ASSIGNMENT 8

MA226 : Monte Carlo Simulation
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1.0 Problem

Consider the multivariate normal,

$$X = \begin{pmatrix} X1 \\ X2 \end{pmatrix} \sim N(\mu, \Sigma)$$

$$\text{where } \mu = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} 1 & 2a \\ 2a & 4 \end{pmatrix}$$

1.1 Problem1

For the case $a = -0.25, 0, 0.25$, generate 1000 values of X and calculate sample means,sample variances and sample correlations. Make empirical contour plots based on above generated samples.

1.1 Solution

1.1 Graph

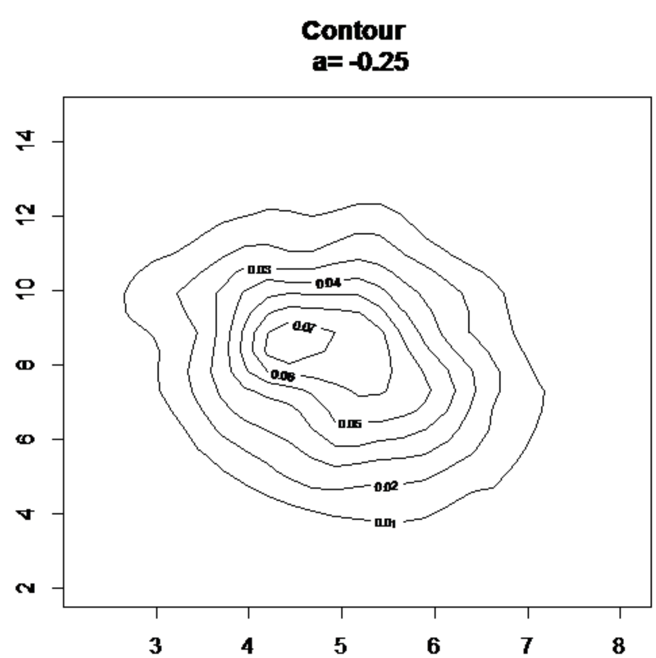

Figure 1: Contour plot for a=-0.25

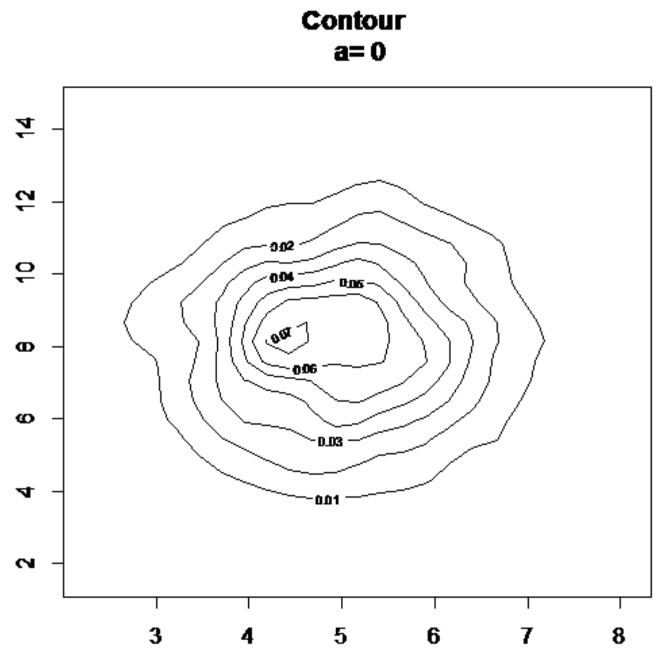

Figure 2: Contour plot for a=0

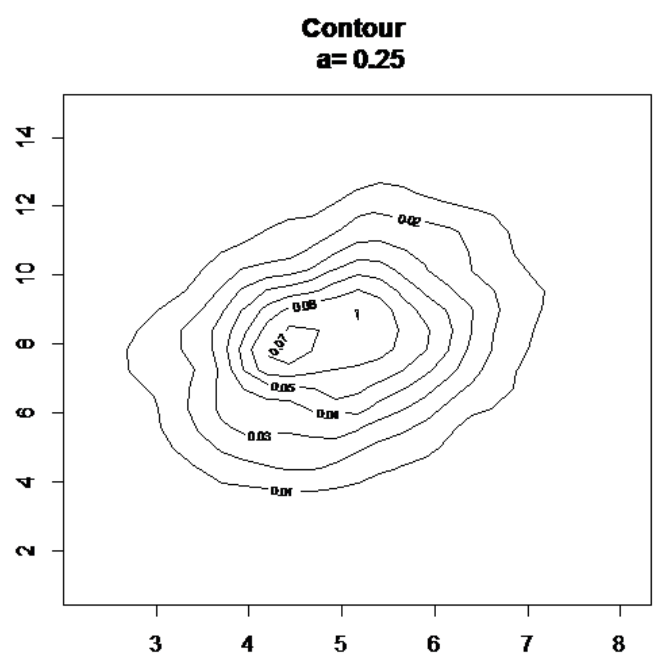

Figure 3: Contour plot for a=0.25

1.1 R Code

```
library("lattice")
library("MASS")
x1<-vector()
x2<-vector()
z1<-vector()
z2<-vector()
x<-vector()
y<-vector()
main<-function(a)
{
    set.seed(2251)
    z1=rnorm(1000)
    z2=rnorm(1000)
    x1<-5+z1
    x2<-8+2*a*z1+2*sqrt(1-a^2)*z2
    x<-c(x1,x2,nrow=1000,ncol=2)
    for(i in 1:1000)
    {
        cat(x1[i],x2[i],"\n")
    }
    cat("the mean of x1 is :",mean(x1),"\n")
    cat("the mean of x2 is :",mean(x2),"\n")
    cat("the var of x1 is :",var(x1),"\n")
    cat("the var of x2 is :",var(x2),"\n")
    cat("the covar  is :",cov(x1,x2),"\n")
    cat("the correlation  is :",cor(x1,x2),"\n")
    y<-kde2d(x1,x2)
    contour(y,main=paste("Contour","\n","a=",paste(a)))
}
main(-0.25)
x11()
main(0)
x11()
main(0.25)
```

Listing 1: R Code which generates the bivariate random numbers and contours

1.1 Results

- For a= -0.25
  - The mean of X_1 is 4.964
  - The mean of X_2 is 8.076
  - The variance of X_1 is 1.0388
  - The variance of X_2 is 4.051
  - The correlation of X_1, X_2 is -0.2486894
- For a= 0
  - The mean of X_1 is 4.964
  - The mean of X_2 is 8.0604
  - The variance of X_1 is 1.0388

- The variance of X_2 is 4.018
- The correlation of X_1, X_2 is 0.01155

- For a= 0.25
  - The mean of X_1 is 4.964
  - The mean of X_2 is 8.04103
  - The variance of X_1 is 1.0388
  - The variance of X_2 is 4.123
  - The correlation of X_1, X_2 is 0.25968

1.1 Explanation of Results

For X_1 the mean and variance came out to be nearer to 5 and 1 respectively. For X_2 the mean and variance came out to be nearer to 8 and 4 respectively. Hence the generated multivariate random numbers are correct.

