Theoretical Information

When two transmission lines are close together, because of the interaction of the electromagnetic fields of each line, power can be coupled between the lines. Those coupled lines are used to construct directional couplers. Generally, in design of directional couplers microstrip and stripline forms are used. Although microstrip transmission lines do not support TEM and named as quasi-TEM, usually they are assumed to operate in TEM mode.

It is important that whether true TEM or not, all parallel line couplers have odd and even mode, and resulting Zoe and Zoo (even and odd mode impedances respectively). In the analysis of the directional couplers we will use also even-odd mode analysis.

There are many kinds of directional couplers in different forms. However directional couplers we will mention here are single and three section microstrip directional couplers.

1.Single-section Microstrip Coupled Line Directional Coupler

Single-section microstrip directional couplers are constructed from the coupled

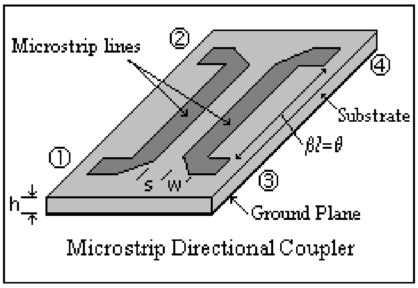


Figure 1

transmission microstrip lines. A single-section microstrip directional coupler is shown in the Figure 1.

1.1.Design:

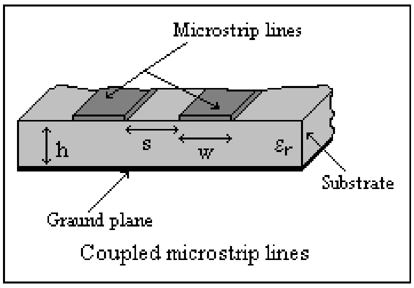
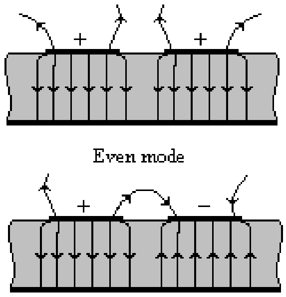


Figure 2

The cross-sectional view of coupled microstrip lines are shown in the Figure 2. Looking at that figure, we say that in the design of coupled lines, we consider following parameters: w/h, s/h, er, eeff , Zoo, Zoe, Zo and Coupling. By using even and odd mode excitation as in the Figure 3, we find even and odd mode impedances Zoe and Zoo.



The electric fields for parallel-coupled microstrip lines

Figure 3

In design of coupled transmission lines when finding Zoo and Zoe we use the following approximate synthesis technique:

- Firstly, with given Zo (single line characteristic impedance), er (relative dielectric constant of the substrate) and C (coupling of the coupled lines) determining shape ratios for equivalent single microstrip lines.
- Secondly, finding the shape ratio w/h and the spacing ratio s/h for the desired coupled microstrip by using the single line shape ratios found in first step.

Since we have in hand C (in dB) and Zo(impedance) let us find Zoe and Zoo as following:

$$c = 10^{-C/20}$$

as c is the coupling coefficient. Now, Zoo and Zoe are;

$$Z_{oe} = Z_o\sqrt{\frac{1+c}{1-c}} \quad \text{and} \quad Z_{oo} = Z_o\sqrt{\frac{1-c}{1+c}}$$

Using Zoe and Zoo if we consider Zoe and Zoo for single line as Zose and Zoso;

$$Z_{ose} = \frac{Z_{oe}}{2} \quad \text{and} \quad Z_{oso} = \frac{Z_{oo}}{2}$$

Now what we should do is to find $(w/h)_{se}$ and $(w/h)_{so}$ from Zose and Zoso. To do that let us use single line equations:

$$\frac{W}{h} = \begin{cases} \dfrac{8e^{A}}{e^{2A}-2} & \text{for W/h } < 2 \\ \dfrac{2}{\pi}\left[B-1-\ln(2B-1)+\dfrac{\varepsilon_{r-1}}{2\varepsilon_r}\left\{\ln(B-1)+0.39-\dfrac{0.61}{\varepsilon_r}\right\}\right] & \text{for W/h } > 2 \end{cases}$$

