4. Let $\{e_t\}$ be a zero-mean white noise process with Variance σ_e^2.

(a) Recall the random walk process we learned about in Week 2: $Y_t = e_1 + e_2 + \dots + e_t$. Based on our work in Video 5, was this process Stationary?

No, it's not.

(b) Let $W_t = \nabla Y_t = Y_t - Y_{t-1}$, where Y_t is the random walk process in (a). Is this process stationary? Show your walk.

1. $E(W_t) = E(Y_t - Y_{t-1}) = E(Y_t) - E(Y_{t-1}) = 0$ for all t. ✓

2. What is the autocovariance for $\{W_t\}$?

1. For $t = s$: $Var(W_t) = Var(Y_t - Y_{t-1}) = Var(Y_t) + Var(Y_{t-1}) - Cov(Y_t, Y_{t-1})$

$= \gamma_0 + \gamma_0 - \gamma_1 = 2\gamma_0 - \gamma_1$

2. For $t \neq s$: $Cov(W_t, W_s) = Cov(Y_t - Y_{t-1}, Y_s - Y_{s-1})$

$= Cov(Y_t, Y_s) - Cov(Y_t, Y_{s-1}) - Cov(Y_{t-1}, Y_s) + Cov(Y_{t-1}, Y_{s-1})$

$= \gamma_{t-s} - \gamma_{t-(s-1)} - \gamma_{t-1-s} + \gamma_{(t-1)-(s-1)}$

$= \gamma_{t-s} - \gamma_{t-s+1} - \gamma_{t-s-1} + \gamma_{t-s}$

$= \dots =$ it's not a function of t & s

$\Rightarrow$ Yes! Stationary.