Analysis of PCB Variations for Impedance Control

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FOR IMPEDANCE CONTROL

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Abstract

Ericsson is a world-leading provider of telecommunications equipment and related services to mobile and fixed network operators globally. Over 1,000 networks in more than 175 countries utilize Ericsson's network equipment and 40 percent of all mobile calls are made through their systems. Ericsson is one of the few companies worldwide that can offer end-to-end solutions for all major mobile communication standards. Tons of PCB is needed in this industry. With more complex hardware designs the need for efficient ways of controlling the impedance in our boards is increasing. Higher data rates and frequencies require better control of the PCB and understanding of the variations in the manufacturing of the PCB. The thesis goal is to study and research on how we can use cheaper PCB materials and handle the variations of these cheap PCB materials. This can be achieved with sorting out the spreading when it comes to conductor width & thickness, dielectric constant & thickness. One more area to explore is the variations of the dielectric constant due to glass cloth. How could statistical methods be used when looking at the variations of the material properties? Mathematica and Advanced Design System (ADS) were used to perform statistical analysis. The result of the thesis can help PCB designer to choose materials more wisely and provide advice to suppliers if they can achieve high quality products.

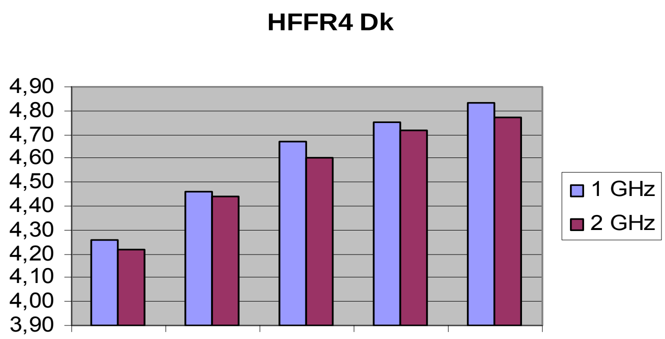
Contents

1 INTRODUCTION

1.1 Background

Characteristic impedance is an important point should be considered when we design PCB. Many factors impact it including trace width, conductor thickness, dielectric constant, and substrate height. The major impact comes from substrate; therefore substrate materials always get peoples' attention. FR-4 and HFFR4 are the most popular on today's market.

FR-4 is the most widely used in PCB industry today. In order to improve flame-resistant properties, FR-4 epoxy resin systems typically contain bromine, a halogen, which is not quite environment-friendly. Under environmental pressure, people begin looking for a new material and HFFR4 (Halogen Free FR4) gets increasingly attention. Though HFFR4 systems are inferior to FR-4 performance in a one-to-one fashion with a Halogen-free replacement, halogen-free materials are often most effective in synergistic combinations. It is possible now to design robust formulations with optimized performance. PCB can achieve the desired performance and get more cost effective combinations by choosing from various non-halogenated systems. The chart below shows how dielectric constant varies between suppliers with 2 different frequencies.



Besides dielectric constant difference between suppliers, they may also have different precision and accuracy under manufacturing. Today's technology can not provide 100% the same product as designed. Small variations can not be avoided. Suppliers may have different standards for their own products. Variations of variables may be small, but the total impact could be big. Another question is how small the variation is or how precise and accurate the finished product can be. High precision and accuracy means high quality, but also means high price. Is it necessary to pay a high price for your design? We PCB designer therefore must choose wisely.

1.2 Thesis purpose

The thesis goal is to study and research on how we can handle the variations of these cheap PCB materials. This can be achieved with sorting out the spreading when it comes to conductor width & thickness, dielectric constant & thickness. One more area to explore is the variations of the dielectric constant due to glass cloth. How could statistical methods be used when looking at the variations of the material properties? Mathematica and Advanced Design System (ADS) were used to perform statistical analysis.

