ASSIGNMENT 6 SOLUTIONS

STAT 485/685 E100/G100: Applied Time Series Analysis
Fall 2020
Simon Fraser University

1. Suppose that a certain time series dataset has the following sample ACF values: $r_1 = 0.3$, $r_2 = 0.4$, $r_3 = -0.04$ and $r_4 = 0$. Obtain the following sample PACF values. Show all your work.

   (a) $\hat{\phi}_{11}$

   Solution:

   From slide 14 of Video 24, we have that

$$\hat{\phi}_{11} = r_1 = 0.3$$

   (b) $\hat{\phi}_{22}$

   Solution:

   From slide 15 of Video 24, we have that

$$\hat{\phi}_{22} = \frac{r_2 - r_1^2}{1 - r_1^2} = \frac{0.4 - (0.3)^2}{1 - (0.3)^2} = 0.340659 \approx 0.34$$

   (c) $\hat{\phi}_{33}$

   Solution:

   We will first need:

$$\hat{\phi}_{21} = \hat{\phi}_{11} - \hat{\phi}_{22}\hat{\phi}_{11} = 0.3 - (0.340659)(0.3) = 0.197802$$

   Then, from slide 15 of Video 24:

$$
\begin{aligned}
\hat{\phi}_{33} &= \frac{r_3 - \hat{\phi}_{21} r_2 - \hat{\phi}_{22} r_1}{1 - \hat{\phi}_{21} r_1 - \hat{\phi}_{22} r_2} \\
&= \frac{-0.04 - (0.197802)(0.4) - (0.340659)(0.3)}{1 - (0.197802)(0.3) - (0.340659)(0.4)} \\
&= -0.275137 \\
&\approx -0.28
\end{aligned}
$$

(d) $\hat{\phi}_{44}$ *(This one will take a bit more work!)*

Solution:

We will first need:

$$\hat{\phi}_{31} = \hat{\phi}_{21} - \hat{\phi}_{33}\hat{\phi}_{22} = 0.197802 - (-0.275137)(0.340659) = 0.291530$$

and

$$\hat{\phi}_{32} = \hat{\phi}_{22} - \hat{\phi}_{33}\hat{\phi}_{21} = 0.340659 - (-0.275137)(0.197802) = 0.395082$$

Then, from the general equation on slide 14 of Video 24 (substituting $k = 4$):

$$
\begin{aligned}
\hat{\phi}_{44} &= \frac{r_4 - \sum_{j=1}^{3} \hat{\phi}_{3,j} r_{4-j}}{1 - \sum_{j=1}^{3} \hat{\phi}_{3,j} r_j} \\
&= \frac{r_4 - \hat{\phi}_{31} r_3 - \hat{\phi}_{32} r_2 - \hat{\phi}_{33} r_1}{1 - \hat{\phi}_{31} r_1 - \hat{\phi}_{32} r_2 - \hat{\phi}_{33} r_3} \\
&= \frac{0 - (0.291530)(-0.04) - (0.395082)(0.4) - (-0.275137)(0.3)}{1 - (0.291530)(0.3) - (0.395082)(0.4) - (-0.275137)(-0.04)} \\
&= -0.085851 \\
&\approx -0.086
\end{aligned}
$$

2. Suppose that a time series dataset, of size $n = 100$, has the following sample PACF values: $\hat{\phi}_{11} = 0.3$, $\hat{\phi}_{22} = 0.9$, $\hat{\phi}_{33} = -0.4$ and $\hat{\phi}_{44} = 0.1$. Based on this information alone, which model do you believe might be appropriate for this dataset? Justify your decision using an argument about the standard errors of the estimates.

Solution:

From slide 16 of Video 24, we know that, in order to test the null hypothesis that an AR(p) model is correct, we can construct our critical limits at any $k > p$ using

$$\pm 2\, SE(\hat{\phi}_{kk}) = \pm \frac{2}{\sqrt{n}},$$

since $1/\sqrt{n}$ is the approximate standard error of $\hat{\phi}_{kk}$.

If an estimate $\hat{\phi}_{kk}$ falls outside of these "dashed lines" (i.e., if it is either $> 2/\sqrt{n}$ or $< -2/\sqrt{n}$), then we can be fairly confident that the true value of ϕ_{kk} is not equal to zero.

