1). $\{e_t\}$ be a white noise, mean zero, variance σ_e^2.

$Y_t = e_t - 2e_{t-1}$.

(a) Mean function for $\{Y_t\}$

$E(Y_t) = E(e_t - 2e_{t-1}) = E(e_t) - 2E(e_{t-1})$

$= 0 - 2\times 0 = 0$

(b) Autocovariance

① if $t = s$: $Cov(Y_t, Y_s) = Var(Y_t) = Var(e_t - 2e_{t-1})$

$= Var(e_t) + 4Var(e_{t-1}) = \sigma_e^2 + 4\sigma_e^2 = 5\sigma_e^2$

② if $|t-s| = 1$

order doesn't matter.

$Cov(Y_t, Y_s) = Cov(Y_t, Y_{t-1}) = Cov(e_t - 2e_{t-1}, e_{t-1} - 2e_{t-2})$

$= \underbrace{\cancel{Cov(e_t, e_{t-1})}}_{0} - 2\underbrace{\cancel{Cov(e_t, e_{t-2})}}_{0} - 2Cov(e_{t-1}, e_{t-1}) + 4\underbrace{\cancel{Cov(e_{t-1}, e_{t-2})}}_{0}$

$= -2Var(e_{t-1}) = -2\sigma_e^2$

③ if $|t-s| > 1$

$Cov(Y_t, Y_s) = Cov(e_t - 2e_{t-1}, e_{t-2} - 2e_{t-2})_0$

$= \cancel{Cov(e_t, e_{t-2})} - 2\cancel{Cov(e_t, e_{t-2})} - 2\cancel{Cov(e_{t-1}, e_{t-2})} + 4\cancel{Cov(e_{t-1}, e_{t-2})}$

$$\gamma_{t,s} = \begin{cases} 5\sigma_e^2 & \text{if } t=s \\ -2\sigma_e^2 & \text{if } |t-s|=1 \\ 0 & \text{if } |t-s|>1. \end{cases}$$

(c) Autocorrelation function for $\{Y_t\}$

$$Corr(Y_t, Y_s) = \frac{Cov(Y_t, Y_s)}{\sqrt{Var(Y_t)\,Var(Y_s)}}$$

$Var(Y_t) = Var(e_t - 2e_{t-1})$
$= 5\sigma_e^2.$

$$\frac{-2\sigma_e^2}{\sqrt{5\sigma_e^2 \times 5\sigma_e^2}}$$

$$\rho_{t,s} = \begin{cases} 1 & \text{if } t=s \ \ (Corr(Y_t, Y_t)=1) \\ -\frac{2}{5} & \text{if } |t-s|=1 \\ 0. & \text{if } |t-s|>1 \end{cases}$$

(d) Is $\{Y_t\}$ Stationary?

1. $\mu_t = 0$
2. Covariance is free of t. ✓

→ Yes!!