Assignment 11

MA226 : Monte Carlo Simulation
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1 Problem

1.1 Statement

The Process $\{S(t)\}$ is a GBM with drift parameter μ, volatility parameter σ and initial value $S(0)$ if

$$S(t) = S(0)\; e^{\left[\mu - \frac{\sigma^2}{2}\right] t + \sigma W(t)}$$

where $\{W(t)\}$ is a standard BM. As with the case of a BM, we have a simple recursiveprocedure to simulate a GBM at $0 = t_0 < t_1 < \cdots < t_n$ as

$$S(t_{i+1}) = S(t_i)\, e^{\left[\mu - \frac{\sigma^2}{2}\right](t_{i+1} - t_i) + \sigma \sqrt{t_{i+1} - t_i}\, Z_{i+1}}$$

where $Z_1, Z_2, \cdots, Z_n$ are independent $N(0, 1)$ variates. In the interval [0, 5], taking both positive and negative values for μ and for at least two different values of σ^2, simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points for each path). Also, by generating a large number of sample paths, compare the actual and simulated distributions of $S(5)$. Calculate expectation and variance of $S(5)$ and match it with their respective theoretical values.

1.2 Solution

1.2.1 Graph

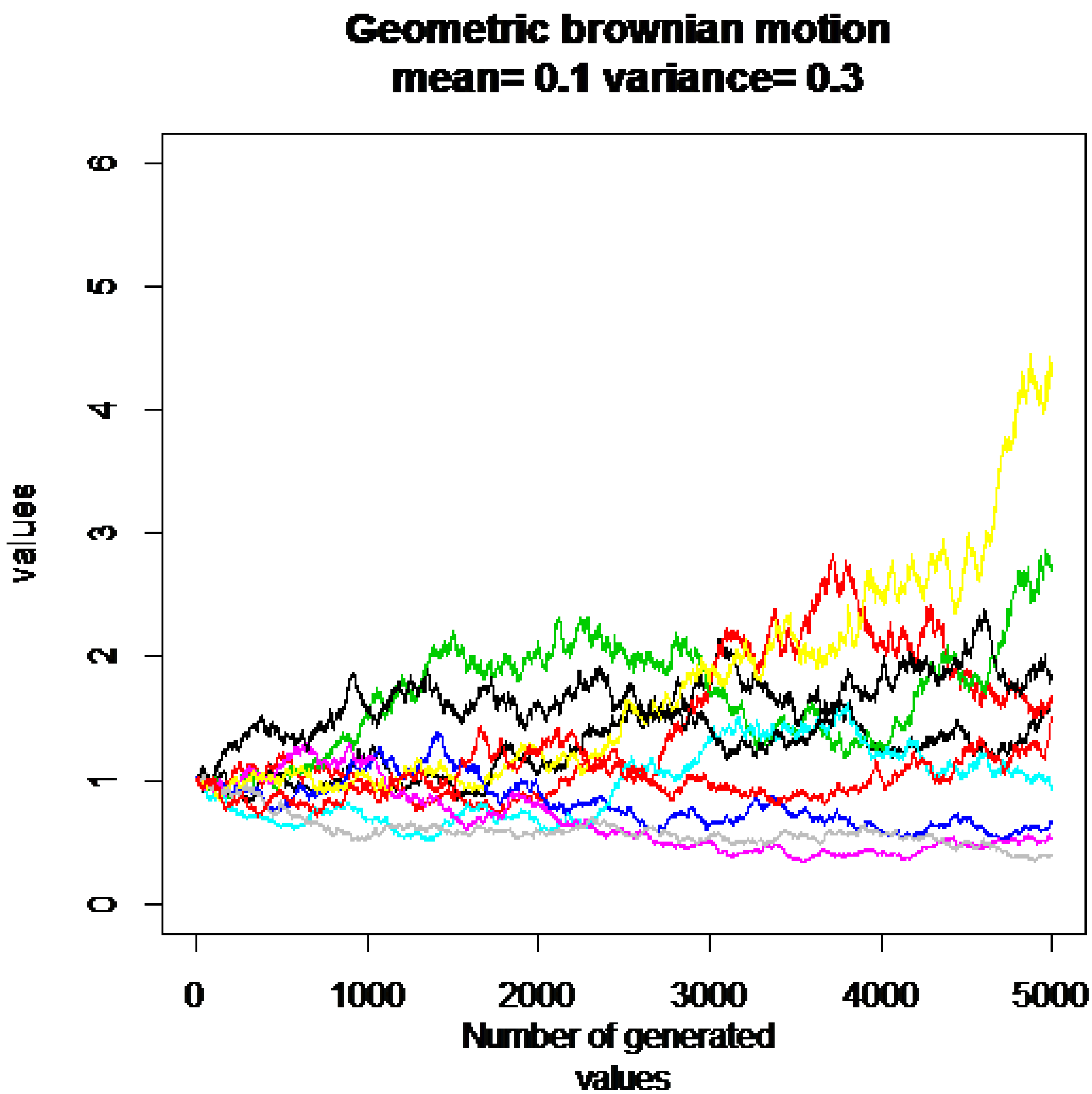

Figure 1: 10 Geometric Brownian Paths with $S(0) = 1$, $\mu = 0.1$ and $\sigma = 0.3$.

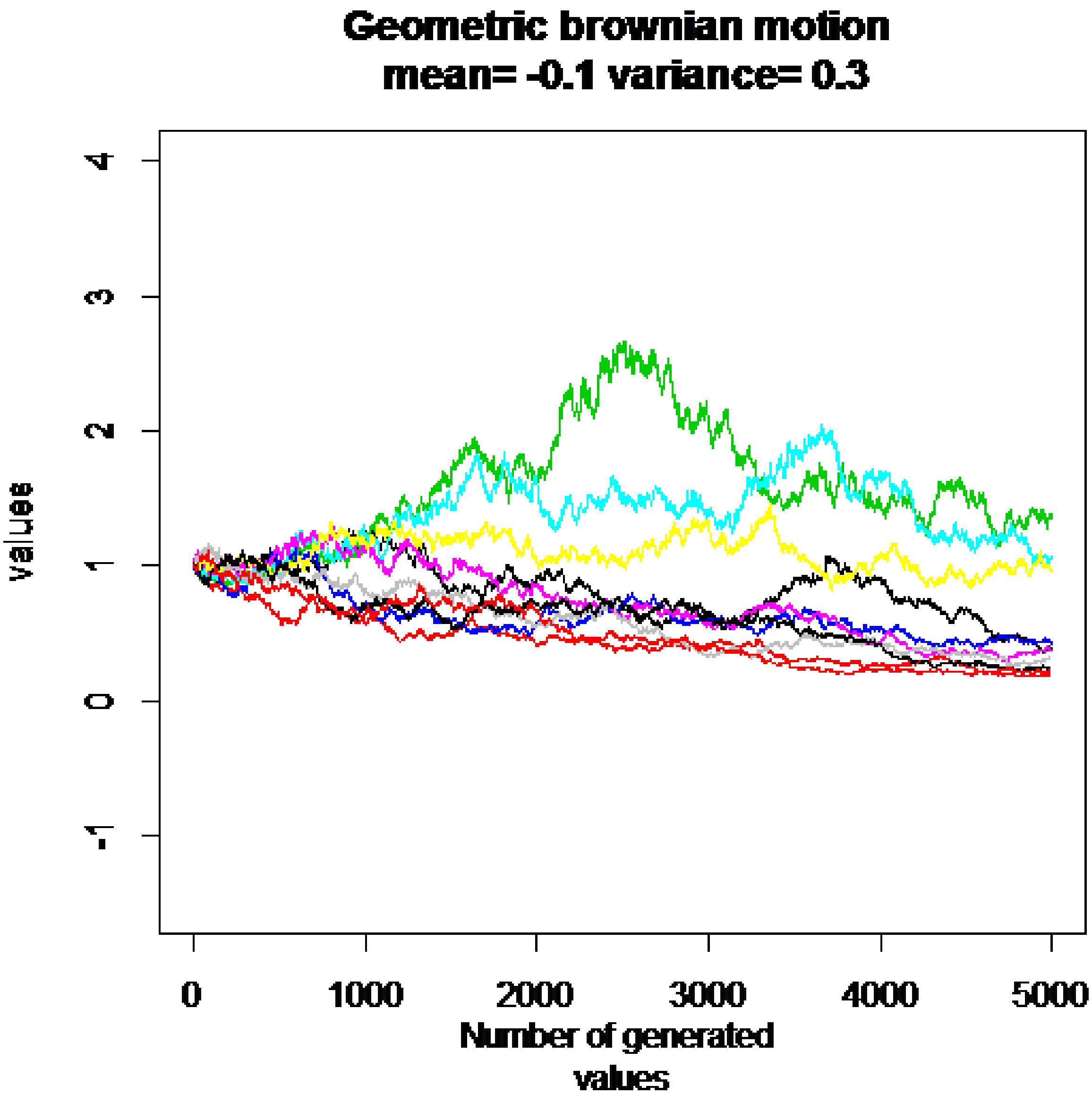

Figure 2: 10 Geometric Brownian Paths with $S(0) = 1$, $\mu = -0.1$ and $\sigma = 0.3$.

