Gradient Boosting

CS109OA Introduction to Data Science

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Photo: Greg Mccutcheon
Chicago

Outline

- Introduction to Boosting
- **Gradient Boosting**
- Mathematical Formulation - Gradient Boosting

Recap: Boosting

"Can a set of weak learners create a single strong learner?"

Leslie Gabriel Valiant

How many jellybeans do you see?

Recap: Boosting

The key intuition behind boosting is that one can take an ensemble of simple models $\{T_h\}_{h \in H}$ and additively combine them into a single, more complex model. Here, T_h, is a weak learner, and H is the collection of all weak learners we allow the algorithm to use(hypothesis space).

Each model T_h might be a poor fit for the data, but a linear combination of the ensemble

$$T = \sum_{h \in H} \lambda_h T_h$$

can be expressive and flexible.

Question: But which models should we include in our ensemble? What should the coefficients or weights (λ_h) in that linear combination be?

Recap: Boosting

We give more weight to models that make fewer mistakes.

$$T = \sum_{h \in H} \lambda_h T_h$$

Let's break it down!

Recap: Boosting

We give more weight to models that make fewer mistakes.

$$T = \sum_{h \in H} \lambda_h T_h$$

T = Final model (the boosted model)

Recap: Boosting

We give more weight to models that make fewer mistakes.

$$T = \sum_{h \in H} \lambda_h T_h$$

λ_h = Weight (importance) we give to that weak learner

T_h = One weak learner

Gradient Boosting

There are two main ideas in gradient boosting:

- Gradient boosting is a method for iteratively building a complex model T by adding simple models.

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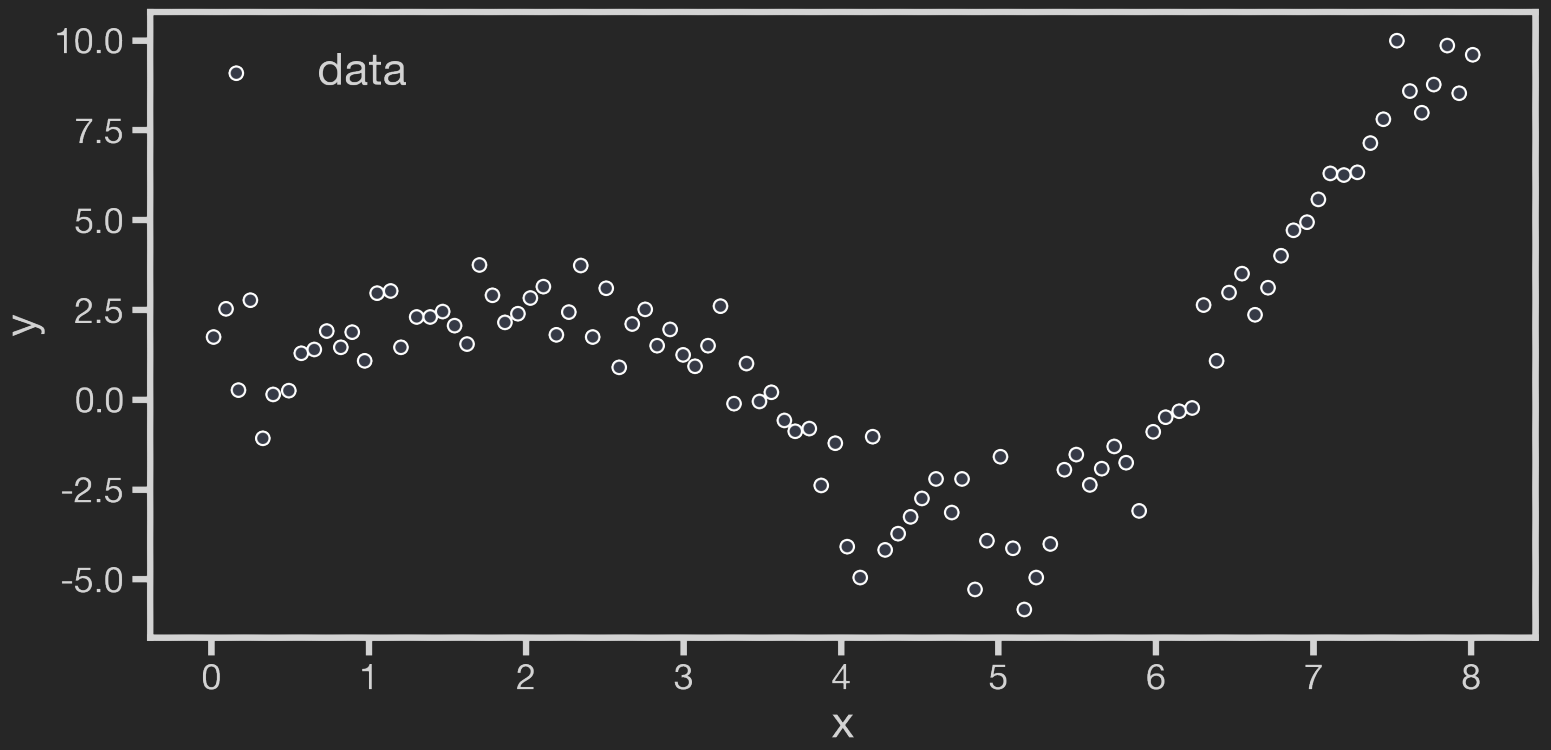
- Gradient boosting is a method for iteratively building a complex model T by adding simple models.

- Each new simple model added to the ensemble compensates for the weaknesses of the current ensemble.