With this dataset, the approximate critical limits for any $\hat{\phi}_{kk}$ are:

$$\frac{2}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$$

Therefore, we test each of the following:

$$\begin{aligned} |\hat{\phi}_{11}| &= |0.3| = 0.3 > 0.2 \\ |\hat{\phi}_{22}| &= |0.9| = 0.9 > 0.2 \\ |\hat{\phi}_{33}| &= |-0.4| = 0.4 > 0.2 \\ |\hat{\phi}_{44}| &= |0.1| = 0.1 < 0.2 \end{aligned}$$

Thus, we conclude that ϕ_{11}, ϕ_{22} and ϕ_{33} are all significantly different from zero, while ϕ_{44} is not.

Since ϕ_{44} is the first PACF value that is not significantly different from zero, we conclude that this is an AR(3) model.

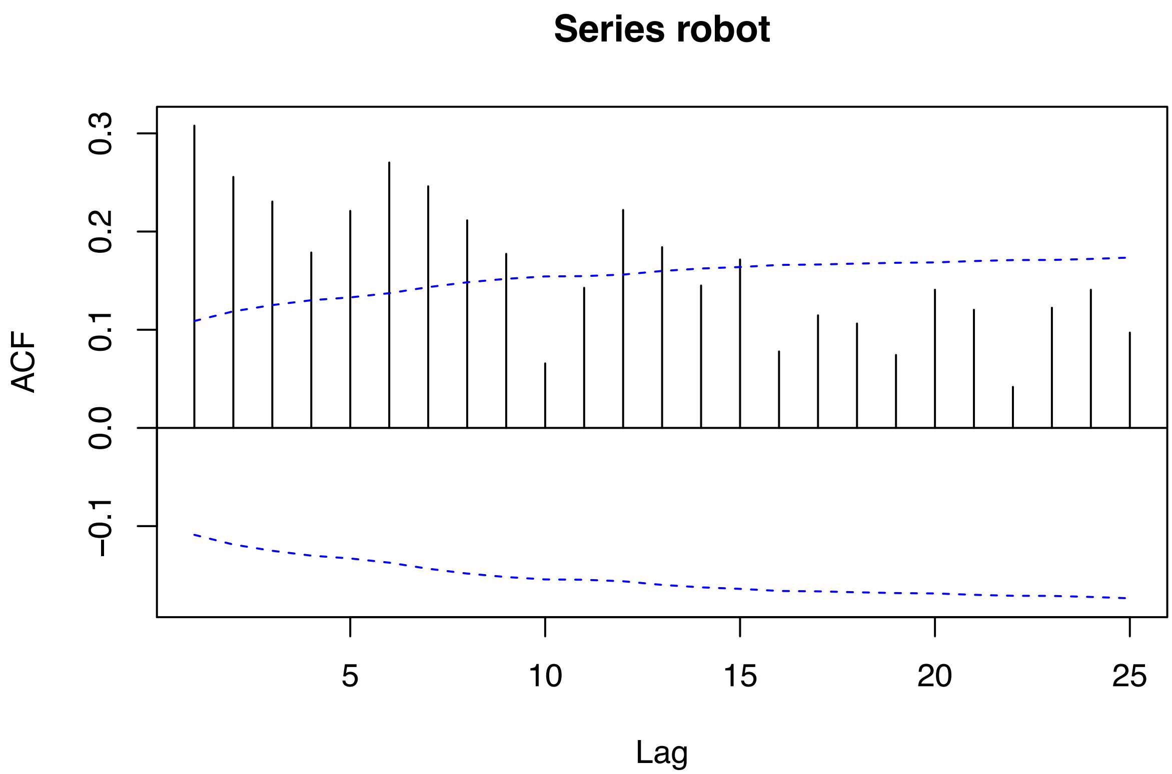
3. The dataset "robot" gives the final position (in the x-direction) of an industrial robot put through a series of planned exercises many times. Read in this dataset, and use it to answer the following questions.
Remember to include all code and plots in your answers below.

a) Create a sample ACF plot for this dataset. Explain what you see, and any conclusions you might be able to make from this plot.

