Math Stories for Children of All Ages

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Introduction

Why Write this Book

Mathematics is an essential aspect of human life. There is a sterotype of people disliking and struggling with mathematics, which I believe is simply because we are, at a young age, often told to try to compute, rather than being challenged with fundamental questions of how to understand the world around us.

How to use this book

Learning and understanding mathematics takes more than reading this book purely for its story, but wrestling with new ideas and questions, and attempting to resolve those questions through reasoning. To this end, after reading the story on each page, have students attempt to develop an answer using their intuition and prior knowledge to build toward the correct answer. Only after they have worked towards the right answer without hints and engaged in discussion, read the solution.

Even beyond the scope of the book, encourage them to ask questions about the world around them. Reason through math in games they know and love, in stories, and in everyday life and present unintuitive problems which they can use the tools from this book and otherwise to solve in genuine scenarios.

Additional Resources

- MathForLove
- `https: //visualize-it.github.io/trig_functions/simulation.html`

Contents

All was quiet in the goose village. It was a sort of place where parent geese could look out and feel their goslings were safe. But one fall evening, there was a sense of despair in the village. All the geese knew that migration was coming, and that they would have to embark on a dangerous journey. Every year, the buildings got taller, there were more and more people along the way, and there were more and more predators.

"Enough is enough." Duckie thought. But what could he do? Tho goose authorities, the leaders of the village, had controlled migrations for as long as anyone in the village could remember. They had kept everyone safe, but every year it had been getting harder.

There were of course the stories of the fabled "golden goose" of legend who lived on the highest mountain, who would save all goose kind, but as far as anyone knew, those were only myth.

"No. I have no choice."

Duckie decided then and there, that he would try to find the golden goose, and would help find the great change to society the hero was meant to bring. Hopefully then, he could save all the geese. He would have to break the goose authority's rules and search for the hero.

Chapter 1

Arithmetic

That night, Duckie decided to leave the village. He knew he would have to do so secretly, because the authorities did not approve of anyone leaving the village, must less searching for the golden goose. After all, they had tried so hard to make people forget about the golden goose altogether. Duckie waited until everyone had gone to sleep, and started on his journey

He began to sneak towards the village border, where he saw two guards snoring loudly. He tiptoed past, and all seemed quiet, but just as he turned his back to the village, he heard alarms ringing.

Honk! Honk! Honk!

They droned. Duckie turned around and saw the two guards he had walked past coming towards him. The goose authorities had been alerted of his mission, and he would have to run. He began flying, and as he looked back, saw the goose police on his tail.

1.1   Addition

On the first night of his journey, Duckie flew **A** km from the village towards the mountain to try to escape the goose police. Unsure of how far the goose police would chase him, he decided to take a quick break to gain his strength, but in a flash, he saw the police on the horizon and began to fly again. From there, he flew another **B** km. Duckie wants to know how much he has traveled to know how far he is from the goose authorities. Where is Duckie?

He first flew **A** km, then changes this amount by **B**. This is $\begin{array}{r} 0 \\ + \ 0 \\ \hline 0 \end{array}$ km
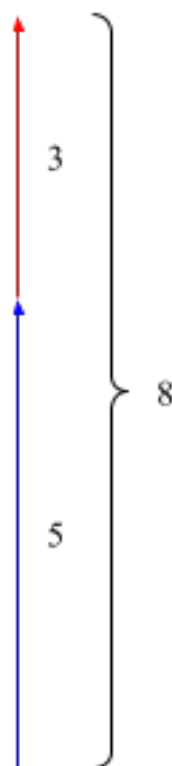
Definition   Addition is the most basic operation. It represents a change by a specific amount. Imagine an arrow in one direction, then putting another arrow on the end of the arrow. Addition lets us know where the end of the second arrow now is.
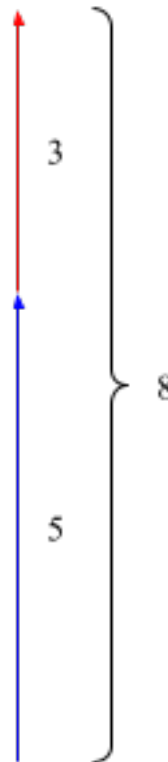
1.2 Multiplication

Duckie did this routine on the first day, second, day, and third day. Duckie wants to know how far he is from the goose village. Where is Duckie?