where $A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$

$$B = \frac{377\pi}{2Z_0\sqrt{\varepsilon_r}}$$

At that point, we are able to find (w/h)se and (w/h)so by applying Zose and Zoso (as Zo) to the single line microstrip equations. Now we can come to a point where we reach the w/h and s/h for the desired coupled microstrip line using a family of approximate equations as following:

$$\frac{s}{h} = \frac{2}{\pi}\cosh^{-1}\left[\frac{\cosh\{(\pi/2)(w/h)_{se}\} + \cosh\{(\pi/2)(w/h)_{so}\} - 2}{\cosh\{(\pi/2)(w/h)_{so}\} - \cosh\{(\pi/2)(w/h)_{se}\}}\right]$$

$$\left(\frac{w}{h}\right)_{se} = \frac{2}{\pi}\cosh^{-1}\left(\frac{2d - g + 1}{g + 1}\right)$$

$$\left(\frac{w}{h}\right)_{so} = \frac{2}{\pi}\cosh^{-1}\left(\frac{2d - g - 1}{g - 1}\right) + \frac{4}{\pi(1 + \varepsilon_r/2)}\cosh^{-1}\left(1 + 2\frac{w/h}{s/h}\right) \qquad \varepsilon_r \le 6$$

$$\left(\frac{w}{h}\right)_{so} = \frac{2}{\pi}\cosh^{-1}\left(\frac{2d - g - 1}{g - 1}\right) + \frac{1}{\pi}\cosh^{-1}\left(1 + 2\frac{w/h}{s/h}\right) \qquad \varepsilon_r \ge 6$$

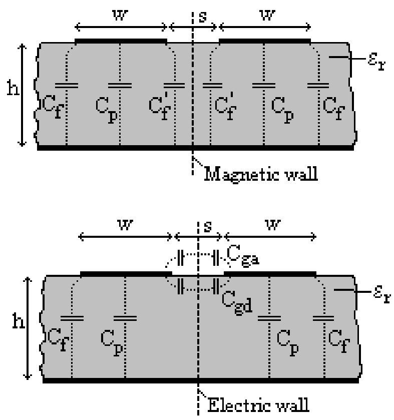
$$g = \cosh\left(\frac{\pi s}{2h}\right) \qquad d = \cosh\left(\pi\frac{w}{h} + \frac{\pi}{2}\frac{s}{h}\right)$$

As a result, we began from parameters C, Zo and er and at the end find w/h and s/h.

$$\lambda = \upsilon_p / f \text{ and } \upsilon_p = c/\sqrt{\varepsilon_{eff}}\text{, here c is the speed of light, } 3.10^8\, m/s$$

Until that point we only cared about the cross-sectional parameters of the coupler. However, as it can be seen from Figure 1 directional coupler has an important parameter, length. In determining length of the directional coupler, in order to have better coupling, we should have a design frequency and arrange the length accordingly. Here for better coupling, our length consideration is l/4 (the reason of having that length will be better explained in the following analysis part).

While calculating the length of the coupler we assume that the phase velocities of the coupler in even and odd mode are approximately equal. That means here is that we will only calculate effective permittivity and for even and odd mode we will consider it as equal. The important point here is to get the eeff by using the cross-sectional parameters in our hand. In achieving this what we can use is the capacitances in our configuration. Although we assumed the even and odd mode phase velocities equal, in capacitance evaluations, even and odd mode permittivities are used and therefore we will use those to find effective permittivity. Capacitances of even and odd mode are shown in the Figure 4.



Seperation of capacitances

Figure 4

Now before introducing the capacitance evaluations, let us think about how we find eeff from capacitances:

The even-mode and odd-mode characteristic impedances are;

$$Z_{oe} = \left(c\sqrt{C_e C_{e1}}\right)^{-1}$$

$$Z_{oo} = \left(c\sqrt{C_o C_{o1}}\right)^{-1}$$

The effective microstrip permittivities are;

$$\varepsilon_{effe} = \frac{C_e}{C_{e1}}$$

$$\varepsilon_{effo} = \frac{C_o}{C_{o1}}$$

If we get C_{e1} and C_{o1} from first two even-odd mode impedance equations and substitute them in above effective permittivity equations what we get is;

$$\varepsilon_{effe} = \left(cC_e Z_{oe}\right)^2 \quad \text{and} \quad \varepsilon_{effo} = \left(cC_o Z_{oo}\right)^2$$