1.3 Limitations of the task

The reliability of my study is all based on the reliability of the data from PCB suppliers. The quantity of their samples is not large enough to find out the real distribution of every variable. It should take hundreds or thousands of samples, but that large amount is not realistic. People assume usually all variables following normal distribution, and my study can only based on this assumption.

1.4 Method

I began my project with pilot study to understand the problem and find a proper way to solve it. Because I did not have any knowledge of PCB impedance, it took over a week. After that I tested Monte Carlo simulation on Advanced Design System, which is required by the customer.

My work consists of 4 parts: distribution study, sensitivity study, methodology creating, and report. Because my project is a research project, and report contains all results that should be delivered to the customer, I listed it in the main part of my work too. Every part took about 10 days, and I worked 8-9 hours a day.

My work place is near the supervisor's office. We can easily contact with each other face to face, or via email. My question can be answered, and useful information can be delivered to me by an efficient way.

The tools used for the study are Advanced Design System and Mathematica. Advanced Design System has algorithms that run in black box to calculate characteristic impedance. Hardware designers in Ericsson use this tool to design circuits. The purpose of using this tool to simulate the outcomes is to create the possibility for future to use this tool to do simple distribution analysis by her/his self (a simple example in appendix c). The sensitivity analysis must be done by more professional math tools. In this project, I chose Mathematica to do the calculation and plotting.

2 BACKGROUND THEORY

2.1 Characteristic impedance

Characteristic impedance is the ratio of the amplitudes of a single pair of voltage and current waves propagating along a uniform transmission line without reflections. It is usually written as

Z0 or **Zc.** Characteristic impedance is determined by 2 variables: the per-unit-length inductance L and the per-unit-length capacitance C of a line.[1]

$$Z_0 = \sqrt{\frac{L}{C}}$$

When the loaded impedance is equal to the characteristic impedance, the wave will not be reflected. An existed circuit means that the value of **Z0** has been determined; therefore we must be very careful when design and produce PCB. Any small changes could impact the result. In order to keep the reflection under an acceptable range, analysis of PCB variations is necessary.

The per-unit-length variables of PCB lands are very difficult to derive, and it will not be discussed in this thesis. However we can use some design tools, ADS for instance, to calculate the characteristic impedance. We can also find formulas in many literatures. Usually, these factors are given in terms of L and C: relative permittivity and thickness of the substrate, width and thickness of the trace, distance between traces. A table of characteristic impedance calculating for microstrip-line stands below. W is width of the strip, t is thickness of the strip, h is the thickness of the substrate and εr is the relative permittivity.

Table 2.6 Microstrip-Line Characteristics

Parameters	Expressions
Characteristic impedance (Ω)	$Z_0 = \begin{cases} \frac{\eta_0}{2\pi\sqrt{\epsilon_e}} \ln\left(\frac{8h}{W'} + 0.25\frac{W'}{h}\right), & \frac{W}{h} \le 1 \\ \frac{\eta_0}{\sqrt{\epsilon_e}}\left[\frac{W'}{h} + 1.393 + 0.667 \ln\left(\frac{W'}{h} + 1.444\right)\right]^{-1}, & \frac{W}{h} \ge 1 \end{cases}$
	$\eta_0 = 120\pi\Omega$
	$\frac{W'}{h} = \frac{W}{h} + \frac{1.25}{\pi}\frac{t}{h}\left(1 + \ln\frac{4\pi W}{t}\right), \quad \frac{W}{h} \le \frac{1}{2\pi}$
	$\frac{W'}{h} = \frac{W}{h} + \frac{1.25}{\pi}\frac{t}{h}\left(1 + \ln\frac{2h}{t}\right), \quad \frac{W}{h} \ge \frac{1}{2\pi}$
Effective dielectric constant	$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}F\left(\frac{W}{h}\right) - \frac{\epsilon_r - 1}{4.6}\frac{t/h}{\sqrt{W/h}}$
	$F\left(\frac{W}{h}\right) = \begin{cases} \left(1 + 12\frac{h}{W}\right)^{-1/2} + 0.04\left(1 - \frac{W}{h}\right)^2, & \frac{W}{h} \le 1 \\ \left(1 + 12\frac{h}{W}\right)^{-1/2} & \frac{W}{h} \ge 1 \end{cases}$

More tables can be found in Appendix A.

