

# motion analysis

Computational Vision

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# introduction

What do we gain if we analyse videos instead than images?

we focus on *motion information*:

- We will now consider low level algorithms (temporal features).
- Later in the course we will introduce higher level (action/activity classification) methods

# introduction

We are observing a scene with one camera acquiring a set of images “close in time”

**image sequence:** series of N images, or *frames*, acquired at discrete time instants

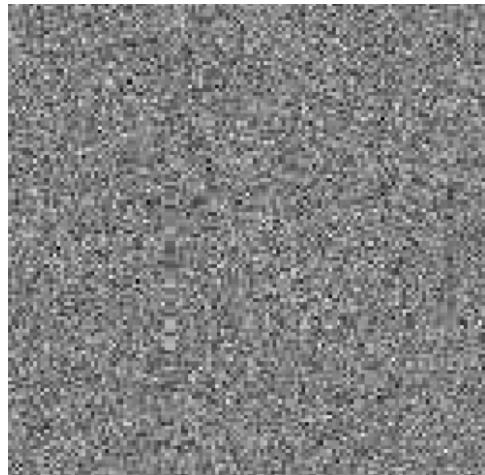
$$t_k = t_0 + k\Delta t$$

fixed time interval (small)



# the importance of motion cues

## example



the presence of a structure is suggested to the visual system only by motion information: the changes we perceive from one frame to the next

# Introduction

## Brightness Constancy assumption

An important assumption to make when analysing image sequences is the following:

*illumination does not vary in the observation interval*

In this case the image changes from  $t_1$  to  $t_2$  are caused by the *relative motion* between scene and camera

# the problems of motion

We may identify different types of questions related with motion analysis

- **correspondence:** which elements of a frame correspond to which elements of the next frame?  
*(very closely related with image matching...)*  
→ motion estimation - optic flow computation
- **reconstruction:** what was the 3D position and 3D velocity of the observed elements?

# the problems of motion (cont'd)

## Other problems related to motion analysis

- **motion segmentation:** what regions of the image plane correspond to different moving parts? in the case the camera is still, motion segmentation is called **change detection**
- **tracking:** estimate the (2D or even 3D) trajectories of points or objects from image sequences  
**Main computational tools: dynamic filters (eg Kalman)**

# Motion analysis

## What we will do

### In summary

Motion segmentation: change detection

Correspondence: optical flow estimation

Feature (corner) tracking

# Motion segmentation: change detection

# Motion segmentation

## Change detection

$I_t$



In the case the camera is still motion segmentation is can be implemented as a difference

Assuming we have a reference image  $I_{ref}$

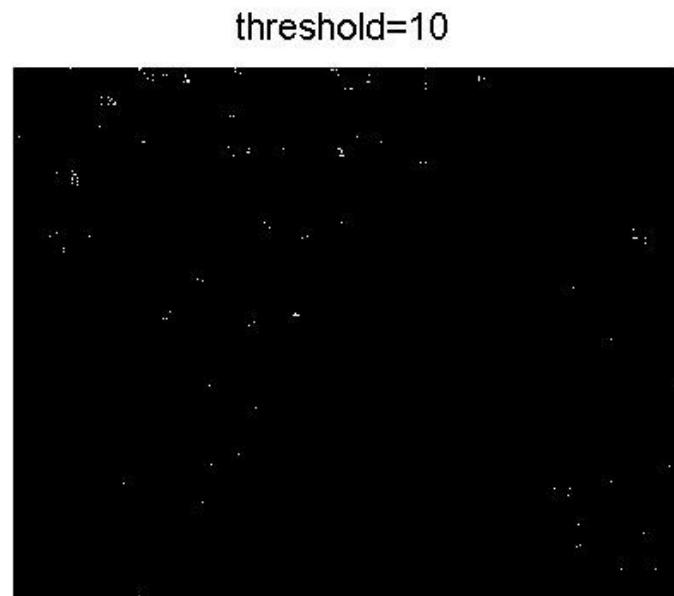
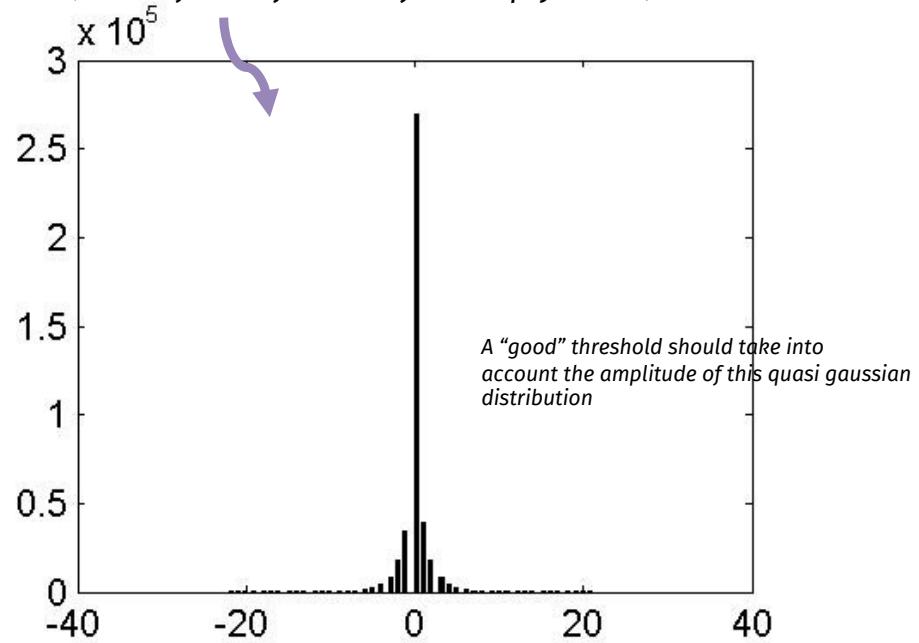
$$M_t(x, y) = \begin{cases} 1 & \text{if } |I_t(x, y) - I_{ref}(x, y)| > \tau \\ 0 & \text{otherwise} \end{cases}$$

the threshold  $\tau$  depends on the acquisition device and the acquisition conditions. It must be chosen considering a trade-off between false positives and false negatives.

# The threshold



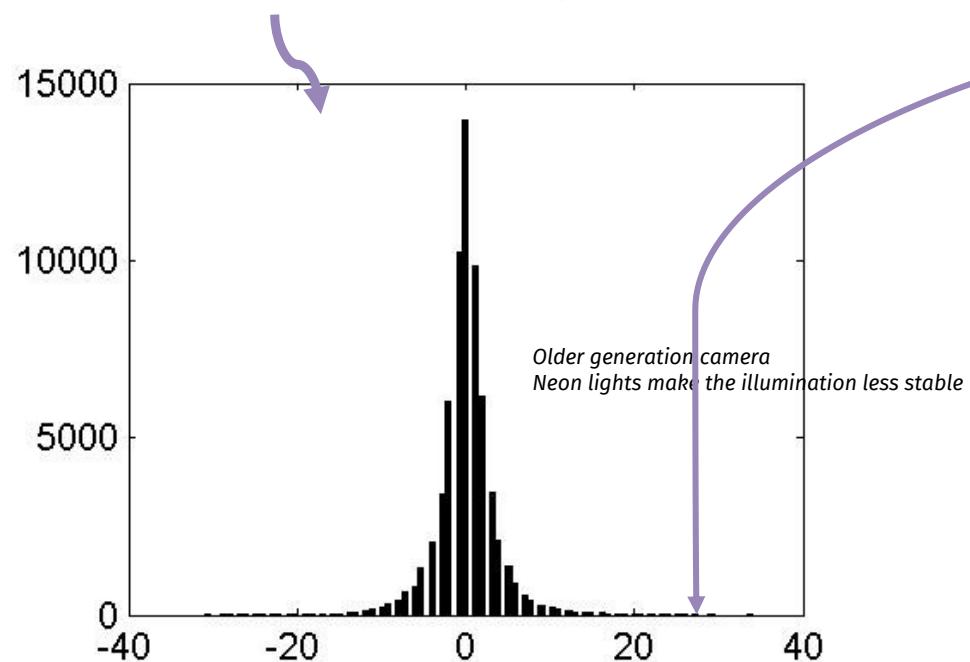
*Histogram of the difference between 2 apparently identical images  
(two adjacent frames of an empty scene)*



# The threshold

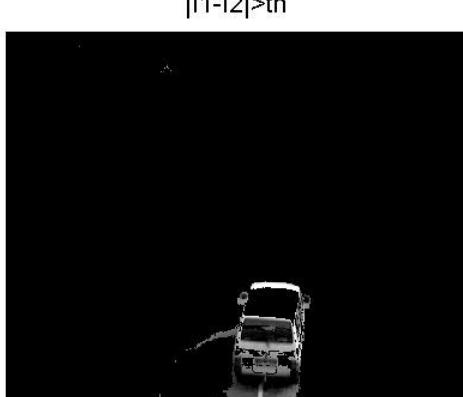
 $I_t$  $I_{t+1}$ 

*Histogram of the difference between 2 apparently identical images  
(two adjacent frames of an empty scene)*