1.2 Problem2

Also plot the actual and empirical marginal cdfs of X_1 and X_2

1.2 Solution

1.2 Graph

- a=-0.25

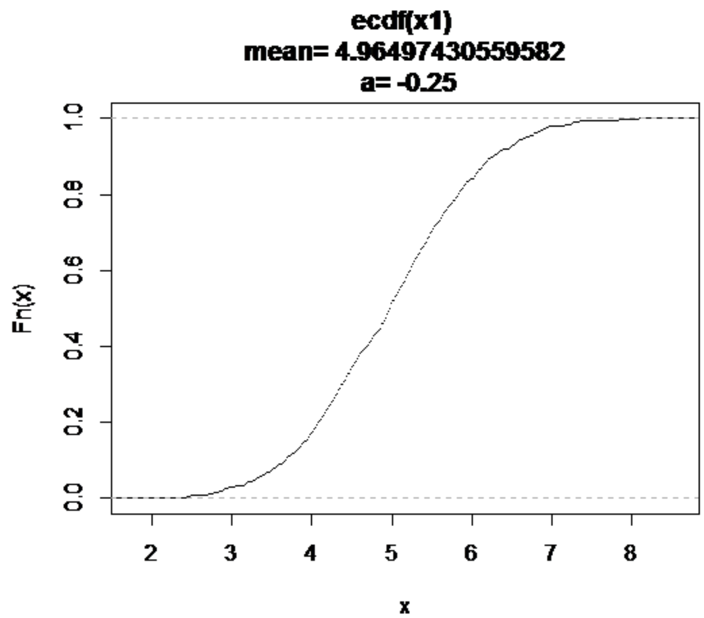

Figure 4: Empirical disribution for X_1

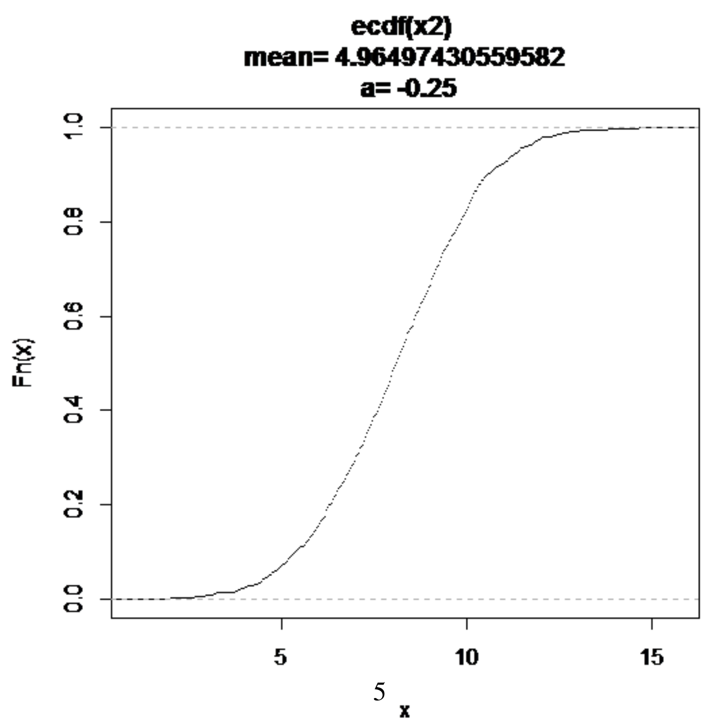

Figure 5: Empirical distribution for X_2

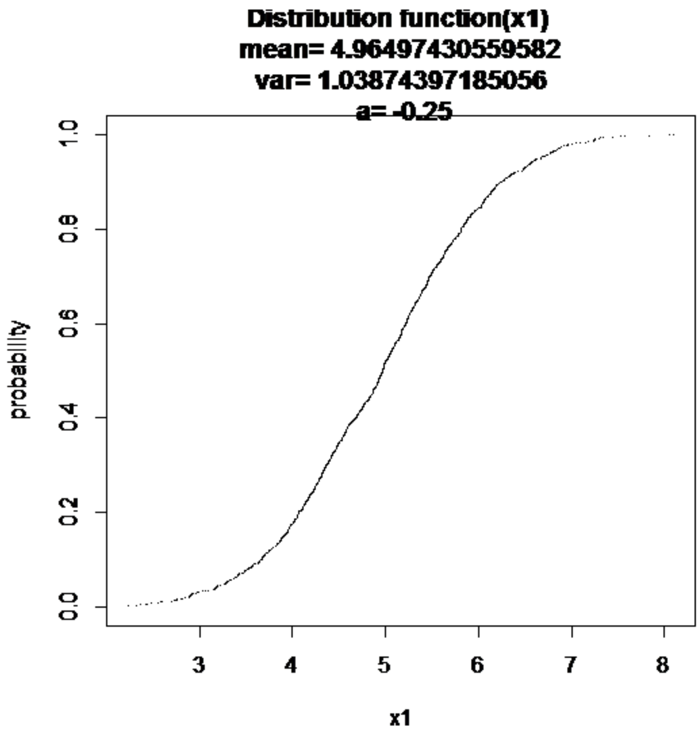

Figure 6: Actual distribution for X_1

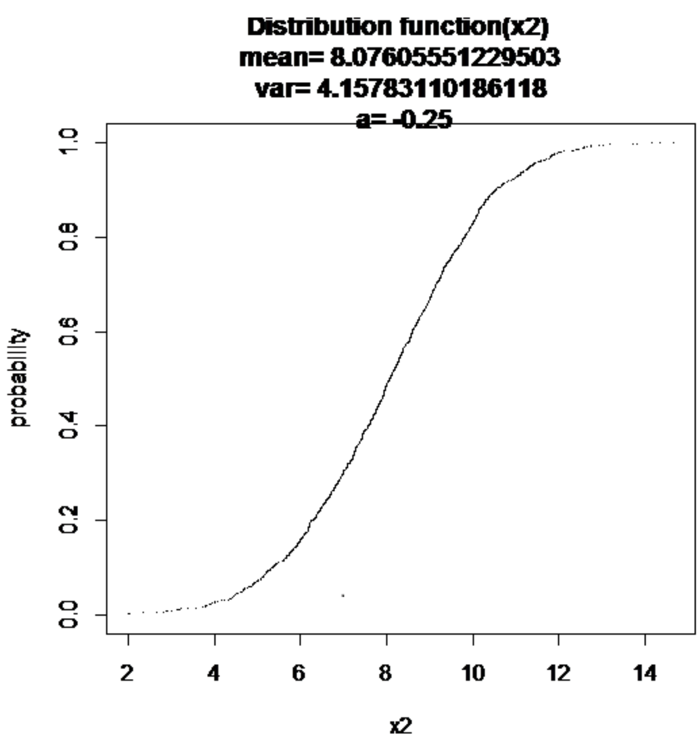

Figure 7: Actual distribution for X_2

- a=0

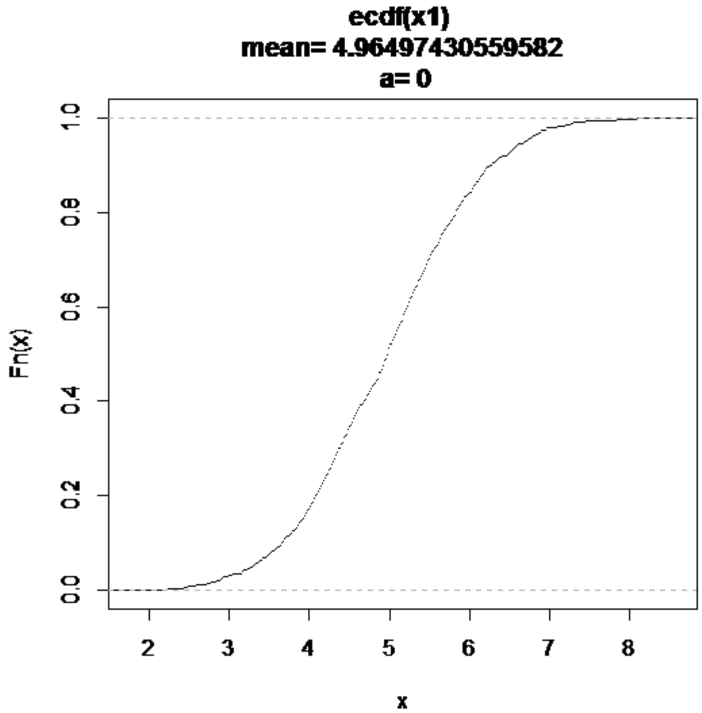

Figure 8: Empirical disribution for X_1

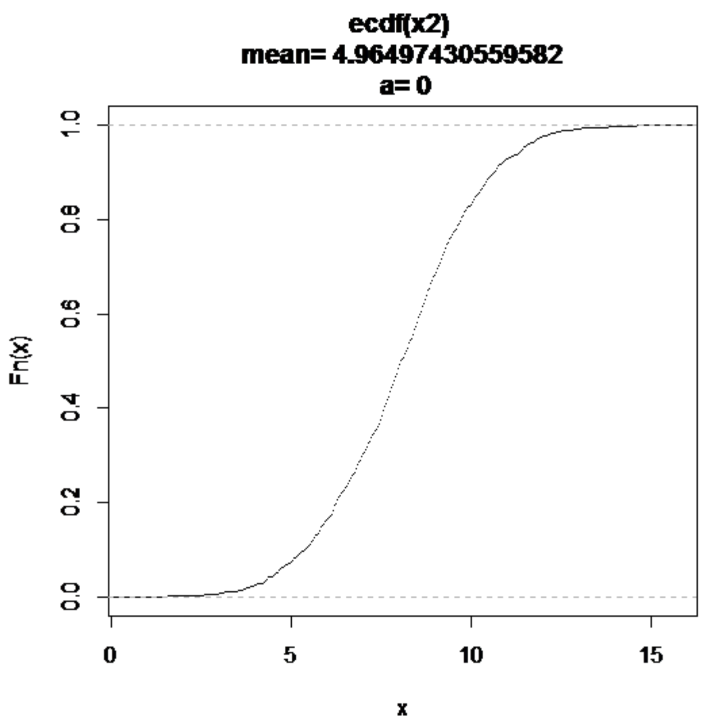

Figure 9: Empirical distribution for X_2

