Analyzing the Card Game BS

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Chapter 1

Rules of the Game

1.1 Objective of the Game

Objective In this game, each player (i) has the objective of disposing of all of their cards.

1.2 Setup of the Game

Let there be n players, $3 \leq n \leq 7, n \in \mathbb{N}$.

Let there be a shared pot into which players may discard cards.

Let there be a global type α, defined as $\alpha_0 = 1; \alpha_t = (\alpha_{t-1} \bmod 13) + 1$.

The game uses a deck of 52 cards consisting of 4 suites. Each suite is $\{x | x \in \mathbb{N} x \in [1, 13]\}$.

Initially, each player is given $\lfloor \frac{52}{n} \rfloor$, and $52 \bmod n$ cards are placed in the pot.

1.3 Defining a Player

Let the hand of i be represented by a set $\{p_{i1}, p_{i2}, ..., p_{i12}, p_{i13}\}$ where p_{ir} is the number of cards of type r, and i is the player.

Assume $\forall_j \sum_{i=1}^{n} p_i j = 4$

1.4 Defining the pot

Let the pot be represented as player γ

1.5 Defining a Turn

Play is conducted counter-clockwise, and no player may be skipped.

In a turn, consider the players i and $j, i \neq j$.

Let a turn (t) consist of the following sequence:

1. Player i discards k_t cards; $k_t \in [1, 4], k_t \in \mathbb{N}$
2. Player j may "call BS"
   (a) $\forall type(k_t) = \alpha_t \rightarrow$ j must take the pot
   (b) $\exists type(k_t) \neq \alpha_t \rightarrow$ i must take the pot

1.6 The Problem

Assume you are player 1 p_{1r}. We must calculate $P(p_{i\alpha_t} \geq k_t)$

Chapter 2

Describing the Initial State with $\forall_{i \in \gamma} i = 0$

2.1 Assumptions

To begin, let us find a formula to compute the relevant probability given the pot is empty and that there is no other information provided.

Relevant Variables The number of cards to distribute between $n-1$ players is $4 - p_{0\alpha_t}$. The number of cards player i places is k_t, and the true amount they hold is $p_{i\alpha_t}$.

In general, to split y items among z people, There are $\binom{y+z-1}{z-1}$ ways.

We must count the number of ways in which $4 - p_{0\alpha_t}$ may be distributed between $n-1$ players. This is $\binom{2-p_{0\alpha_t}+n}{n-2}$

We must count the number of ways in which players j such that $j \neq i$ can hold, which is $\binom{1-p_{0\alpha_t}-k_t+n}{n-3}$

We must count the number of ways in which player i can hold k_t cards, which is $\binom{n-1}{k_t}$

This means that in the most naive case, the probability of player i having p cards is $\frac{\binom{n-1}{k_t} * \binom{1-p_{0\alpha_t}-k_t+n}{n-3}}{\binom{2-p_{0\alpha_t}+n}{n-2}}$