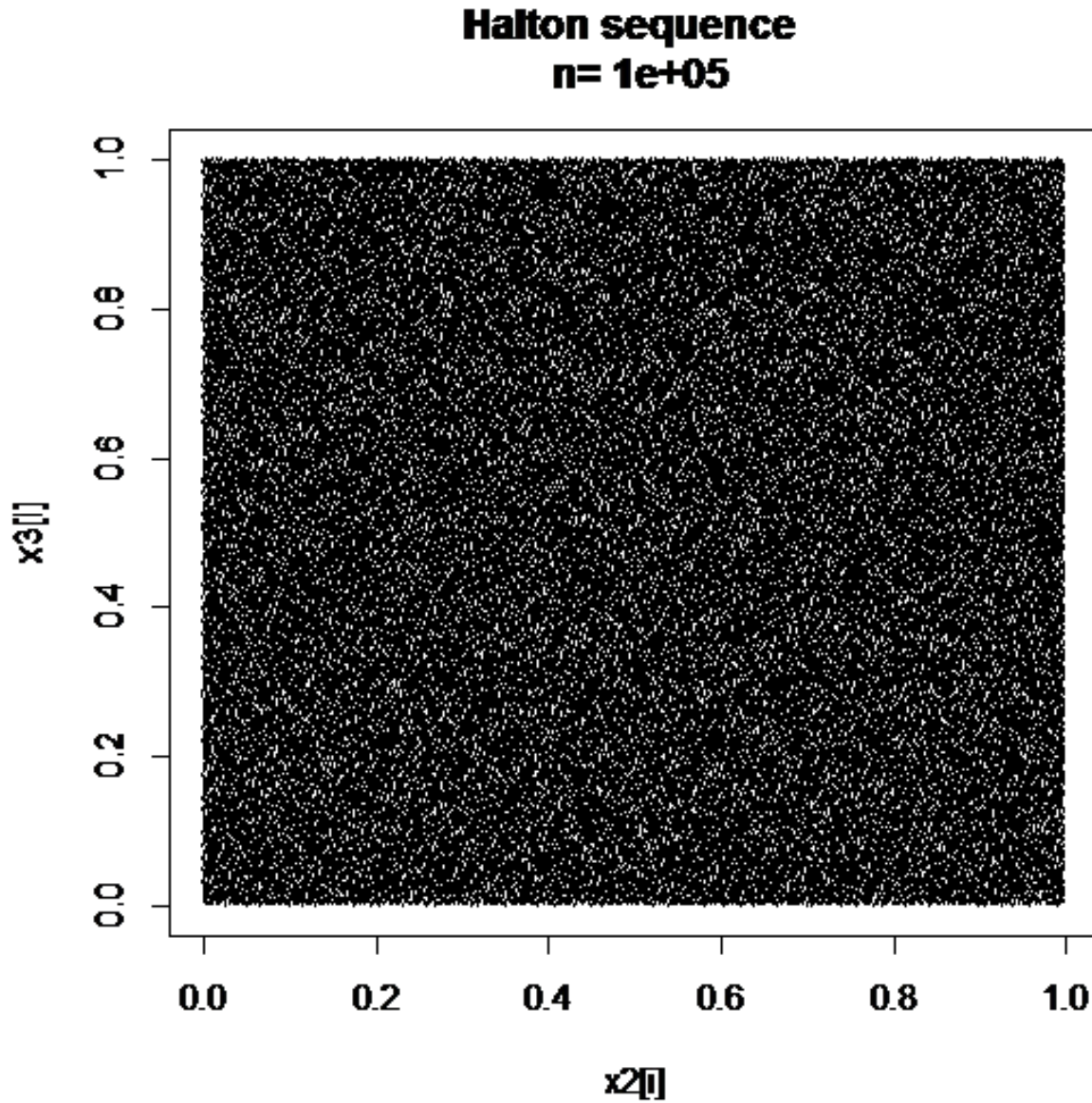
2.2.1 Graphs

Figure 6: Halton Sequence for N = 100.

Figure 7: Halton Sequence for N = 100000.

2.2.2 Result

From Figure 6 and Figure 7, we see that the Halton sequence defined by $x_i = (\phi_2(i), \phi_3(i))$, we see that the Halton sequence is uniformly distributed across the set $[0, 1) \times [0, 1)$.

2.3 R Code

```r
a<-vector()
b<-vector()
x<-vector()
y<-vector()
c<-vector()
convert<-function(n,base){
    i=1
    while(n!=0){
        a[i]=n%%base
        n=floor(n/base)
        i=i+1
    }
    return(a)
}
radical<-function(b,base){
    t=length(b)
    sum1=0
    for(i in 1:t){
        sum1=sum1+b[i]*((1/base)^i)
    }
    return(sum1)
}
main<-function(m){
    for(i in 1:m){
        b=convert(i,2)
        x[i]=radical(b,2)
        c=convert(i,3)
        y[i]=radical(c,3)
    }
    print(x)
    print(y)
    plot(x,y,cex=0.1,xlab="x2[i]",ylab="x3[i]",main=paste("Halton sequence",
        "\n","n=",paste(m)))
}
main(100)
x11()
main(100000)
```

Listing 2: R code which generates the Halton sequence.