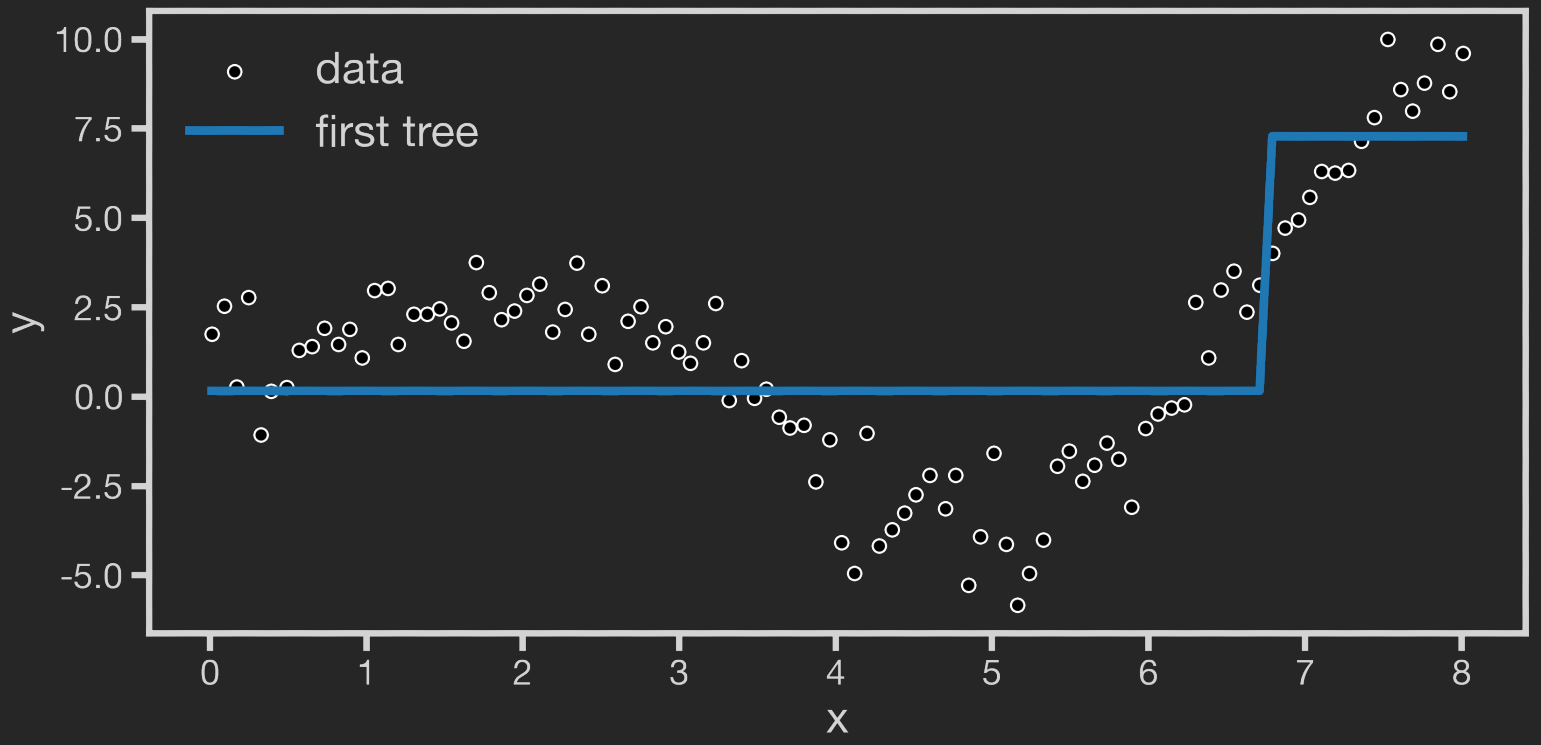
Gradient Boosting: Illustration

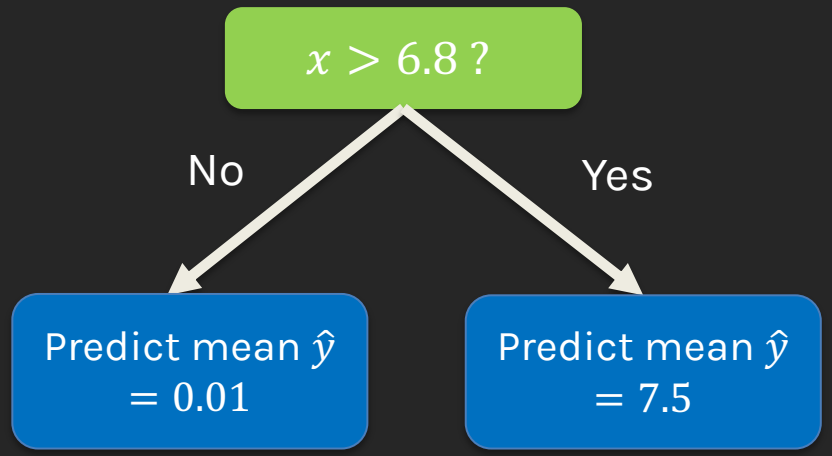
Consider the following dataset:



Gradient Boosting: Illustration

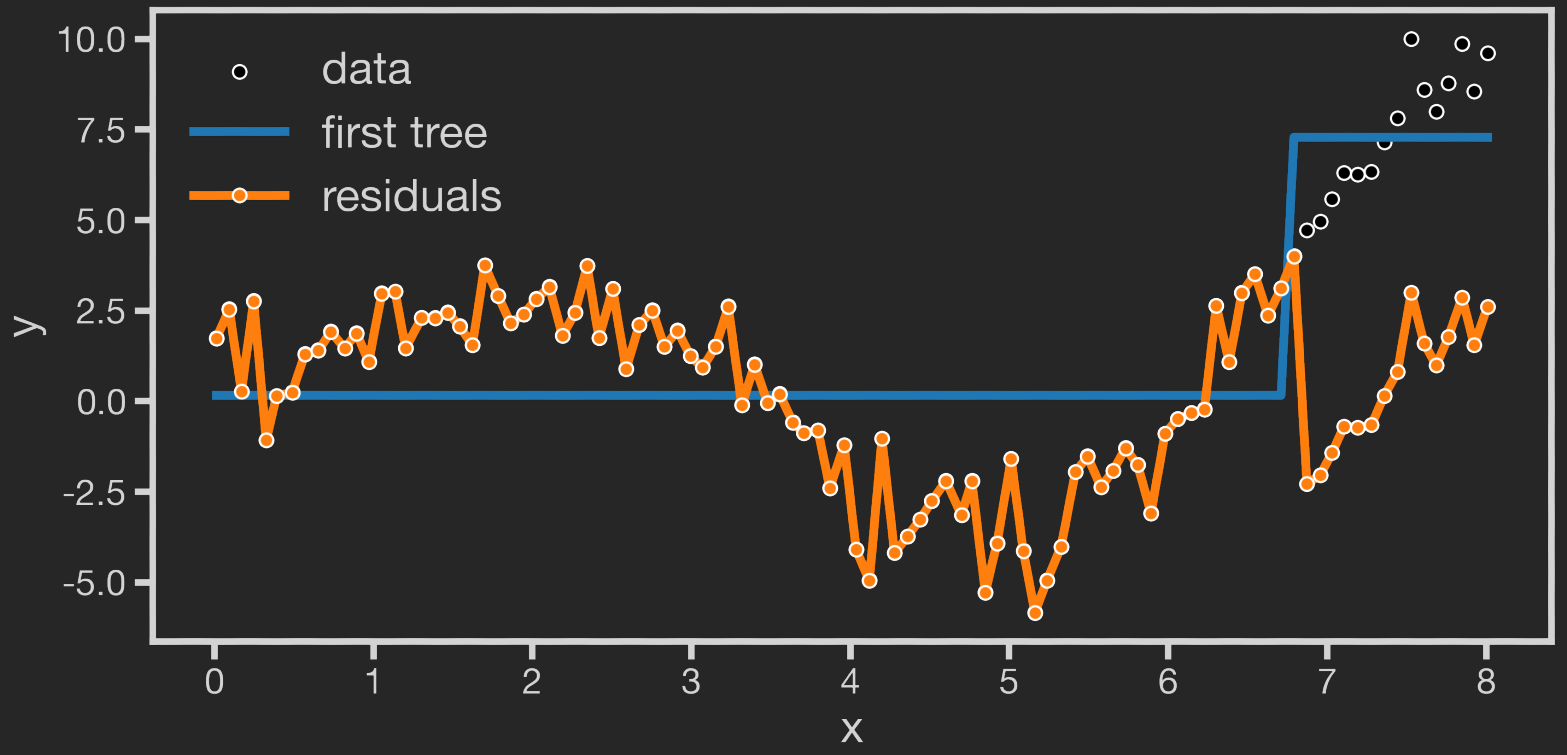
Step 1: Fit a simple model $T^{(0)}$ on the training data: $\{(x_1, y_1), \ldots, (x_N, y_N)\}$.

