Analyzing the Card Game BS

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Chapter 1

Rules of the Game

1.1 Objective of the Game

Objective In this game, each player (i) has the objective of disposing of all of their cards.

1.2 Setup of the Game

Let there be n players, $3 \leq n \leq 7$.

Let there be a shared table into which players may discard cards.

Let there be a global number α, defined as $\alpha_0 = 1; \alpha_t = (\alpha_{t-1} \bmod 13) + 1$.

The game uses a deck of 52 cards consisting of 4 suites, with exactly 13 cards in each suite.

Initially, each player is given $\lfloor \frac{52}{n} \rfloor$, and the remaining $52 \bmod n$ cards are placed on the table.

1.3 Defining a Player

Let the hand of player i be represented by a set $\{p_{i,1}, p_{i,2}, ..., p_{i,12}, p_{i,13}\}$ where $p_{i,r}$ is the number of cards of type r.

Assume there are 4 cards of each number: $\forall_j \sum_{i=1}^{n} p_{ij} = 4$

1.4 Defining the table

Let the table be represented as player $n+1$

1.5 Defining a Turn

Play is conducted counter-clockwise, and no player may be skipped.

In a turn, consider the players i and $j, i \neq j$.

Let a turn (t) consist of the following sequence:

1. Player i discards k_t cards; $k_t \in [1,4], k_t \in \mathbb{N}$
2. Player j may "call BS"
   (a) $\forall_{i \in k_t} type(i) = \alpha_t \rightarrow$ j must take the table
   (b) $\exists_{i \in k_t} type(i) \neq \alpha_t \rightarrow$ i must take the table

1.6 The Problem

Assume you are player 1 p_{1r}. We must calculate $P(p_{i\alpha_t} \geq k_t)$

1.7 Levels of Thought

1. The table is empty and we have no knowledge of the player's cards.
2. If a player i tells the truth and player j calls bluff in turn t, player j definitely holds k_t cards of type α_t.
3. The table is treated as another player to whom cards are given in each turn
4. The table definitely holds the cards that you have placed into it in your previous turns, and when the table is taken by a player i, i definitely holds those cards.

Chapter 2

Describing the Initial State with $\forall_{i \in \gamma} i = 0$

2.1 Assumptions

To begin, let us find a formula to compute the relevant probability given the table is empty and that there is no other information provided.

Relevant Variables The number of cards to distribute between $n-1$ players is $4 - p_{0\alpha_t}$. The number of cards player i places is k_t, and the true amount they hold is $p_{i\alpha_t}$.

In general, to split y items among z people, There are $\binom{y+z-1}{z-1}$ ways.

We must count the number of ways in which $4 - p_{0\alpha_t}$ may be distributed between $n-1$ players. This is $\binom{2-p_{0\alpha_t}+n}{n-2}$

We must count the number of ways in which players j such that $j \neq i$ can hold, which is $\binom{1-p_{0\alpha_t}-k_t+n}{n-3}$

We must count the number of ways in which player i can hold k_t cards, which is $\binom{n-1}{k_t}$

This means that in the most naive case, the probability of player i having exactly k_t cards is $\frac{\binom{n-1}{k_t} * \binom{1-p_{0\alpha_t}-k_t+n}{n-3}}{\binom{2-p_{0\alpha_t}+n}{n-2}}$

We must compute the probability from k_t up to 4, since a player may have more cards than what they play. $\sum_{k_t}^{4-p_{0\alpha_t}} \frac{\binom{n-1}{k_t} * \binom{1-p_{0\alpha_t}-k_t+n}{n-3}}{\binom{2-p_{0\alpha_t}+n}{n-2}}$

$f(r) = 0$

Based on these bounds, this gives us the formula as follows. $\frac{\sum_{r=\max(known,k)}^{min(known_i, 4-\sum known_{\neq i}, 4)} f(r)}{\sum_{r=\max(known_i-1,0)}^{min(held_i, 4-\sum known_{\neq i}, 4)} f(r)}$

Chapter 3

Including the table

The rules of the table