# The reference image

the simplest way of detecting moving objects is to compare consecutive (or close) frames



In general it is not a good choice

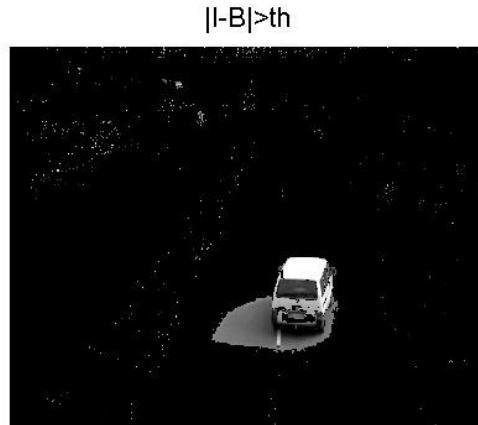
Artifacts

Difficult to detect slow motion

Noisy localization

# The reference image

The reference image or, more in general the *background model*, is a “picture” of the empty scene, containing all the parts which are not moving



# change detection

## The reference image – version 1

The reference image is a “picture” of the empty scene, containing all the parts which are not moving

The simplest way of computing it is to average the first N frames (assuming that at the beginning the scene is stationary)

$$I_{ref} = B = \frac{1}{N} \sum_{t=1}^N I_t$$

# Change detection algorithm

## Version 1

INPUT – image sequence  $\{I_0, \dots, I_T\}$

Initialize  $I_{ref}$  with the first N images

$$I_{ref} = B = \frac{1}{N} \sum_{t=1}^N I_t$$

For each new frame  $I_t \in \{I_{N+1}, \dots, I_T\}$

**detect changes:**

$$M_t(x, y) = \begin{cases} 1 & \text{if } |I_t(x, y) - I_{ref}(x, y)| > \tau \\ 0 & \text{otherwise} \end{cases}$$

OUTPUT – binary maps sequence  $\{M_{N+1}, \dots, M_T\}$

### PROS

- Very simple

### CONS

- It assumes the background will not change in all the observation span (almost impossible in practice!)

*Look at the door!*



# Change detection algorithm

## Version 2 – running average

INPUT – image sequence  $\{I_0, \dots, I_T\}$

Initialize the background  $B_0$  as in version 1

For each new frame  $I_t \in \{I_{N+1} \dots I_T\}$

1. detect changes

$$M_t(x, y) = \begin{cases} 1 & \text{if } |I_t(x, y) - B_{t-1}(x, y)| > \tau \\ 0 & \text{otherwise} \end{cases}$$

2. Update the background

$$B_t(x, y) = \begin{cases} B_{t-1}(x, y) & \text{if } |I_t(x, y) - B_{t-1}(x, y)| > \tau \\ (1 - \alpha)B_{t-1}(x, y) + \alpha I_t(x, y) & \text{otherwise} \end{cases}$$

OUTPUT – binary maps sequence  $\{M_{N+1}, \dots, M_T\}$

NOTICE : this method deals with  
slowly changing background (e.g,  
illumination changing in the scene)



notice: the background model is  
updated at each frame

# Change detection

## The reference image – version 2 – running average

$$B_t(x, y) = \begin{cases} B_{t-1}(x, y) & \text{if } |I_t(x, y) - B_{t-1}(x, y)| > \tau \\ (1 - \alpha)B_{t-1}(x, y) + \alpha I_t(x, y) & \text{otherwise} \end{cases}$$

### PROS

- it is quite robust to moving objects in the scene
- it incorporates smooth stable changes  
(at a speed which is proportional to  $\alpha$ )
- it is simple and computationally efficient



### CONS

- it does not deal with repetitive (and uninteresting) motion



# Change detection algorithm

## Version ... 3 – running average

INPUT – image sequence  $\{I_0, \dots, I_T\}$

Initialize the background  $B_0$  as in version 1

For each new frame  $I_t \in \{I_{N+1}, \dots, I_T\}$

1. detect changes

$$M_t(x, y) = \begin{cases} 1 & \text{if } |I_t(x, y) - B_{t-1}(x, y)| > \tau \\ 0 & \text{otherwise} \end{cases}$$

2. Update the background

$$B_t(x, y) = \begin{cases} B_{t-1}(x, y) & \text{if } |I_t(x, y) - I_{t-\Delta t}(x, y)| > \tau' \\ (1 - \alpha)B_{t-1}(x, y) + \alpha I_t(x, y) & \text{otherwise} \end{cases}$$

OUTPUT – binary maps sequence  $\{M_{N+1}, \dots, M_T\}$

With this version we try to deal with abrupt persistent variations (objects moved, a blade of light that enters, ...)



notice: the background model is updated at each frame

# Motion segmentation

## General case

If the camera is moving, we need to consider the presence of a **dominant motion (the ego motion)**

A possible approach is to first obtain a rough estimate of the ego motion and then compensate it to generate a synthetic datum *as if the camera were still*

Much harder and computationally intensive (as an alternative we often look for semantic info: people detection, car detection, ...)

# blob tracking



## Blob detection

- $M_t$  is a binary map where all pixels are still individual entities
- To obtain larger "entities" we compute **connected components** (objects or parts of objects) which we normally refer to as "**blobs**"
- Connected components are image segments where each pair of pixel can be connected by a path remaining inside the segment

## Blob descriptors

- Position
- velocity (direction, magnitude)
- shape descriptors (area, perimeter, aspect ratio, higher order moments)
- texture, color, ...

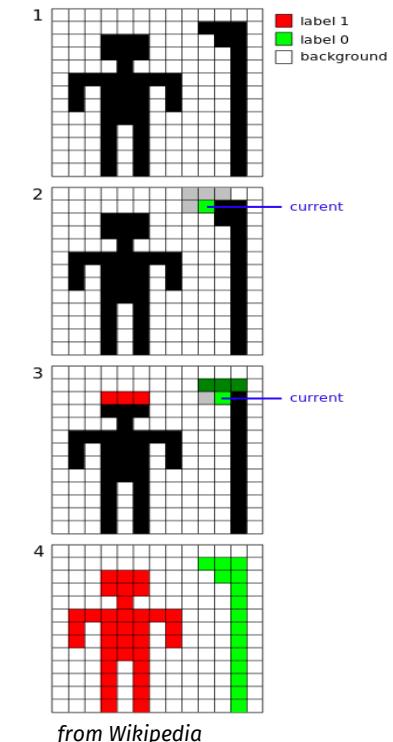


## Blob tracking

The goal of tracking is to associate each blob at time  $t+1$  with one of the blobs observed in previous instants  $\{0, \dots, t\}$

The two main ingredients are

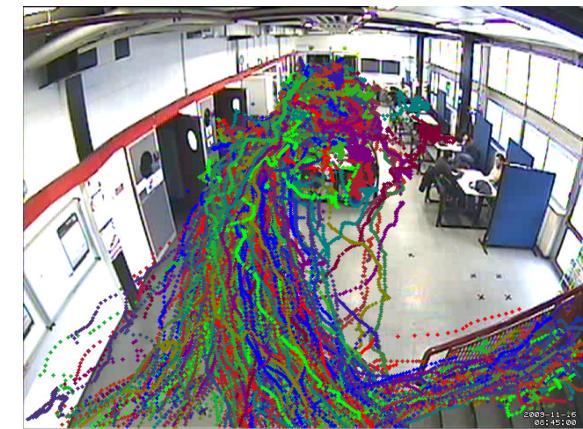
- similarity between blobs (in adjacent time instants): we may extend what we know on features matching
- temporal continuity (dealing with temporary gaps and integrating observations along longer time frames). This is related to dynamic filtering



# from tracking to temporal sequences

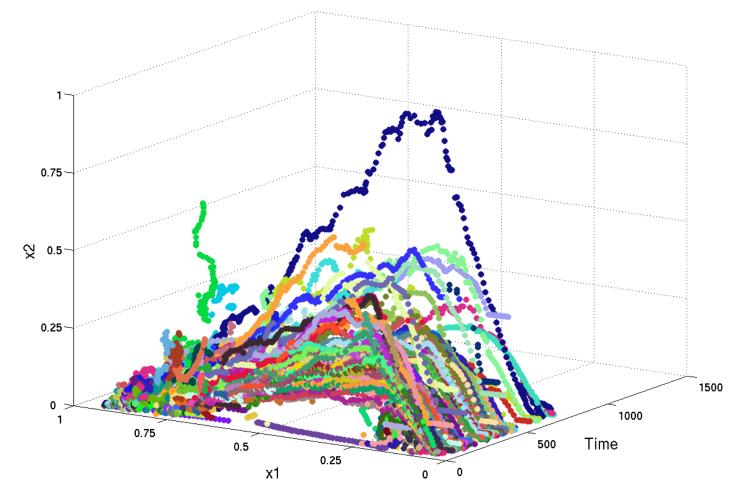


After a few hours we may  
Gather quite a lot of measurements



for each moving object in the scene we  
may collect a temporal sequence of  
instantaneous observations

$$\{\mathbf{x}_i\}_{i=1}^N \text{ with } \mathbf{x}_i = (x_i^1, x_i^2, \dots, x_i^{k_i})^\top$$



