Analyzing the Card Game BS

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Chapter 1

Rules of the Game

1.1 Objective of the Game

Objective In this game, each player (i) has the objective of disposing of all of their cards.

1.2 Setup of the Game

Let there be n players, $3 \leq n \leq 7, n \in \mathbb{N}$.

Let there be a shared pot into which players may discard cards.

Let there be a global type α, defined as $\alpha_0 = 1; \alpha_t = (\alpha_{t-1} \bmod 13) + 1$.

The game uses a deck of 52 cards consisting of 4 suites. Each suite is $\{x | x \in \mathbb{N} x \in [1, 13]\}$.

Initially, each player is given $\lfloor \frac{52}{n} \rfloor$, and $52 \bmod n$ cards are placed in the pot.

1.3 Defining a Player

Let the hand of i be represented by a set $\{p_{i1}, p_{i2}, ..., p_{i12}, p_{i13}\}$ where p_{ir} is the number of cards of type r, and i is the player.

Assume $\forall_j \sum\limits_{i=1}^{n} p_{ij} = 4$

1.4 Defining the pot

Let the pot be represented as player γ

1.5 Defining a Turn

Play is conducted counter-clockwise, and no player may be skipped.

In a turn, consider the players i and $j, i \neq j$.

Let a turn (t) consist of the following sequence:

1. Player i discards k_t cards; $k_t \in [1, 4], k_t \in \mathbb{N}$
2. Player j may "call BS"
   (a) $\forall_{i \in k_t} type(i) = \alpha_t \rightarrow$ j must take the pot
   (b) $\exists_{i \in k_t} type(i) \neq \alpha_t \rightarrow$ i must take the pot

1.6 The Problem

Assume you are player 1 p_{1r}. We must calculate $P(p_{i\alpha_t} \geq k_t)$

1.7 Levels of Thought

1. The pot is empty and we have no knowledge of the player's cards.
2. If a player i tells the truth and player j calls bluff in turn t, player j definitely holds k_t cards of type α_t.
3. The pot is treated as another player to whom cards are given in each turn
4. The pot definitely holds the cards that you have placed into it in your previous turns, and when the pot is taken by a player i, i definitely holds those cards.

Chapter 2

Describing the Initial State with $\forall_{i\in\gamma} i = 0$

2.1 Assumptions

To begin, let us find a formula to compute the relevant probability given the pot is empty and that there is no other information provided.

Relevant Variables The number of cards to distribute between $n-1$ players is $4 - p_{0\alpha_t}$. The number of cards player i places is k_t, and the true amount they hold is $p_{i\alpha_t}$.

In general, to split y items among z people, There are $\binom{y+z-1}{z-1}$ ways.

We must count the number of ways in which $4 - p_{0\alpha_t}$ may be distributed between $n-1$ players. This is $\binom{2-p_{0\alpha_t}+n}{n-2}$

We must count the number of ways in which players j such that $j \neq i$ can hold, which is $\binom{1-p_{0\alpha_t}-k_t+n}{n-3}$

We must count the number of ways in which player i can hold k_t cards, which is $\binom{n-1}{k_t}$

This means that in the most naive case, the probability of player i having exactly k_t cards is $\frac{\binom{n-1}{k_t}*\binom{1-p_{0\alpha_t}-k_t+n}{n-3}}{\binom{2-p_{0\alpha_t}+n}{n-2}}$

We must compute the probability from k_t up to 4, since a player may have more cards than what they play. $\sum_{k_t}^{4} \frac{\binom{n-1}{k_t}*\binom{1-p_{0\alpha_t}-k_t+n}{n-3}}{\binom{2-p_{0\alpha_t}+n}{n-2}}$

Chapter 3

Considering information about a player

Throughout the game, we constantly gain information about each player. The first case in which you gain information is as follows. Consider a turn where player i puts down k_t cards of type α_t, and player j calls bluff. If player j must take the pot, they definitely are holding k_t cards of type α_t. Their player count can be updated such that $p_{j\alpha_t}$ becomes $(p + k_t)_{j\alpha_t}$. Let this turn be denoted as $t*$

We need to use this information to update our previous formula.

Instead of $p_{0\alpha_t}$, we must use $\sum\limits_{i=0}^{n} p_{i\alpha_{t*}}$

Remember, we must distribute the remaining cards in the same manner, because the player may have been holding some cards of type α_t before the turn.

$$\sum_{k_t}^{4} \frac{\binom{n-1}{k_t} * \binom{1-\sum_{i=0}^{n} p_{i\alpha_{t*}} - k_t + n}{n-3}}{\binom{2-\sum_{i=0}^{n} p_{i\alpha_{t*}} + n}{n-2}}$$

Of course, this ignores if the player making the move is also player j, since they definitely have those cards. To fix this, we must add back k_{t*} back.