Trends:

Introduction to Trends in Time Series

Week III: Video 7

STAT 485/685, Fall 2020, SFU

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Our Roadmap

1. **Key Ideas:**
   - **Intro and fundamental concepts (Ch. 1-2):** Means, autocovariances and autocorrelations of time series, and the concept of stationarity.
   - **Estimating trends (Ch. 3):** Temporarily ignoring the rest of the variability in the series, how do we just estimate the mean trend? Can a regression be useful here?
   - **Models for stationary time series (Ch. 4)**
   - **Models for non-stationary time series (Ch. 5)**
2. **Building a Model:** Model specification (Ch. 6), Parameter estimation (Ch. 7), Model diagnostics (Ch. 8)
3. **Forecasting (Ch. 9)**
4. Other topics, as time permits.

Video 7 Learning Objectives

By the end of this video, we should be able to:

- Define the *trend* of a time series
- Identify several different types of time series trends
- Estimate a constant trend term, using a given time series dataset

Trends in Time Series

Suppose our process of interest, $\{Y_t\}$, has some mean function μ_t (which may or may not be a function of t).

We can separate out the mean from the rest of the process by writing:

$$Y_t = \mu_t + X_t$$

where $\{X_t\}$ is the "de-trended" version of the process, i.e. $E(X_t) = 0$.

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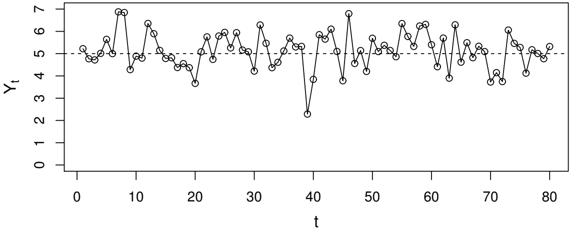
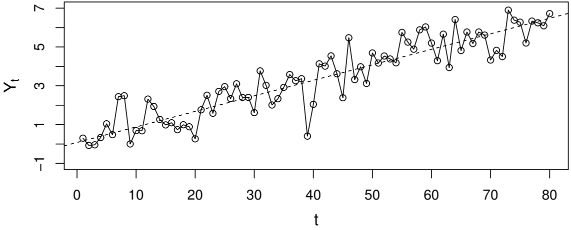
Some examples of how μ_t may look:

- Constant trend: $\mu_t = \mu$ for all t

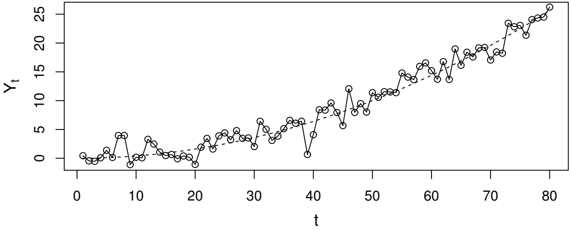


Trends in Time Series (cont'd)

- Linear trend: $\mu_t = \beta_0 + \beta_1 t$

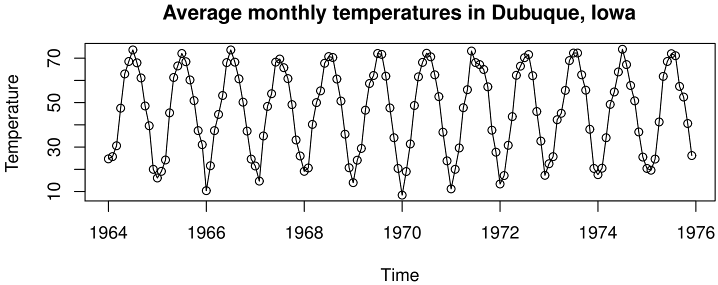


- Quadratic trend: $\mu_t = \beta_0 + \beta_1 t + \beta_2 t^2$



Trends in Time Series (cont'd)

- Cyclical/seasonal trend: e.g., $\mu_t = \mu_{t-12}$ for all t



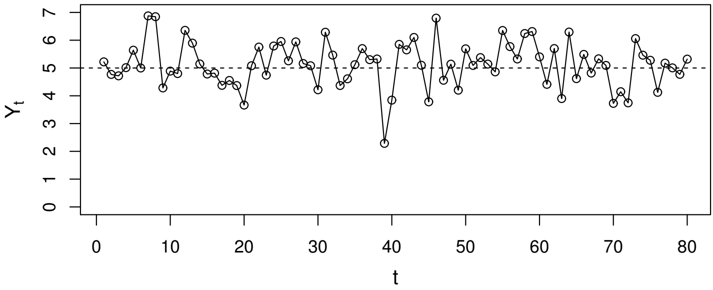
Constant Mean

Suppose:

$$Y_t = \mu + X_t$$

In other words, the mean μ is constant over time.