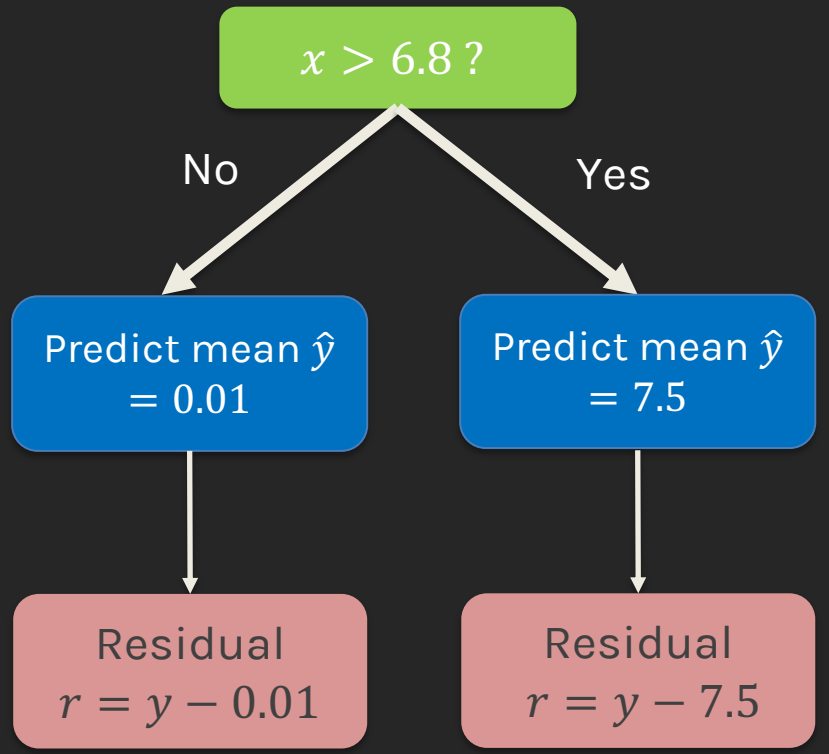




Gradient Boosting: Illustration

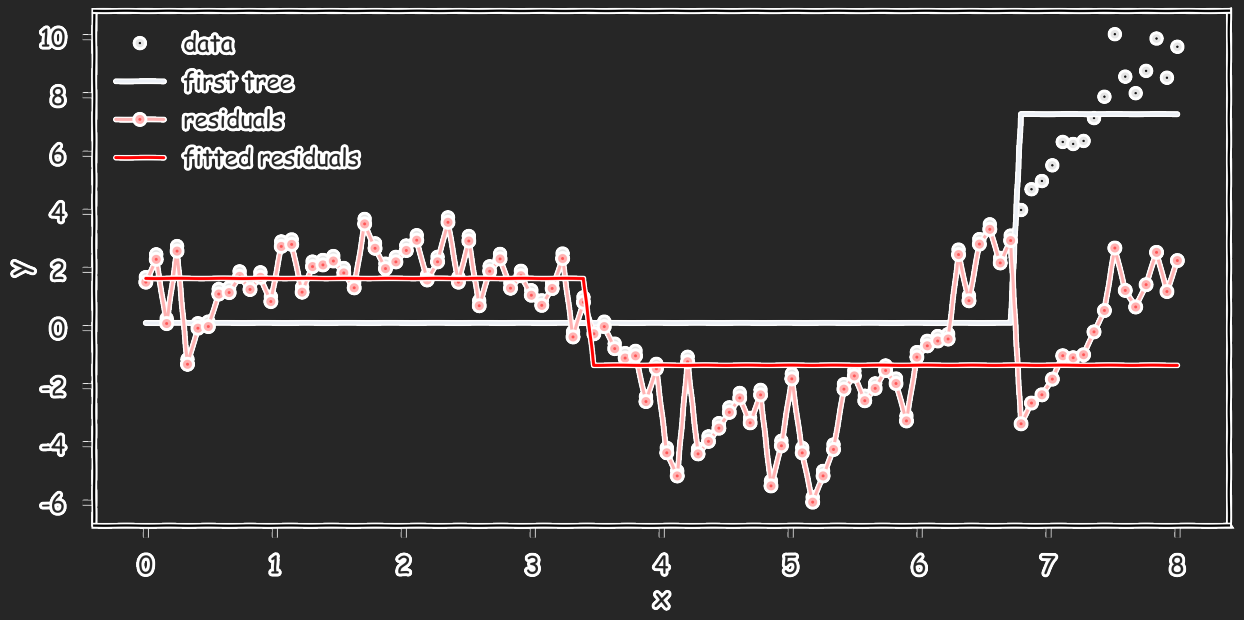
Step 2: Compute the residuals $\{r_1, \ldots, r_N\}$ for $T^{(0)}$. Set $T \leftarrow T^{(0)}$.

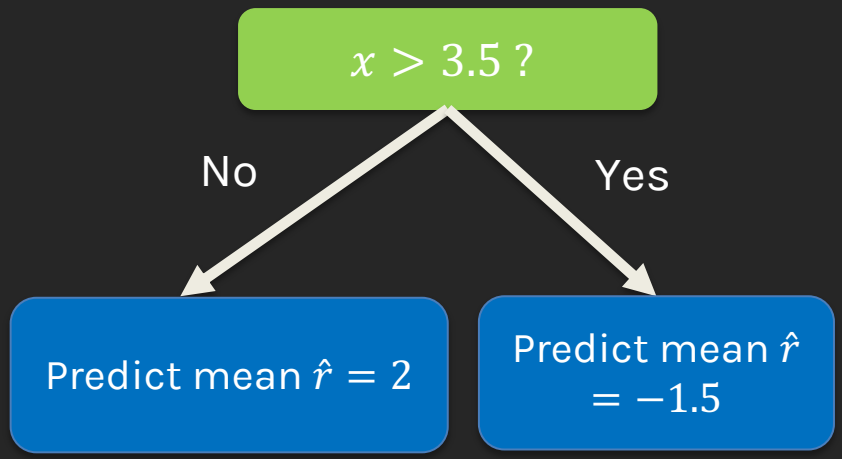




Gradient Boosting: Illustration

Step 3: Fit another model $T^{(1)}$ on: $\{(x_1, r_1), \dots, (x_N, r_N)\}$.

