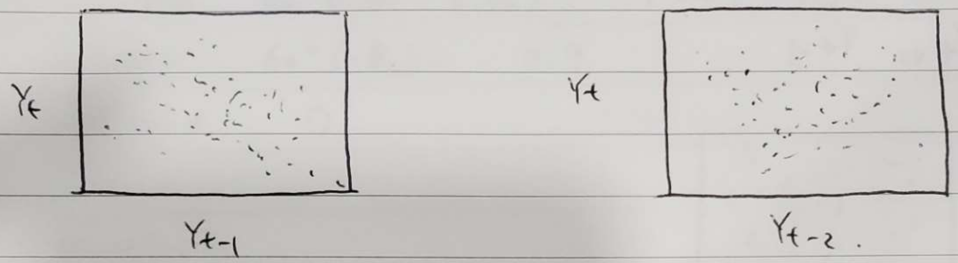
4. Consider an MA(1) model that results in the following plots of Y_t vs. Y_{t-1} and Y_t vs Y_{t-2}:



(a) Based on the above plots, give an estimate of the lag-1 autocorrelation ρ_1 for this model.

it looks like have a week negative autocorrelation

so, $\rho_1 \approx -0.4$

(b) Use your estimate in (a) to obtain two possible estimates of the parameter θ. (Hint: You will need the equation for the solution of a quadratic function: if $ax^2+bx+c=0$, then $x = (-b \pm \sqrt{b^2-4ac})/2a$).

$$\rho_1 = \frac{-\theta}{1-\theta^2} = -0.4$$

164
82
$2\sqrt{?}$

$$\Rightarrow \quad \theta = 0.4(1-\theta^2)$$

$$4\theta^2 + 10\theta - 4 = 0$$

$$x = \frac{-10 \pm \sqrt{100+64}}{8} = \frac{-10 \pm 2\sqrt{41}}{8}$$

(c) If we knew the value θ was restricted to be $-1 < \theta < 1$, which estimate from (b) would you choose?

$$\frac{-10+2\sqrt{41}}{8} \quad \because \quad -1 < \frac{-10+2\sqrt{41}}{8} < 1$$