Solution:

```
library(TSA)
```

```
data(robot)
acf(robot, ci.type='ma')
```



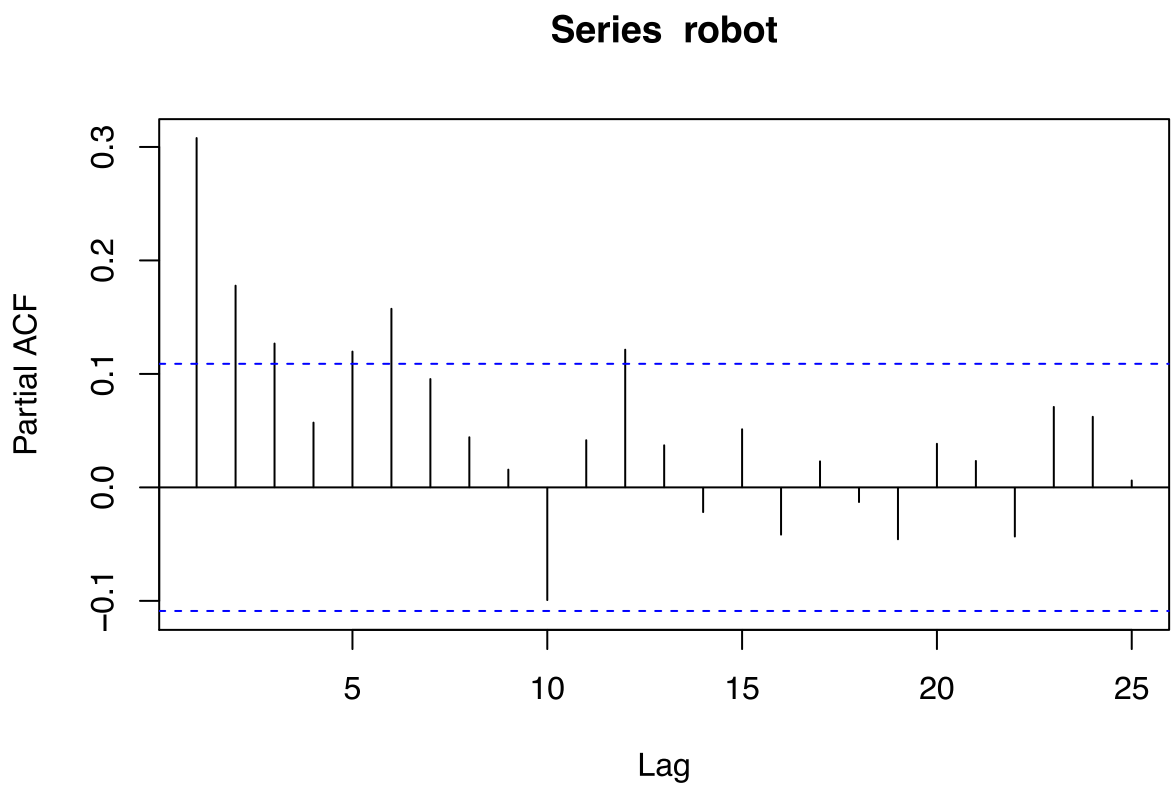
It looks like the values are decaying exponentially, and not cutting off anywhere. Therefore, this does not appear to be an MA model.

Note: We have to make sure to include ci.type='ma', to ensure that we are using the correct "dashed lines", since we are testing for an MA model here.

b) Create a sample PACF plot for this dataset. Explain what you see, and any conclusions you might be able to make from this plot.

Solution:

```r
pacf(robot)
```



This could potentially be an AR(3) or AR(6) model, since this is where the values potentially cut off. However, it is not entirely clear, as there might be some exponential decay.

c) Create a sample EACF table for this dataset. Explain what you see, and any conclusions you might be able to make from this table.

Solution:

```r
eacf(robot)
```

```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x x x x x x x x o x  x  x  x
## 1 x o o o o o o o o o o  o  o  o
## 2 x x o o o o o o o o o  o  o  o
## 3 x x o o o o o o o o o  o  o  o
## 4 x x x x o o o o o o o  o  x  o
## 5 x x x o o o o o o o o  o  x  o
## 6 x o o o o x o o o o o  o  o  o
## 7 x o o x o x x o o o o  o  o  o
```

There appears to be a triangle with an upper-left corner indicating an ARMA(1,1) model.

d) Based on the above results, make a conclusion about a model that may be appropriate for this dataset. Explain your reasoning.

Solution:

Based on the above plots, we would choose an ARMA(1,1) model.

The reasoning is as follows:

- The ACF plot showed exponential decay, indicating that this is not a pure MA model.
- The PACF plot also showed some potential exponential decay, suggesting that this might not perhaps be a pure AR model.
- The EACF table, on the other hand, showed a very clear choice of an ARMA(1,1) model.

All of these observations support the conclusion that this appears to be an ARMA(1,1) model.