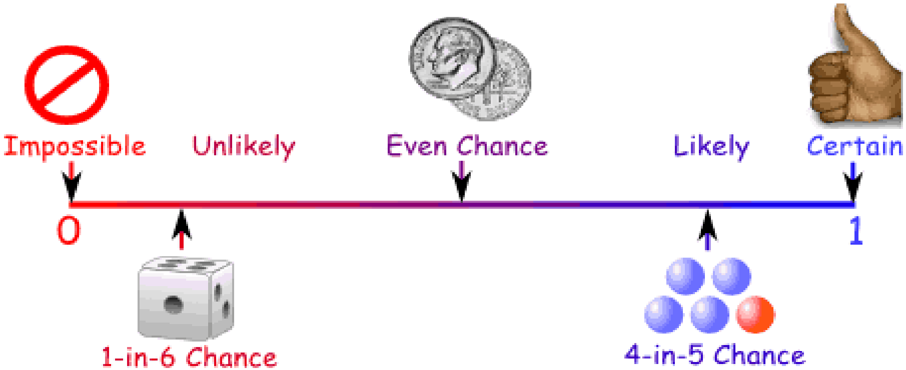
Business Analytics

Probability

Proschool
An IMS Initiative

Probability



- Probability is the measure of the likeliness that an event will occur.
- Probability is quantified as a number between 0 and 1 (where 0 indicates impossibility and 1 indicates certainty).
- The higher the probability of an event, the more certain we are that the event will occur.

Probability

- Probability does not tell us exactly what will happen, it is just a guide.

  Example: toss a coin 100 times, how many heads will come up?
- Probability says that heads have a ½ chance,

  so we can expect 50 Heads.
- But when we actually try it we might get 48

  heads, or 55 heads ... or anything really, but in

  most cases it will be a number near 50.

Probability

Probability is written as,

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

Example: the chances of rolling a "4" with a die

Number of ways it can happen: 1 (there is only 1 face with a "4" on it)

Total number of outcomes: 6 (there are 6 faces altogether)

$$\text{So the probability} = \frac{1}{6}$$

Example: there are 5 marbles in a bag: 4 are blue, and 1 is red. What is the probability that a blue marble gets picked?

Number of ways it can happen: 4 (there are 4 blues)

Total number of outcomes: 5 (there are 5 marbles in total)

$$\text{So the probability} = \frac{4}{5} = \mathbf{0.8}$$

Important terms in Probability

- **Experiment or Trial:**

  An action where the result is uncertain.

e.g Tossing a coin, throwing dice, seeing what pizza people choose.

- **Sample Space:**

  The set of all possible outcomes of an experiment.

  Example:

  Selecting a card from a deck.

  There are 52 cards in a deck (excluding Jokers)

  Hence, the Sample Space is all 52 possible cards:

  {Ace of Hearts, 2 of Hearts, etc... }

- **Sample Points:**

  The Sample Space is made up of Sample Points.

  The elements of sample space are sample points.

Important terms in Probability

Example:

In the deck of cards- the 5 of Clubs is a sample point,
the King of Hearts is a sample point
"King" is not a sample point. As there are 4 Kings that is 4 different sample points.

- **Event:**

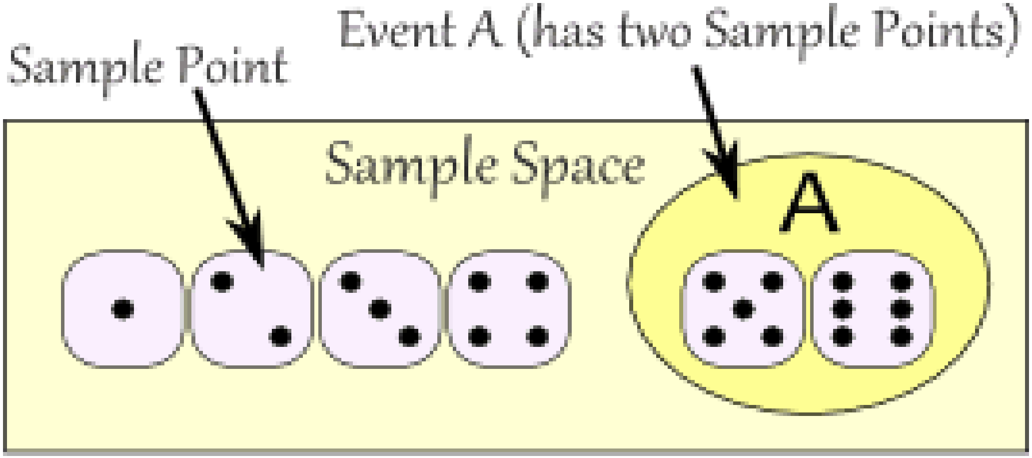
  A single result of an experiment

  Example : Getting a Tail when tossing a coin is an event, getting a "5" when a die is rolled is an event.