2.2 Monte Carlo Simulation

Monte Carlo Simulation is a method that repeatedly takes random samples to compute the properties of some phenomenon or behavior. It is very useful when inputs have significant uncertainty. Monte Carlo simulation is widely used in science and engineering, including astrophysics, physical chemistry, biology, finance and many other areas. In electronics engineering, it is applied to analyze any correlated variations in circuits.

[1] Clayton R.Paul, (2010), *Transmission Lines in Digital and Analog Electronic Systems: Signal Integrity and Crosstalk.* Wiley, ISBN 978-0-470-59230-4

Monte Carlo Simulation works this way:

- Get random value for each variable
- Compute the outcome
- Repeat step 1 and step 2 as many times as possible to produce hundreds or thousands of possible outcomes.
- Analyze the results to get probabilities of different outcomes occurring.

The "random value" is not always required to be truly random. It is more useful to generate pseudorandom sequences that appear random enough and approximate the properties of these variables. That means that the known distributions of variables should pass statistical tests and be proved. Shlomo Sawilowsky lists the requirements for a high quality Monte Carlo Simulations:[2]

- The (pseudo-random) number generator has certain characteristics (*e.g.*, a long "period" before the sequence repeats)
- The (pseudo-random) number generator produces values that pass tests for randomness
- There are enough samples to ensure precise results
- The proper sampling technique is used
- The algorithm used is valid for what is being modeled
- It simulates the phenomenon in question.

In my study, Process Capability Index Values, which indicates how much "natural variation" a process experiences relative to its specification, are available form PCB suppliers, and is considered reliable. All results depend on the data from suppliers.

The main advantage of Monte Carlo Simulation is simplicity. Specific knowledge of the solution is not required. It is also easy to implement on a computer. The main disadvantage is slowness. In order to get precise result, thousands or even millions samples may be needed, and surely that will take time.

[2] Sawilowsky, Shlomo S.; Fahoome, Gail C. (2003). *Statistics via Monte Carlo Simulation with FORTRAN*. Rochester Hills, MI: JMASM. ISBN 0-9740236-0-4)

2.3 A simple example for how PCB variations affect the characteristic impedance

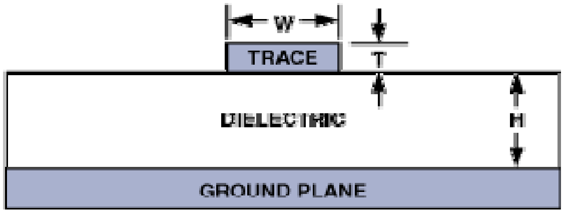


Figure 2.1[3]

In this section, a simple case is used to reveal the problem and show how Monte Carlo Simulation works. All the information needed stands in table: (For instance, W, short for Width, has value 0.246 and tolerance +/- 0.030)

	Nom	Positive Tolerance	Negative Tolerance
Width (W)	0.246	+0.030	-0.030
Height (H)	0.150	+0.030	-0.030
Thick (T)	0.050	+0.020	-0.020
Dielectric Constant (Dk)	4.53	+0.31	-0.31
Imp.	50.000		

Let us see how much the result will be changed when variables reach their own limits:

Width	Height	Thick	Dk	Imp.	Diff (%)
0.276	0.150	0.050	4.53	47.03	-5.9
0.216	0.150	0.050	4.53	53.53	7.1
0.246	0.120	0.050	4.53	43.77	-12.5
0.246	0.180	0.050	4.53	55.49	11.0
0.246	0.150	0.070	4.53	49.11	-1.8
0.246	0.150	0.030	4.53	51.22	2.4
0.246	0.150	0.050	4.83	48.68	-2.6
0.246	0.150	0.050	4.22	51.61	3.2
0.276	0.120	0.070	4.83	39.13	-21.7
0.216	0.180	0.030	4.22	62.44	24.9