Duckie flew **C** km, **3** times. This

$$
\begin{array}{r}
3 \\
\times\ 0 \\
\hline
\text{is}\quad 0
\end{array}
$$

km

Definition   Multiplication is the second basic operation. It represents stretching or squishing another number by a certain amount. Imagine a rubber band. Multiplying its length by P is the same as stretching the band so it is P long for every 1 length of the band.
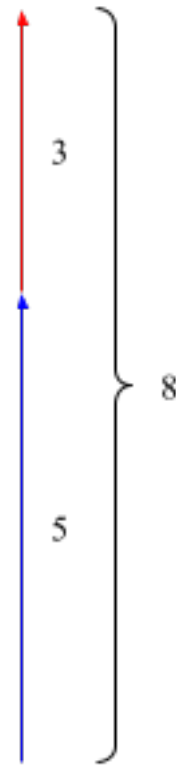
1.3   Negative Numbers

On the morning of the 4th day, after Duckie had traveled **P1** kms, he decided he had to take evasive maneuvers to try to confuse the goose police. In order to do this, he decided he should do the one thing they would not expect: go towards the village. He traveled backwards **C** km. This is called –**C** km. – means opposite direction. Duckie wants to know how far he is from the village. Where is Duckie?

He first flew **P1** km, then goes **-C** km. This is $\dfrac{-\dfrac{8}{1}}{7}$ km

Definition   A number less than 0 is called "negative", and is in the opposite direction. If it added to another number is 0, such as 5 and -5, then they are called "additive inverses", becuase when added together their changes cancel each other out.

1.4 Multiples of a negative
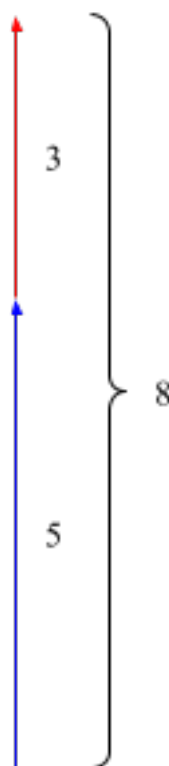
While Duckie was travelling in the negative direction, he took a break every **-D** km. He flew that distance $\frac{C}{D}$ times. How far does Duckie travel while taking evasive maneuvers?

Duckie traveled C km D times.

This is $\dfrac{\times \dfrac{1}{3}}{3}$ km

Definition Multiplying by a negative number shows stretching or squishing another number in the opposite direction

1.5   Negative of a Negative

On the fifth day, Duckie thought he had confused the goose police, and that they were no longer following him. To get to the golden goose, he knew he would have to travel away from the village, and so turned around again and flew for E km. Duckie traveled in the opposite direction of –E, or -(-E). Duckie traveled E km forward. Duckie wants to know how far he is from the goose village. Where is Duckie?

He was at **P3** and then traveled

$$+\begin{array}{r} 0 \\ 0 \\ \hline 0 \end{array}$$

−**E**. This is   0 km.
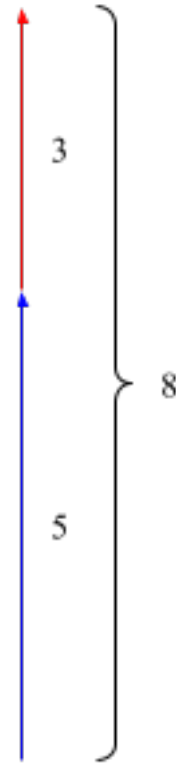
Definition   A number in the opposite direction of the opposite direction of a number is in the same direction of that number. It is written as -(-number), which is the same as that number.

1.5.1   Multiplication Table

	0	1	2	3	4	5	6	7	8	9	10
0	**0**	0	0	0	0	0	0	0	0	0	0
1	0	**1**	2	3	4	5	6	7	8	9	10
2	0	2	**4**	6	8	10	12	14	16	18	20
3	0	3	6	**9**	12	15	18	21	24	27	30
4	0	4	8	12	**16**	20	24	28	32	36	40
5	0	5	10	15	20	**25**	30	35	40	45	50
6	0	6	12	18	24	30	**36**	42	48	54	60
7	0	7	14	21	28	35	42	**49**	56	63	70
8	0	8	16	24	32	40	48	56	**64**	72	80
9	0	9	18	27	36	45	54	63	72	**81**	90
10	0	10	20	30	40	50	60	70	80	90	**100**

Observations about the Multiplication Table   :

0   Any number multiplied by 0 is 0. This makes sense, because any number repeated 0 times is the same as not having it at all.