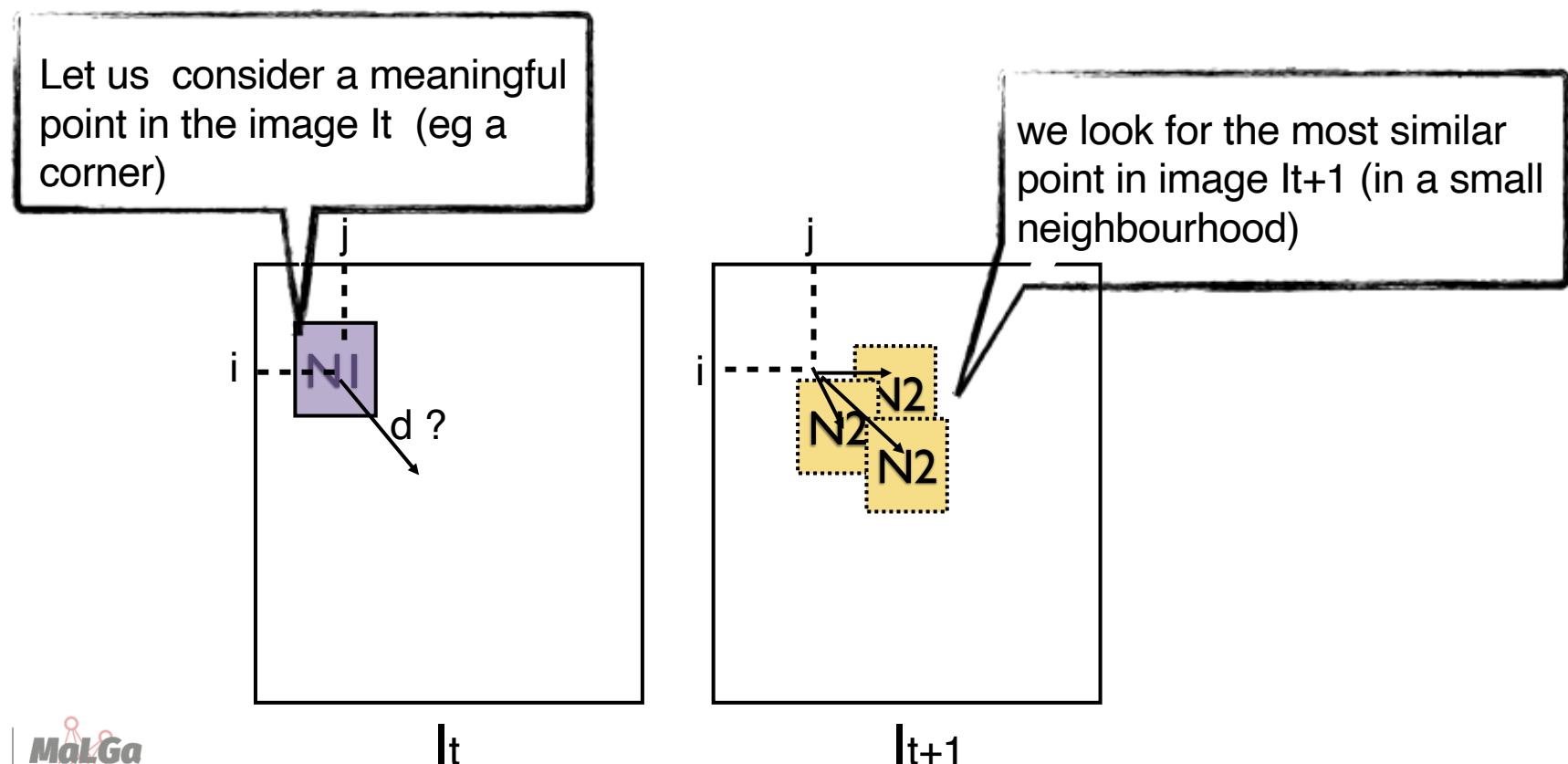
instantaneous observation (may include  
different types of information, such as position,  
shape, speed, ...)

# Correspondence: optical flow

# Motion analysis as a correspondence problem

correspondence: which elements of a frame correspond to which elements of the next frame?

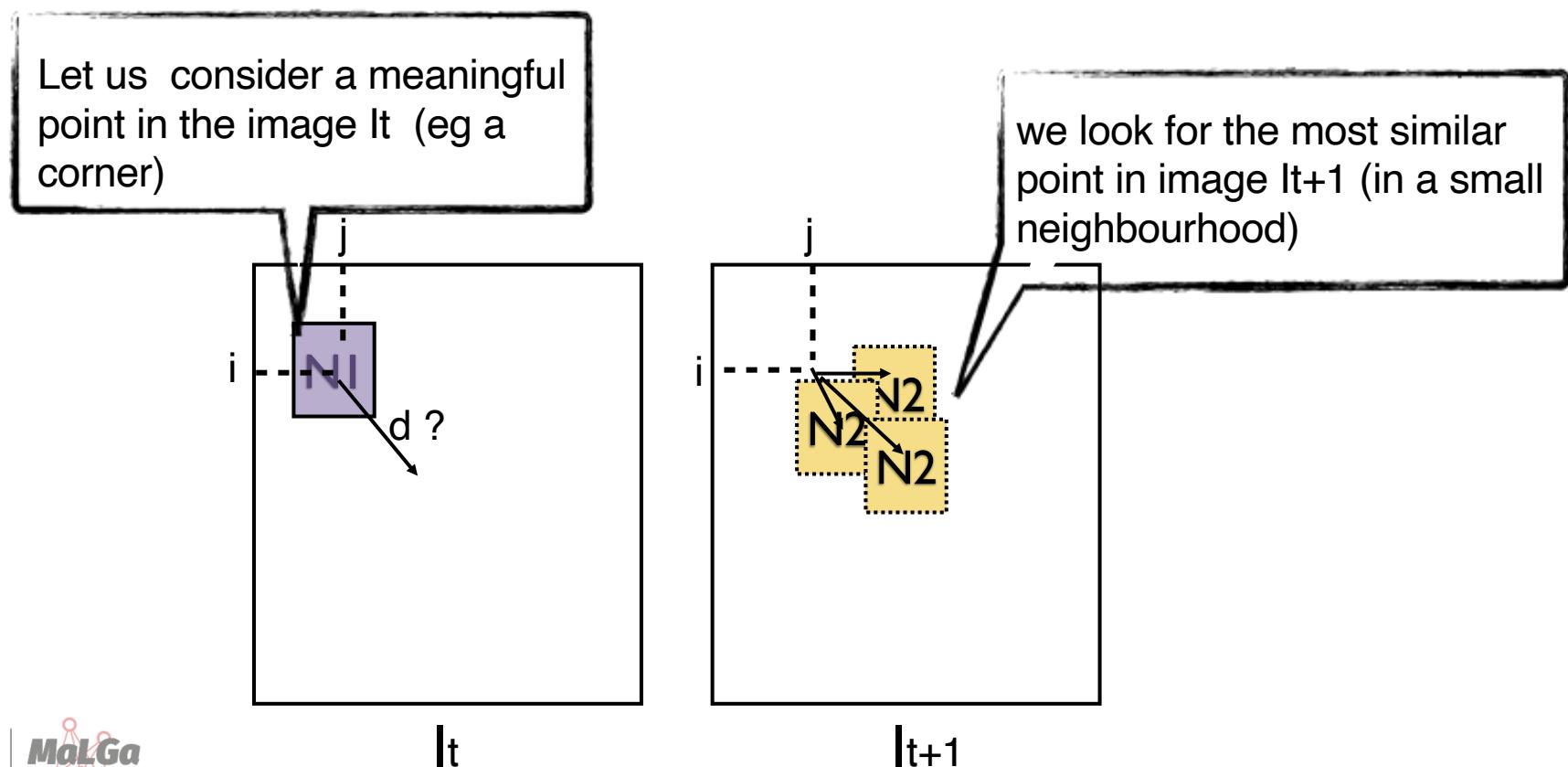
We take advantage of the high similarity between adjacent frames



# Motion analysis as a correspondence problem

We take advantage of the high similarity between adjacent frames

1. Correlation-based approaches
2. Gradient-based approaches



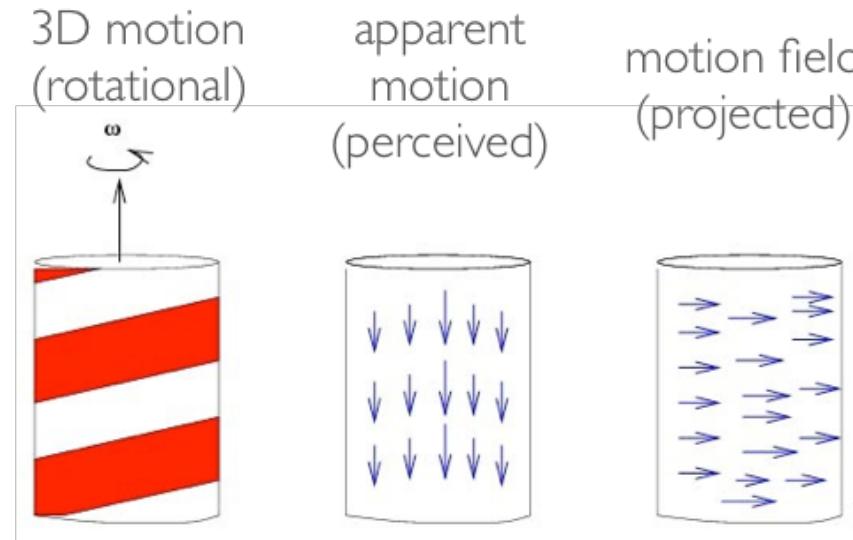
# Motion analysis as a correspondence problem

