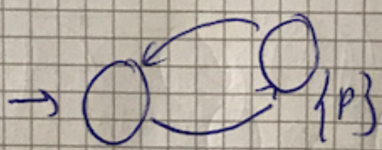
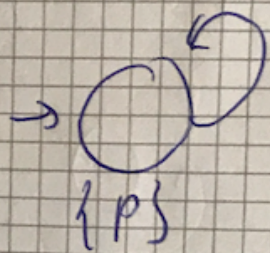
Esercizio 4

1.

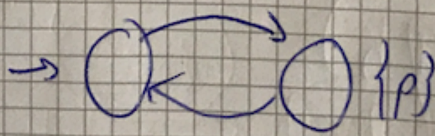
i	0	1	2	3	4	5	6
σ_i	$\emptyset$	$\{p\}$	$\{p,q\}$	$\{q\}$	$\{p\}$	$\emptyset$	$\{p,q\}$
$p \wedge q$	F	F	T	F	F	F	T
$F(p \wedge q)$	T	T	T	T	T	T	T
$p \, U \, q$	F	T	T	T	F	F	T

2.1. $GF\,p$

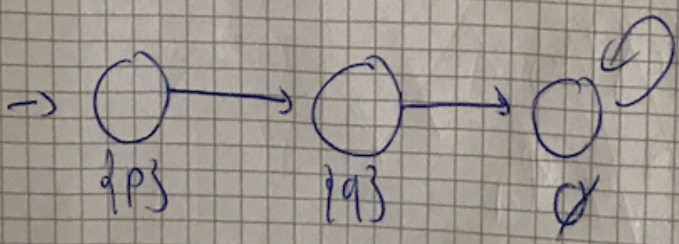




2.2. $GF\,p \wedge GF\,\neg p$



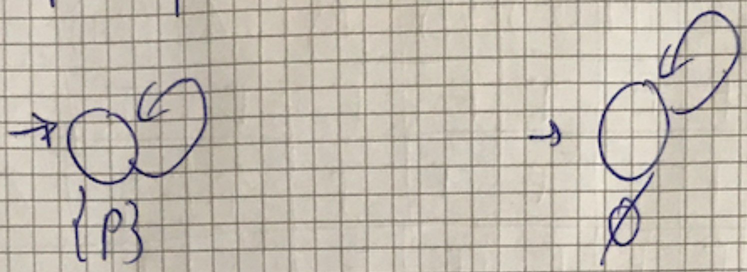
2.3 $(p \, U \, q) \wedge (p \, U \neg q)$



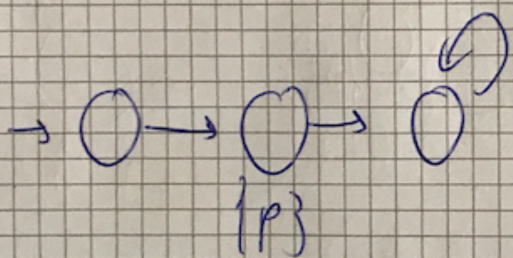
2.4 $G p \wedge G \neg p$

↳ Non può essere soddisfatta

2.5 $G p \vee G \neg p$



2.6. $F p \wedge F \neg p$



3. $\varphi_1 = G(b \Rightarrow XF b)$

$\Pi_1 \not\models \varphi_1$ Si on prend la trace

σ: $\{a,b\}\ \{c\}\ \{c\} \ldots$

$\sigma \not\models \varphi_1$

Mais $\Pi_2 \models \varphi_1$ car depuis 1 toutes les traces voient un jour b et depuis 2 où il y a b, on retourne à 1.

3. $\varphi_1: G(b \Rightarrow XFb)$

$\Pi_1 \not\models \varphi_1$ Se prendiamo la traccia

$\pi = \{a,b\}\{c\}\{c\} \dots \{c\} \dots \not\models \varphi_1$

~~Pa~~ $\Pi_2 \models \varphi_1$ con da 1' tutte le traccie vedono b

e da 2', ritorna in 1'