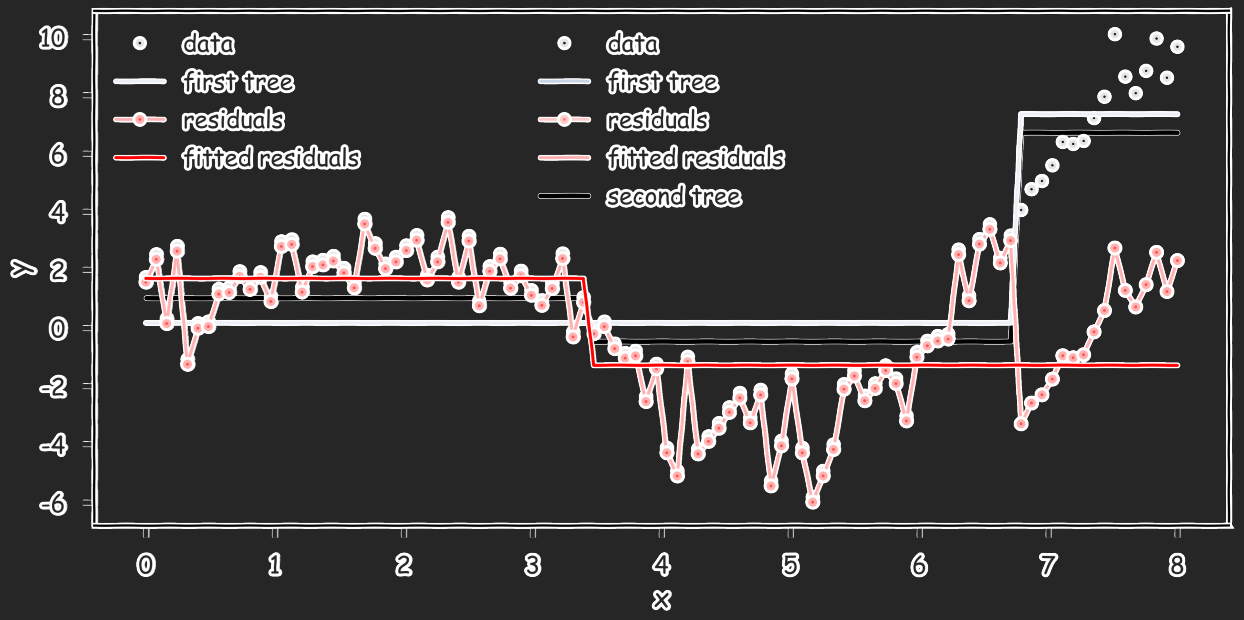


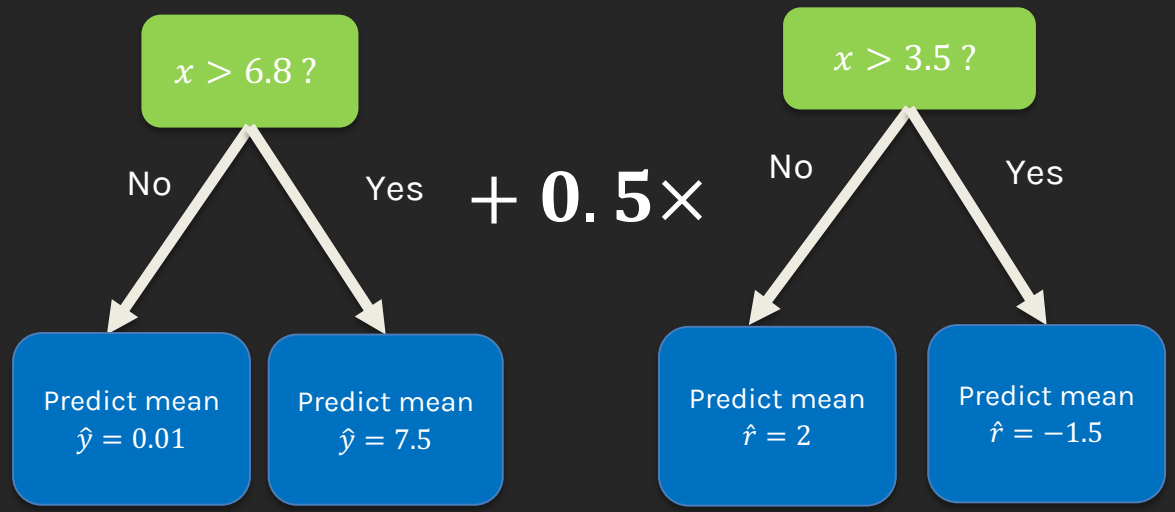


Gradient Boosting: Illustration

Step 4: Combine the two trees in step 1 and 3 by setting $T \leftarrow T + \lambda T^{(1)}$.

Assume $\lambda = 0.5$.





