

Bluff

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1 Introduction

2 Rules

2.1 Setup

Let there be n players, $3 \leq n \leq 7$.

Objective Each player (i)'s objective is to dispose of all of their cards.

Let there be a shared table into which players may discard cards.

Let there be a global counter α , defined as $\alpha_0 = 1$

$$\alpha_t = (\alpha_{t-1} \bmod 13) + 1.$$

The game uses a standard deck of playing cards without jokers.

A deck consists of 52 cards consisting of 4 suites, with 13 cards in each suite numbered from 1 to 13.

Initially, the cards are divided evenly between all players s.t. each player has $\lfloor \frac{52}{n} \rfloor$. The excess $52 \bmod n$ cards are placed on the table

3 Defining a Player

Let the true distribution of cards for card α held be $T(\alpha)$.

Let the known distribution of cards for card α held be $K(\alpha)$

Assume there are 4 cards of each number: $\forall_{a \in [1,13]} \sum T(a) = 4$

4 Defining the table

Let the table be represented as player $i = 0$, and the remaining players be $i = 1, 2, \dots, n$

5 Defining a Turn

Play is conducted counter-clockwise, and no player may be skipped.

In a turn, consider the distinct players i and j .

Let a turn (t) consist of the following sequence:

1. Player i discards p_t cards. $p_t \in [1, 4]$
2. Player j may "call BS"
 - (a) $\forall_{i \in p_t} type(i) = \alpha_t \iff j$ must take the table
 - (b) $\exists_{i \in p_t} type(i) \neq \alpha_t \iff i$ must take the table

When i takes the table, $\forall_{a \in [1, n]}$
 $T(a)_i = T(a)_i + T(a)_0; T(a)_0 = 0$

6 The Problem

We must find $P(T(\alpha_t)_i \geq p_t)$

Equation 1 The number of solutions to the equation $\sum a_r = b, a_r \geq r = \binom{b - \sum_{r=1}^n r + |a| - 1}{|a| - 1}$

7 Initial Game State

The number of cards at the table can be seen as $\sum K(\alpha_t) + \sum T(\alpha_t) = 4$

s.t. $\forall_i K(\alpha_t)_i \geq K(\alpha_t)_i$

Applying equation 1, we may solve the equation with unknowns of T for the $n+1$ players.

Let the function $\eta(S) = \binom{4 - \sum_{i=1}^n S_i + |S| - 1}{|S| - 1}$ where S is the set of known cards.

Let $K(\alpha_t)' = \{K(\alpha_t)_0, K_1, \dots, K(\alpha_t)_{i-1}, p_t, K(\alpha_t)_{i+1}, \dots, K(\alpha_t)_{n-1}, K(\alpha_t)_n\}$

$P(T(\alpha_t)_i \geq p_t) = \frac{\eta(K')}{\eta(K)}$

8 Additional Information

To gain more information about players (and thus update the set K to have non-zero values), there are exactly 2 cases in which information is revealed.

1. If a player j calls "BS" and the player i was truthful, the condition holds that $\forall_{i \in p_t} type(i) = \alpha_t \iff j$ must take the table. At this point, $K(\alpha_t)_j$ must increase by p_t , as j is definitely holding their cards.

2. If you, the player, place a card of value r in the the table, then until someone takes the table, $K(r)_0$ must increase by 1. Similarly, if the table is taken by another player j , those cards are now held by j instead of 0/the table.

9 Conclusion