Inferential Analysis

Part 2

Probability

- Probability is a branch of mathematics that deals with calculating the likelihood of a given event's occurrence.
- Probability is the measure of the likeliness that an event will occur.
- Probability is quantified as a number between 0 and 1 (where 0 indicates impossibility and 1 indicates certainty).
- The higher the probability of an event, the more certain that the event will occur.

- Probability is written as,

$$\text{Probabilit (Event happening)} = \frac{\text{Number of ways it can happen}}{\text{Total Number of Outcomes}}$$

Key Terms in Probability

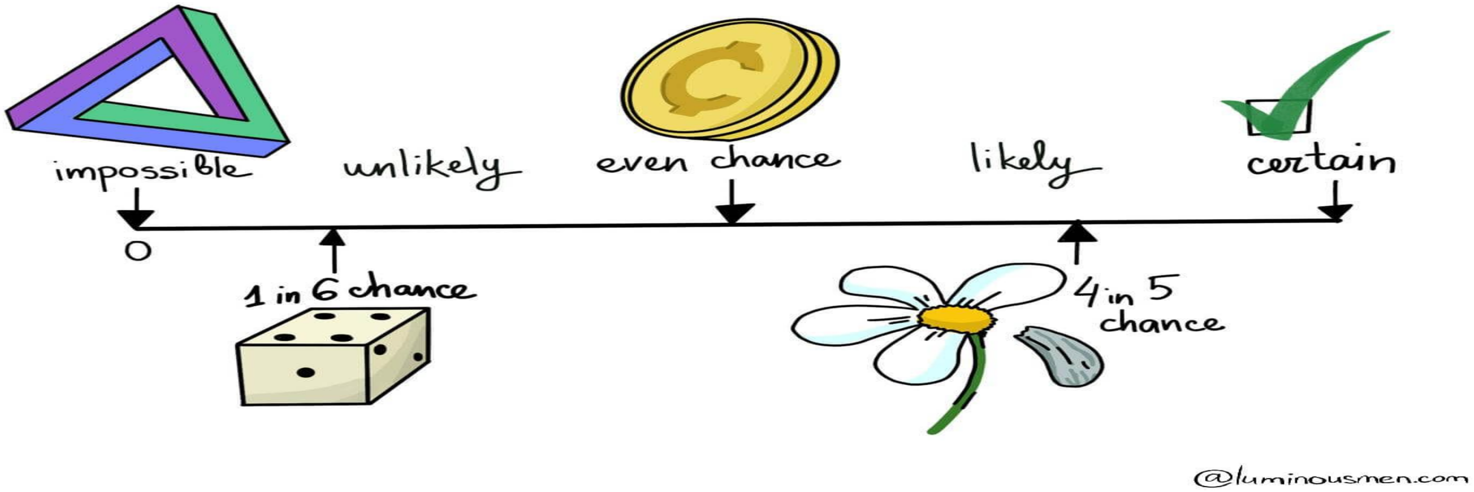
- **Probability: A number between 0 and 1 that indicates the chance or likelihood of an event happening.**
- **Probability Experiment: A situation where number of trials are conducted to determine probability.**
- **Sample spaces: Set of all possible of outcomes of any trial in a probability experiment.**
- **Sample point: One element of the sample spaces.**
- **Outcome: Result of the one trial in probability experiment.**
- **Equally likely outcomes: Outcomes with the same probability.**
- **Expected frequency: Number of times a specific outcomes is expected to occur when a probability experiment or a trial is repeated number of times.**
- **Observed frequency: Number of times that specific outcome did occur.**
- **Event: One or more favourable outcomes of probability experiment.**
- **Complement of an Event: All outcomes that are not the Event.**

Key Terms in Probability

- **Independent Events:** Events that don't affect or are not affected by another event or events. Each event has exactly the same probability.

- **Dependent Events:** Events that affect or are affected by another event or events.

- **Mutually Exclusive Events:** Events that cannot happen at same time. The probability of mutually exclusive events is Zero.