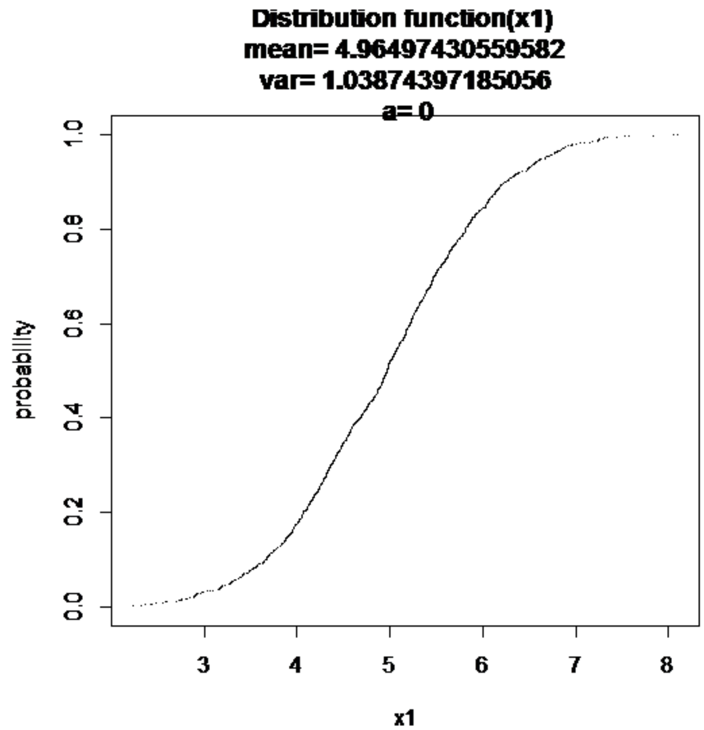

Figure 10: Actual distribution for X_1

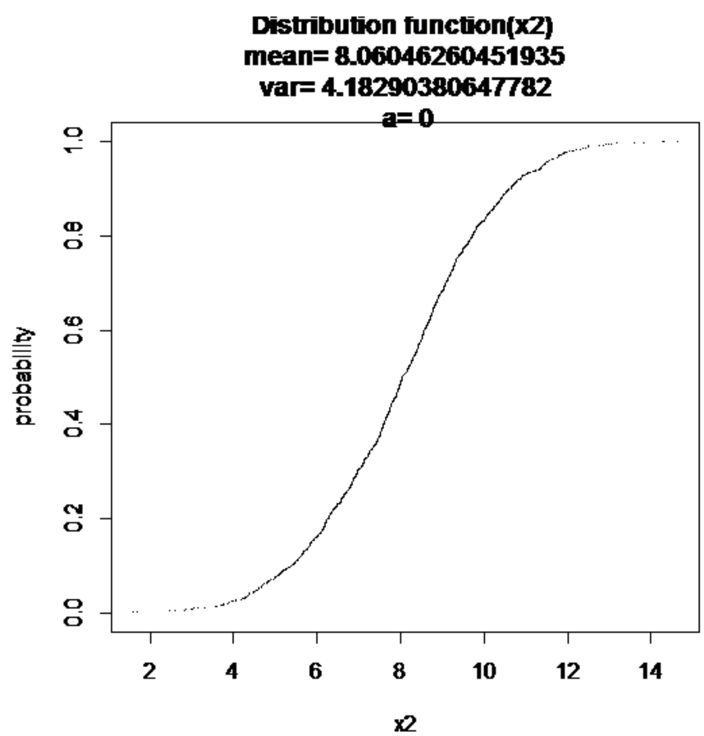

Figure 11: Actual distribution for X_2

- a=0.25

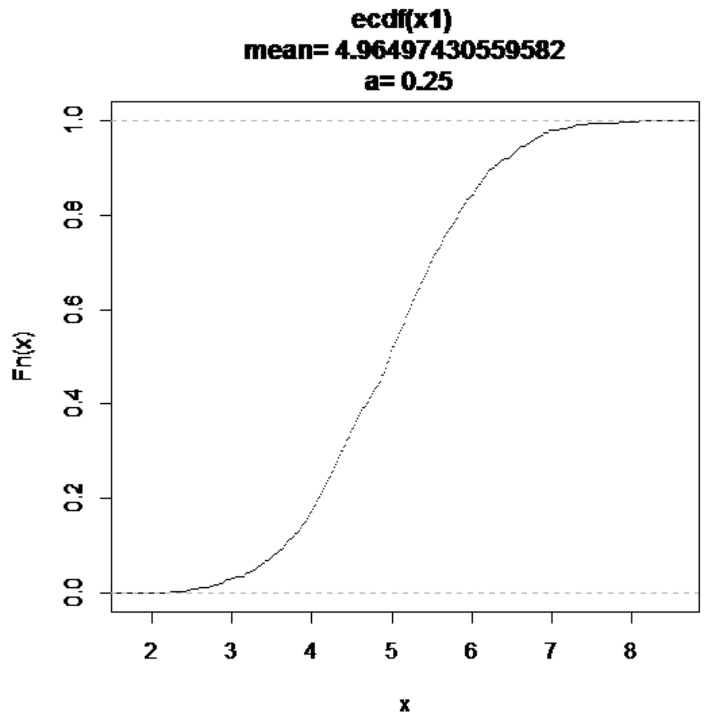

Figure 12: Empirical disribution for X_1

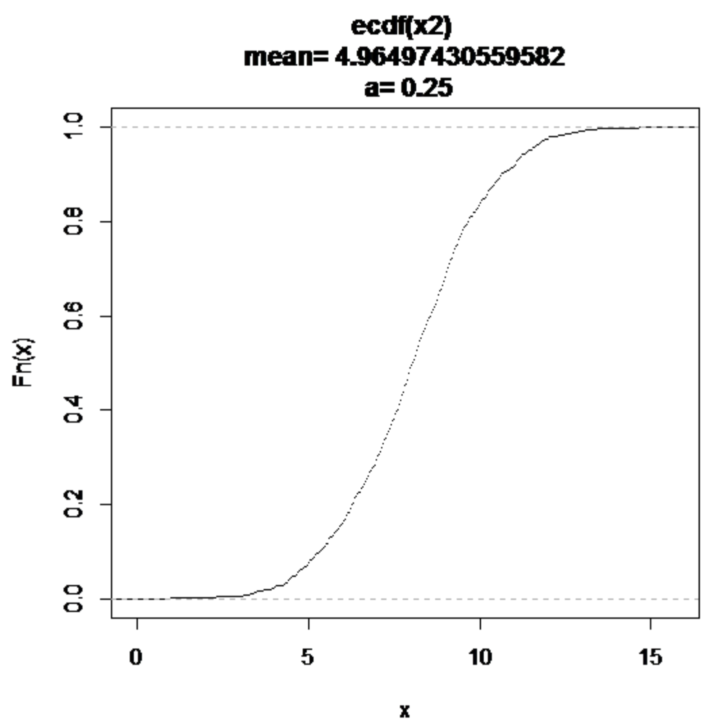

Figure 13: Empirical distribution for X_2

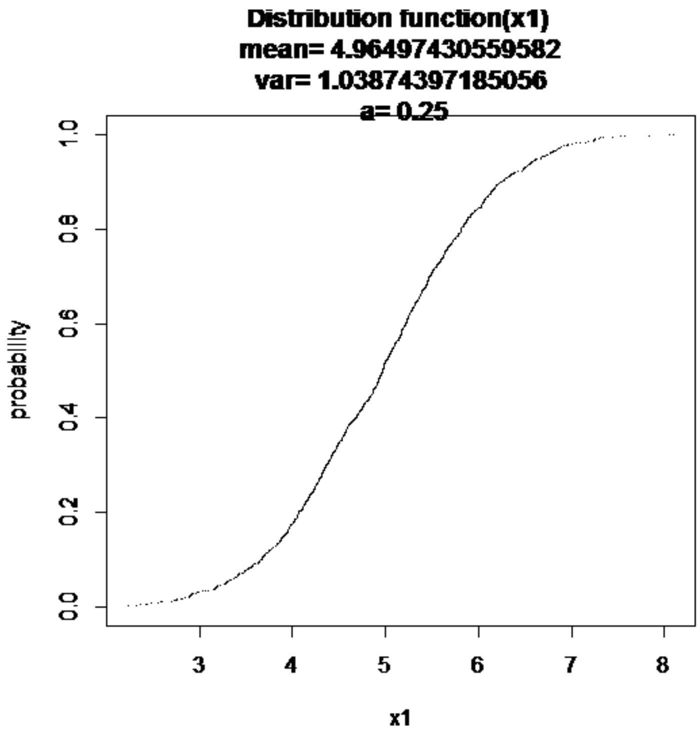

Figure 14: Actual distribution for X_1

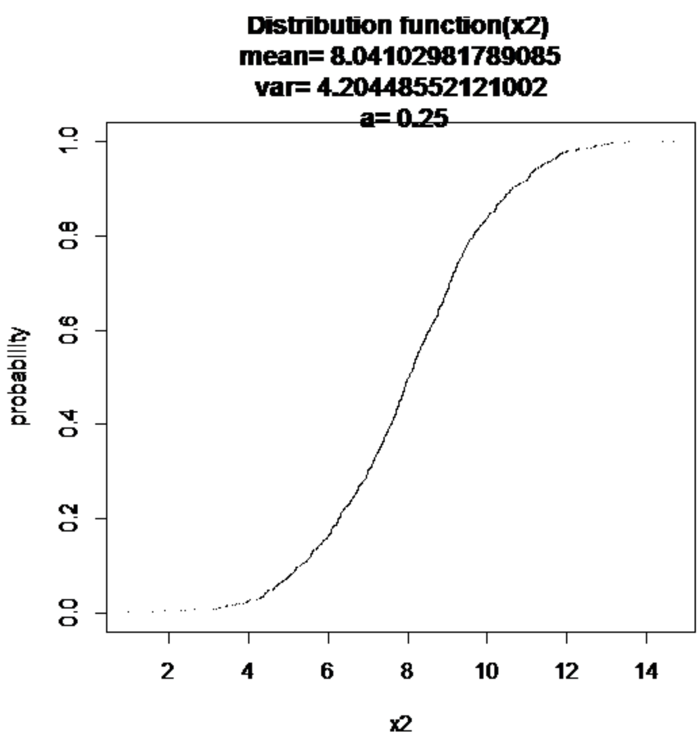

Figure 15: Actual distribution for X_2

1.2 R Code

```
library("lattice")
library("MASS")
x1<-vector()
x2<-vector()
z1<-vector()
z2<-vector()
x<-vector()