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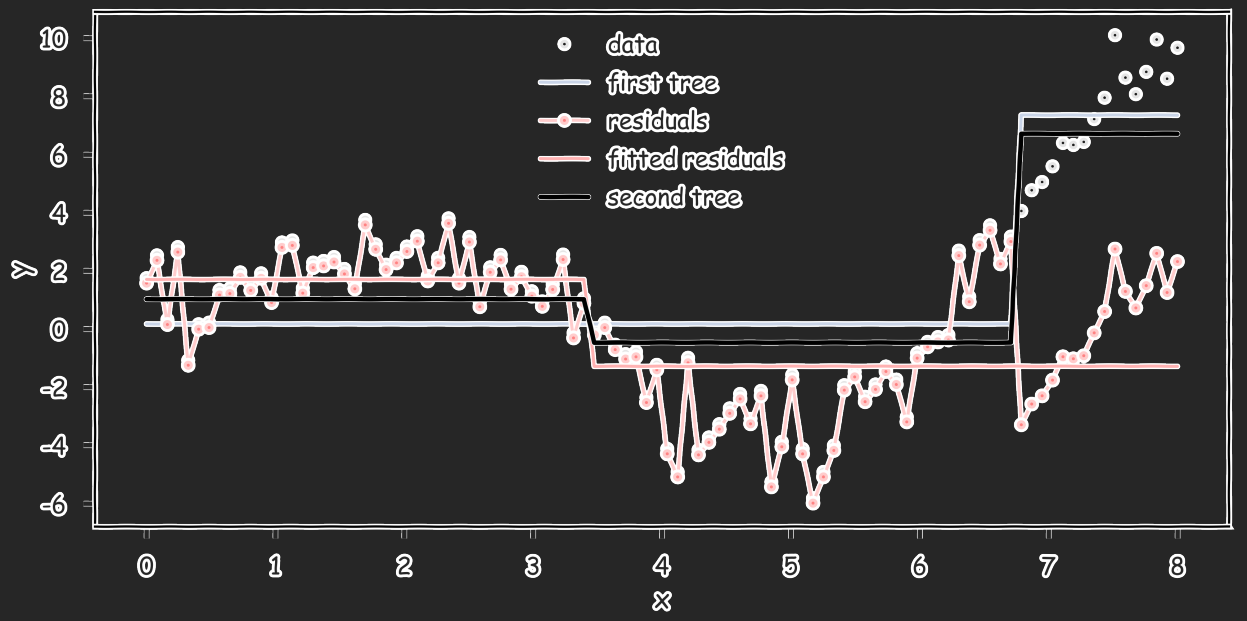
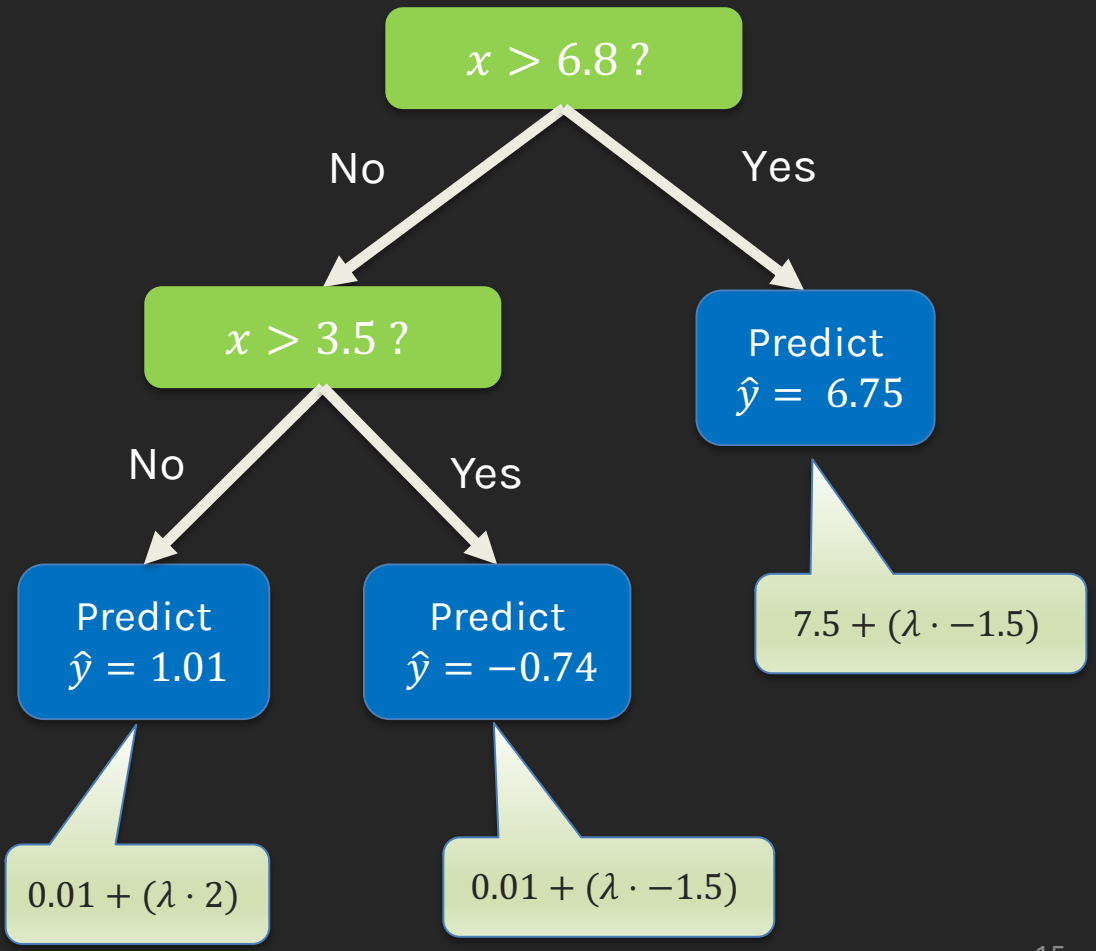
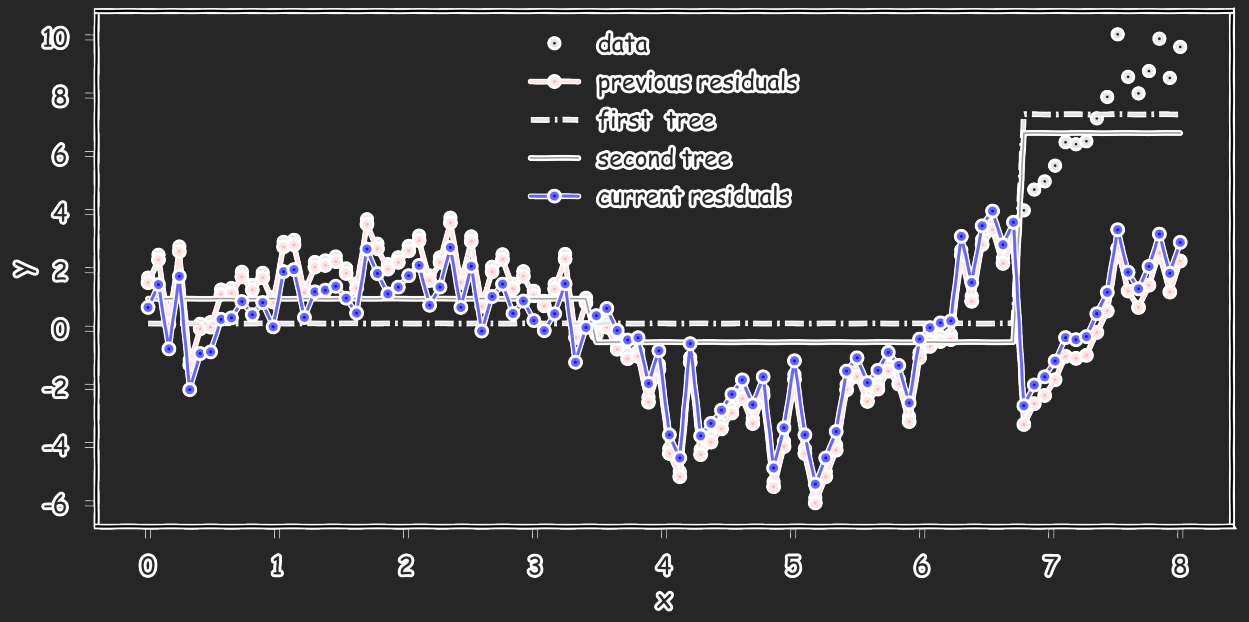
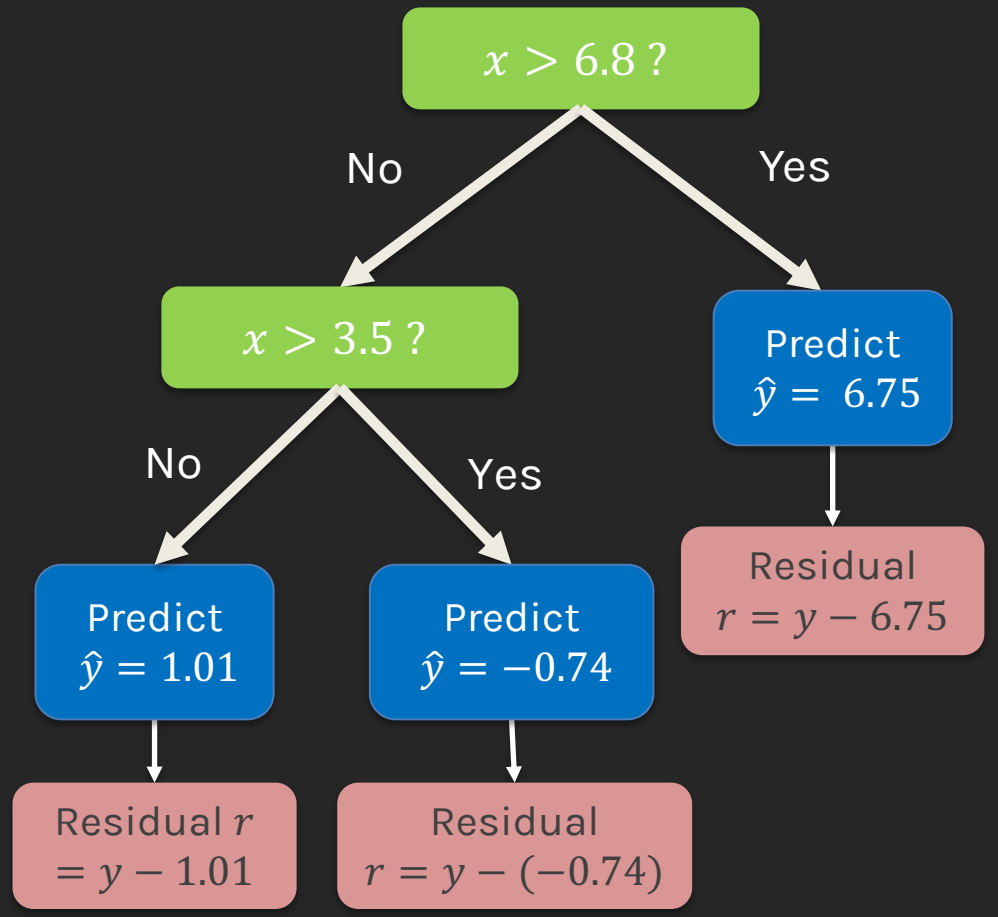




