ASSIGNMENT 1

STAT 485/685 E100/G100: Applied Time Series Analysis

Fall 2020

Simon Fraser University

This week's assignment will be based on fundamental concepts in stochastic processes. The topics here have been covered in the videos of Weeks 1 and 2, as well as Chapter 2 of the textbook *Time Series Analysis with Applications in R (2nd ed.)* by Cryer & Chan.[1]

Due date: **Friday, Sept. 25th at 12:00 pm** (Pacific Time).

Marks: 10.

We will be submitting assignments via Crowdmark. Please see Canvas for instructions on setting up an account in Crowdmark and submitting assignments.

Important policies on assignment submissions:

- Please write each question on a **separate page**! This is important for the purposes of Crowdmark.
- Please **show all your work**, in order to get full marks.
- Please upload your complete answers as a PDF file, Word document or high-resolution images. The answers can be typed or hand-written.
- If you're hand-writing answers, please make sure they are **neat and clearly readable**, and that the photo is high resolution. There may be penalties if the marker cannot clearly read your answers.
- Please **clearly label the question numbers**.

[1] Cryer, J. D., & Chan, K. S. (2008). *Time series analysis: with applications in R.* Springer Science and Business Media.

1. Let X and Y be random variables such that $E(X) = 5$, $Var(X) = 1$, $E(Y) = 3$, $Var(Y) = 4$ and $Corr(X, Y) = 0.5$. Find:

   (a) $Cov(X, Y)$.

   (b) $Cov(2X + Y, X - 7Y + 1)$.

2. *(Exercise 2.4 in Cryer & Chan.)* Let $\{e_t\}$ be a zero-mean white noise process with a variance denoted by σ_e^2. Suppose that the observed process is $Y_t = e_t + \theta e_{t-1}$, where θ is some constant value.

   (a) Find the autocovariance function for $\{Y_t\}$, in terms of θ. *(Hint: Remember to consider all possible combinations of s and t: when they are equal, when they are 1 unit apart, etc.)*

   (b) Find the autocorrelation function for $\{Y_t\}$, in terms of θ.

   (c) Suppose that θ may be equal to either 3 or 1/3. Evaluate the autocorrelation function for each of these cases.

   (d) Suppose you observe a time series $\{Y_1, Y_2, \ldots, Y_n\}$, and use it obtain an estimate of the autocorrelation function (using methods we will learn later in the course). Would you be able to use this estimate of $\rho_{t,s}$ to determine whether θ is equal to 3 or 1/3? Why or why not?

3. *(Exercise 2.11 in Cryer & Chan.)* Suppose $Cov(X_t, X_{t-k}) = \gamma_k$, which is free of t, but that $E(X_t) = 3t$.

   (a) Is the process $\{X_t\}$ stationary?

   (b) Let $Y_t = 7 - 3t + X_t$. Is $\{Y_t\}$ stationary?

4. Let $\{e_t\}$ be a zero-mean white noise process with variance σ_e^2.

   (a) Recall the random walk process we learned about in Week 2: $Y_t = e_1 + e_2 + \ldots + e_t$. Based on our work in Video 5, was this process stationary? (You don't have to re-do the derivations here.)

   (b) Let $W_t = \nabla Y_t = Y_t - Y_{t-1}$, where Y_t is the random walk process in (a). Is this process stationary? Show your work.