Fundamental Concepts for Time Series: Review & Exercises

Week II: Video 6

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What Have We Learned?

In the Week 2 videos, we have seen a few examples of how to derive, for a given stochastic process:

- the mean function
- the autocovariance function
- the autocorrelation function

We've also seen the definition of stationarity, and how to check whether a process is weakly stationary:

1. Check: The mean function does not depend on t.
2. Check: The covariance between any Y_t and Y_{t-k} does not depend on t. (i.e., The autocovariance function does not depend on the time points, just the difference between them.)

In this video, we will briefly go over a few more examples of these derivations. More examples will be presented in the Week 3 tutorial and practice session.

Example 1: Average of White Noises

Let $e_1, e_2, e_3, \ldots$ be a white noise process with mean zero and variance σ_e^2. Then, suppose our stochastic process of interest is:

$$Y_t = \frac{e_t + e_{t-1}}{2}.$$

1. What is the mean function for this process?

Example 1: Average of White Noises (cont'd)

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

2. What is the autocovariance function for this process?

Example 1: Average of White Noises (cont'd)

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

3. What is the autocorrelation function for this process?

Example 1: Average of White Noises (cont'd)

$$Y_t = \frac{e_t + e_{t-1}}{2}$$

4. Is this process weakly stationary?

Example 2: A Linear Function

Suppose there is some stochastic process $\{X_t\}$ with a zero mean and autocovariance function γ_k. Our stochastic process of interest is:

$$Y_t = 5 + 2t + X_t.$$

1. What is the mean function for this process?

2. What is the autocovariance function for this process?

Example 3: Differencing

Let's go back to Example 2:

$$Y_t = 5 + 2t + X_t,$$

where $\{X_t\}$ is a process with zero mean and autocovariance function γ_k.

Now, define

$$W_t = \nabla Y_t = Y_t - Y_{t-1}.$$

This is referred to as the **differenced series** of Y_t.

1. What is the mean function for $\{W_t\}$?

Example 3: Differencing (cont'd)

$$W_t = \nabla Y_t = Y_t - Y_{t-1}$$

3. What is the autocovariance function for $\{W_t\}$?

4. Is $\{W_t\}$ weakly stationary?

Final Comments

That's all for now!

In this video, we've gone over a few examples from Chapter 2 of the textbook.

In Example 3, we saw how *differencing* can be used to turn a non-stationary process into a stationary one. More on this in Chapter 5.

You'll find more useful examples in the Week 3 tutorial and practice session. Please also try out some of the exercises from the textbook if you have time.

Next Week in STAT 485/685: Estimating trends!

References

[1] Chan, K. S., & Ripley, B. (2020). TSA: Time Series Analysis. R package version 1.2.1.

[2] Cryer, J. D., & Chan, K. S. (2008). *Time series analysis: with applications in R.* Springer Science and Business Media.