Gradient Boosting: Illustration

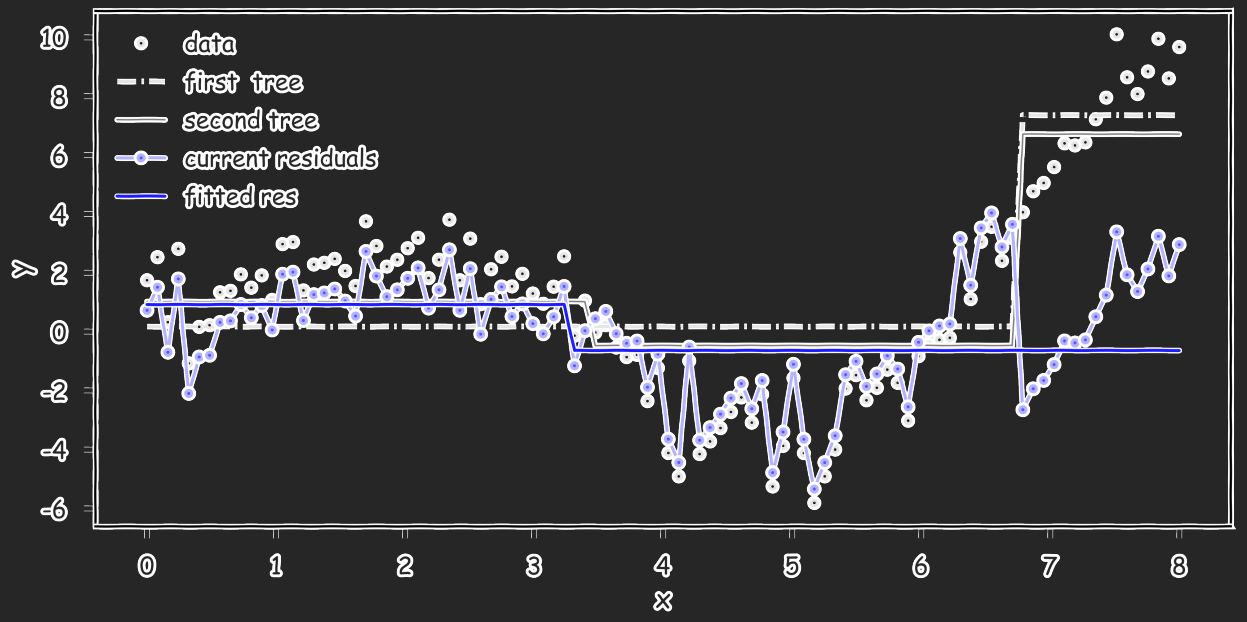
Step 5: Repeat step 2 on the new model by calculating the residuals $\{r_1, \ldots, r_N\}$ for current model T.