Example:



Estimating the Constant Mean

Suppose we are trying to estimate the constant mean μ, using our observed time series $Y_1, Y_2, \ldots, Y_n$.

The most common estimate of a constant trend is:

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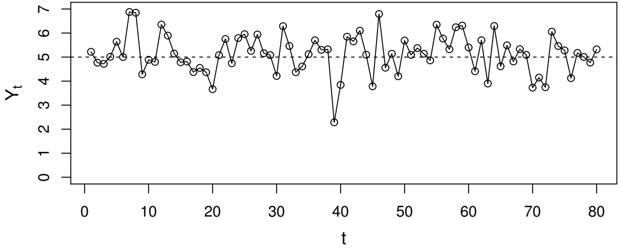
Example: In the dataset below, taking the average of the time series values would give us an estimate of the underlying constant mean:



Properties of $\bar{Y}$

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In general, if $\{X_t\}$ is any *stationary* process, with autocorrelation function ρ_k:

$$Var(\bar{Y}) = \frac{\gamma_0}{n}\left[1 + 2\sum_{k=1}^{n-1}\left(1 - \frac{k}{n}\right)\rho_k\right]$$

Some special cases:

- If $\rho_1 < 0$ and $\rho_k = 0$ for all $k > 1$ (i.e. time series oscillates a lot back and forth over the mean): $Var(\bar{Y}) < \gamma_0/n$.

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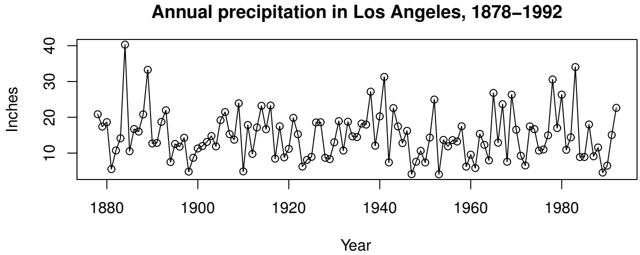
For non-stationary processes, determining $Var(\bar{Y})$ can be a bit more difficult.

R Example



R Code:

```
data(larain)
model.const <- lm(larain~1)
summary(model.const)
```

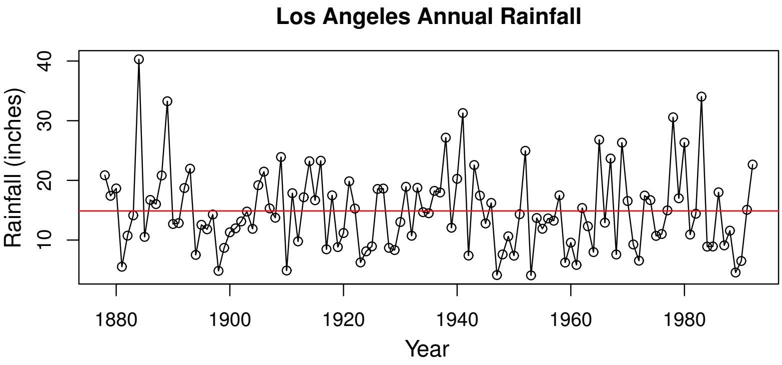
Output:

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  14.8884     0.6416    23.2   <2e-16 ***
```

R Example (cont'd)

Plotting:

```
plot(larain, type='o', xlab='Year', ylab='Rainfall (inches)',
     main='Los Angeles Annual Rainfall')
abline(model.const, col='red')
```



Final Comments

That's all for now!

In this video, we've learned what we mean when we talk about "trends" in time series analysis, and we've seen how to estimate a constant trend term using a time series dataset.

Coming Up Next: Linear trends, and regression methods.

References

[1] Cryer, J. D., & Chan, K. S. (2008). *Time series analysis: with applications in R.* Springer Science and Business Media.

[2] Chan, K. S., & Ripley, B. (2020). TSA: Time Series Analysis. R package version 1.2.1.