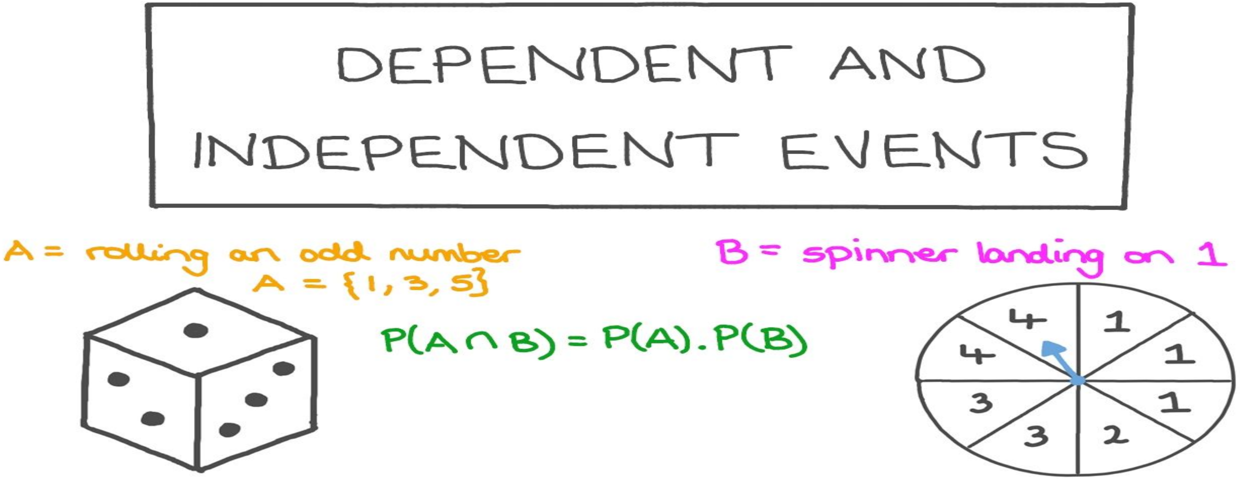
Types of Events

• **Dependent Event:** Dependent event also called "Conditional", where one event is affected by other events

• Example: • Drawing 2 Cards from a Deck. After taking one card from the deck there is one card less than the previous, so the probability changes!

• **Independent Event:** One event is not affected by any other events.

• Example: You toss a coin three times and it comes up "Heads" each time. what is the chance that the next toss will also be a "Head"? The chance is simply 1/2, or 50%, just like ANY OTHER toss of the coin. What it did in the past will not affect the current toss.



Conditional Probability

• A Conditional probability is a probability whose sample space has been limited to only those outcomes that fulfill a certain condition.

• A rule that can be used to determine a conditional probability from unconditional probabilities is:

P(A | B) = $P(A \cap B) / P(B)$

where:

• P(A | B) = Conditional probability that event A will occur given that event B has occurred already

• $P(A \cap B)$ = Unconditional probability that event A and event B both occur

• P(B) = Probability that event B occurs

• The usual notation for "event A occurs given that event B has occurred" is "A | B" (A given B).

• The symbol | is a vertical line and does not imply division.

$$\underbrace{P(A \mid B)}_{\text{Probability of } A \text{ given } B} = \frac{\overbrace{P(A \cap B)}^{\text{Probability of } A \text{ and } B}}{\underbrace{P(B)}_{\text{Probability of } B}}$$

Bayes theorem

- Bayes theorem shows the relationship between one conditional probability and its inverse
- It provides a mathematical rule for revising an estimate of experience and observation

Related terms in Bays theorem

P(A):The probability of event A not concerning its associated event B.