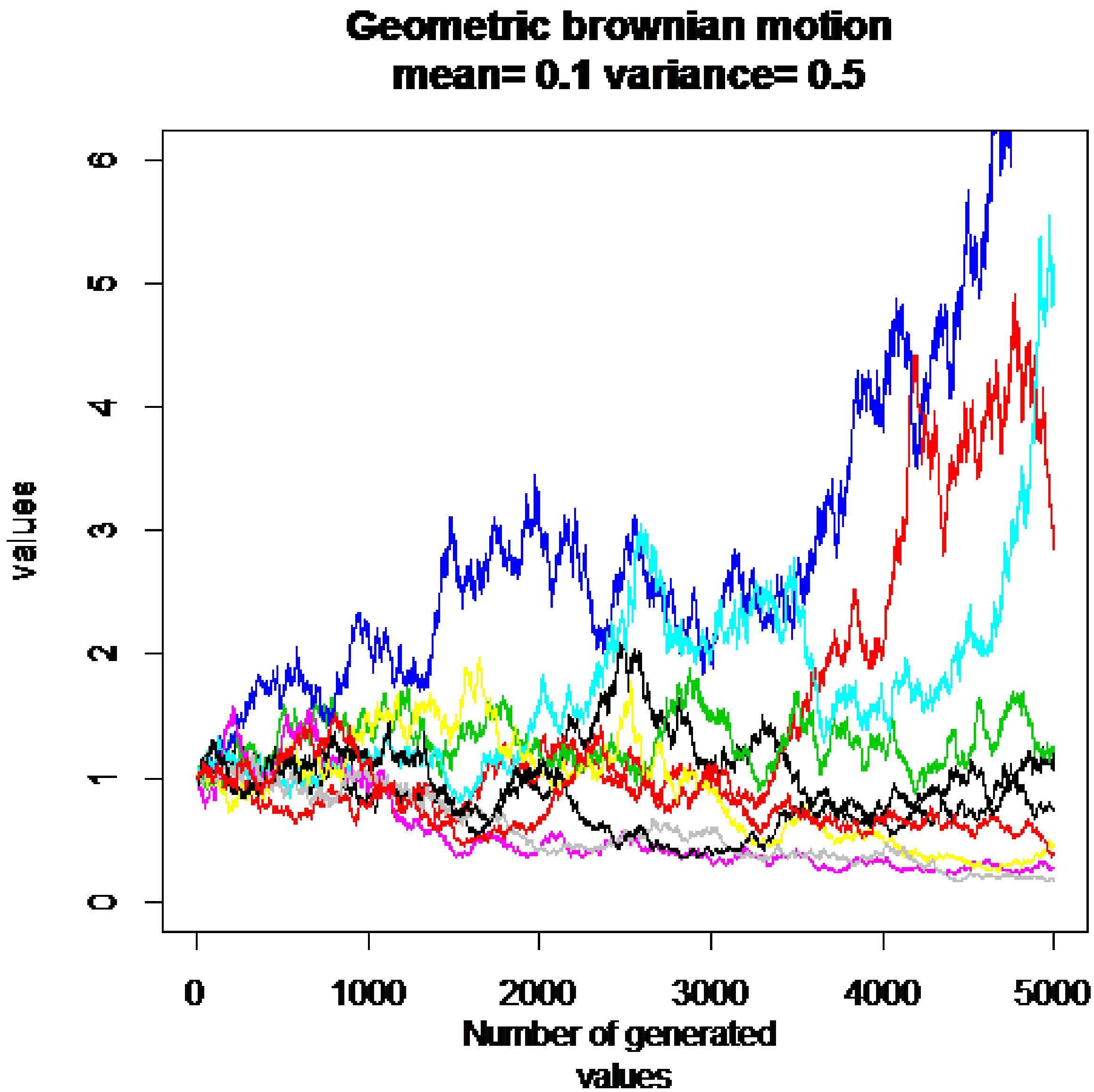

Figure 3: 10 Geometric Brownian Paths with $S(0) = 1$, $\mu = 0.1$ and $\sigma = 0.5$.