In the case of motion, correspondence may also be seen as a problem of *estimating the apparent motion of the brightness pattern* (the so called **optical flow**)

**Why "apparent"?** Because it's not the real motion, but the one we perceive

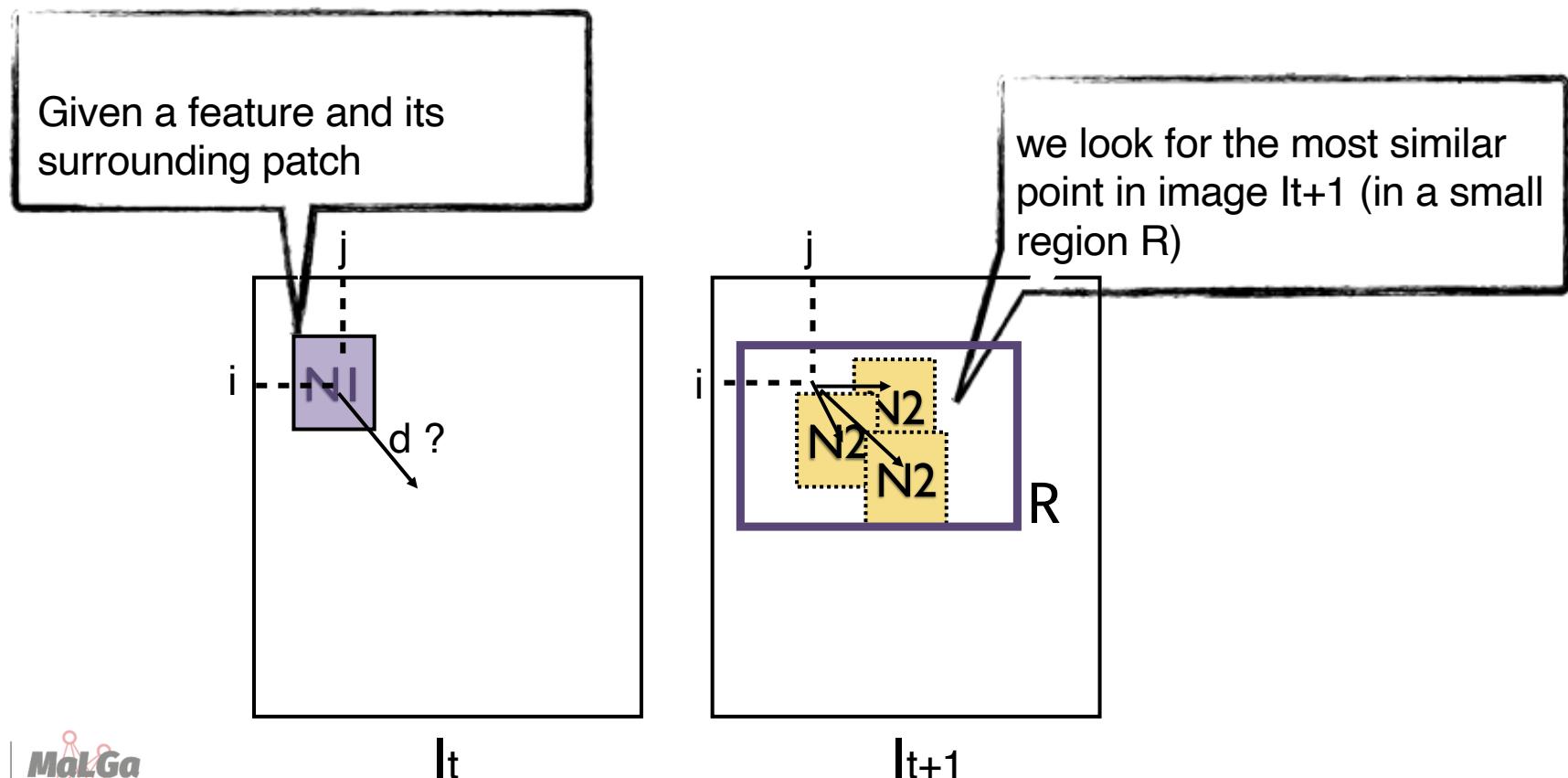
**Example1.** A ball is bouncing in a dark room: what do we perceive? Nothing!

**Example2:** the barber pole



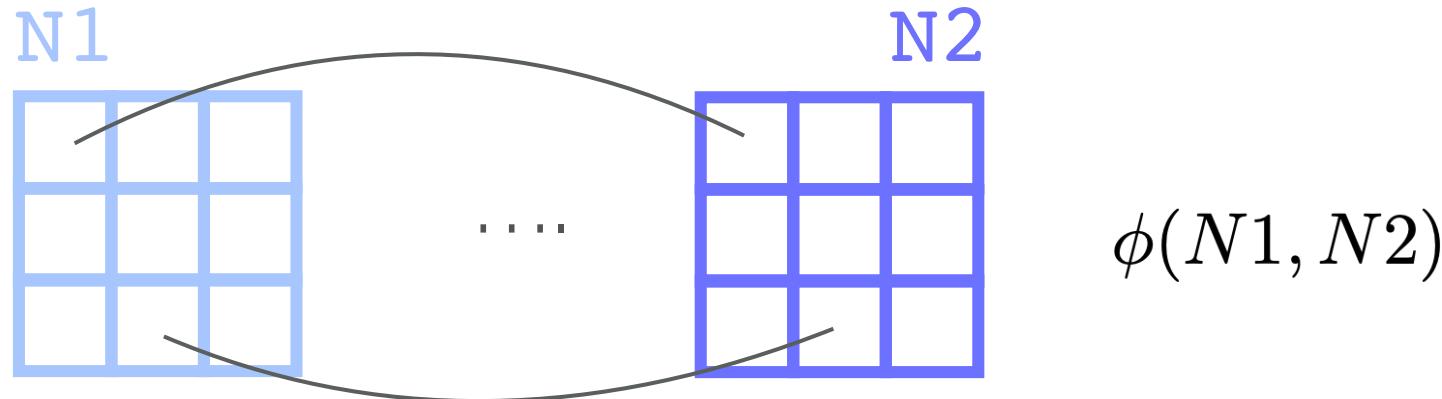
# Motion analysis as a correspondence problem

## Correlation-based approaches



# Correlation-based approaches

We can measure similarity between pairs of patches in a pixel-wise manner



# Similarity measures for image patches

## SUM OF SQUARED DIFFERENCES

$$\phi_{SSD}(N1, N2) = - \sum_{k,l=-\frac{W}{2}}^{\frac{W}{2}} (N1(k, l) - N2(k, l))^2$$

## NORMALIZED CROSS CORRELATION

$$\phi_{NCC}(N1, N2) = - \sum_{k,l=-\frac{W}{2}}^{\frac{W}{2}} \frac{(N1(k, l) - \mu_1)(N2(k, l) - \mu_2)}{W^2 \sigma_1 \sigma_2}$$