y<-vector()
z5<-vector()
z6<-vector()
dstbnfunc<-function(x1,x2,a)
{
    x1=sort(x1)
    x2=sort(x2)
    t=1000
    for(i in 1:t)
    {
        z5[i]=i/t
        z6[i]=i/t
    }
    #windows.options(reset=TRUE)
    plot(x1,z5,"p",pch='.',xaxt="s",main=paste("Distribution function(x1)","\n",
    "mean=",paste(mean(x1)),"\n","var=",paste(var(x1),"\n","a=",paste(a)))
    ,xlab="x1",ylab="probability")
    axis(side = 1, at = x)
    x11()
    plot(x2,z6,"p",pch='.',xaxt="s",main=paste("Distribution function(x2)","\n",
    "mean=",paste(mean(x2)),"\n","var=",paste(var(x2),"\n","a=",paste(a))),
    xlab="x2",ylab="probability")
    axis(side = 1, at = x)
}
main<-function(a)
{
    set.seed(2251)
    z1=rnorm(1000)
    z2=rnorm(1000)
    x1<-5+z1
    x2<-8+2*a*z1+2*sqrt(1-a^2)*z2
    plot(ecdf(x1),main=(paste("ecdf(x1)","\n","mean=",paste(mean(x1),"\n",
    "a=",paste(a)))))
    x11()
    plot(ecdf(x2),main=(paste("ecdf(x2)","\n","mean=",paste(mean(x1),"\n",
    "a=",paste(a)))))
    x11()
    dstbnfunc(x1,x2,a)
}
main(-0.25)
x11()
main(0)
x11()
main(0.25)
```

Listing 2: R Code which generates the actual and empirical marginal cdf's

1.3 Problem3

Let us recall generating a bivariate normal with the help of conditional distributions . Suppose that $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ and the conditional distribution of X_2 given $X_1 = x$ is $N(\mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1), \sigma_2^2(1 - \rho^2))$ where $|\rho| < 1$ is the correlation coefficient between X_1 and X_2.The vector (X_1, X_2) is said to have a bivariate normal distribution. Simulate the vector for a particular set of parameter values, using this idea of conditional distribution . Estimate the sample quantities(mean,etc.) and compare the actual values. Take same $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$.