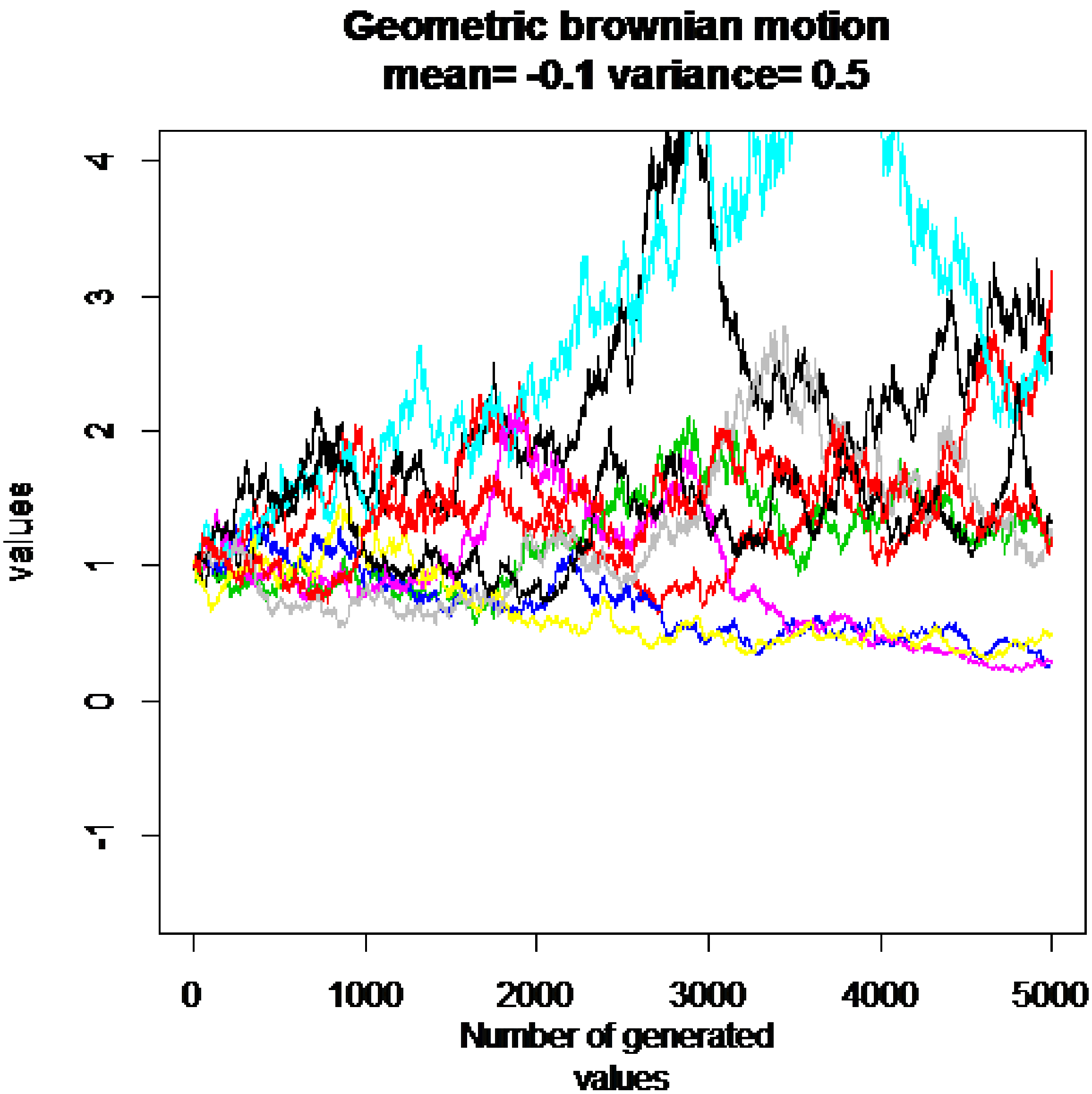

Figure 4: 10 Geometric Brownian Paths with $S(0) = 1$, $\mu = -0.1$ and $\sigma = 0.5$.