This is also called as Prior probability of A

P(B):The probability of event B not concerning its associatec

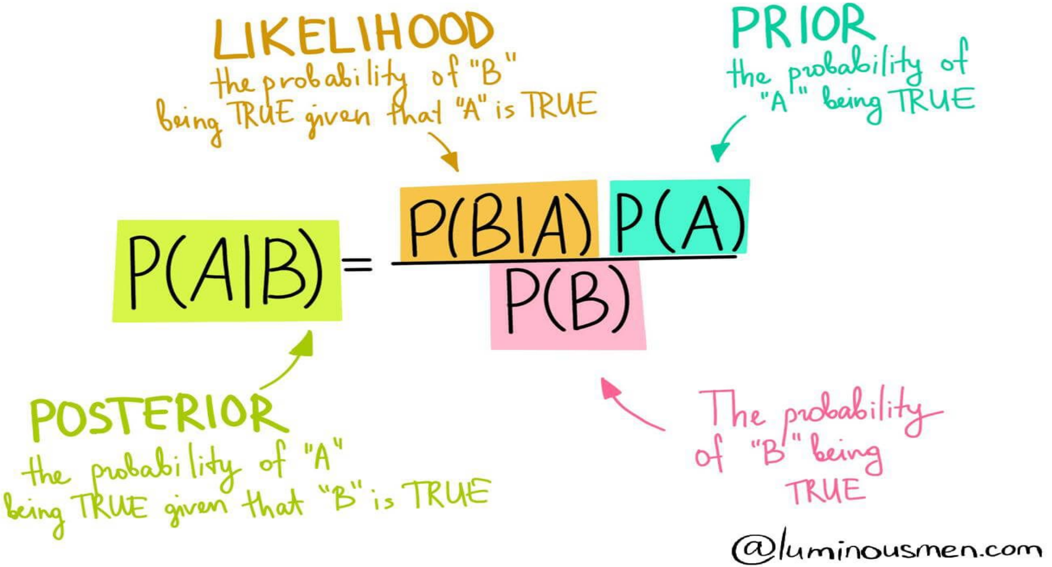
This is also called as Prior probability of A

P(B|A) : Conditional probability of B given A.

This is also called as likelihood

P(A|B) : Conditional probability of A given B.

This is also called as posterior probability



1.1 Discrete Distributions

	Notation[1]	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\text{Unif}\{a,\ldots,b\}$	$\begin{cases} 0 & x<a \\ \frac{\lfloor x \rfloor - a+1}{b-a} & a \le x \le b \\ 1 & x>b \end{cases}$	$\frac{I(a<x<b)}{b-a+1}$	$\frac{a+b}{2}$	$\frac{(b-a+1)^2-1}{12}$	$\frac{e^{as}-e^{-(b+1)s}}{s(b-a)}$
Bernoulli	$\text{Bern}(p)$	$(1-p)^{1-x}$	$p^x(1-p)^{1-x}$	p	$p(1-p)$	$1-p+pe^s$
Binomial	$\text{Bin}(n,p)$	$I_{1-p}(n-x,x+1)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^s)^n$
Multinomial	$\text{Mult}(n,p)$		$\frac{n!}{x_1!\ldots x_k!}p_1^{x_1}\cdots p_k^{x_k} \quad \sum_{i=1}^k x_i = n$	np_i	$np_i(1-p_i)$	$\left(\sum_{i=0}^k p_i e^{s_i}\right)^n$
Hypergeometric	$\text{Hyp}(N,m,n)$	$\approx \Phi\left(\frac{x-np}{\sqrt{np(1-p)}}\right)$	$\frac{\binom{m}{x}\binom{m-x}{n-x}}{\binom{N}{x}}$	$\frac{nm}{N}$	$\frac{nm(N-n)(N-m)}{N^2(N-1)}$	
Negative Binomial	$\text{NBin}(n,p)$	$I_p(r,x+1)$	$\binom{x+r-1}{r-1}p^r(1-p)^x$	$r\frac{1-p}{p}$	$r\frac{1-p}{p^2}$	$\left(\frac{p}{1-(1-p)e^s}\right)^r$
Geometric	$\text{Geo}(p)$	$1-(1-p)^x \quad x\in\mathbb{N}^+$	$p(1-p)^{x-1} \quad x\in\mathbb{N}^+$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^s}$
Poisson	$\text{Po}(\lambda)$	$e^{-\lambda}\sum_{i=0}^{x}\frac{\lambda^i}{i!}$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$e^{\lambda(e^s-1)}$

