1 BNF Grammar

l	$::=$	memory location	
Γ	$::=$	$\emptyset \mid \Gamma, x : \tau$	Typing environment
S	$::=$	$\emptyset \mid S, l := e$	Store
E	$::=$	$\emptyset \mid E, x := l$	Variable environment
$\varphi_{i,j,k}$	$::=$	E_i, S_j, Γ_k	General environment
B	$::=$	$Int \mid \tau \to \tau$	Base Types
τ	$::=$	X	Type Variables
		$\mid Lazy\langle\tau\rangle$	
		$\mid Lazy^{*}\langle\tau\rangle$	
		$\mid \lambda X.\tau$	Type abstraction
		$\mid (\lambda X.\tau)\ \tau$	Type Application
		$\mid \{\exists X_1, x : X_1, \varphi : X_1 \to X_2\}$	Existential triplet
		$\mid B$	
n	$::=$	number literals	
x	$::=$	variables	
f	$::=$	$\lambda x : \tau.e$	
e	$::=$	n	
		$\mid f$	
		$\mid (e)$	
		$\mid$ ‘e‘	Deferred Expressions
		$\mid e - e$	
		$\mid e < e$	
		$\mid e\ e$	
		$\mid x$	
		$\mid \langle e, \varphi_{i,j,k}\rangle$	Closures
		$\mid$ if e then e else e	
a	$::=$	$\tau\ x := e; a$	Declaration
		$\mid x := e; a$	Assignment
		$\mid nop$	

2 Typing Rules

$$\frac{\Gamma(x) = \tau \quad E(x) = l_x \quad S(l_x) = e}{\varphi_{i,j,k} \vdash x : \tau[e]}\ \text{EL} \qquad \frac{\varphi_{i,j,k}, x : \tau_1 \vdash e : \tau_2}{\varphi_{i,j,k} \vdash (\lambda x : \tau_1 . e) : \tau_1 \to \tau_2}\ \text{E-Abs} \qquad \frac{}{(\lambda X.t)(\tau) : [X \mapsto \tau]t}\ \text{T-App}$$

$$\frac{\varphi_{i,j,k} \vdash e : \tau}{\varphi_{i,j,k} \vdash `e` : Lazy\langle\tau\rangle}\ \text{Defer} \qquad \frac{\varphi_{i,j,k} \vdash x : \tau}{\varphi_{i,j,k} \vdash formula(e) : \tau}\ \text{Formula} \qquad \frac{}{bt(\tau) : (B :: \tau)}\ \text{BT}$$

$$\frac{\tau : type}{\tau <: \tau}\ \text{STR} \qquad \frac{\tau_1 <: \tau_2 \quad \tau_2 <: \tau_3}{\tau_1 <: \tau_3}\ \text{STT} \qquad \frac{\tau_1 <: \tau_2 \quad \tau_2 <: \tau_1}{\tau_1 \equiv \tau_2}\ \text{STA} \qquad \frac{T_1 <: S_1 \quad S_2 <: T_2}{S_1 \to S_2 <: T_1 \to T_2}\ \text{F-ST}$$

$$\frac{\tau : B}{\tau <: Lazy\langle\tau\rangle}\ \text{VL-ST} \qquad \frac{}{Lazy\langle\tau\rangle <: Lazy\langle Lazy\langle\tau\rangle\rangle}\ \text{LL-ST} \qquad \frac{bt(\tau_1) = \tau_2}{\tau_1 <: Lazy^*\langle\tau_2\rangle}\ \text{LL*-ST}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 : \tau_1 \to \tau_2 \quad \varphi_{i,j,k} \vdash e_2 : \tau_1}{\varphi_{i,j,k} \vdash e_1(e_2) : \tau_2}\ \text{E-App} \qquad \frac{}{\varphi_{i,j,k} \vdash \bot : \tau}\ \bot \qquad \frac{\varphi_{i,j,k} \vdash e : \tau}{\varphi_{i,j,k} \vdash \langle e, \varphi_{i',j',k'}\rangle : \tau}\ \text{Closure}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 : \tau_1 \to \tau_2 \quad \varphi_{i,j,k} \vdash e_2 : \tau_1 \to \tau_2}{\varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : \tau_1 \to \tau_2}\ \text{F-RT} \qquad \frac{\varphi_{i,j,k} \vdash e_1 : Int \quad \varphi_{i,j,k} \vdash e_2 : Int}{\varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : Int}\ \text{I-RT}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 : Lazy\langle Lazy\langle\tau\rangle\rangle \quad \varphi_{i,j,k} \vdash e_2 : Lazy\langle\tau\rangle}{\varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : Lazy\langle\tau\rangle}\ \text{LN-RT}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 : Lazy\langle B\rangle \quad \varphi_{i,j,k} \vdash e_2 : B}{\varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : B}\ \text{L-RT} \qquad \frac{\varphi_{i,j,k} \vdash e_1 : Lazy^*\langle\tau\rangle \quad \varphi_{i,j,k} \vdash e_2 : Lazy^*\langle\tau\rangle}{\varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : Lazy^*\langle\tau\rangle}\ \text{L*-RT}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 : \tau \quad bt(\tau) = b}{\varphi_{i,j,k} \vdash feval(e_1) : b}\ \text{FE}$$

