ASSIGNMENT 9

MA226 : Monte Carlo Simulation
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1.0 Problem1

Use the following Monte Carlo estimator to approximate the expected value

$$I = E(exp(\sqrt{U}))$$

where $U \sim (0,1)$, $I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i$ *where* $Y_i = exp(\sqrt{U_i})$ *with* $U_i = \sim(0,1)$. Take all values of M to be 10^2 , 10^3 , 10^4, 10^5 . Determine the 95% confidence interval for I_M for all the four values of M that you have taken.

1.1 solution

1.2 R code

```
u<-vector()
y<-vector()
main<-function(m)
{
    set.seed(104729)
    u=runif(m)
    y=exp(sqrt(u))
    thta=mean(y)
    quant=qnorm(0.975)
    lowerci=thta-(sqrt(var(y))*quant)/(sqrt(m))
    upperci=thta+(sqrt(var(y))*quant)/(sqrt(m))
    cat("The lower confidence interval for" ,m ," is : ",lowerci,"\n")
    cat("The higher confidence interval for" ,m ," is : ",upperci,"\n")
}
main(100)
main(1000)
main(10000)
main(100000)
```

Listing 1: R Code which gives confidence intervals and variance reductions

1.3 Results

- The lower confidence interval for 100 is : 1.961859
- The higher confidence interval for 100 is : 2.129071
- The lower confidence interval for 1000 is : 1.986139

- The higher confidence interval for 1000 is : 2.040825
- The lower confidence interval for 10000 is : 1.995547
- The higher confidence interval for 10000 is : 2.012833
- The lower confidence interval for 100000 is : 1.996873
- The higher confidence interval for 100000 is : 2.002341

2.0 Problem2

Repeat the above exercise using antithetic variates via the following estimator and calculate the percentage of variance reduction

$$\hat{I_M} = \frac{1}{M}\sum_{i=1}^{M} \hat{Y}_i$$

where

$$\hat{Y}_i = \frac{exp(\sqrt{U_i})+exp(\sqrt{1-U_i})}{2}$$

with $U_1 \sim (0, 1)$

2.1 Solution

2.1 R Code

```
u<-vector()
v<-vector()
y<-vector()
y1<-vector()
main<-function(m)
{
    set.seed(104729)
    u=runif(m)
    v=1-u
    y=(exp(sqrt(u))+exp(sqrt(v)))/2
    y1=exp(sqrt(u))
    thta=mean(y)
    quant=qnorm(0.975)
    lowerci=thta-(sqrt(var(y))*quant)/(sqrt(m))
    upperci=thta+(sqrt(var(y))*quant)/(sqrt(m))
    cat("The lower confidence interval for" ,m ," is : ",lowerci,"\n")
    cat("The higher confidence interval for" ,m ," is : ",upperci,"\n")
    vredtn=100*(var(y)-var(y1))/var(y1)
    cat("The reduction in variance for ",m, "is : ",vredtn,"%","\n")

}
main(100)
main(1000)
main(10000)
main(10^5)
```

Listing 2: R Code which gives confidence intervals and variance reductions

2.2 Results

- The lower confidence interval for 100 is : 1.995761
- The higher confidence interval for 100 is : 2.007882
- The reduction in variance for 100 is : -99.47455
- The lower confidence interval for 1000 is : 1.997865
- The higher confidence interval for 1000 is : 2.001914
- The reduction in variance for 1000 is : -99.45168

- The lower confidence interval for 10000 is : 1.999303
- The higher confidence interval for 10000 is : 2.000581
- The reduction in variance for 10000 is : -99.45378
- The lower confidence interval for 100000 is : 1.999751
- The higher confidence interval for 100000 is : 2.000157
- The reduction in variance for 100000 is : -99.44659

3.0 Problem3

Use $\sqrt{U}$ to construct control variate estimate and repeat the above exercise.calculate the percentage of variance reduction.

3.1 Solution

3.1 R Code

```
u<-vector()
w<-vector()
y<-vector()
x<-vector()
main<-function(m)
{
    set.seed(104729)
    u=runif(m)
    y=sqrt(u)
    x=exp(sqrt(u))
    c=(-1)*(cov(x,y)/var(y))
    w=x+c*(y-mean(y))
    thta=mean(w)
    quant=qnorm(0.975)
    lowerci=thta-(sqrt(var(w))*quant)/(sqrt(m))
    upperci=thta+(sqrt(var(w))*quant)/(sqrt(m))
    cat("The lower confidence interval for" ,m ," is : ",lowerci,"\n")
    cat("The higher confidence interval for" ,m ," is : ",upperci,"\n")
    vredtn=100*(var(w)-var(x))/var(x)
    cat("The percentage reduction for ",m,"is found to be ",vredtn,"\n")
}
main(100)
main(1000)
main(10000)
main(100000)
```

Listing 3: R Code which gives confidence intervals and variance reductions

3.2 Results

- The lower confidence interval for 100 is : 2.035605
- The higher confidence interval for 100 is : 2.055325
- The percentage reduction for 100 is found to be -98.60913
- The lower confidence interval for 1000 is : 2.010213
- The higher confidence interval for 1000 is : 2.016751
- The percentage reduction for 1000 is found to be -98.5707
- The lower confidence interval for 10000 is : 2.003176
- The higher confidence interval for 10000 is : 2.005204
- The percentage reduction for 10000 is found to be -98.62342

- The lower confidence interval for 100000 is : 1.999284
- The higher confidence interval for 100000 is : 1.99993
- The percentage reduction for 100000 is found to be -98.60686