1   Any number multiplied by 1 is itself. This makes sense, because any number repeated one time is itself. This has a special name: Identity

10   Any number multiplied by 10 is shifted to the left by one digit.

Symmetry   The table is identical across the bolded diagonal. This shows that 4×2 is the same as 2×4. This property also has a special name : *Commutativity*

Chapter 2

Properties

After 5 days of running form the goose police, Duckie was certain that he had escaped. He decided to stop for dinner, but just as he took off, **Squak!** He ran headfirst into another goose.

"Hold it right there! You need to come back with me to the village!" the policegoose demanded, slightly dazed.

"It's dangerous in the outside world. This is for your own safety!"

Duckie knew that the policegoose, named Güs, was a fair goose, and so considered this for a moment. "Of course its dangerous! But I have to do this. I am looking for the golden goose of legend. The one who is supposed to save us, change all of goose society and help us through our migrations."

"Oh. Hmm, that makes sense. But why are the goose authorities trying to stop you? Isn't your cause noble?" Güs asked.

"They are afraid. If I can find the golden goose, then they may not be able to stay in power. You joined the goose police to help people, and if you let me go, you can do just that!"

"Your cause is noble, and I would like to help, but if I just let you go without at least trying to stop you, I will get in trouble! Ill tell you what. I will have a series of challenges with you. If you beat me in all the challenges, I will let you go. If not, you have to let me take you back. Sound fair?"

Duckie was reluctant, but knew that he had no other options. "Fine. Let's do the challenges."

2.1   Associativity

Güs and Duckie both thought they were good flyers, so they decided to compete in a three-part flying distance competition. The first leg of the race was F km, the second leg was G km, and the third leg was H km. The distance flying competition rules stated that any goose would be allowed to take one break in between laps. They both want to win, so create strategies to try to win. Güs decided to travel the first leg (F km) of the race, takes his break then travel the second and third legs (G km and H km). Duckie instead traveled the first leg, (F km) of the race and the second leg (G m) of the race, then took his break, and traveled the third leg (H km). Did they tie?
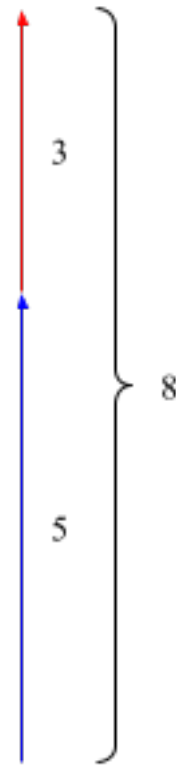
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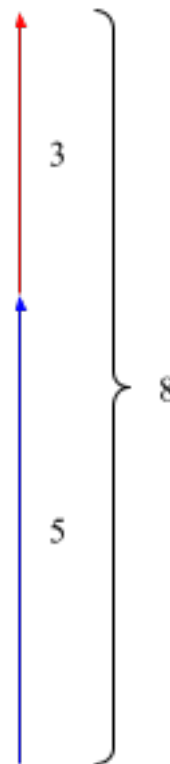
Yes! Duckie and Güs traveled the same amount

Definition   When adding, computation can be grouped in any way. In other words: (A+B)+C = A+(B+C)

2.2   Additive Commutativity

Duckie and Güs decided that to break the tie, they needed do another competition.
Because geese are water animals, long-distance swimming is a very important skill, so they decided that that should be their next competition. They each have a different strategy. Güs swam I m, dried off, then swam J m. Duckie instead swam J m, dried off, then swam I m. Did they tie?
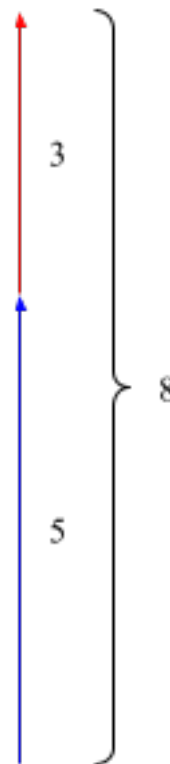
Yes! Duckie and Güs swam the same amount

Definition   The order of adding does not matter. In other words:
A+B = B+A

2.3 Distributivity

Duckie and Güs were starting to get frustrated. No matter what happened, the seemed to tie every time! Duckie had thought that if he competed long enough, Güs would get tired and he would be able to win, so challenged him to another flying contest. In the third contest, Güs traveled K m, then L m, and repeated that pattern M times. Duckie instead traveled K m M times, then L m M times. Did they tie?

