Hwk 06 Solutions L8

Due on Oct 23, 2020

1 Concepts

1.1 Question 1

(a) In equation 1, the parameter β_0 measures the value of $f(X)$ (the mean value of Y) when each of the indicators is zero (i.e. for the baseline [first] region).

(b) The parameter β_K measures the difference in $f(X)$ (mean values of Y between the Kth (last) region and the baseline (first) region.

2 Applications

2.1 Question 1

The medians of wind speed and temperature are 9.7 and 79 respectively.

2.2 Question 2

The following is a list of all wind speed and temperature values, along with indicators for both variables which are true iff the corresponding variable exceeds its median. It would also be acceptable to use indicator values, 0 and 1, instead of the logical values `FALSE'' and`TRUE", respectively. It your further be acceptable to use the cut() function to define a factor instead. Finally, it is also fine to reverse the direction of the regions, with 1 for below the median and 0 for above.

```
##      Wind Temp wind.hilo temp.hilo
## 1    7.4   67     FALSE     FALSE
## 2    8.0   72     FALSE     FALSE
## 3   12.6   74      TRUE     FALSE
## 4   11.5   62      TRUE     FALSE
## 5    8.6   65     FALSE     FALSE
## 6   13.8   59      TRUE     FALSE
## 7   20.1   61      TRUE     FALSE
## 8    9.7   69     FALSE     FALSE
## 9    9.2   66     FALSE     FALSE
## 10  10.9   68      TRUE     FALSE
## 11  13.2   58      TRUE     FALSE
## 12  11.5   64      TRUE     FALSE
## 13  12.0   66      TRUE     FALSE
## 14  18.4   57      TRUE     FALSE
## 15  11.5   68      TRUE     FALSE
```

```
## 16   9.7   62   FALSE   FALSE
## 17   9.7   59   FALSE   FALSE
## 18  16.6   73    TRUE   FALSE
## 19   9.7   61   FALSE   FALSE
## 20  12.0   61    TRUE   FALSE
## 21  12.0   67    TRUE   FALSE
## 22  14.9   81    TRUE    TRUE
## 23   5.7   79   FALSE   FALSE
## 24   7.4   76   FALSE   FALSE
## 25   9.7   82   FALSE    TRUE
## 26  13.8   90    TRUE    TRUE
## 27  11.5   87    TRUE    TRUE
## 28   8.0   82   FALSE    TRUE
## 29  14.9   77    TRUE   FALSE
## 30  20.7   72    TRUE   FALSE
## 31   9.2   65   FALSE   FALSE
## 32  11.5   73    TRUE   FALSE
## 33  10.3   76    TRUE   FALSE
## 34   4.1   84   FALSE    TRUE
## 35   9.2   85   FALSE    TRUE
## 36   9.2   81   FALSE    TRUE
## 37   4.6   83   FALSE    TRUE
## 38  10.9   83    TRUE    TRUE
## 39   5.1   88   FALSE    TRUE
## 40   6.3   92   FALSE    TRUE
## 41   5.7   92   FALSE    TRUE
## 42   7.4   89   FALSE    TRUE
## 43  14.3   73    TRUE   FALSE
## 44  14.9   81    TRUE    TRUE
## 45  14.3   80    TRUE    TRUE
## 46   6.9   81   FALSE    TRUE
## 47  10.3   82    TRUE    TRUE
## 48   6.3   84   FALSE    TRUE
## 49   5.1   87   FALSE    TRUE
## 50  11.5   85    TRUE    TRUE
## 51   6.9   74   FALSE   FALSE
## 52   8.6   86   FALSE    TRUE
## 53   8.0   85   FALSE    TRUE
## 54   8.6   82   FALSE    TRUE
## 55  12.0   86    TRUE    TRUE
## 56   7.4   88   FALSE    TRUE
## 57   7.4   86   FALSE    TRUE
## 58   7.4   83   FALSE    TRUE
## 59   9.2   81   FALSE    TRUE
## 60   6.9   81   FALSE    TRUE
## 61  13.8   81    TRUE    TRUE
## 62   7.4   82   FALSE    TRUE
## 63   4.0   89   FALSE    TRUE
## 64  10.3   90    TRUE    TRUE
## 65   8.0   90   FALSE    TRUE
## 66  11.5   86    TRUE    TRUE
## 67  11.5   82    TRUE    TRUE
## 68   9.7   80   FALSE    TRUE
## 69  10.3   77    TRUE   FALSE
```

```
## 70    6.3   79      FALSE       FALSE
## 71    7.4   76      FALSE       FALSE
## 72   10.9   78       TRUE       FALSE
## 73   10.3   78       TRUE       FALSE
## 74   15.5   77       TRUE       FALSE
## 75   14.3   72       TRUE       FALSE
## 76    9.7   79      FALSE       FALSE
## 77    3.4   81      FALSE        TRUE
## 78    8.0   86      FALSE        TRUE
## 79    9.7   97      FALSE        TRUE
## 80    2.3   94      FALSE        TRUE
## 81    6.3   96      FALSE        TRUE
## 82    6.3   94      FALSE        TRUE
## 83    6.9   91      FALSE        TRUE
## 84    5.1   92      FALSE        TRUE
## 85    2.8   93      FALSE        TRUE
## 86    4.6   93      FALSE        TRUE
## 87    7.4   87      FALSE        TRUE
## 88   15.5   84       TRUE        TRUE
## 89   10.9   80       TRUE        TRUE
## 90   10.3   78       TRUE       FALSE
## 91   10.9   75       TRUE       FALSE
## 92    9.7   73      FALSE       FALSE
## 93   14.9   81       TRUE        TRUE
## 94   15.5   76       TRUE       FALSE
## 95    6.3   77      FALSE       FALSE
## 96   10.9   71       TRUE       FALSE
## 97   11.5   71       TRUE       FALSE
## 98    6.9   78      FALSE       FALSE
## 99   13.8   67       TRUE       FALSE
## 100 10.3   76       TRUE       FALSE
## 101 10.3   68       TRUE       FALSE
## 102  8.0   82      FALSE        TRUE
## 103 12.6   64       TRUE       FALSE
## 104  9.2   71      FALSE       FALSE
## 105 10.3   81       TRUE        TRUE
## 106 10.3   69       TRUE       FALSE
## 107 16.6   63       TRUE       FALSE
## 108  6.9   70      FALSE       FALSE
## 109 14.3   75       TRUE       FALSE
## 110  8.0   76      FALSE       FALSE
## 111 11.5   68       TRUE       FALSE
```