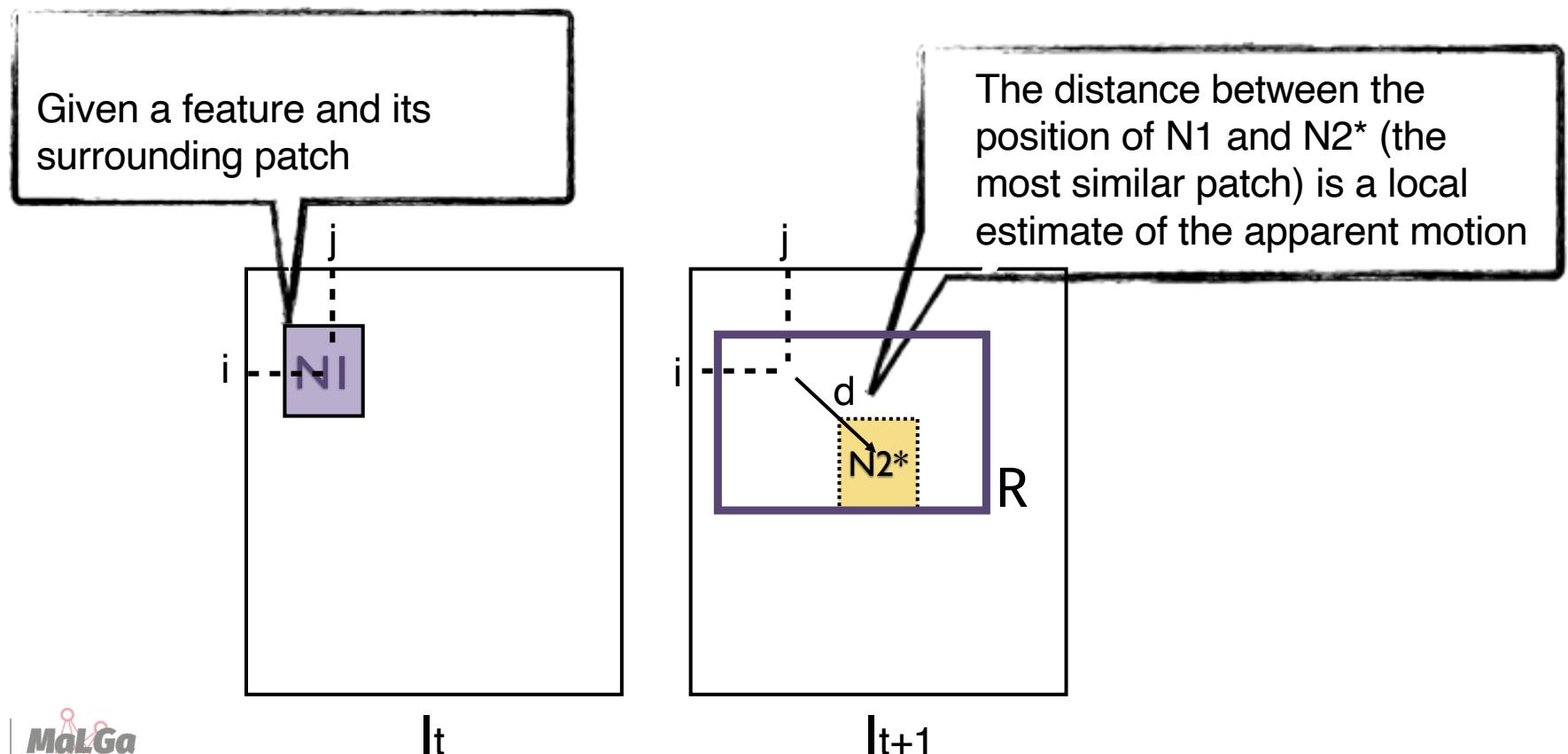
$$\mu_i = \frac{1}{W} \sum_{k,l=1}^W Ni(k, l)$$

NCC: values in  
the range [-1, 1]

$$\sigma_i = \sqrt{\frac{1}{W} \sum_{k,l=1}^W (Ni(k, l) - \mu_i)^2} \quad \text{with } i = 1, 2$$

# Motion analysis as a correspondence problem

## Correlation-based approaches



# Motion analysis as a correspondence problem

## Gradient-based derivation

It's based on the image brightness constancy equation: from one frame to the next the image appearance does not change

$$I(x, y, t) = I(x + u, y + v, t + 1)$$

If the motion is small, we can use the following Taylor approximation

$$I(x + u, y + v, t + 1) = I(x, y, t) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} + H.O.$$

We thus obtain  $\frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} = 0$

Or more compactly

$$I_x u + I_y v + I_t = 0$$

$$(\nabla I)^\top \mathbf{u} + I_t = 0$$

# Optical Flow

## Gradient-based derivation

The optical flow is a vector field subject to the constraint

$$(\nabla I)^\top \mathbf{u} + I_t = 0$$

This constraint gives us 1 equation per each image point

Notice that one constraint is not enough to compute the optical flow (2 unknowns)

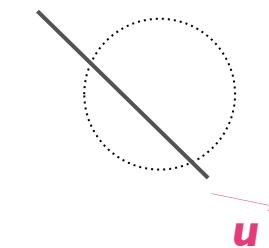
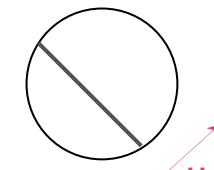
# Optical Flow

## The aperture problem

the image brightness constancy equation allows us to determine the optical flow component parallel to the spatial image gradient (normal flow)

Analytically

$$u_n = \frac{(\nabla I)^\top \mathbf{u}}{\|\nabla I\|} = \frac{-I_t}{\|\nabla I\|}$$



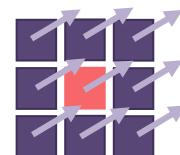
# Optical Flow Algorithms

## Lucas Kanade Algorithm

many algorithms start from the idea of adding constraints to the underdetermined system obtained by the brightness constancy equation

we will see a simple way of doing so: the Lucas-Kanade algorithm

- assumption:  $\mathbf{u}$  is constant in a small neighbourhood of a point

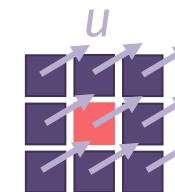


# Optical Flow Algorithms

## Lucas Kanade Algorithm

the assumption allows us to obtain a system of equations with one equation for each point in the neighbourhood

$$(\nabla I(\mathbf{x}_i, t))^\top \mathbf{u} + I_t(\mathbf{x}_i, t) = 0 \quad \mathbf{x}_i \in N$$



we then obtain a linear system  $\mathbf{Au}=\mathbf{b}$  with

$$A = \begin{bmatrix} \nabla I(\mathbf{x}_1, t)^\top \\ \nabla I(\mathbf{x}_2, t)^\top \\ \vdots \\ \vdots \\ \nabla I(\mathbf{x}_m, t)^\top \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -I_t(\mathbf{x}_1, t) \\ -I_t(\mathbf{x}_2, t) \\ \vdots \\ \vdots \\ -I_t(\mathbf{x}_m, t) \end{bmatrix}$$

m elements  
in the N  
neighbour.

# Optical Flow Algorithms

## Lucas Kanade Algorithm

The linear system may be solved with the pseudo-inverse

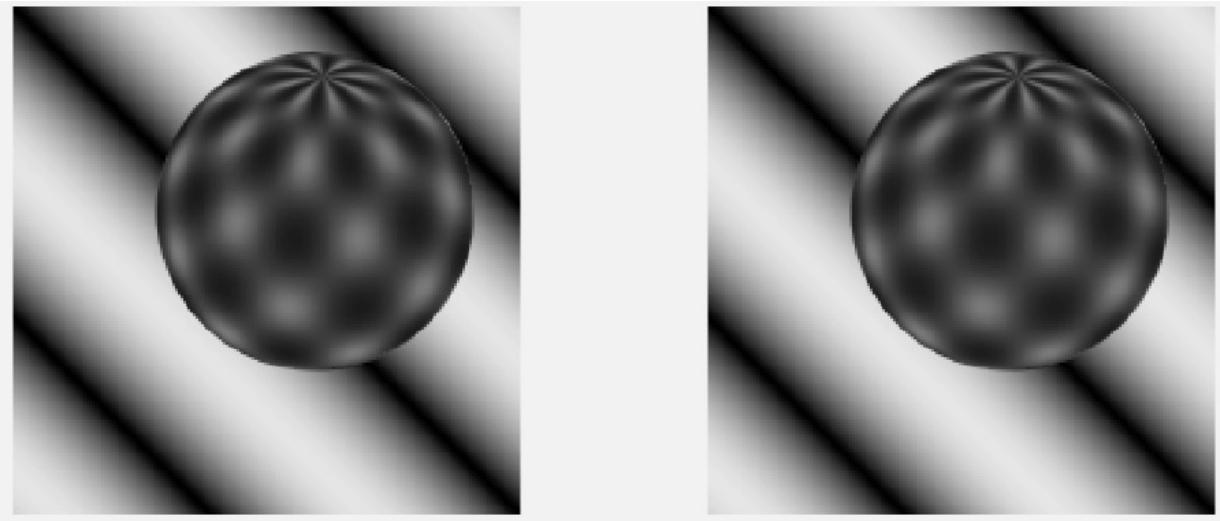
$$\mathbf{u} = A^\dagger \mathbf{b} \quad \text{with} \quad A^\dagger = (A^\top A)^{-1} A^\top$$

notice that the inversion of the matrix will be ill-posed if the matrix is not full rank

The matrix is full rank in the proximity of corners (points who do not suffer from the aperture problem)