Yes! Duckie and Güs flew the same amount

Definition   Multiplying by a full expression is the same as multiplying by each part of the expression. In other words:
A*(B+C) = (A*B) + (A*C)

2.4   Multiplicative Commutativity

At this point, Güs realized that Duckie was a very capable goose. When he first challenged Duckie, he expected winning to be a cakewalk, but his opponent ended up posing quite a challenge. Being an honorable goose, this made him respect Duckie more. He decided that the next contest should be a weightlifting competition, where whoever lifted the most amount of total weight would win. Duckie lifted M kg N times, whereas Güs lifted N kg but does it M times. Did they tie?

Yes! Duckie and Güs lifted the same amount
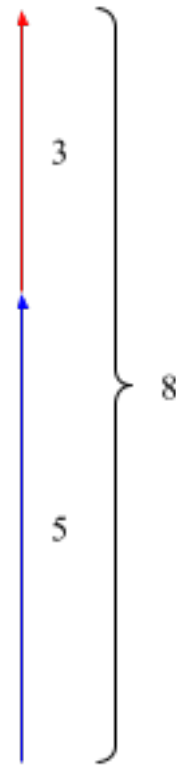
Definition   The order of multiplication does not affect the result. In other words: A * B = B * A

After the competition, Duckie and Güs sat down, exhausted. They had both realized that, no matter what competition they had, they were equally matched. Neither one would be able to beat the other, and they had won each other's respect.

"Why don't you join me?" asked Duckie.

"I can't!" Güs cried. "What about my life in the goose police?"

"Well, do you believe the golden goose is out there?"

"Honestly, I'm not sure, but if the stories are true, it could change the world"

Duckie agreed. He knew that there was the possibility the golden goose was a hoax, but believed the positive implications outweighed the dangers of disobeying the goose authorities.

"You joined the goose police to help people", he reasoned. "You are clearly strong and capable of making the dangerous journey, and if you can join me, you could do exactly that"

Güs knew he was right. While he was afraid of what could happen along the journey, he decided to be brave and join Duckie.

ou"All right. I will join you!", he said resolutely.

Chapter 3

Essential Algebra

Meanwhile, Duckie and Güs's various competitions have made the goose authorities a bit confused. Because of what their "she-goose-igans", they have lost track of where Duckie is. They decide that they need to review their records of Duckie and Güs's locations, and try to figure out what they were doing on the 5th day.

3.1   Equality

Because Duckie and Güs are travelling together, are always at the same position at any time.

Güs's position = Duckie's position. Duckie's position is P4, so Güs's position is also P4

Definition   When two things are the same, we write this in mathematics as "="
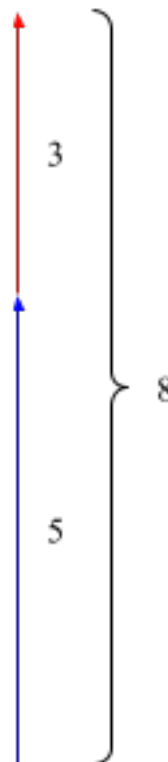
3.2   The Equality Rule

The goose authorities, having trained Güs in the police academy, know he is a very fast flyer. In fact, he won the village racing competition for the past three years! Because Duckie and Güs are travelling together now, they realize that every time Güs flies ahead of Duckie, Duckie has to increase his position by the same amount to keep up.

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Duckie's position $+ \Delta$x = Güs's position $+ \Delta$x
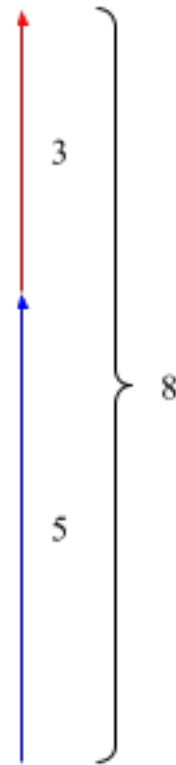
Definition   What happens to one side of an equation must happen to the other side so that they are still equal.