2.3   Question 3

(a) The following is a summary of the linear model of ozone on the indicators for wind speed and temperature.

```
##
## Call:
## lm(formula = Ozone ~ wind.hilo + temp.hilo, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -53.256 -13.065  -1.874   9.435  98.744
```

```
##
## Coefficients:
##              Estimate Std. Error t value       Pr(>|t|)
## (Intercept)    36.216      4.547   7.966 0.0000000000018 ***
## wind.hiloTRUE  -21.342      4.973  -4.292 0.0000387938694 ***
## temp.hiloTRUE   33.040      4.970   6.648 0.0000000012523 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.82 on 108 degrees of freedom
## Multiple R-squared:  0.4538, Adjusted R-squared:  0.4437
## F-statistic: 44.86 on 2 and 108 DF,  p-value: 0.00000000000000657
```

(b) Both predictors have statistically significant effects on ozone (wind speed: $p = 3.88\text{e-}05$, temperature: $p = 1.25\text{e-}09$).

(c) See Figure 1 for a plot of the predicted values from our model. We see that ozone increases for large values of temperature, and decreases for large values of wind speed.

2.4 Question 4

(a) The following is a summary of the linear model of ozone on the indicators for wind speed and temperature with interactions.

```
##
## Call:
## lm(formula = Ozone ~ wind.hilo * temp.hilo, data = data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -56.865 -10.220  -0.583  10.135  95.135
##
## Coefficients:
##                             Estimate Std. Error t value    Pr(>|t|)
## (Intercept)                   29.857      5.322   5.610 0.00000016038 ***
## wind.hiloTRUE                -11.274      6.697  -1.683       0.0952 .
## temp.hiloTRUE                 43.008      6.664   6.454 0.00000000326 ***
## wind.hiloTRUE:temp.hiloTRUE  -21.532      9.794  -2.198       0.0301 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.39 on 107 degrees of freedom
## Multiple R-squared:  0.4774, Adjusted R-squared:  0.4627
## F-statistic: 32.58 on 3 and 107 DF,  p-value: 0.000000000000004851
```

(b) The interaction term has a statistically significant effect on ozone ($p = 3.01\text{e-}02$).