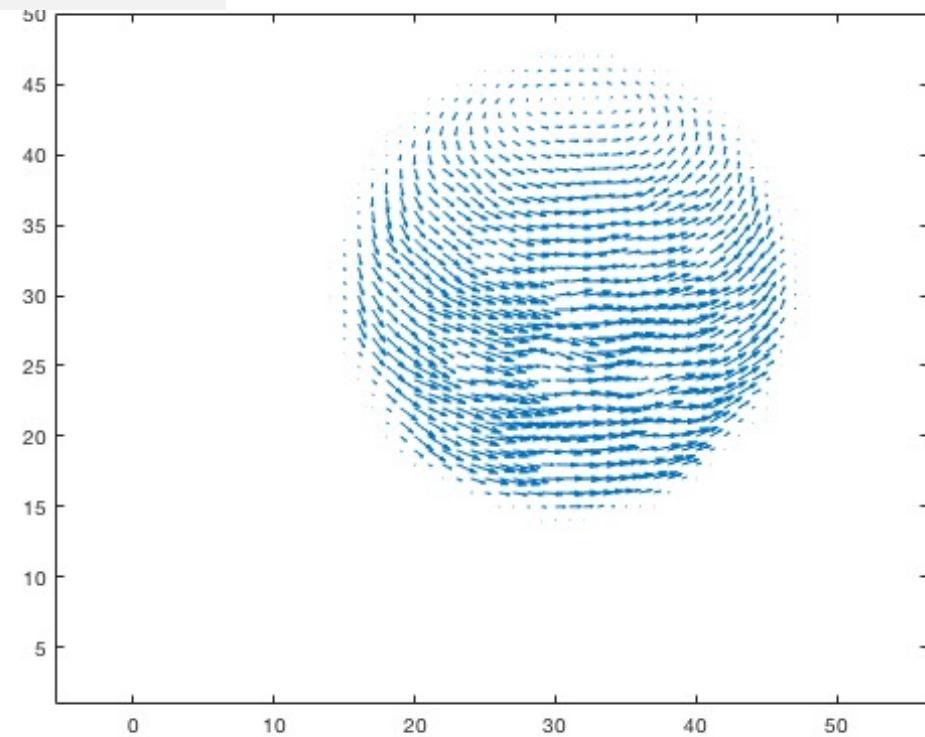
This is why often times Lucas Kanade is implemented as a *sparse* algorithm (after a corner detection stage) – see for instance OpenCV implementation