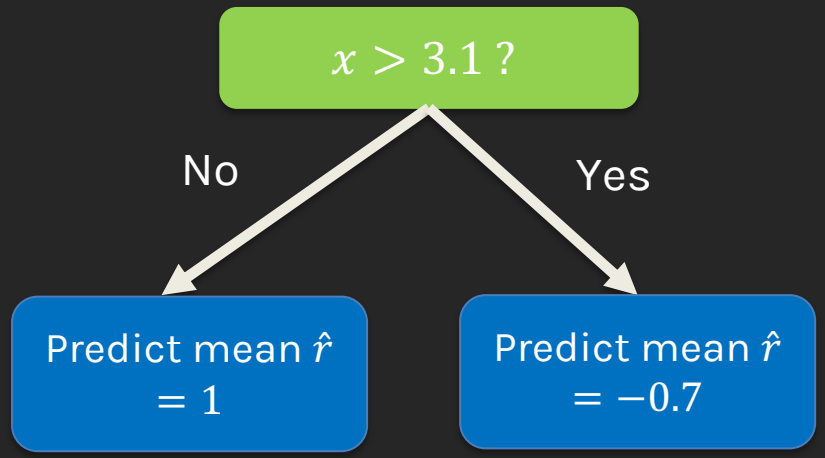




Gradient Boosting: Illustration

Step 6: Repeat step 3 and fit another model $T^{(2)}$ on the new residuals: $\{(x_1, r_1), \ldots, (x_N, r_N)\}$.

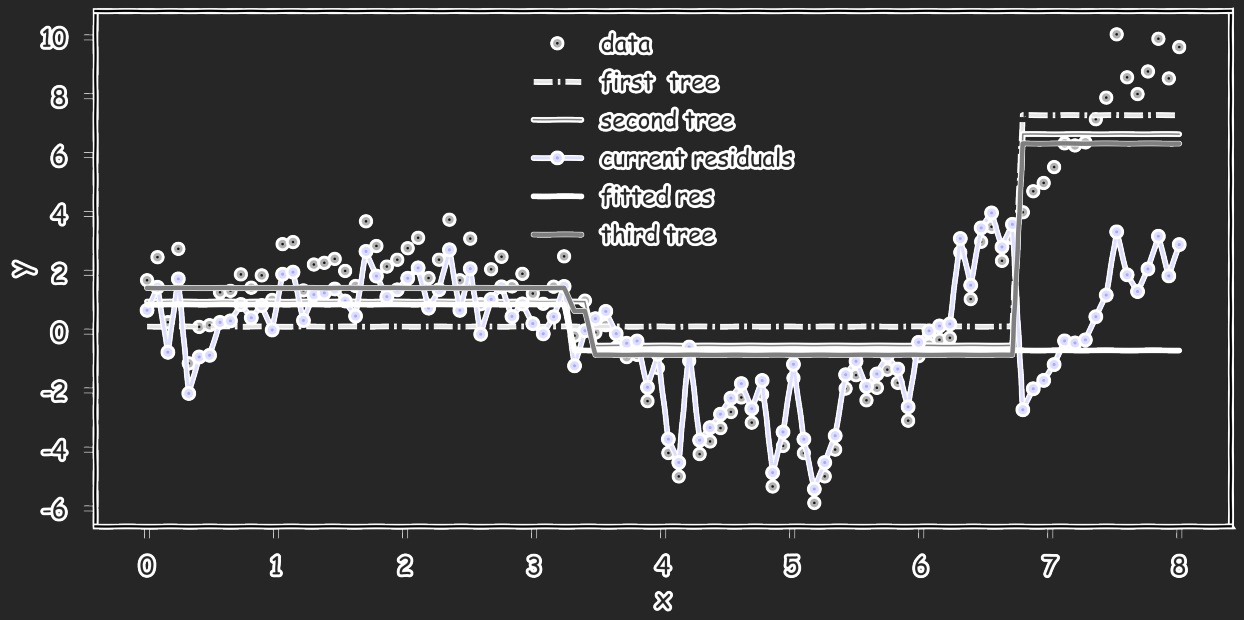


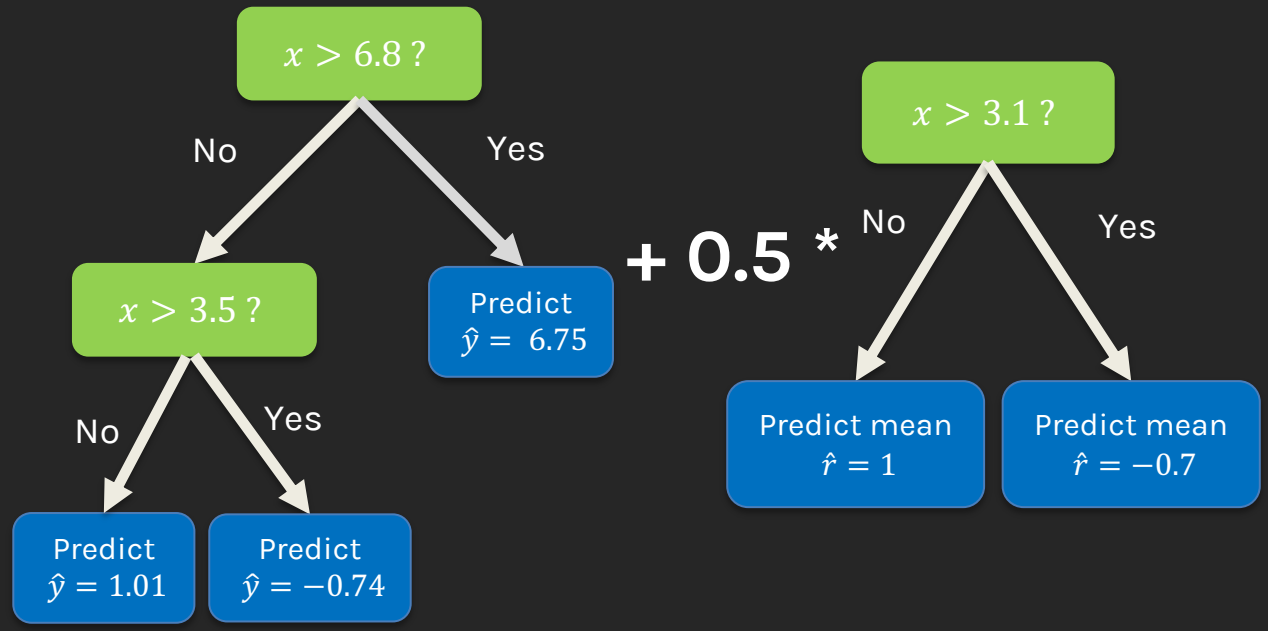


Gradient Boosting: Illustration

Step 7: Combine the two trees in step 4 and 6 by setting $T \leftarrow T + \lambda T^{(2)}$.

Assume $\lambda = 0.5$.