(c) See Figure 2 for a plot of the predicted values from our model. The interaction appears to lessen the effect of temperature on ozone at large values of wind speed compared to at small values of wind speed. Alternatively, the effect of wind speed is smaller at low values of temperature than at high values of temperature.
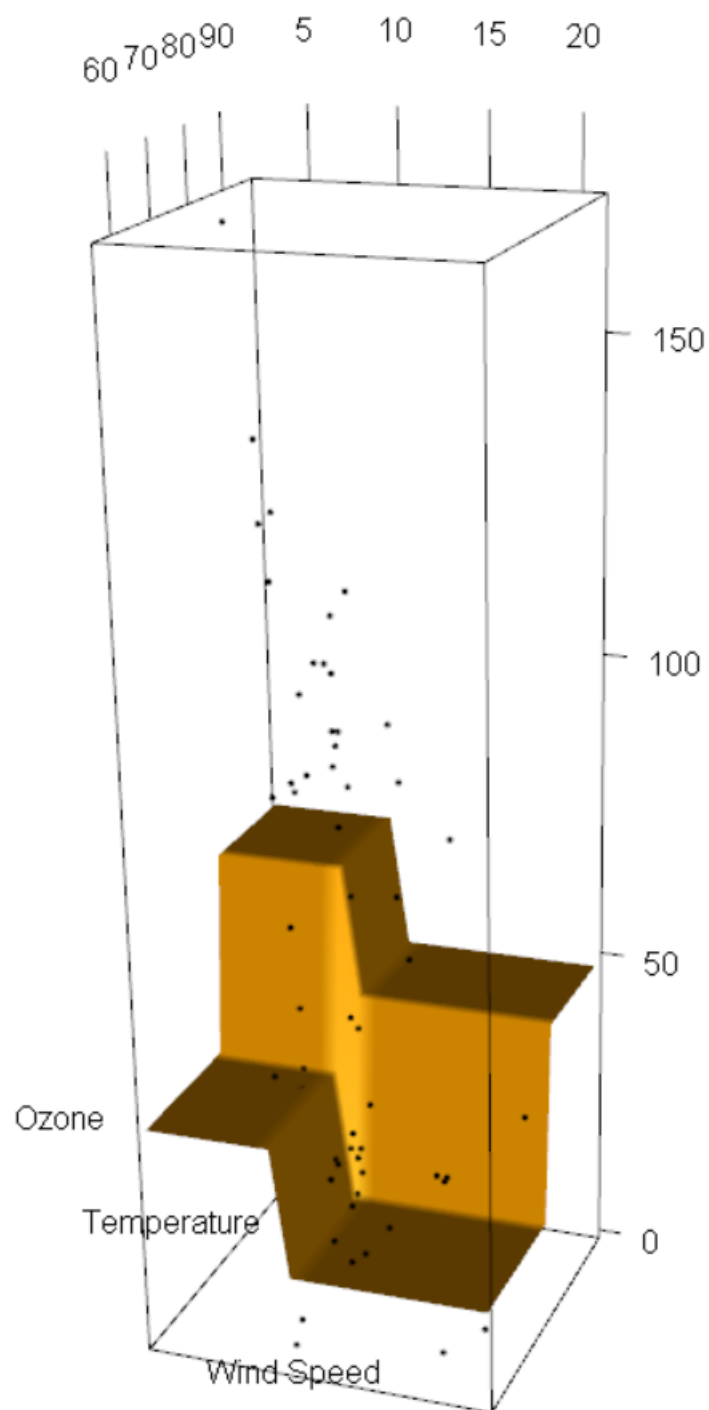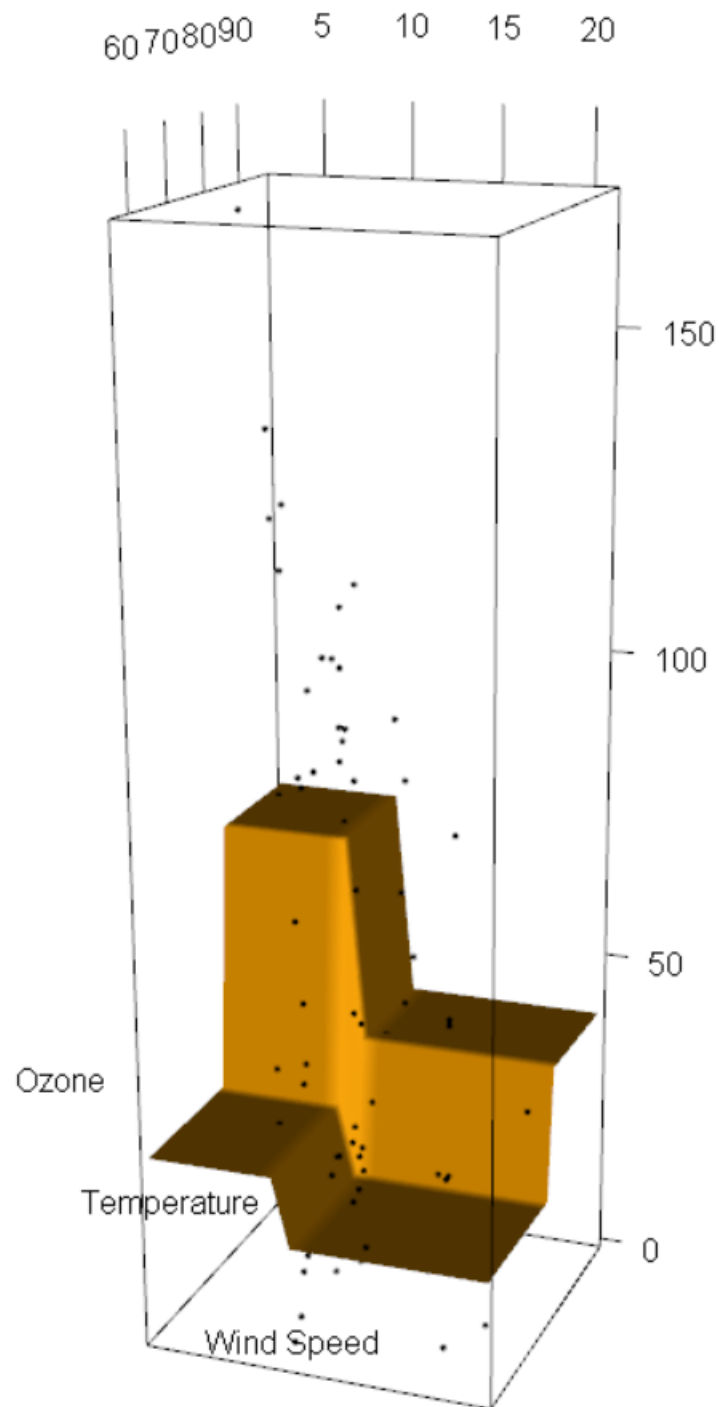
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Figure 1: Step function with only main effects.

Figure 2: Step function with interaction.