Bluff

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**Abstract**

# 1 Introduction

# 2 Rules

## 2.1 Setup

Let there be $n$ players, $3 \leq n \leq 7$.

**Objective** Each player $(i)$'s objective is to dispose of all of their cards.

Let there be a shared table into which players may discard cards.

Let there be a global counter $\alpha$, defined as $\alpha_0 = 1$
$\alpha_t = (\alpha_{t-1} \bmod 13) + 1$.

The game uses a standard deck of playing cards without jokers.

A deck consists of 52 cards consisting of 4 suites, with 13 cards in each suite numbered from 1 to 13.

Initially, the cards are divided evenly between all players s.t. each player has $\lfloor \frac{52}{n} \rfloor$. The excess 52 mod $n$ cards are placed on the table

# 3 Defining a Player

Let the true distribution of cards for card $\alpha$ held be $T(\alpha)$.

Let the known distribution of cards for card $\alpha$ held be $K(\alpha)$

Assume there are 4 cards of each number: $\forall_{a \in [1,13]} \sum T(a) = 4$

## 4 Defining the table

Let the table be represented as player $i = 0$, and the remaining players be $i = 1, 2, ..., n$

## 5 Defining a Turn

Play is conducted counter-clockwise, and no player may be skipped.

In a turn, consider the distinct players $i$ and $j$.

Let a turn $(t)$ consist of the following sequence:

1. Player $i$ discards $p_t$ cards. $p_t \in [1, 4]$
2. Player $j$ may "call BS"
   (a) $\forall_{i \in p_t} type(i) = \alpha_t \iff j$ must take the table
   (b) $\exists_{i \in k_t} type(i) \neq \alpha_t \iff i$ must take the table

   When $i$ takes the table, $\forall_{a \in [1,n]}$
   $T(a)_i = T(a)_i + T(a)_0; T(a)_0 = 0$

## 6 The Problem

We must find $P(T(\alpha_t)_i \geq p_t)$

## 7 Initial Game State

**Equation 1** The number of solutions to the equation $\sum a_r = b, a_r \geq r = \binom{b - \sum r + |a| - 1}{b}$ Thus
$\sum K(\alpha_t) + \sum T(\alpha_t) = 4$
$N(S) = \binom{4 - \sum S + |S| - 1}{|S| - 1}$

Let $K(\alpha_t)' = \{K(\alpha_t)_0, K_1, ..., K(\alpha_t)_{i-1}, p_t, K(\alpha_t)_{i+1}, ..., K(\alpha_t)_{n-1}, K(\alpha_t)_n\}$
$P(T(\alpha_t)_i \geq p_t) = \frac{f(K')}{f(K)}$

## 8 Conclusion