1.2 Continuous Distributions

	Notation	$F_X(x)$	$f_X(x)$	$\mathbb{E}[X]$	$\mathbb{V}[X]$	$M_X(s)$
Uniform	$\mathrm{Unif}(a,b)$	$\begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a<x<b \\ 1 & x>b \end{cases}$	$\frac{I(a<x<b)}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{sb}-e^{sa}}{s(b-a)}$
Normal	$\mathcal{N}(\mu,\sigma^2)$	$\Phi(x)=\int_{-\infty}^{x}\phi(t)\,dt$	$\phi(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$	μ	σ^2	$\exp\left\{\mu s+\frac{\sigma^2 s^2}{2}\right\}$
Log-Normal	$\ln\mathcal{N}(\mu,\sigma^2)$	$\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left[\frac{\ln x-\mu}{\sqrt{2\sigma^2}}\right]$	$\frac{1}{x\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(\ln x-\mu)^2}{2\sigma^2}\right\}$	$e^{\mu+\sigma^2/2}$	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$	
Multivariate Normal	$\mathrm{MVN}(\mu,\Sigma)$		$(2\pi)^{-k/2}\lvert\Sigma\rvert^{-1/2}e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$	μ	Σ	$\exp\left\{\mu^T s+\frac{1}{2}s^T\Sigma s\right\}$
Student's t	$\mathrm{Student}(\nu)$	$I_x\left(\frac{\nu}{2},\frac{\nu}{2}\right)$	$\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-(\nu+1)/2}$	0	0	
Chi-square	χ^2_k	$\frac{1}{\Gamma(k/2)}\gamma\left(\frac{k}{2},\frac{x}{2}\right)$	$\frac{1}{2^{k/2}\Gamma(k/2)}x^{k/2}e^{-x/2}$	k	$2k$	$(1-2s)^{-k/2}\ s<1/2$
F	$\mathrm{F}(d_1,d_2)$	$I_{\frac{d_1x}{d_1x+d_2}}\left(\frac{d_1}{2},\frac{d_1}{2}\right)$	$\frac{\sqrt{\frac{(d_1x)^{d_1}d_2^{d_2}}{(d_1x+d_2)^{d_1+d_2}}}}{x\mathrm{B}\left(\frac{d_1}{2},\frac{d_1}{2}\right)}$	$\frac{d_2}{d_2-2}$	$\frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$	
Exponential	$\mathrm{Exp}(\beta)$	$1-e^{-x/\beta}$	$\frac{1}{\beta}e^{-x/\beta}$	β	β^2	$\frac{1}{1-\beta s}\ (s<1/\beta)$
Gamma	$\mathrm{Gamma}(\alpha,\beta)$	$\frac{\gamma(\alpha,x/\beta)}{\Gamma(\alpha)}$	$\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta s}\right)^\alpha\ (s<1/\beta)$
Inverse Gamma	$\mathrm{InvGamma}(\alpha,\beta)$	$\frac{\Gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma(\alpha)}$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}e^{-\beta/x}$	$\frac{\beta}{\alpha-1}\ \alpha>1$	$\frac{\beta^2}{(\alpha-1)^2(\alpha-2)^2}\ \alpha>2$	$\frac{2(-\beta s)^{\alpha/2}}{\Gamma(\alpha)}K_\alpha\left(\sqrt{-4\beta s}\right)$
Dirichlet	$\mathrm{Dir}(\alpha)$		$\frac{\Gamma\left(\sum_{i=1}^k\alpha_i\right)}{\prod_{i=1}^k\Gamma(\alpha_i)}\prod_{i=1}^k x_i^{\alpha_i-1}$	$\frac{\alpha_i}{\sum_{i=1}^k\alpha_i}$	$\frac{\mathbb{E}[X_i](1-\mathbb{E}[X_i])}{\sum_{i=1}^k\alpha_i+1}$	
Beta	$\mathrm{Beta}(\alpha,\beta)$	$I_x(\alpha,\beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$1+\sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{s^k}{k!}$
Weibull	$\mathrm{Weibull}(\lambda,k)$	$1-e^{-(x/\lambda)^k}$	$\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}e^{-(x/\lambda)^k}$	$\lambda\Gamma\left(1+\frac{1}{k}\right)$	$\lambda^2\Gamma\left(1+\frac{2}{k}\right)-\mu^2$	$\sum_{n=0}^{\infty}\frac{s^n\lambda^n}{n!}\Gamma\left(1+\frac{n}{k}\right)$
Pareto	$\mathrm{Pareto}(x_m,\alpha)$	$1-\left(\frac{x_m}{x}\right)^\alpha\ x\geq x_m$	$\alpha\frac{x_m^\alpha}{x^{\alpha+1}}\quad x\geq x_m$	$\frac{\alpha x_m}{\alpha-1}\ \alpha>1$	$\frac{x_m^\alpha}{(\alpha-1)^2(\alpha-2)}\ \alpha>2$	$\alpha(-x_m s)^\alpha\Gamma(-\alpha,-x_m s)\ s<0$