# Lucas Kanade example

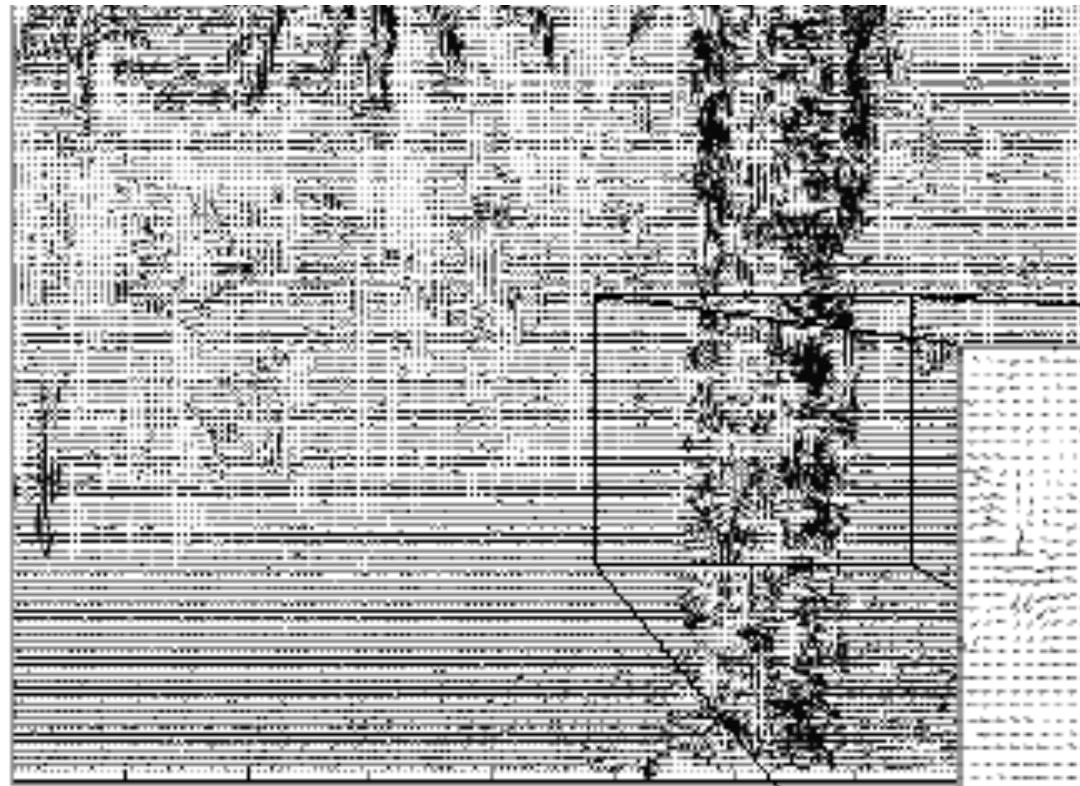


Lucas Kanade on a synthetic highly textured sequence of a slowly rotating sphere

↑  
*Small displacements!*



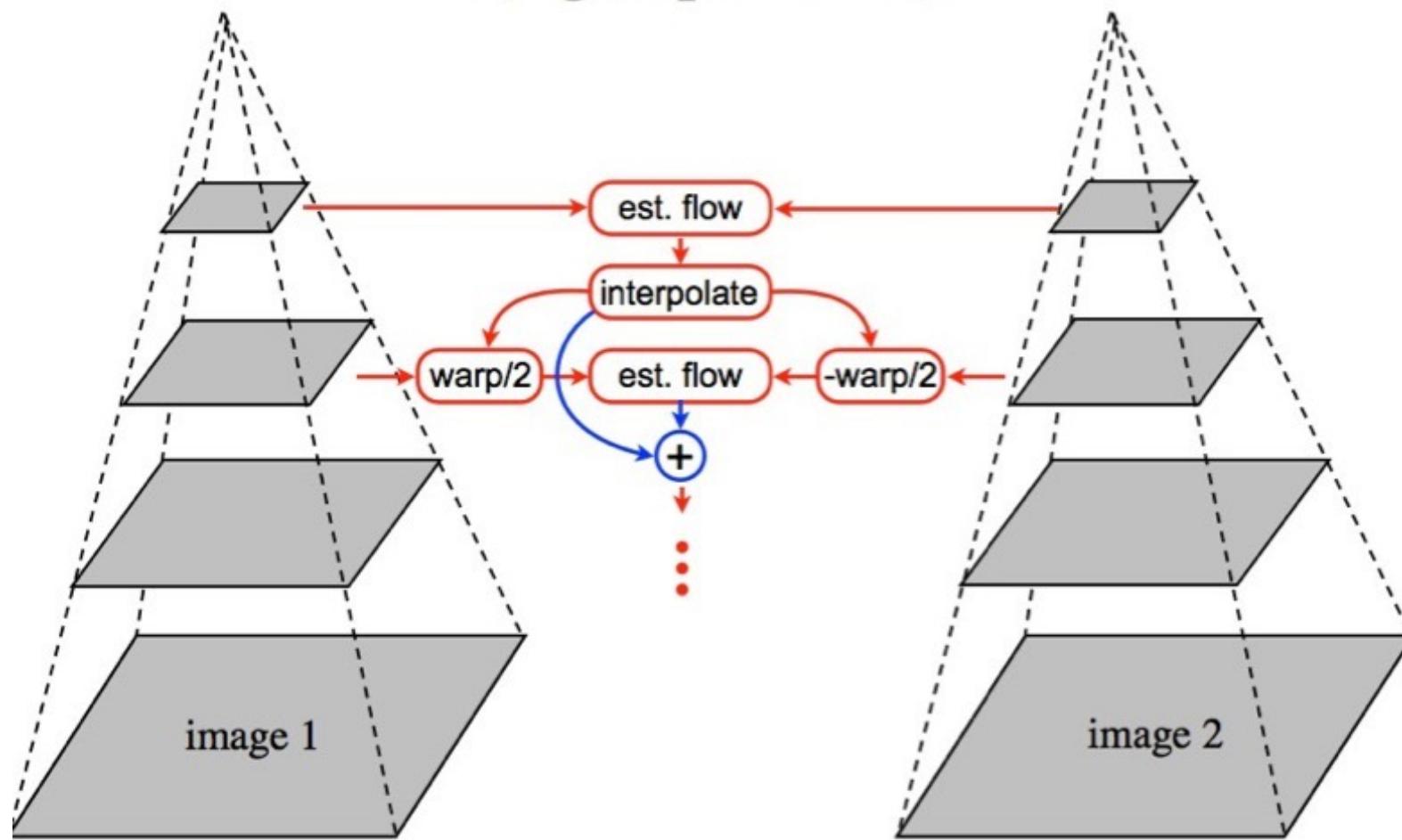
# Lucas Kanade example



*Fails in the case of large motion*



# coarse to fine estimation

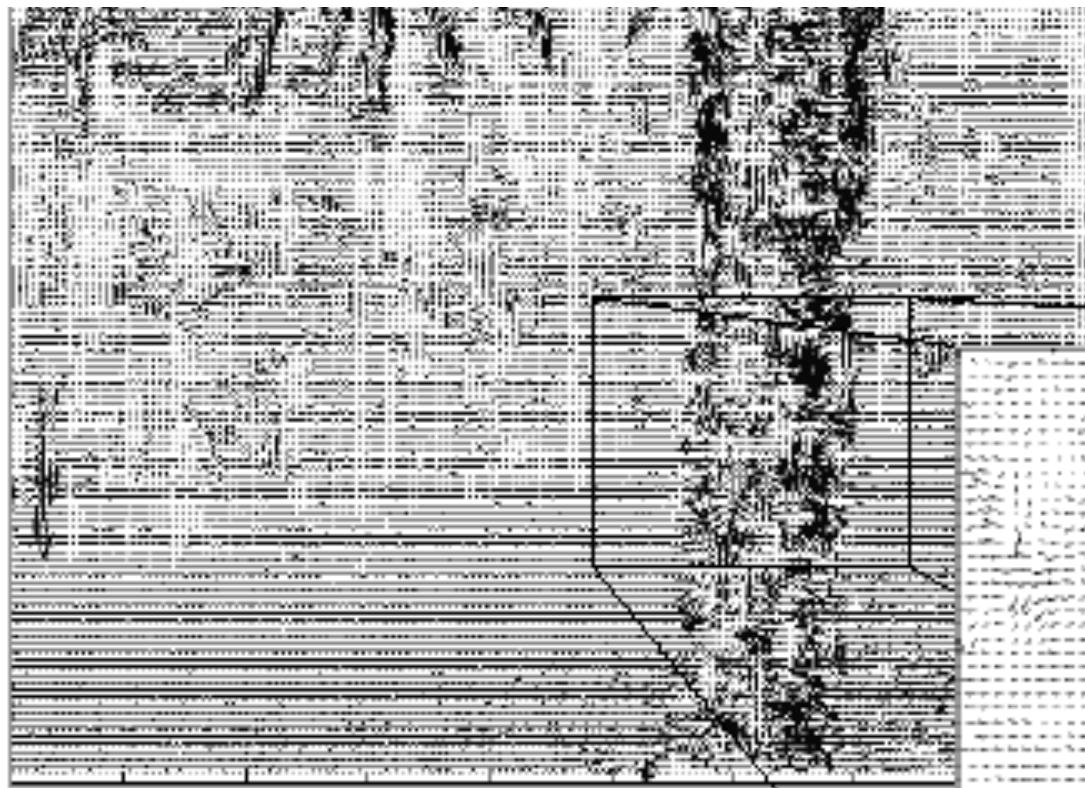


# Coarse to fine estimation algorithm

At each level  $i$

- Take flow estimated in the previous level:  $u(i-1)$
- Upsample the flow and create a first estimate of  $u(i)$   
(double the size of the matrices and multiply the values by 2)
- Warp  $I(t)$  w.r.t  $u(i)$  and apply LK to  $I_{\text{warp}}(t)$  and  $I(t+1)$  obtaining a correction  $u'(i)$
- Update  $u(i)$  by adding  $u'(i)$

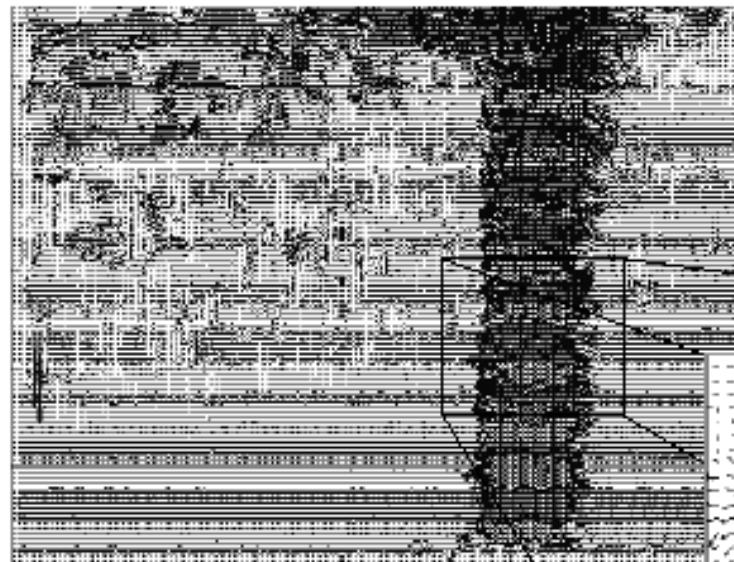
# Lucas Kanade example



Fails in areas of large motion



# Multi-resolution Lucas Kanade example

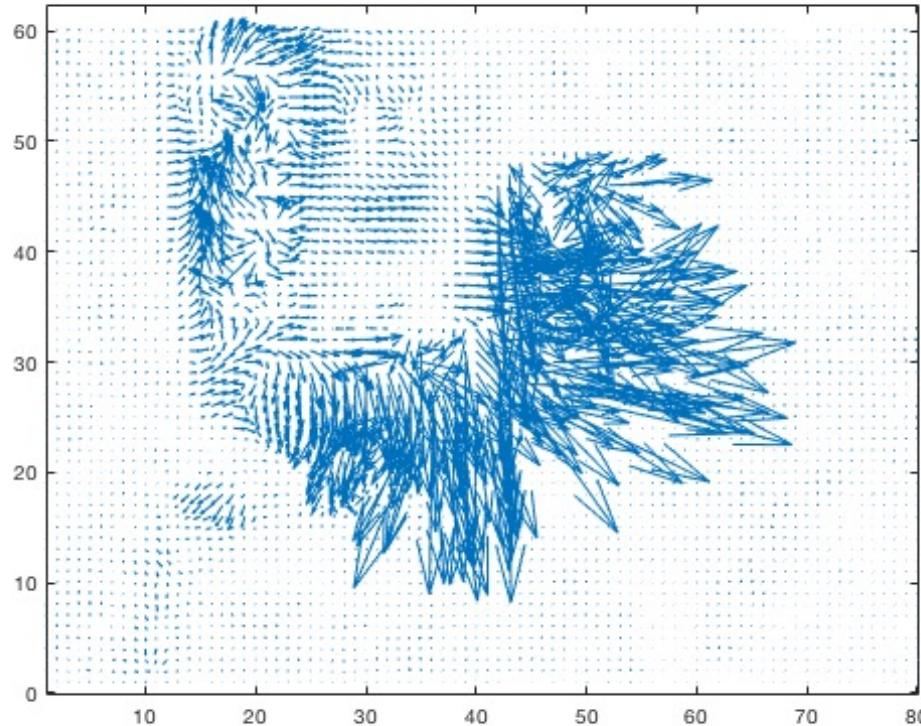


Lucas-Kanade with Pyramids

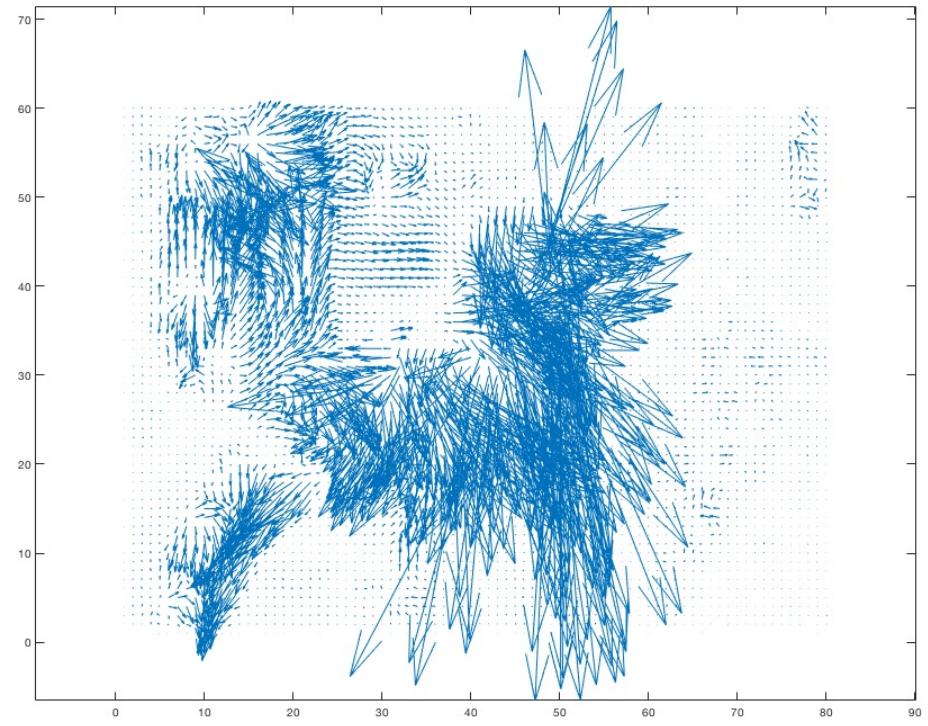
# lucas kanade another example (less texture)



Lucas Kanade



Lucas Kanade hierarchical version



# lucas kanade - sparse implementation (opencv)

Optical flow estimated only on points not suffering from the aperture problem (corners)

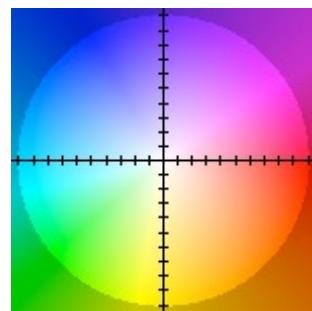


Port: 23,0 (min:16,7 max:39,3) fps

Display: 10,0 (min:9,8 max:10,0) fps

/objectViewerLeft

# example - dense estimates



these results have been obtained with the **Färneback method** available in OpenCv

this method applies a more complex parametrization (polynomial) over the pixel neighbourhood,  
but still maintain a very high efficiency.