- An event can include one or more possible outcomes:

  Choosing a "King" from a deck of cards (any of the 4 Kings) is an event

  Rolling an "even number" (2, 4 or 6) is also an event


Event A (has two Sample Points)
Sample Point
Sample Space
A

Types of Events

- **Independent Event:**

One event is not affected by any other events.
Example: You toss a coin three times and it comes up "Heads" each time ... what is the chance that the next toss will also be a "Head"?
The chance is simply 1/2, or 50%, just like ANY OTHER toss of the coin.
What it did in the past will not affect the current toss!

- **Dependent Event:**

Also called "Conditional", where one event is affected by other events

Dependent Events

Example: Drawing 2 Cards from a Deck
After taking one card from the deck there is one card less card less than the previous, so the probability changes!

Let's look at the chances of getting a King.
For the 1st card the chance of drawing a King is 4 out of 52
But for the 2nd card:
If the 1st card was a King, then the 2nd card is less likely to be a King, as only 3 of the 51 cards left are Kings.
If the 1st card was not a King, then the 2nd card is slightly more likely to be a King, as 4 of the 51 cards left are King.
This is because we are removing cards from the deck.

Independent Events

- **Mutually Exclusive Event:**

If there is no common sample point between two events, then the events are mutually exclusive (i.e. the events do not occur together.

- **Sure or Certain event:**

An event containing all the sample points of the sample space is a sure or certain events.

- **Impossible event:**

An event containing no sample point is called an impossible event.

Conditional Probability

- In many situations, once more information becomes available, we are able to revise our estimates for the probability of further outcomes or events happening.
- For example, suppose you go out for lunch at the same place and time every Friday and you are served lunch within 15 minutes with probability 0.9.
- However, given that you notice that the restaurant is exceptionally busy, the probability of being served lunch within 15 minutes may reduce to 0.7.
- This is the conditional probability of being served lunch within 15 minutes given that the restaurant is exceptionally busy.

Conditional Probability

- The usual notation for "event A occurs given that event B has occurred" is "A | B" (A given B).
- The symbol | is a vertical line and does not imply division.
- P(A | B) denotes the probability that event A will occur given that event B has occurred already.

Conditional Probability

- A rule that can be used to determine a conditional probability from unconditional probabilities is:

  $P(A \mid B) = P(A \cap B) / P(B)$

  where:
- $P(A \mid B)$ = the (conditional) probability that event A will occur given that event B has occurred already
- $P(A \cap B)$ = the (unconditional) probability that event A and event B both occur
- $P(B)$ = the (unconditional) probability that event B occurs

Conditional Probability

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Conditional Probability

Example 1:

- A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

Analysis:

Solution: $$P(\text{Second}|\text{First}) = \frac{P(\text{First and Second})}{P(\text{First})} = \frac{0.25}{0.42} = 0.60 = 60\%$$

- This problem describes a conditional probability since it asks us to find the probability that the second test was passed given that the first test was passed. In the last lesson, the notation for conditional probability was used in the statement of Multiplication Rule 2.

Conditional Probability

Example 2:

The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

Solution: $$P(\text{Absent}|\text{Friday}) = \frac{P(\text{Friday and Absent})}{P(\text{Friday})} = \frac{0.03}{0.2} = 0.15 = 15\%$$

Conditional Probability

Example 3

Why are some mutual fund managers more successful than others?

- One possible factor is where the manager earned his or her MBA.

17% of mutual funds outperform the market. 40% of mutual funds are managed by a graduated from a top 20 MBA.11% of mutual funds outperform the market and are managed by a graduated from a top 20 MBA program.

- What is the probability that a mutual fund outperforms the market given that the manager graduated from a top 20 program?

Conditional Probability

Solution

Let A be the event that a mutual fund outperforms the market.

Let B be the event that a mutual fund is managed by

a graduate from a top 20 MBA program.

$P(A) = 0.17$

$P(B) = 0.40$

$P(A \text{ and } B) = 0.11$

Conditional Probability

Probability that a mutual fund outperforms the market given that the manager graduated from a top 20 program:

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$= \frac{0.11}{0.40}$$

$$= 0.275$$

Thank You