3 Operational Semantics of Expressions

$$\frac{}{bt(B) = B}\ \text{BT-B} \qquad \frac{}{bt(Lazy\langle B\rangle) = B}\ \text{BT-LB} \qquad \frac{bt(\tau_1) = \tau_2}{bt(Lazy\langle \tau_1\rangle) = \tau_2}\ \text{BT-LL}$$

$$\frac{\varphi_{i,j,k} \vdash n : Int}{\varphi_{i,j,k} \vdash n \Downarrow n}\ \text{E-Int} \qquad \frac{\varphi_{i,j,k} \vdash f : \tau_1 \to \tau_2}{\varphi_{i,j,k} \vdash f \Downarrow \langle f, \varphi_{i,j,k}\rangle : \tau_1 \to \tau_2}\ \text{E-Fun}$$

$$\frac{\varphi_{i,j,k} \vdash \langle f, \varphi_{i',j',k'}\rangle : \tau_1 \to \tau_2}{\varphi_{i,j,k} \vdash \langle f, \varphi_{i',j',k'}\rangle \Downarrow \langle f, \varphi_{i',j',k'}\rangle : \tau_1 \to \tau_2}\ \text{E-FClosure}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : B}{\varphi_{i,j,k} \vdash feval(e_1) \Downarrow e_2 : B}\ \text{feval-1}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 : Lazy\langle \tau_1\rangle \qquad \varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : \tau_1 \qquad \varphi_{i,j,k} \vdash feval(e_2) \Downarrow e_3 : \tau_2}{\varphi_{i,j,k} \vdash feval(e_1) \Downarrow e_3 : \tau_2}\ \text{feval-2}$$

$$\frac{\varphi_{i,j,k} \vdash x : \tau_1[e_1] \qquad \varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : \tau_2}{\varphi_{i,j,k} \vdash x \Downarrow e_2 : \tau_2}\ \text{Var-RValue} \qquad \frac{\varphi_{i,j,k} \vdash e : \tau}{\varphi_{i,j,k} \vdash \text{`}e\text{`} \Downarrow \langle e, \varphi_{i,j,k}\rangle : \tau}\ \text{D-Eval}$$

$$\frac{\dfrac{\tau_1 \not\equiv \tau_3 \to \tau_4}{\varphi_{i,j,k} \vdash \langle e_1, \varphi_{i',j',k'}\rangle : \tau_1} \qquad \begin{array}{l} S_y = S_j, S_{j'} \\ \Gamma_z = \Gamma_{j'}, \Gamma_j \\ \varphi_{i',y,z} \vdash e_1 \Downarrow e_2 : \tau_2 \end{array}}{\varphi_{i,j,k} \vdash \langle e_1, \varphi_{i',j',k'}\rangle \Downarrow e_2 : \tau_2}\ \text{C-Eval}$$

$$\frac{\varphi_{i,j,k} \vdash x : \tau[e]}{\varphi_{i,j,k} \vdash formula(x) \Downarrow e : \tau}\ \text{F-eval} \qquad \frac{\varphi_{i,j,k} \vdash e : B}{\varphi_{i,j,k} \vdash \{B, e, \lambda x.\ formula(x)\} : Lazy^*\langle B\rangle}\ \text{E-Pack-1}$$

$$\frac{\varphi_{i,j,k} \vdash e : Lazy\langle \tau_1\rangle \qquad bt(\tau_1) = \tau_2}{\varphi_{i,j,k} \vdash \{Lazy^n\langle \tau_1\rangle, e, \lambda x.\ feval(formula(x))\} : Lazy^*\langle \tau_2\rangle}\ \text{E-Pack-2}$$

$$\frac{\varphi_{i,j,k} \vdash \{X, e, f\} : Lazy^*\langle \tau\rangle \qquad \varphi_{i,j,k} \vdash f(e) \Downarrow e_1 : \tau}{\varphi_{i,j,k} \vdash feval(\{X, e, f\}) \Downarrow e_1 : \tau}\ \text{feval-3}$$