# **Correspondence over time: feature tracking**

# Feature tracking: multi-frames algorithm

input:

- a video sequence  $I(0), I(1), \dots, I(T)$

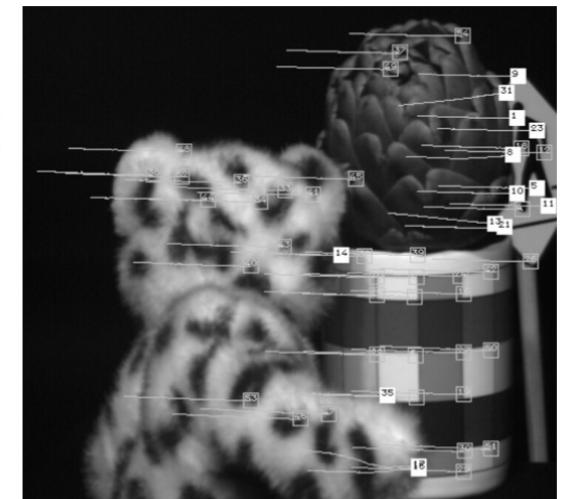
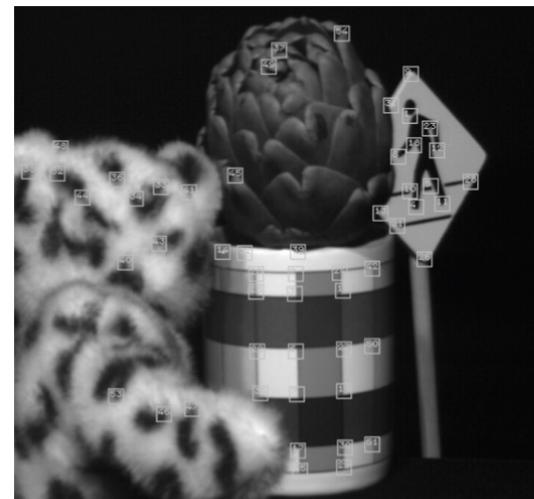
Here we only extract keypoints on the first frame (we could add subsequent extractions)

Extract corners from frame  $I(0)$ :  $c_i$  with  $i=1, \dots, N$

for each pair of adjacent frames  $I(t) I(t+1)$

- for each corner  $c_i(t)$  on  $I(t)$ 
  - Estimate its optical flow and obtain an updated position  $c_i(t+1)$
  - Update the  $c_i$  track

Output:  $N$  corner tracks

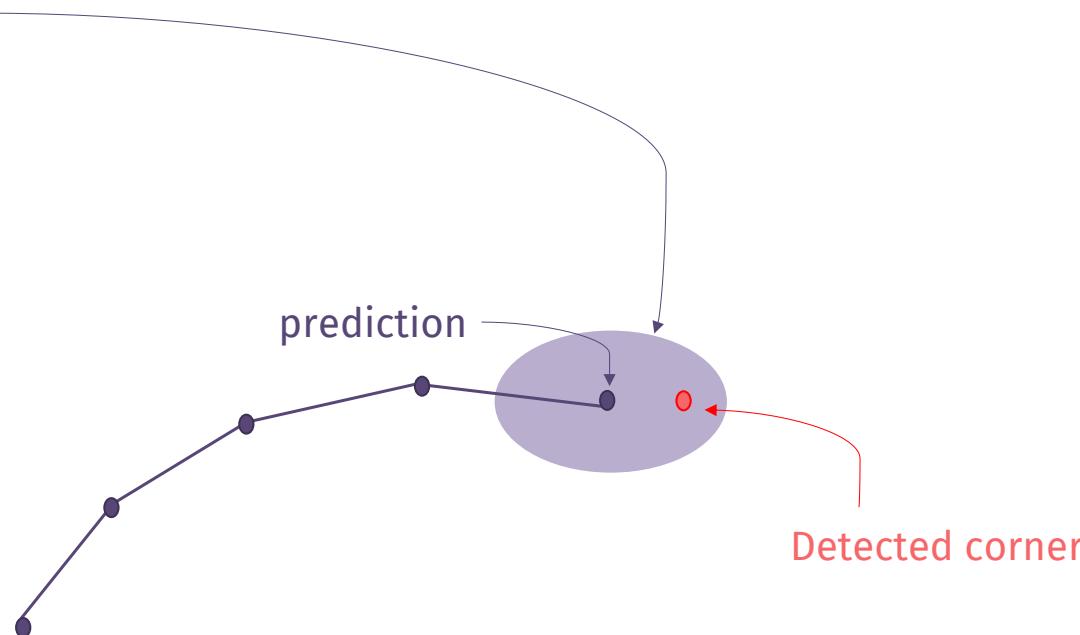


# Feature tracking: the role of Kalman filter

Kalman filter allows us to smooth the noisy trajectories

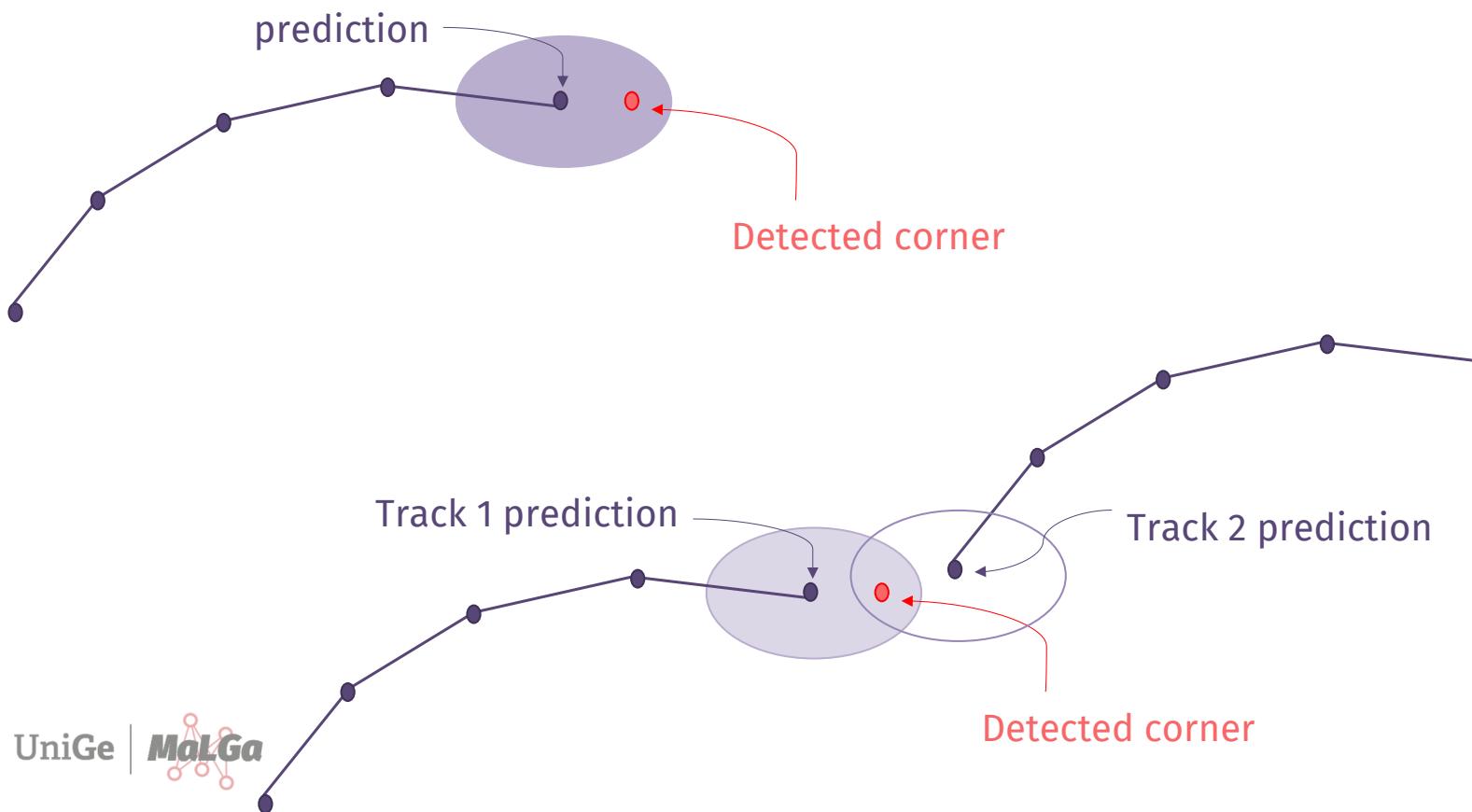
Further, it can be used to fill trajectory gaps: we may use the estimated state as a hypothesis of the missing measurement

The  $P_t$  state covariance matrix provides us with a measure of the state uncertainty ellipse



# Feature tracking: data association problem

When we are tracking multiple corners, we often face data association problems



# UniGe

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