About the eeff , it is the arithmetic mean of the even and odd mode effective permittivities;

$$\sqrt{\varepsilon_{eff}} = \frac{\sqrt{\varepsilon_{effe}} + \sqrt{\varepsilon_{effo}}}{2}$$

and then using the derived equations of the even and odd mode effective permittivities, what we reach is

$$\sqrt{\varepsilon_{eff}} = \frac{cC_e Z_{oe} + cC_o Z_{oo}}{2}$$

At this point, what we need is the even and odd mode impedances and even and odd mode capacitances. We show above how we can calcualate even and odd mode impedances with Zo and coupling in hand and here we will look how we can calculate even and odd mode capacitances.

Writing the total capacitances for each mode;

$$C_e = C_p + C_f + C_f{'}$$
$$C_o = C_p + C_f + C_{ga} + C_{gd}$$

The capacitance Cp simply relates to the parallel-plate line value is given by

$$C_p = \varepsilon_o \varepsilon_r \frac{w}{h}$$

And considering the fringing capacitance Cf because of each microstrip taken alone as if for a single strip;

$$2C_f = \frac{\sqrt{\varepsilon_{eff}}}{cZ_o} - C_p$$

$$C_f{'} = \frac{C_f}{1 + A(h/s)\tanh(8s/h)} \sqrt{\frac{\varepsilon_r}{\varepsilon_{eff}}}$$

$$A = \exp\{-0.1\exp(2.333 - 2.53w/h)\}$$

where c is the free space velocity. As it can be seen from Figure 4, there is odd-mode fringing field capacitances for the air and dielectric regions across the coupling gap, which are Cga and Cgd, respectively. Determining Cga, it is obtained by using an equivalent coplanar strip geometry calculation,

$$C_{ga} = \varepsilon_0 \frac{K(k')}{K(k)}$$

where

$$k = \frac{s/h}{s/h + 2w/h}$$
$$k' = \sqrt{1 - \mathrm{k}^2}$$

the ratio of the elliptic functions is

For $0 \le \mathrm{k}^2 \le 0.5$

$$\frac{K(k')}{K(k)} = \frac{1}{\pi}\ln\left(2\frac{1+\sqrt{k'}}{1-\sqrt{k'}}\right)$$

For $0.5 \le \mathrm{k}^2 \le 1$

$$\frac{K(k')}{K(k)} = \frac{\pi}{\ln\left\{2\left(1+\sqrt{k}\right)/\left(1-\sqrt{k}\right)\right\}}$$

For the C_{gd} our way is different. The modified capacitance for coupled striplines give C_{gd}

$$C_{gd} = \frac{\varepsilon_0 \varepsilon_r r}{\pi} \ln\left\{\coth\left(\frac{\pi}{4}\frac{s}{h}\right)\right\} + 0.65C_f\left(\frac{0.02}{s/h}\sqrt{\varepsilon_r} + 1 - \varepsilon_r^{-2}\right)$$

In achieving our goal, what we should do is to put those capacitances in the following mentioned formula;

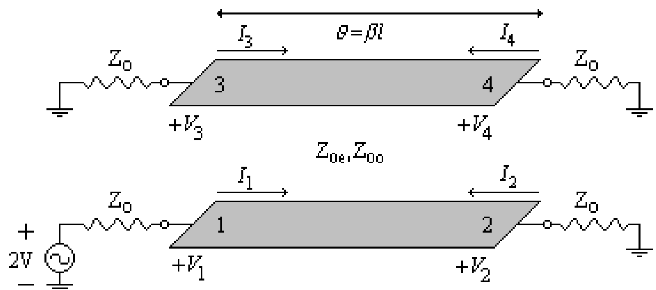
$$\sqrt{\varepsilon_{eff}} = \frac{cC_e Z_{oe} + cC_o Z_{oo}}{2}$$

and solve the above equation for e_{eff}.