$$\frac{\varphi_{i,j,k} \vdash \{X_1, e_1, f_1\} : Lazy^*\langle \tau\rangle \qquad \varphi_{i,j,k} \vdash e_1 : X_1 \Downarrow e_2 : X_2}{\varpi_{i,j,k} \vdash \{X, e, f\} \Downarrow \{X_2, e_2, feval\} : Lazy^*\langle \tau\rangle}\ \text{E-Eval}$$

$$\frac{\varphi_{i,j,k} \vdash f \Downarrow \langle \lambda x.t, \varphi_{i',j',k'}\rangle : \tau_1 \to \tau_2 \qquad \varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : \tau_2 \qquad \begin{array}{l} l'_x = newLoc(E') \\ E_x = E_{i'}[l'_x/x] \\ S_y = S_{j'}[e_2/l'_x] \\ \Gamma_z = \Gamma_{k'}[\tau_1/x] \\ \varphi_{x,y,z} \vdash t \Downarrow t' : \tau_2 \end{array}}{\varphi_{i,j,k} \vdash f(e_1) \Downarrow \langle t', \varphi_{x,y,z}\rangle : \tau_2}\ \text{E-App}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 \Downarrow n_1 : Int \qquad \varphi_{i,j,k} \vdash e_2 \Downarrow n_2 : Int}{\varphi_{i,j,k} \vdash e_1 - e_2 \Downarrow [\![n_1 - n_2]\!] : Int} \text{ Minus}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 \Downarrow n_1 : Int \qquad \varphi_{i,j,k} \vdash e_2 \Downarrow n_2 : Int \qquad [\![n_1 < n_2]\!]}{\varphi_{i,j,k} \vdash e_1 < e_2 \Downarrow 1 : Int} \text{ Less-True}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 \Downarrow n_1 : Int \qquad \varphi_{i,j,k} \vdash e_2 \Downarrow n_2 : Int \qquad [\![n_1 \geq n_2]\!]}{\varphi_{i,j,k} \vdash e_1 < e_2 \Downarrow -1 : Int} \text{ Less-False}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 \Downarrow n : Int \qquad \varphi_{i,j,k} \vdash e_2 \Downarrow e_2' : \tau \qquad \varphi_{i,j,k} \vdash e_3 : \tau \qquad \varphi_{i,j,k} \vdash e_1 < 1 \Downarrow -1}{\varphi_{i,j,k} \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow e_2'} \text{ If-True}$$

$$\frac{\varphi_{i,j,k} \vdash e_1 \Downarrow n : Int \qquad \varphi_{i,j,k} \vdash e_2 : \tau \qquad \varphi_{i,j,k} \vdash e_3 \Downarrow e_3' : \tau \qquad \varphi_{i,j,k} \vdash e_1 < 1 \Downarrow 1}{\varphi_{i,j,k} \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 \Downarrow e_3'} \text{ If-False}$$

$$\frac{\varphi_{i,j,k} \vdash t : S \qquad S <: T}{\varphi_{i,j,k} \vdash t : T} \text{ Subtype-Replace}$$

4 Operational Semantics of Actions

$$\frac{\begin{array}{l} l_x = newLoc(E_i) \\ E_a = E_i[l_x/x] \\ \Gamma_c = \Gamma_k[\tau_2/x] \end{array} \qquad \varphi_{a,j,c} \vdash e_1 : \tau_1 \qquad \begin{array}{r} \varphi_{a,j,c} \vdash e_1 \Downarrow e_2 : \tau_2 \\ S_b = S_j[e_2/l_x] \end{array}}{\varphi_{i,j,k} \vdash \langle \tau_1\ x := e_1, \varphi_{a,b,c} \rangle} \text{ Decl}$$

$$\frac{\varphi_{i,j,k} \vdash x : e_2 \qquad \varphi_{i,j,k} \vdash e_1 : \tau_1 \qquad \varphi_{i,j,k} \vdash e_1 \Downarrow e_2 : \tau_2 \qquad \begin{array}{l} l_x = E_i(x) \\ S_b = S_j[e_2/l_x] \end{array}}{\varphi_{i,j,k} \vdash \langle x := e, \varphi_{i,b,c} \rangle} \text{ Assign}$$

$$\frac{\varphi_{i,j,k}, \Gamma \vdash \langle a_1, \varphi_{i',j',k'} \rangle \qquad \varphi_{i',j',k'} \vdash \langle a_2, \varphi_{a,b,c} \rangle}{\varphi_{i,j,k} \vdash \langle a_1; a_2, \varphi_{a,b,c} \rangle} \text{ Seq}$$