The change is up to 24.9%. This is big difference but does it happen that often so becomes a serious problem for us? We can check with Monte Carlo Simulation. Considering the worst case, which means all variables' values are following uniform distribution within their own interval, the simulation result is as below: (the simulation runs in Mathematica)

[3] http://www.analog.com/library/analogDialogue/archives/39-09/3909_13.gif, 20120620

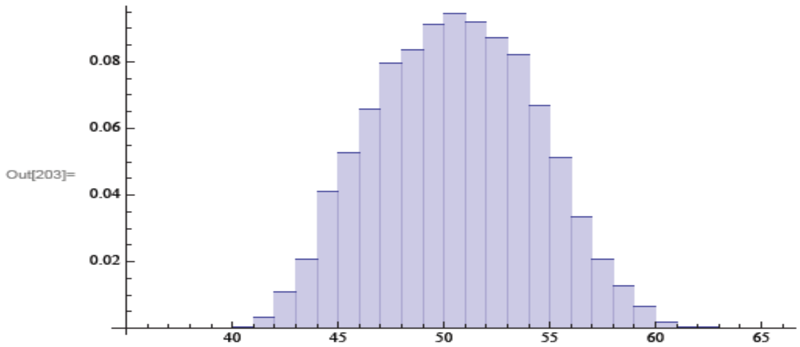


Figure 2.2

From the histogram of PDF, we see that the possibility that characteristic impedance has value 62.44 is very small. Most results are between 45 and 55 ohm. With help of Monte Carlo simulation, the distribution of characteristic impedance can be estimated, and further more we can estimate the passing rate of product from every supplier. In this example, characteristic impedance is normally distributed. This can be verified by probability plot:

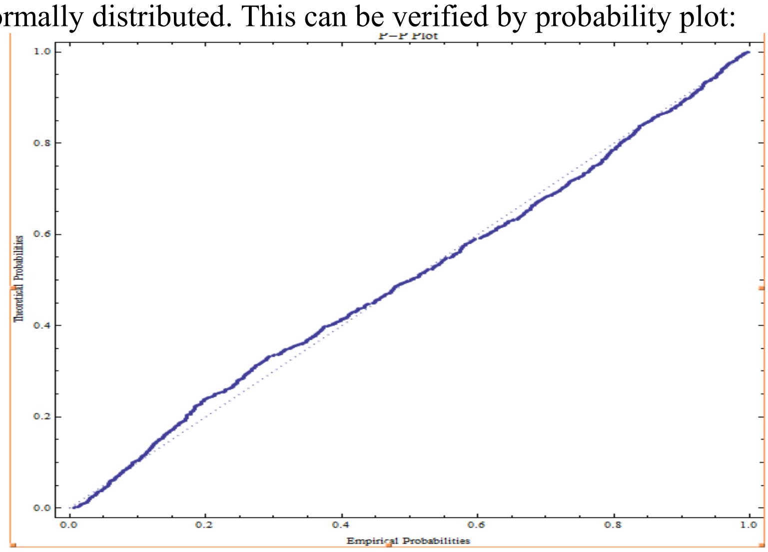


Figure 2.3

If the tolerance of characteristic impedance is +/- 8 ohm, the passing rate would be ca 95%.

3 FACTORS THAT AFFECT THE DISTRIBUTION OF CHARACTERISTIC IMPEDANCE AND PASSING RATE

3.1 Variables' distribution

The most apparent factor must be variables' distribution. If income values were as we expected, all problems would not exist. We can also simply notice the difference when a variable changes from a highly concentrated distribution to uniform distribution. This may seem simple, but we actually can dig little deeper.