$\varphi_2: a \cup b$

$\Pi_1 \models \varphi_2$ perché tutte le traccie iniziano con $\{a,b\}$

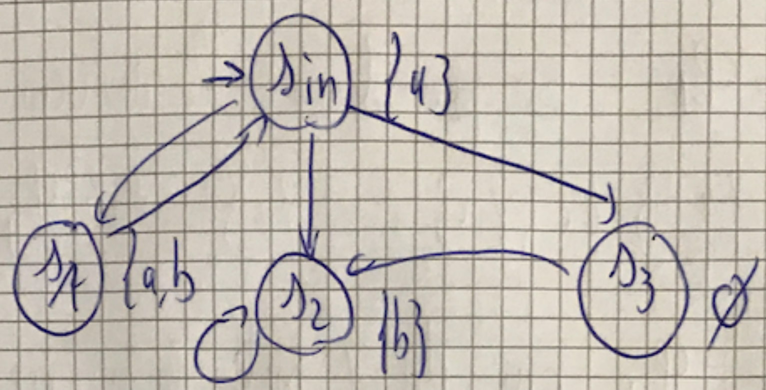
$\Pi_2 \not\models \varphi_2$ perché abbiamo la traccia $\{a\} \emptyset \{b\}\{b\} \dots \{b\} \dots \not\models \varphi_2$

$\varphi_3 = GFc$

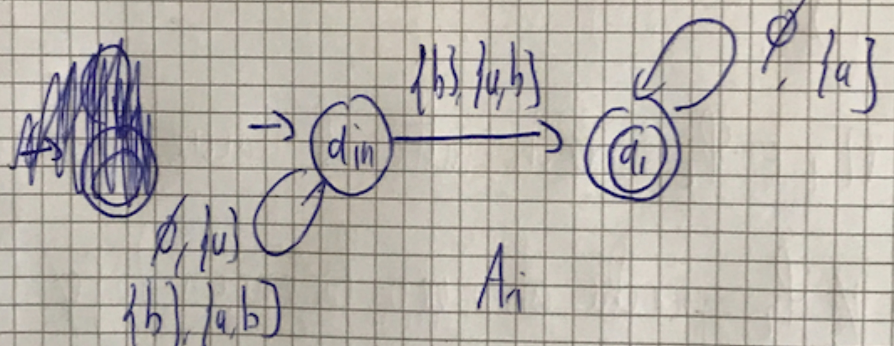
$\Pi_1 \models \varphi_3$ perché tutte le traccie vedono infinitamente spesso o $\{b,c\}$ o $\{c\}$

$\Pi_2 \not\models \varphi_3$ perché c non c'è in Π_2!

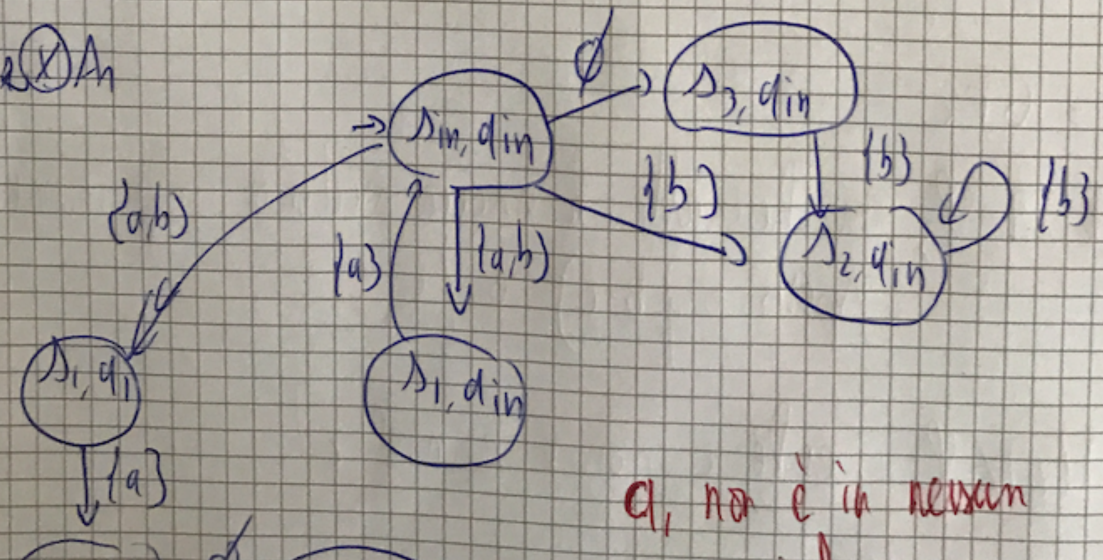
Dimostriamo più formalmente perché $M_2 \models \varphi_1$.



Sia A_1 t.c. $\mathcal{L}_\omega(A_1) = (2^{PA})^\omega \setminus Seq(\varphi_1)$ (Prendiamo $PA = \{a,b\}$).



$M_2 \otimes A_1$



q_1 non è in nessun ciclo

$\Rightarrow M_2 \models \varphi_1$