1.3 Solution

1.3 Graph

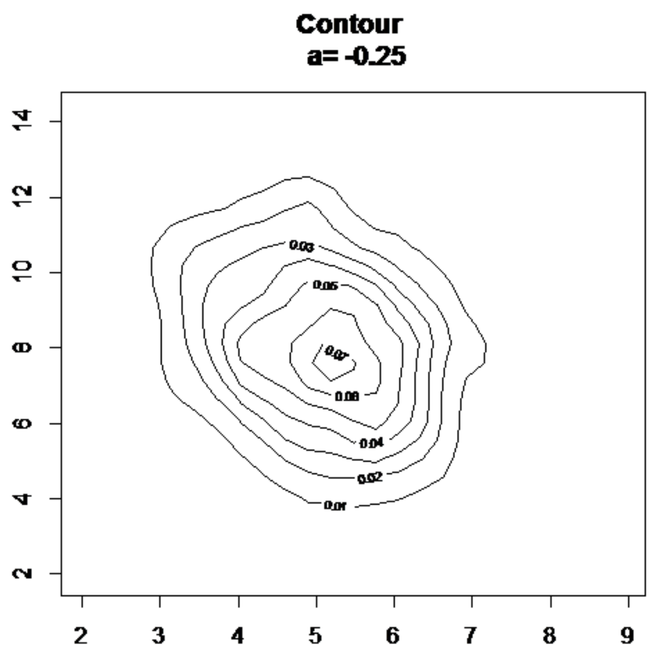

Figure 16: Contour plot for a=-0.25

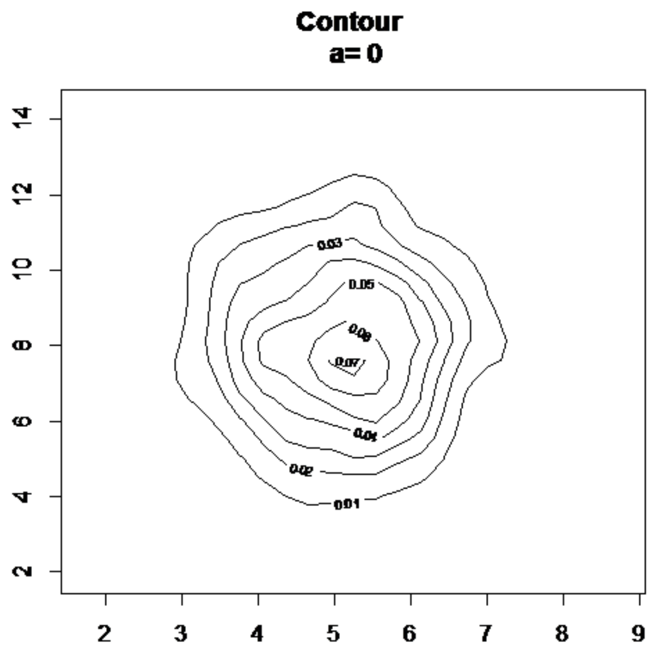

Figure 17: Contour plot for a=0

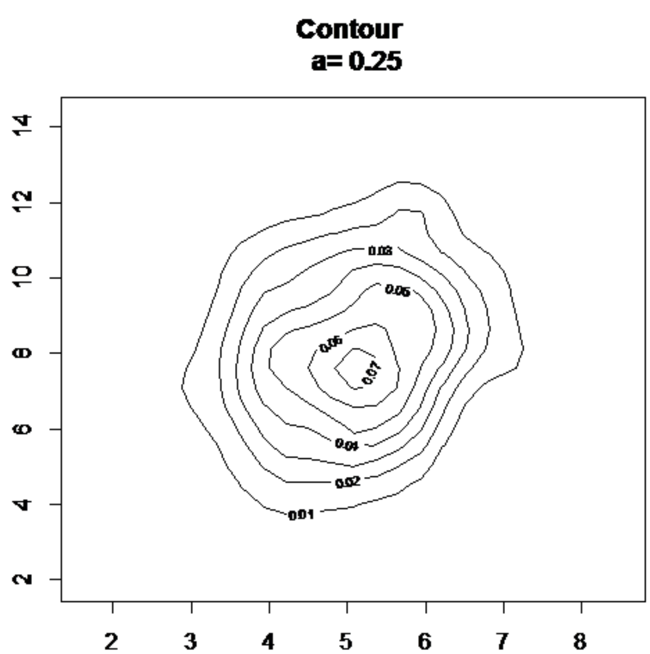

Figure 18: Contour plot for a=0.25

1.3 R Code

```
library("lattice")
library("MASS")
x1<-vector()
x2<-vector()
z1<-vector()
z2<-vector()
x<-vector()
y<-vector()
z5<-vector()
z6<-vector()
dstbnfunc<-function(x1,x2,a)
{
    x1=sort(x1)
    x2=sort(x2)
    t=1000
    for(i in 1:t)
    {
        z5[i]=i/t
        z6[i]=i/t
    }
    #windows.options(reset=TRUE)
    plot(x1,z5,"p",pch='.',xaxt="s",main=paste("Distribution function(x1)","\n",
    "mean=",paste(mean(x1)),"\n","var=",paste(var(x1),"\n","a=",paste(a)))
    ,xlab="x1",ylab="probability")
    axis(side = 1, at = x)
    x11()
    plot(x2,z6,"p",pch='.',xaxt="s",main=paste("Distribution function(x2)","\n",
    "mean=",paste(mean(x2)),"\n","var=",paste(var(x2),"\n","a=",paste(a))),
    xlab="x2",ylab="probability")
    axis(side = 1, at = x)
}
main<-function(a)
{
    set.seed(2251)
    z1=rnorm(1000)
    z2=rnorm(1000)
    x1<-5+z1
    x2<-8+2*a*z1+2*sqrt(1-a^2)*z2
