Concepts

1. Suppose you fit a model, $f(X) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 (X_1/X_2)$

   (a) For a fixed value of $X_2 = c$, what is the "slope" of X_1 (i.e., how much does $f(X)$ change for a 1-unit change in X_1)? **Write the answer in terms of parameters and *c*.**

   -> f(X) = $\beta_0 + X1(\beta_1 + \beta_3/X2) + \beta_2 X2$

   -> $\beta_1 + \beta_3/c$

   (b) For a fixed value of $X_1 = d$, what is the "slope" of X_2 (i.e., how much does $f(X)$ change for a 1-unit change in X_2)? **Write the answer in terms of parameters and *d*.**

$$f(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 \left(\frac{x_1}{x_2}\right)$$

$$\left[\cancel{\beta_0} + \cancel{\beta_1 x_1} + \beta_2 (x_2+1) + \beta_3 \left(\frac{x_1}{x_2+1}\right)\right] - \left[\cancel{\beta_0} + \cancel{\beta_1 x_1} + \beta_2 x_2 + \beta_3 \left(\frac{x_1}{x_2}\right)\right]$$

$$= \beta_2 + \beta_3 \left(\frac{x_1}{x_2+1} - \frac{x_1}{x_2}\right)$$

$$= \beta_2 + \beta_3 \left(\frac{x_1 x_2 - x_1(x_2+1)}{x_2(x_2+1)}\right)$$

$$= \beta_2 + \beta_3 \left(\frac{-x_1}{x_2(x_2+1)}\right)$$

$$= \beta_2 - \frac{\beta_3 d}{x_2(x_2+1)}$$

2. Suppose that X is numerical and Z is categorical (factor) with two levels. Someone has shown you a model where they fit lm(Y~X + Z + X:Z).**Write the model that R fits as a single regression model of the form** $f(x)$ = **....** Use variables z_q, $q = 1, \dots, Q$, to represent indicators for level q of Z

$$Y = \beta_0 + \beta_1 \times X + \begin{cases} \beta_2 + \beta_3 \times X & \text{if } q=2. \\ 0. & \text{if } q=1 \end{cases}$$