After finding the e_{eff} using capacitances, it is very easy to find the length of the directional coupler:

$$l = \lambda / 4 = \frac{c}{4f\sqrt{\varepsilon_{eff}}}$$ where f is the design frequency and c is the speed of light in free space

1.2.Analysis



The schematic circuit of the coupled line

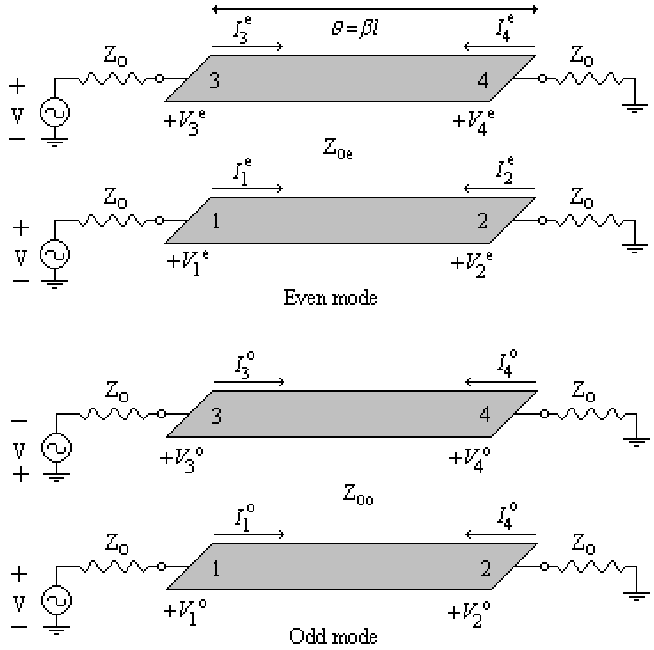
Figure 5

The even and odd mode characteristic impedances are mentioned in the design part. At this point, in order to analyze the coupled line directional coupler we need to use an even-odd mode analysis. The schematic circuit for the coupled lines are shown in Figure 5. As it can be seen from the figure that our coupler which we analyze is a four port network. In our analysis, we terminate three of ports with the impedance Z_0 and drive with a voltage generator of 2V and impedance of Z_0 at port 1. Z_0 here is the characteristic impedance of single microstrip line. Moreover mentioning about the ports, port 1 is the input port, port 2 is the through port. port 3 is the coupled port and the port 4 is the isolated port. In addition in grounds in the Figure 5 are common.

Applying the even–odd mode analysis to the coupler, we consider our coupler as in the Figure 6, in even mode feeding the port 1 and 3 with V and in odd mode feeding port 3 with –V and port 1 with V. If we use superposition technique and add all port voltages what we get is 0 for all three and 2V for port1. Due to the symmetry of the coupler voltages and currents shown in the Figure 6 are as following:

For the even mode $I_1^e = I_3^e$, $I_2^e = I_4^e$, $V_1^e = V_3^e$ and $V_2^e = V_4^e$

For the odd mode $I_1^o = -I_3^o$, $I_2^o = -I_4^o$, $V_1^o = -V_3^o$ and $V_2^o = -V_4^o$



Decomposition of the coupled line coupler in even and odd mode

Figure 6

Then input impedance at port 1 of the coupler can be expressed as

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o}$$

Letting Z^e_{in} be the input impedance at port 1 for the even mode and Z^o_{in} be the input impedance for the odd mode, then what we have is the following since, for each mode the line looks like a transmission line of characteristic impedance Z_{0o} or Z_{0e} with a terminated load impedance of Z_0:

$$Z_{in}^e = Z_{0e} \frac{Z_0 + jZ_{0e} \tan\theta}{Z_{0e} + jZ_0 \tan\theta}$$

$$Z_{in}^o = Z_{0o} \frac{Z_0 + jZ_{0o} \tan\theta}{Z_{0o} + jZ_0 \tan\theta}$$

At this point when we make a voltage division;

$$V_1^e = V \frac{Z_{in}^e}{Z_{in}^e + Z_0}, \quad I_1^e = \frac{V}{Z_{in}^e + Z_0}$$

$$V_1^o = V \frac{Z_{in}^o}{Z_{in}^o + Z_0}, \quad I_1^o = \frac{V}{Z_{in}^o + Z_0}$$