    plot(ecdf(x1),main=(paste("ecdf(x1)","\n","mean=",paste(mean(x1),"\n",
    "a=",paste(a)))))
    x11()
    plot(ecdf(x2),main=(paste("ecdf(x2)","\n","mean=",paste(mean(x1),"\n",
    "a=",paste(a)))))
    x11()
    dstbnfunc(x1,x2,a)
}
main(-0.25)
x11()
main(0)
x11()
main(0.25)
```

Listing 3: R Code which generates the bivariate random numbers and contours

1.3 Results

- For a= -0.25
  - The mean of X_1 is 5.0743
  - The mean of X_2 is 8.004
  - The variance of X_1 is 1.0120
  - The variance of X_2 is 4.051
  - The correlation of X_1, X_2 is -0.2268183
- For a= 0
  - The mean of X_1 is 5.077
  - The mean of X_2 is 8.0049
  - The variance of X_1 is 1.024808
  - The variance of X_2 is 4.078
  - The correlation of X_1, X_2 is 0.0276
- For a= 0.25
  - The mean of X_1 is 5.075619
  - The mean of X_2 is 8.004974
  - The variance of X_1 is 1.0397
  - The variance of X_2 is 4.1336
  - The correlation of X_1, X_2 is 0.276986

1.3 Explanation of Results

For X_1 the mean and variance came out to be nearer to 5 and 1 respectively. For X_2 the mean and variance came out to be nearer to 8 and 4 respectively. Hence the generated multivariate random numbers are correct.