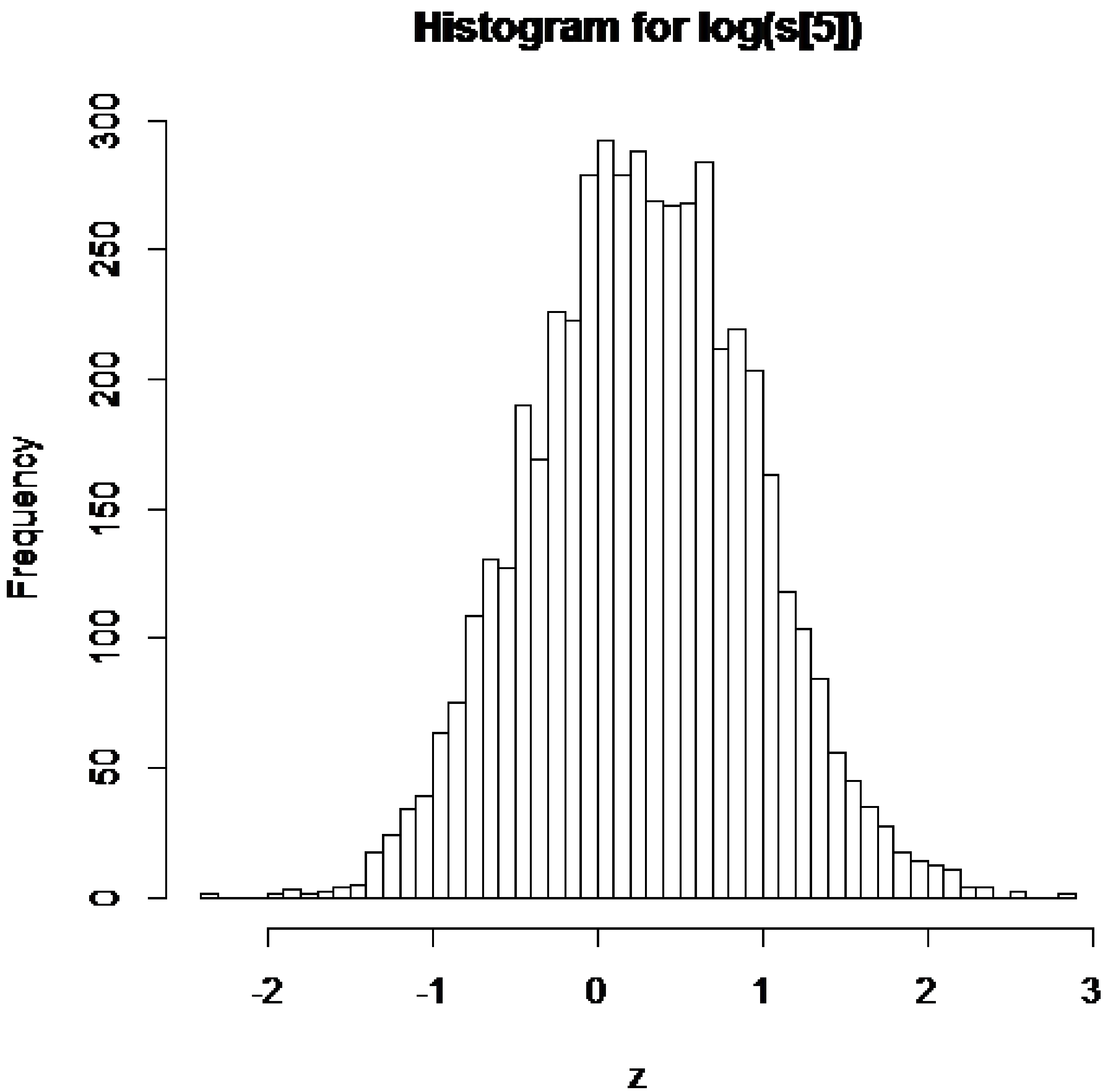

Figure 5: Histogram for $\log(S(5))$ with $S(0) = 1$, $\mu = 0.1$ and $\sigma = 0.3$ for 5000 values of $S(5)$.

