

# MFEM CODE FOR STREAMERS\*

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**Abstract.** Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

**Key words.** Numerical modeling, finite element method, discontinuous Galerkin method, electric discharges.

**MSC codes.** 35K55

**2. Introduction.** Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

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**Algorithm 2.1** Pseudocode for the streamers code

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**INPUT:** The mesh, initial conditions (Gaussian seed for  $n_i$  and background value for  $n_e$ ), BCs for  $n_i, n_e$  and the voltage  $v$ .

**OUTPUT:** Electrostatic fields and the concentration of different species.

**procedure** STREAMER CODE

1. Define parameters used for the resolution and for mesh refinement.
2. Read the mesh and perform uniform refinement (if needed).
3. Choose time steppers for  $n_e$  and  $n_i$ .
4. Generate initial refined mesh using the initial value  $n_{i0}$ .
5. Define FS :  $H^1(\Omega_{\text{ele}} : \mathbb{R})$ ,  $H^1(\Omega_{n_i} : \mathbb{R})$ ,  $H^1(\Omega_{n_e} : \mathbb{R})$ ,  $\mathbf{H}(\text{curl}; \Omega_{\text{ele}} : \mathbb{R}^3)$  and  $L^2(\Omega_{\text{ele}} : \mathbb{R})$ .
6. Define BCs, the BilinearForm and the LinearForm **for the electrostatic problem**.
7. Define ICs for  $n_e$  and  $n_i$ , and instantiate *AdvectionDiffusionOperators* objects used for the resolution of ADR equations. For a given ADR PDE:

$$\partial_t n = \text{div} \left( \mu(E) \mathbf{E} n + D(E) \mathbf{grad} n \right) + \bar{\alpha}(E) n,$$

this object represents the RHS of the discretized ODE :

$$\frac{dn_h}{dt} = \mathbf{M}^{-1} \mathbf{f}(t, n_h),$$

9. Define the list of estimators, the **ThreshHoldRefiner** and the **ThreshHoldDerefiner**.
  10. Define DataCollections (ParaViewDataCollection and GmshDataCollection).
  11. Initialize OdeSolvers for  $n_e$  and  $n_i$ .
  12. **Time loop**.
  - for** ( $n \leftarrow 1$  To  $N_{\text{TS}}$ ) **do**  $\triangleright$  the time loop
    - 12.1. Set Time and  $\Delta\text{Time}$ .
    - 12.2. Refiner.Reset() and Derefiner.Reset().
    - 12.3. **Refinement loop**.
    - for** ( $l \leftarrow 1$  To  $N_{\text{Ref}}$ ) **do**  $\triangleright$  The refinement loop
      - 12.3.1. **Solve the electrostatic problem** (compute total electric charge, compute the BilinearForm and the LinearForm, Linear solve, RecoverSolution, saveSolutions).
      - 12.3.2. Apply the refiner  $\rightarrow$  compute errors. **If Refiner.Stop()**  $\rightarrow$  exit the refinement loop, **else**, update objects (FunctionSpace, BilinearForms LineaForms, Vectors, ...)
    - end for**
    - 12.4. **Derefinement**  $\rightarrow$  update objects.
    - 12.5. Solve the ADR equations for  $n_e$  and  $n_i$  (project  $\mathbf{grad} v$  onto  $\mathbf{H}(\text{curl}; \Omega_{\text{ele}} : \mathbb{R}^3)$ , update parameters used in the ADR for  $n_e \rightarrow$  solve for  $n_e$ , update parameters used in the ADR for  $n_i \rightarrow$  solve for  $n_i$ , compute  $n_i - n_e$  and  $\bar{\alpha}_{\mu_e}(E) E n_e$  and use them in the error estimators, Save solutions).
  - end for**
  - end procedure**
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