3.3 Variables

The goose authorities are still a bit confused about Duckie and Güs's progress. They know that there were 4 competitions, and that they started at P4. They also know that they ended at P5. How much did they travel in each competition?
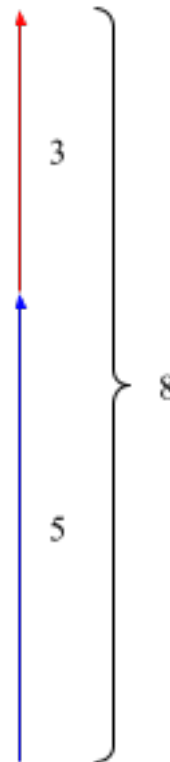
$$4x + P4 = P5$$
$$4x + P4 - P4 = P5 - P4$$
$$\frac{4x}{4} = \frac{P5-P4}{4}$$
$$x = \frac{P5-P4}{4}$$

Definition Letters and Symbols can be used to represent a number which isn't already known, such as X. $2x + 3 = 6$ means that 2*(some number) + 3 = 6.

Chapter 4

Ratios and Fractions

It took alot of work, but the authorities finally figured out where Duckie is. They notice that they have made quite a bit of progress, which is very bad news for them. They want to use this information to try to stop them. Because of Güs defecting to join Duckie's journey, they realize that sending other policegeese may weaken their power in the village.

Because of their past migrations, they know of lots of bounty hunters along the way, and they believe that they can have them set traps. Bounty hunters are famously clever, and decide to use specific strategies to help them catch Duckie and Güs.

4.1   Ratios

The first bounty hunter decides to look at the entire journey, and set up traps every P4 km. He estimates that the entire journey is R km long. Duckie hears of this scheme, and thinks that if he knows how many traps there will be, he can avoid them. For every km in the journey, there will be a trap every P4 km

For every P4 km Duckie has to travel out of (R) km overall, there will be a trap. That means that there is a trap every $\frac{R}{P4}$ km of the journey. For every one km in P4, there are S km in the journey (R), which means that there are S lengths of size P4 within R. There are $\frac{S}{1}$, or S traps
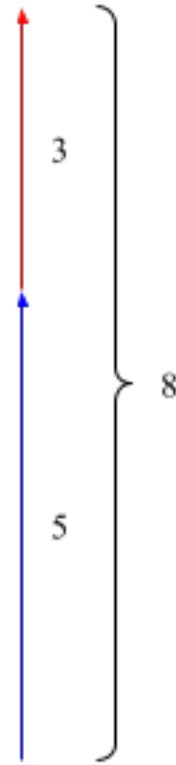
Definition   A ratio is a certain amount for each of another amount, and is written as $\frac{A}{B}$. Break up A into B equal parts

4.2 Multiplying with Fractions

Using this knowledge, Duckie figures out how to avoid the first bounty hunter's traps. The second bounty hunter decides to prepare better than the first hunter. He realizes that km are a very large unit for geese, so decide to start measuring the distance Duckie and Güs have flown in a unit they create called goosemeters. This means that he need to change their position in kilometers into goosemeters. There are U gm for every V m. To help beat the second bounty hunter, Duckie and Güs decide to also measure using gms. How many gms are in P4 km?

$$\frac{U\,gm}{V\,m} * \frac{R\,m}{1} * \frac{1}{S} = \frac{UR}{VS}gm$$

Definition   Like any number, multiplying by a ratio represents scaling by a certain amount. Imagine a rubber band which is P long, which you can stretch and squish. Multiplication represents scaling the band such that $P * \frac{A}{B}$ is the new length of the band. This is the same as $\frac{P*A}{B}$, or $\frac{P}{B} * A$. This can be seen as dividing the band into B equal parts, then taking one of those parts A times.

Notice that that $\frac{A}{B}$ can create numbers in between 0, 1, 2, etc.

