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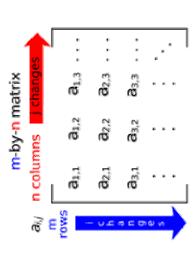
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# **Quick Tutorial in Matrices**

entries), which may be numbers or, more generally, any abstract quantities that In mathematics, a matrix (plural matrices) is a rectangular table of elements (or can be added and multiplied. Matrices are commonly used to describe linear equations.



columns. A matrix with m rows and n columns is called an m-by-n matrix (written The horizontal lines in a matrix are called rows and the vertical lines are called m x n) and m and n are called its dimensions. The dimensions of a matrix are always given with the number of rows first, then the number of columns.



# **Quick Tutorial in Matrices**

#### Matrix addition

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 5 \\ 7 & 5 & 0 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 & 1+5 \\ 1+7 & 0+5 & 0+0 \\ 1+2 & 2+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 8 & 5 & 0 \\ 3 & 3 & 3 \end{bmatrix}.$$

#### Matrix multiplication

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (1 \times 3 + 0 \times 2 + 2 \times 1) & (1 \times 1 + 0 \times 1 + 2 \times 0) \\ (-1 \times 3 + 3 \times 2 + 1 \times 1) & (-1 \times 1 + 3 \times 1 + 1 \times 0) \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & 2 \end{bmatrix}$$

 $3 \times 2$ 

2 × 3

$$\begin{bmatrix}
1 & 0 & 2 \\
-1 & 3 & 1
\end{bmatrix} = \begin{bmatrix}
(1 \times 3 + 0 \times 2 + 2 \times 1) & (1 \times 3 + 3 \times 2 + 1 \times 1) & (-1 \times 3 + 3 \times 2$$

1) 
$$(1 \times 1 + 0 \times 1 + 2 \times 0)$$
  
  $\times 1)$   $(-1 \times 1 + 3 \times 1 + 1 \times 0)$ 

 $2 \times 2$ 



# **Quick Tutorial in Matrices**

The following is a system of equations with two equations and two unknowns.

$$2 x + 5 y = 16$$

$$x + 3 y = 9$$

This can be rewritten in matrix form

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{cases} 16 \\ 9 \end{Bmatrix}$$
$$\begin{cases} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{Bmatrix} 16 \\ 9 \end{Bmatrix}$$
$$= \begin{cases} 3 \\ 2 \end{cases}$$



### Offensive Power Rating

("sw293") in his April 2006 posting. I think he first coined this term OPR and explained how it is calculated in the Chief Delphi post: From the Chief Delphi forum, the earliest I found the use of the http://www.chiefdelphi.com/forums/showpost.php?p=484220&postcount=19 term Offensive Power Rating (OPR) was by Scott Weingart

calculation and called it Calculated Contribution. It seems that he had been using that number even before April 2006 but never Karthik Kanagasabapathy from Team 1114 did the same published it until 2008.

implemented the calculation of OPR from "sw293" and published a lot of results on Chief Delphi before the Championship in 2008. "Bongle" from Team 1281 and Guy Davidson from Team 5



### **How to Calculate OPR?**

Assume team i, j and k are three teams in an alliance and they scored p points in that match. Then we can write  $x_i + x_j + x_k = p$ , where  $x_i$  is the score contributed by team i

Assume team i played with team m and n in another alliance and they score q points in that match. Then we can write

$$x_{i} + x_{m} + x_{n} = q$$

If we add all the matches that team i was involved in, we get

$$2x_i + x_j + x_k + x_m + x_n = p + q = B_i$$

number of teams in that regional, and repeat that for each team, If we put them in row i of an N x N matrix A, where N is the total we get



### **How to Calculate OPR?**

$$2x_i + x_j + x_k + x_m + x_n = p + q = B_i$$

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$$[A]\{x\} = \{B\}$$



### **How to Calculate OPR?**

contribution of each team to each of their alliance. The number is Since the matrix A is symmetric and positive definite, we can use Cholesky decomposition to solve for x. The result x is the known as the Offensive Power Rating of each team.



# A Proposed New Method

The drawback of the Offensive Power Rating is that it completely ignores the contribution of defense. Jay Lundy from Team 254 http://www.chiefdelphi.com/forums/showpost.php?p=733759&postcount=160 has proposed another method that takes into account both defense and offense. Please refer to Chief Delphi post

However it will result in a rectangular matrix which is harder to solve. Also the offense and defense numbers may be hard to interpret. Hence I am proposing a new method that takes into account both offense and defense directly and still have a symmetric and positive definite matrix.



# A Proposed New Method

each match rather than the points scored. So instead of adding up all the points of all the matches and put into B<sub>i</sub>, you add up all the Once you understand how to calculate OPR, it is fairly simple to calculate this new rating. It is based on the winning margin of winning margins and put into B<sub>i</sub>. I call this new rating CCWM which simply stands for Calculated Contribution to Winning Margın.