Using those and the Z_{in} equation mentioned above;

$$Z_{in}^e Z_{in}^o = Z_{0e} Z_{0o} = Z_0^2 \quad \text{and so finding} \quad Z_{in} = Z_0$$

that results in

$$Z_{in} = \frac{Z_{in}^{o}(Z_{in}^{e} + Z_0) + Z_{in}^{e}(Z_{in}^{o} + Z_0)}{Z_{in}^{e} + Z_{in}^{o} + 2Z_0} = Z_0 + \frac{2(Z_{in}^{o} Z_{in}^{e} - Z_0^2)}{Z_{in}^{e} + Z_{in}^{o} + 2Z_0}$$

and letting $Z_0 = \sqrt{Z_{0e} Z_{0o}}$

those reduce Z_{in}^{e} and Z_{in}^{o} to the followings ;

$$Z_{in}^{e} = Z_{0e} \frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}$$

$$Z_{in}^{o} = Z_{0o} \frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}} \tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}} \tan\theta}$$

Therefore if above equations are satisfied then, port 1 and due to the symmetry all the other ports will be matched. The above equation can be thought as the input match condition.

Writing the voltage at port 3:

$$V_3 = V_3^{e} + V_3^{o} = V_1^{e} - V_1^{o} = V\left[\frac{Z_{in}^{e}}{Z_{in}^{e} + Z_0} - \frac{Z_{in}^{o}}{Z_{in}^{o} + Z_0}\right]$$

putting Z_{in}^{e} and Z_{in}^{o} in the above equation of V_3 we get the following :

$$V_3 = V\frac{j(Z_{0e} - Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta} = V\frac{j(Z_{0e} - Z_{0o})\sin\theta}{2Z_0\cos\theta + j(Z_{0e} + Z_{0o})\sin\theta}$$

now defining the midband coupling coefficient,c

$$c = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \qquad \text{and using } Z_0 = \sqrt{Z_{0e} Z_{0o}} \text{ we know that } \sqrt{1 - c^2} = \frac{2Z_0}{Z_{0e} + Z_{0o}}$$

therefore,

$$V_3 = V\frac{jc\sin\theta}{\sqrt{1-c^2}\cos\theta + j\sin\theta}$$

and here V_3/V is the actual voltage coupling coefficient. Considering q=p/2 (at midband in which *l*=l/4 where l is the wavelength at the center frequncy) what we get is $V_3/V=c$. Moreover if we want to determine the characteristic of voltage coupling coefficient versus frequency we can use the above equation.

$$V_4 = V_4^{e} + V_4^{o} = V_2^{e} - V_2^{o} = 0$$

Similar to V_3 consideration, we can think of V_4 as following:

Determining the Coupling in dB,

$$Coupling = C = -20\log\left|\frac{jc\sin\theta}{\sqrt{1-c^2}\cos\theta + j\sin\theta}\right| \text{ dB}$$

$$\text{and in midband } C = -20\log|c| = -20\log\left|\frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}}\right| \text{ dB}$$

$$Z_{0e} = Z_0\sqrt{\frac{1+c}{1-c}}$$

$$Z_{0o} = Z_0\sqrt{\frac{1-c}{1+c}}$$

in midband therefore Z_{0e} and Z_{0o}can be derived easily from above equations as

In the analysis we dealt with here, we made some necessary assumptions as following:

- The quasi-static TEM mode of operation holds and so voltage and current are meaningful quantities on the lines,
- For even and odd modes, transmission lines have the same e_{eff}. With this assumption, the even and odd modes will have the same phase velocity although in practice they are slightly different and this makes some degradation in the performance of the coupler.
- The microstrip transmission lines here posses an appropriate plane of symmetry.

To summarize, with the mentioned assumptions:

i) Port 4 which is named also as Isolated port, always has a zero output, without being dependent of the electrical length of the coupling region. Major cause of poor isolation in practical circuits are unequal phase velocities of even and odd mode.

ii) The input at each port is matched to the feed line characteristic impedance, Z_0, irrespective of the electrical length.

iii) The total output power equals the input power.

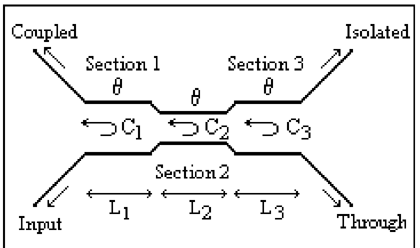
iv) The maximum coupling to port 2 occurs at the frequency that gives a quarter wave coupling length which is l/4. This is the mid-band frequency and due to this property, this couplers are also known as quarter-wave couplers.

v) At the maximum coupling frequency, there is a 90° phase difference between the voltages at port 2 and port 3. Such a coupler can also be described as quadrature coupler.

vi)At the frequencies other than maximum coupling frequency (design frequency)

the coupling of the coupler can be found from $|V_3(q)|$.