1.2.2 R Code

```r
w<-matrix(,nrow=5000,ncol=10)
z<-vector()
main<-function(mean1,var1,x1,x2,x)
{
    exp_w5=0
    for(j in 1:x)
    {
        w[j,1]=1
    }
    for(j in 1:x)
    {
        i=1
        while(i!=5000)
        {
            w[j,i+1]=w[j,i]*exp(((mean1-(var1*var1)/2)*.001)+
                (sqrt(.001)*rnorm(1)*var1))
            #print(w[j,i+1])
            i=i+1
        }
        print(w[j,])
        plot(w[j,],type="l",col=j,ylim=c(x1,x2),xlab="Number of generated
        values",ylab="values",main=paste("Geometric brownian motion","\n",
        "mean=",paste(mean1),"variance=",paste(var1)))
        par(new=TRUE)
    }
    x11()
    for(j in 1:x)
    {
        exp_w5=exp_w5+w[j,5000]
        z[j]=log(w[j,5000])
    }
    hist(z,breaks=49,main="Histogram for log(s[5])")
    cat("The expected value of w[5] is ",exp_w5/x,"\n")
    cat("The variance of w[5] is ",var(w[,5000]),"\n")
    par(new=FALSE)
}
main(0.1,0.3,0,6,10)
x11()
main(-0.1,0.3,-1.5,4,10)
x11()
main(0.1,0.5,0,6,10)
x11()
main(-0.1,0.5,-1.5,4,10)
x11()
#for large no of paths change the row size of matrix..and apply
main(0.1,0.3,0,6,5000)
```

Listing 1: R code which generates the path for the Geometric Brownian Motion.

1.2.3 Result

We take $S(0) = 1$, $\mu = 0.01, \sigma = 0.5$ and time interval $[0, 5]$ to generate 5000 paths, using which we calculate the expectation, variance and the distribution of $S(5)$.

From Figure 5, we observe that $\log(S(5))$ follows a normal distribution, which agrees with the fact that $S(5)$ follows a log-normal distribution.

Theoretical Values:-
Since :
$[S(5)] = S(0)\, e^{5\mu}$ and $\mathrm{Var}[S(5)] = [S(0)]^2\, e^{10\mu}\left(e^{5\sigma^2} - 1\right)$

- The expected value of w[5] for $\mu = 0.1$ and $\sigma = 0.3$ is 1.646
- The variance of w[5] for $\mu = 0.1$ and $\sigma = 0.3$ is 1.5842
- The expected value of w[5] for $\mu = -0.1$ and $\sigma = 0.3$ is 0.607
- The variance of w[5]for $\mu = -0.1$ and $\sigma = 0.3$ is 0.20888
- The expected value of w[5] for $\mu = 0.1$ and $\sigma = 0.5$ is 1.646
- The variance of w[5] is for $\mu = 0.1$ and $\sigma = 0.5$ 6.71
- The expected value of w[5] for $\mu = -0.1$ and $\sigma = 0.5$ is 0.607
- The variance of w[5] for $\mu = -0.1$ and $\sigma = 0.5$ is 0.9140
- The expected value of w[5] for $\mu = 0.1$ and $\sigma = 0.3$ for large no of paths is 1.65808
- The variance of w[5]for $\mu = 0.1$ and $\sigma = 0.3$ for large no of paths is is 1.610813

Actual Values:-

- The expected value of w[5] for $\mu = 0.1$ and $\sigma = 0.3$ is 1.623875
- The variance of w[5] for $\mu = 0.1$ and $\sigma = 0.3$ is 1.449356
- The expected value of w[5] for $\mu = -0.1$ and $\sigma = 0.3$ is 0.5538226
- The variance of w[5]for $\mu = -0.1$ and $\sigma = 0.3$ is 0.1783642
- The expected value of w[5] for $\mu = 0.1$ and $\sigma = 0.5$ is 1.936501
- The variance of w[5] is for $\mu = 0.1$ and $\sigma = 0.5$ 5.506025
- The expected value of w[5] for $\mu = -0.1$ and $\sigma = 0.5$ is 1.452208
- The variance of w[5] for $\mu = -0.1$ and $\sigma = 0.5$ is 1.057466
- The expected value of w[5] for $\mu = 0.1$ and $\sigma = 0.3$ for large no of paths is 1.65808
- The variance of w[5]for $\mu = 0.1$ and $\sigma = 0.3$ for large no of paths is is 1.610813

We can see from the above data that the theoretical values are in good agreement with the actual values of expectation and variance for large number of paths generated .