ASSIGNMENT 8

STAT 485/685 E100/G100: Applied Time Series Analysis
Fall 2020
Simon Fraser University

This week's assignment will be about parameter estimation in time series models. The topics here have been covered in the Week 11 videos, as well as Chapter 7 of the textbook *Time Series Analysis with Applications in R (2nd ed.)* by Cryer & Chan.[1]

Due date: **Tuesday, Dec. 1st at 11:59 pm (end of day)** (Pacific Time).

Marks: 5.

Please include your R code for each of questions that requires it below. Some ideas for how you can most easily do this:

- Copy-paste your code/plots into a Word document along with your responses to the questions, and save as a PDF.
- Take images of your code/plots and upload to Crowdmark, along with your responses to the questions.
- Save your code and responses together in an RMarkdown document and save as PDF (if you've worked with RMarkdown before).

Other important policies on assignment submissions:

- Please write each question on a **separate page**!
- Please **show all your code and work**, in order to get full marks.
- Upload your complete answers as PDF files or high-resolution images.
- If you're hand-writing answers, please make sure they are **neat and clearly readable**, and that the photo is high resolution.
- Please **clearly label the question numbers**.

[1] Cryer, J. D., & Chan, K. S. (2008). *Time series analysis: with applications in R.* Springer Science and Business Media.

For this assignment, we will be using some datasets and functions in the `TSA` package in R. For instructions on how to install and load the package, please see the Week 2 module on Canvas.

Comments on R code:

- Before starting this assignment, it is recommended that you look through the R code from the Week 12 tutorial, which can be found on the page *Missed Something in Week 12?* in the Week 12 module on Canvas. It contains lots of examples that are similar to the questions in this assignment.
- More R code examples can be found in § 7.5 of the textbook.
- We will be using the `ar()` function, examples of which are found in the tutorial. The function asks for `order.max` (which is the value of p in our AR model), and for the argument `method`. We also see an additional argument: `aic=FALSE`. This additional argument ensures that R accepts the value of p we gave it to be the "correct value"; otherwise, it will try to find a "better" value on its own.
- If you'd like to read more about the function, type `?ar` to open up the Help file.
- You will be asked to extract some values (e.g., parameter estimates) from the output. In order to see which values are available, how they are extracted and what they mean, you can read through the "Values" section within the Help file for the function.

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1. *(2 marks)* The `color` dataset in the `TSA` package gives the values of a colour property from 35 consecutive batches in an industrial process. Suppose we decide to fit an AR(1) model to this dataset.
   (Note: If we were to investigate more closely, we would see that this might not be the most appropriate model for this dataset. However, we will consider it here.)

   (a) Fit the AR(1) model to this dataset, using the Maximum Likelihood Estimation approach within the `ar()` function. Give the estimates of ϕ and μ.
   (Hint: We will need the argument `method="mle"` in the `ar()` function.)

   (b) Write out the full equation(s) you could use to estimate σ_e^2, using the estimates of ϕ and μ. Make sure to plug the estimates of ϕ and μ into the equation. You do not have to actually evaluate this estimate, since the dataset is somewhat large.

2. *(3 marks)*

   (a) Fit the AR(1) model to the `color` dataset, using the Method of Moments approach within the `ar()` function. Give the estimates of ϕ and μ.
   (Hint: We will need the argument `method="yw"` in the `ar()` function. This stands for "Yule-Walker", because the Yule-Walker equations need to be solved to get the MOM parameter estimates.)

   (b) Using equation(s) we have learned about in Video 29, obtain an estimate of the process variance γ_0.
   (Hint: You may have to explore a bit to find a function in R that can give you the sample variance of a dataset.)

   (c) Using the above results, and equation(s) we have learned about in Video 29, obtain an estimate of the white noise variance σ_e^2.
   (Hint: If you need some sample correlations from the dataset: For any dataset `mydata`, the vector of r_k-values is given by `acf(mydata)$acf`.)

The remaining questions below are just for practice, and will not be marked.

3. **(0 marks)**

   (a) Fit the AR(1) model to the `color` dataset, using the (Conditional) Least Squares approach within the `ar()` function. Give the estimates of ϕ and μ.
   (Hint: We will need the argument `method="ols"` in the `ar()` function.)

   (b) How does the estimate of ϕ compare to the lag-1 autocorrelation for this dataset?

   (c) Obtain the estimate of μ in a different way – by calculating the sample mean of the data. How does this compare to the estimate of μ you obtained in part (a)?

4. **(0 marks)** Suppose we have a time series dataset of size $n = 3$, as follows: $Y_1 = 4$, $Y_2 = 3$ and $Y_3 = 7$.

   (a) Evaluate the lag-1 autocorrelation for this dataset. Show your work.

   (b) Suppose we wish to fit an MA(1) model to this dataset. Calculate the Method of

Moments estimate of the parameter θ, by hand. Show your work.

(c) Is this solution invertible? Explain your reasoning.

(d) Calculate an estimate of the process variance γ_0. Show your work.

(e) Calculate an estimate of the white noise variance σ_e^2. Show your work.

(f) Calculate an estimate of the process mean μ. Show your work.

(g) Using the parameter estimates you have obtained above, write out an equation for the model for $\{Y_t\}$.

(h) Re-write the equation from part (g), in terms of an intercept term θ_0 (instead of the mean value μ).