4.3   Comparing Fractions

Duckie thinks that if he travels enough in one day, the bounty hunter will become confused and leave them alone. On the eighth day, Duckie and Güs travel an additional W/X of the journey. If Duckie travels more on the eighth day than the combined previous 7 days, the bounty hunter will become confused and stop chasing Duckie. Has Duckie managed to confuse the bounty hunter?
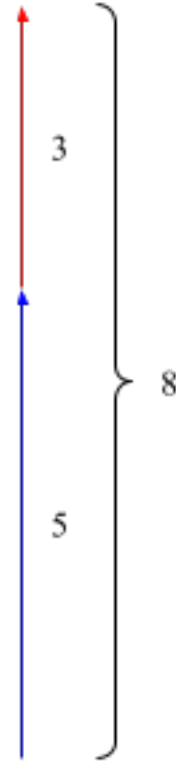
Duckie realizes that

$$\frac{X}{X} = 1$$
$$\frac{UR}{VS} * 1 = \frac{UR}{VS}$$
$$\frac{UR}{VS} * \frac{X}{X} = \frac{UR}{VS} = \frac{URX}{VSX}$$

Which means $\frac{UR}{VS}$ is same as $\frac{URX}{VSX}$

$$similarly \ \frac{W}{X} = \frac{WVS}{VSX}$$

Now, we can compare the equal size parts together. $\frac{WVS}{VSX} < \frac{URX}{VSX}$

Definition   We can only deal with fractions together using same-size parts. We can chop up one fraction by multiplying it by another fraction equal to one. If the *denominator*s (the bottom number of the fractions) are the same, the fractions have same sized parts
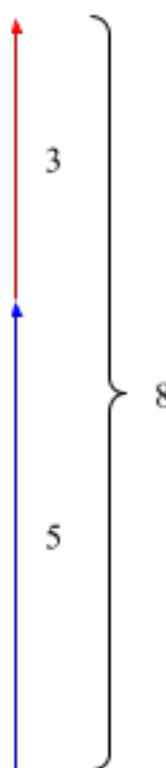
4.4 Adding Fractions

Despite Duckie's best efforts, he is unfortunately not able to confuse the bounty hunter. He is starting to get worried, but Güs has a genius idea. If they can find out where they are along the journey, they may be able find a new route, which will take them away from the hunter's grasp! They have traveled $\frac{WVS}{VSX}$, then they traveled $\frac{URX}{VSX}$. How much of the journey have they traveled?

Duckie and Güs have traveled
$$\frac{WVS}{VSX} + \frac{URX}{VSX}$$
$$= \frac{WVS+URX}{VSX}$$
$$= \frac{WVS+URX}{VSX} \text{ths of the journey.}$$

Definition When adding fractions, break up each fraction so that they are in terms of same-size parts (or having a common denominator). Then, add the numerators as usual.

Duckie and Güs sighed in relief. After running for bounty hunters for two days and two nights, they were exhausted. They decided to travel slower for a few days and enjoy the scenery.

"I can't believe how persistent the goose authorities are." Duckie sighed. "I knew this journey would be tough, but I wasn't expecting bounty hunters!"

Güs considered this. He had worked with the goose authorities, and knew their patterns. He realized how unusual this was. "I agree, the authorities must really be desparate".

Just as Duckie opened his beak to reply, he felt a sudden weight on his shoulder's, and began spiraling out of control. It was a net! Another bounty hunter must have set a trap! He started falling down fast, gaining speed as he went. **Crash!**. Duckie fell to the ground with a thud. Güs hurriedly landed next to him.

"Are you alright Duckie?" he cried in alarm.

"I'm all right, but I think I need to rest my wing for a few days", Duckie winced. He felt that he hadn't broken a wing, but saw some bruising, and knew he would need time to heal.

"The forest is dangerous though! We can't stay here in the open with all the predators!"

"I think my friend Bessie the cow's farm is close enough to walk to from here," Duckie said. "I think she will be able to help keep us safe from the goose authorities."

Together, they waddled to Bessie's farm.

Chapter 5

Rectangles, Perimeter, and Area

After walking all afternoon, Duckie and Güs finally arrived at Bessie's farm. Despite the bad luck with the net, it was a beautiful afternoon, and the pair couldn't help but feel a sense of calm as the walked up to Bessie's barn.

Bessie was sitting outside, chewing on some grass

"Hey there Duckie! Hello Güs! How's it going?" She moo'd

"Hi Bessie", Duckie greeted. "I'm looking for the golden goose of legend, and I need some help"

"Of course! What do you need?" asked Bessie.

"I hurt my wing, and I need to rest here for a few days. The goose authorities sent some bounty hunters after me, and I need you to keep me safe from them", Duckie asked hesitantly.

"Wow, that sounds serious. Feel free to stay as long as you like" Bessie turned around to look at Güs. "While Duckie's is getting better, I could use some help on the farm. It's grass-planting season, and it would be much appreciated if you could hlep me out on the farm"

Güs, enthusiastically agreed, "That sounds great!"