2.Three-section Microstrip Coupled Line Directional Coupler



Three-section Directional Coupler

Figure 7

The single-section microstrip directional coupler is mentioned in previous section. In single-section couplers there exists a limitation of coupling. In single section the coupling can be maximum when the length of the coupler is l/4. However sometimes better coupling is required. This can be increased using multiple sections. Since the phase characteristics are usually better, multi-section coupled line couplers are generally made with an odd number of sections. In this part we will examine only three-section microstrip directional coupler since as a multiple-section, it is widely used. The Figure 7 shows the three-section microstrip directional coupler.

2.1.Design

Generally, approximating the results in analysis of single-section for multiples section the following equation is obtained:

$$V_3 = (jc_1 \sin\theta e^{-j\theta})V_1 + (jc_2 \sin\theta e^{-j\theta})V_1 e^{-2j\theta} + \ldots + (jc_N \sin\theta e^{-j\theta})V_1 e^{-2j(N-1)\theta}$$

where c_n is the voltage coupling coefficien t. With the assumption of symmetric coupler wh ich is $c_1 = c_N, c_2 = c_{N-1}$ etc.

then $V_3 = 2jV_1 \sin\theta e^{-jN\theta}\left[c_1\cos(N-1)\theta + c_2\cos(N-3)\theta + \ldots\ldots + \frac{1}{2}c_M\right]$

where $M = (N+1)/2$

At the center frequency, we define the voltage coupling factor c_0:

$$c_0 = \left|\frac{V_3}{V_1}\right|_{\theta=\pi/2}$$

at N = 3 $\quad c = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[c_1\cos 2\theta + \frac{1}{2}c_2\right]$

For a maximally flat response for this three section coupler, solving for c_1, c_2 and c_3;

we require that : $\quad \left.\frac{d^n}{d\theta^n}c(\theta)\right|_{\theta=\pi/2} = 0 \qquad$ for $n = 1,2$

then

At this point deriving the equation of three-section;

$$c = 2\sin\theta\left[c_1\cos 2\theta + \frac{1}{2}c_2\right]\Bigg|_{\theta=\pi/2} = c_2 - 2c_1$$

$$\frac{dc}{d\theta} = \left[3c_1\cos 3\theta + (c_2 - c_1)\cos\theta\right]_{\pi/2} = 0$$

$$\frac{d^2c}{d\theta^2} = \left[-9c_1\sin 3\theta - (c_2 - c_1)\sin\theta\right]\Big|_{\pi/2} = 10c_1 - c_2 = 0 \Rightarrow 10c_1 = c_2$$

$$\text{solving for } c_1 \text{ and } c_2 \Rightarrow \quad c_1 = c_3 = c/8 \quad \text{and} \quad c_2 = 10c/8$$

Now we can calculate the coupling coefficient of each section with given coupling coefficient of the whole coupler. At this point of the design, if we consider w/h and s/h values, since we have all coupling coefficient values of each section, if we know Z_0 (characteristic impedance of the system) and e_r of the system then we can solve w/h and s/h values of each single-section by using the design calculations in the previously mentioned single-section coupled line coupler design part.

What about the length of each section. It can be seen from Figure 7 that all sections should have an equal electrical length. Here since we think the q=p/2, then the physical length of each section should be l/4. However, since all sections have different w/h and s/h ratios (section 1 and 3 are equal and section 2 is different due to symmetry), each section will have different e_{eff} (e_{eff} here can be calculated for microstrip geometry as mentioned in the single-section part) and therefore different l for the same frequency. That results in different physical length for each section since length=l/4.

2.2.Analysis

The general concepts about multi-section directional coupler, and also for our focus three-section are mentioned in the previous design part. As an analysis of a three-section coupler given c_1=c_3, c_2 values we can calculate the coupling coefficient of the whole coupler as following:

$$c = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[c_1\cos 2\theta + \frac{1}{2}c_2\right]$$

For all coupling coefficients mentioned before we always can calculate coupling in dB as following:

Coupling=C=20log (c) dB

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