Notice that some teams have negative CCWM and if you add up all the CCWM of all the teams in the regional, you will get zero.



# A Proposed New Method

work is if your team is allowed to intentionally score points for your games where your team's Match Ranking Points is based on your opposing alliance's score, this should still be valid since you want unless there is a very big lead and you know you will win for sure. However in a two minutes game, after establishing a big lead, the amount of time left to intentionally score points for your opponent to score as many points as possible. The only time it does not This CCWM gives credit to teams that play good defense. In opponents' alliance. Even so, this does not occur very often s limited.



# Comparison between OPR and CCWM

Using 2008 Regional data, I found that CCWM is as good as OPR in terms of predicting the outcome of the elimination matches.

teams selected as alliances even though there are many factors CCWM seems to correlate better than OPR in terms of actual that affect how teams are selected.

play defense, the first pick should probably be one who can score For a game like the one in 2008 where there are only two balls to as many points as possible. Hence OPR can be a good criteria. For the second pick, I think that using CCWM will have a better chance than using OPR to unearth a gem that is overlooked by hurdle and the third team can either run laps to score points or other teams.

My conclusion is OPR and CCWM both have advantages and disadvantages. It depends on the game and how the match ranking points are scored.



and familiar with that format. Underneath that skin, everything was format with Karthik's database from Team 1114. I chose to make it look and feel similar not just because Karthik did a good job in designing it. I did it because a lot of people are already using it The user interface of the database I developed looks similar in developed independently. Here are a number of differences.

- 1) The color scheme is changed to blue because it is our school color.
- 2) The pick order in the alliance selection is calculated instead of relying on information from teams who were there to minimize error.
- performance instead of multiple world ranking based on multiple regional 3) Each team can have only one world ranking based on their best
- 4) A picture is added instead of information on other awards.
- 5) Both CCWM and OPR are reported.
- 6) Also contains sortable table of results of all teams that can be filtered.



#### Karthik Kanagasabapathy (Team 1114)

Team Number 217	Full Name	Ford Motor	Company/FA	NUC Roboti	cs America/B	&K Corporation	Ford Motor Company/FANUC Robotics America/B&K Corporation & Utica Community Schools	unity Schools	
	Nick Name	ThunderChickens	ckens						
1	Location	MI, USA							
Enter the team number	Division	Galileo							
here. Do not modify any	Regional 1	St Louis Regional	egional			Awards 1	GM Industrial Design Award	Design Award	
office cens.	Regional 2	Detroit Regional	ional						
	Regional 3	Great Lakes Regional	s Regional						
	Finish 1	Regional W	inner						
	Finish 2	Regional Winner	inner			Awards 2	GM Industrial Design Award	esign Award	
	Finish 3	Finalist							
	Record 1	6-2-1							
	Record 2	8-2-0							
	Record 3	5-3-0				Awards 3	GM Industrial Design Award	esign Award	
	Seed 1	9							
	Seed 2	m							
	Seed 3	16	76.2%						
	Draft Position	1 1st pick							
	Draft Position 2	2 1st pick							
	Draft Position 3	3 1st pick							
- (		Team	Regional	Regional Rank	Regional Percentile	Champs Rank	Champs Percentile	World Rank	World Percentile
	Average Offensive Score	1 57.1	41.4	9	88.9%	96	81.9%	127	93.5%
	Average Offensive Score 2	2 70.0	44.3	1	100.0%	28	94.9%	29	98.5%
	Average Offensive Score 3	3 77.8	47.4	5	93.7%	11	98.1%	11	99.5%
	Calculated Contribution 1	31.6	13.8	8	95.6%	82	84.6%	103	94.7%
	Calculated Contribution 2		14.8	-	100.0%	23	95.8%	24	98.8%
0	Calculated Contribution 3	50.7	15.8	1	100.0%	6	98.5%	9	89.6%



Team Number	Full Name		ny/FANUC Robotic	s America/B&	Ford Motor Company/FANUC Robotics America/B&K Corporation & Utica Community Schools	a Community Scho	ols	
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	Finish 2	Regional Winner						-
	Finish 3	Finalist						
	Finish 4	Regional Winner			V			
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	Seed 2	3 of 32						
	Seed 3	16 of 63				1		
	Seed 4	12 of 86			Salar Sa	TO ASSESSED TO	1	4
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	Alliance 2	#1 pick					1	-
	Alliance 3	#1 or 16 pick				l		
	Alliance 4	#1 or 16 pick						
CCWM World		Average	Calculated	CCWM	Average Score	Offensive Power	OPR	OPB World
Rank		Winning Margin Per Match	Contribution to Winning Margin	Regional Rank		Rating	Regional Rank	Rank
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