5.1   Area of Rectangles

On the 9th day of the journey, Güs began working on the farm. His first task was to fly around and scatter grass seeds on the farm. As he went into the store room to find the bags of seeds, he realized he needed to know how many bags of seeds he needs to bring with him. Bessie told him that each bag of seeds can cover a square of grass that is 1 m wide and 1 m across. Güs also knows that the farm is Y m wide and Z m across. How many bags of seeds does Güs need to carry?

There are Y columns of grass, where each column is made of Z 1x1 squares. This means that there are Y*Z = Y*Z meters squared of grass, and that Güs needs to carry Y*Z bags

Definition   Area is how much space an object takes up. When measuring a 2d shape, we find how many 1x1 squares the object fills. When measuring the area of a 3d shape, we find how many 1x1x1 squares the object fills. Multiplying the length of each side together for "square" shapes gives us the area. (We will discuss area further with integrals)
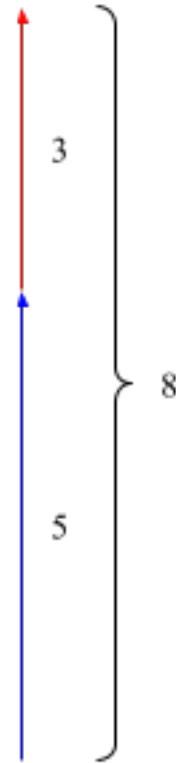
5.2   Area of Square Edge Triangles

As Güs takes off, he notices that the farm is split into two equal parts along the diagonal to make space for Bessie's pet human Farmer John. He realizes that here, he cannot plant grass, as Farmer John needs it for corn. How many bags of grass should Güs plant?

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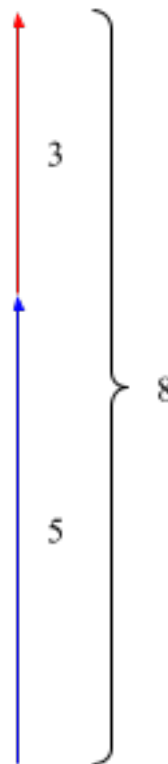
The farm is divided in half, so Güs only needs to plant half of his bags. $\frac{1}{2} * Y * Z = \frac{1}{2} * Y * Z$

Definition   Right triangles (triangles with square edges), are rectangles that have been split in half along the diagonal. This means that the area of a right triangle with side lengths x and y has an area of $\frac{X*Y}{2}$.

5.3 Perimeter of a Shape

After Güs finished planting grass, he decided to take a break for lunch.

"Aaaah", he sighed, contented. As he looked out at the farm, he noticed a discontented look on Bessie's face.

"What's the issue?" he asked.

"It's this fence," Bessie replied. "It's falling apart and we have to look at it all day." Gus decided that to thank her for letting them stay for the night, he would buy her a new fence. How many meters of fence does Güs need to buy?

Two sides of the fence are Y m long, and two sides of the fence are Z m long. That means that there are Y m + Y m + Z m + Z m = 2*Y m+ 2*Z = 2(Ym+Zm) = 2*Z = 2(Y+Z) m of fence

Definition Surface is the size of the edge of an object. When measuring a 2d shape, we find how many 1 long lines fit around the object. When measuring a 3d shape, we find how many 1x1 squares fit around the object. (We will discuss this further with integrals) The perimeter is the sum of all of the side lengths. Because 2 of the side length of the sides of a rectangle are always the same, 2x+2y is its perimeter, which can also be written as 2(x+y)
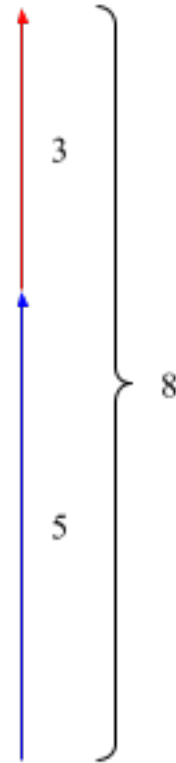
5.4 Shoelace Theorem

The shoelace formula is a tool which lets us find the area of any polygon. Go around the vertex's of the polygon, where the current vertex's coordinates are x_i, y_i and the next vertex's coordinates are x_{i+1}, y_{i+1}

$A := \frac{1}{2} \sum_{cyc} (x_i y_{i+1} - x_{i+1} y_i)$