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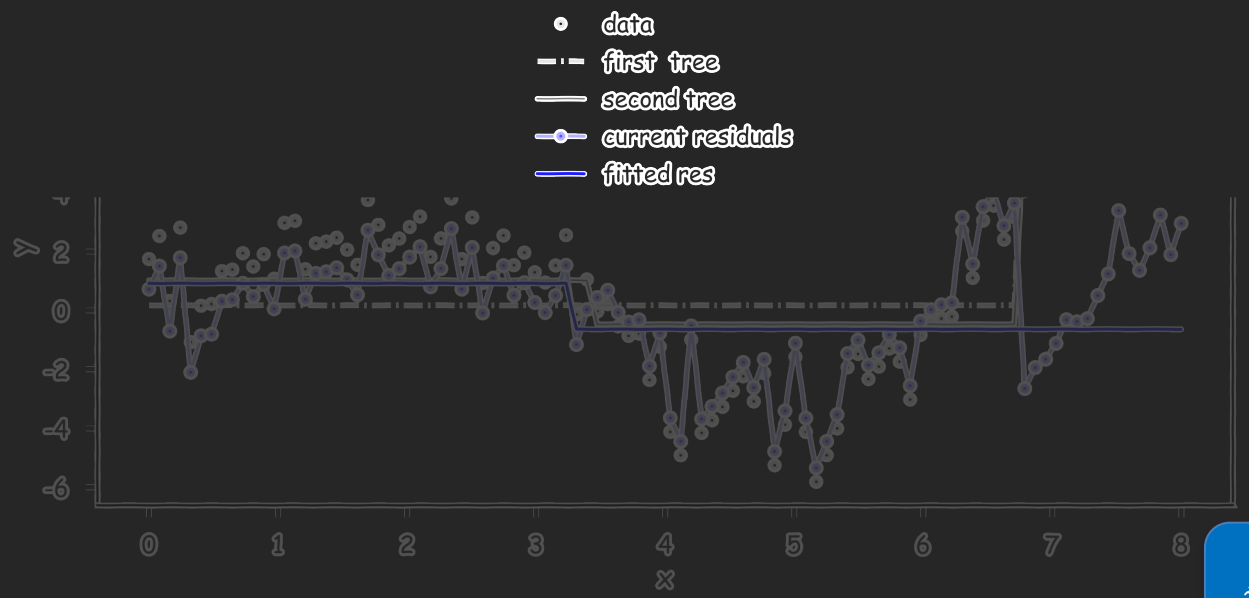

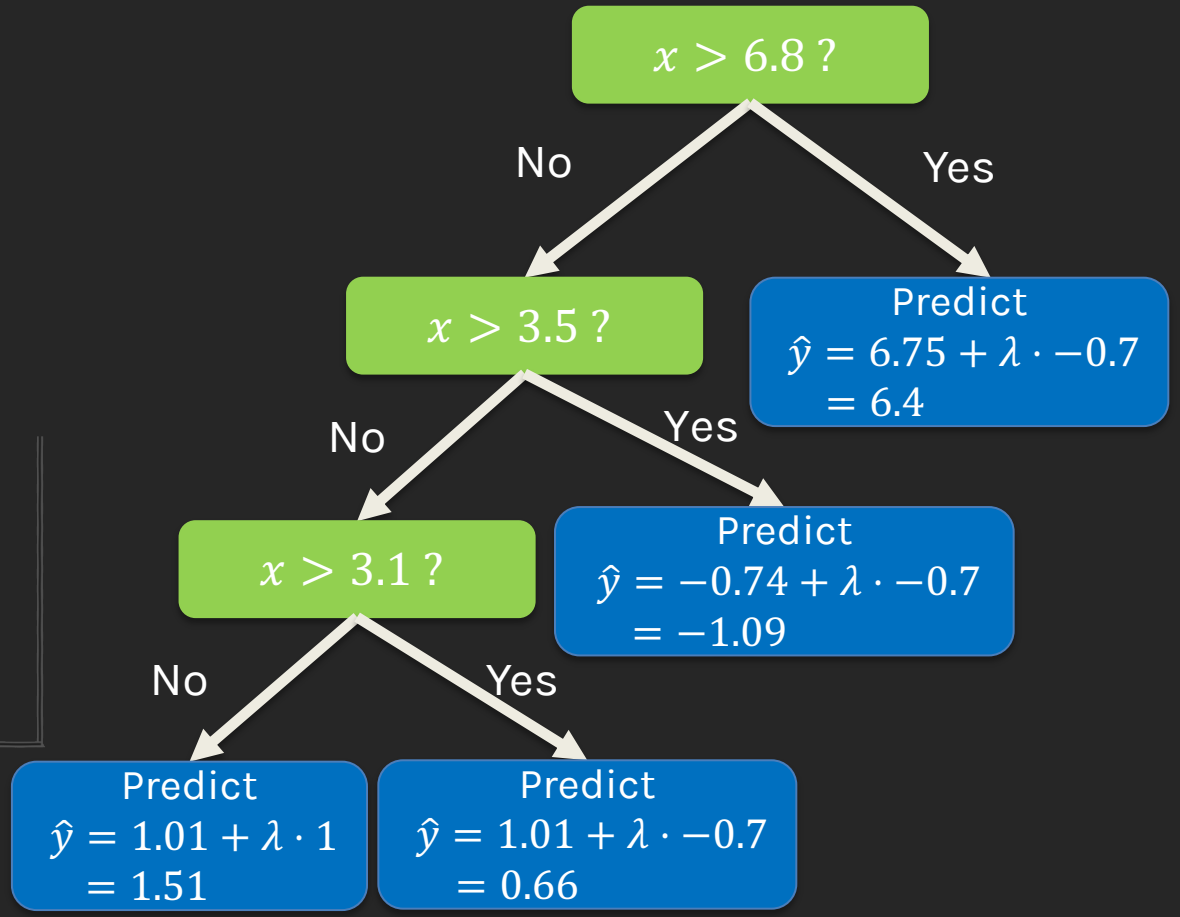

Gradient Boosting: Algorithm

Step 1: Fit a simple model $T^{(0)}$ on the training data $\{(x_1, y_1), \ldots, (x_N, y_N)\}$. Set $T \leftarrow T^{(0)}$.

Step 2: Compute the residuals $\{r_1, \ldots, r_N\}$ for T.

For $i = 1 \ldots$ until stopping condition is met:

Step 3: Fit a simple model, $T^{(i)}$, to the current **residuals**.

That is, train $T^{(i)}$ using $\{(x_1, r_1), \ldots, (x_N, r_N)\}$

Step 4: Set the current model $T \leftarrow T + \lambda T^{(i)}$.

Step 5: Compute residuals at step i, set $r_n \leftarrow r_n - \lambda T^{(i)}(x_n), \ n = 1, \ldots, N$

where λ is a constant called the **learning rate**.

Clarifying Gradient Boosting Implementation

Final Thought:

In practice, we do not explicitly combine the trees as described in these slides. That approach was presented purely for educational purposes. In reality, we use the outlined steps directly to make predictions without constructing a combined model.