3.1.1 Mean, Standard deviation and Expectation

Suppose we have sample space $\{a_1, \ldots, a_n\}$. Then mean A is defined via the equation[4]

$$A := \frac{1}{n} \sum_{i=1}^{n} a_i .$$

For instance, we collected a set of trace width: 0.246mm, 0.242mm, 0.250mm, 0.257mm, 0.245mm, then the mean of this set was 0.248mm. Every single element in this set can be approved according to the requirement 0.246 ± 0.030 mm, but the mean does not have to be the same as the expected value 0.246. The difference does cause difference between characteristic impedances' mean and its expected value, and this may lead to lower passing rate. The example in Chapter 2 is used as reference here to show the difference:

The only thing changed here is the mean of trace width. Run again Monte Carlo Simulation we see the change.

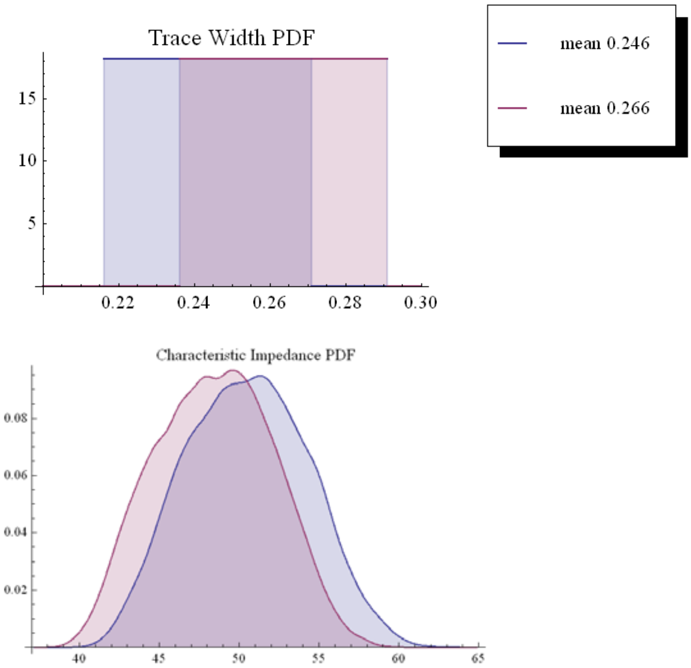


Figure 3.1

If the tolerance of characteristic impedance was ± 5 ohm, the passing rate would decrease from 81% to 79%.

Standard deviation is another factor that will be discussed here. Small value of standard deviation means small variation and high precision. This affects also result. (Figure 3.2)

[4]Mean : http://en.wikipedia.org/wiki/Arithmetic_mean, 20120620

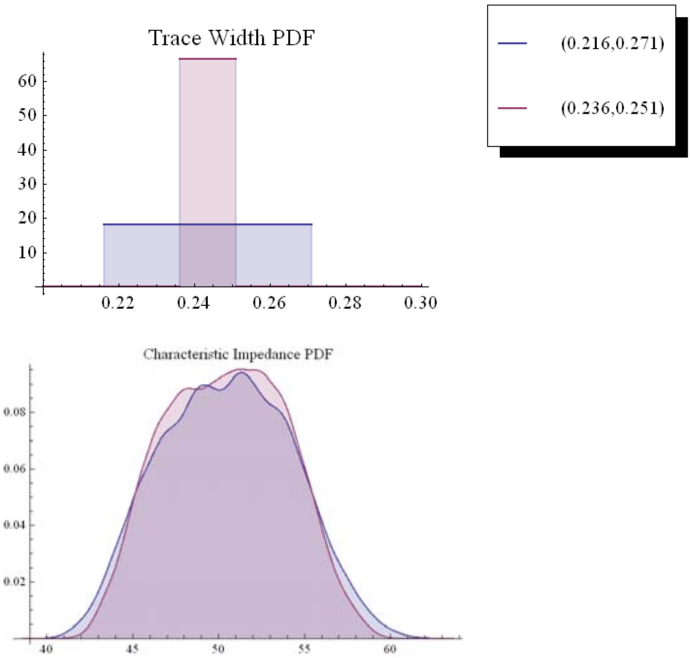
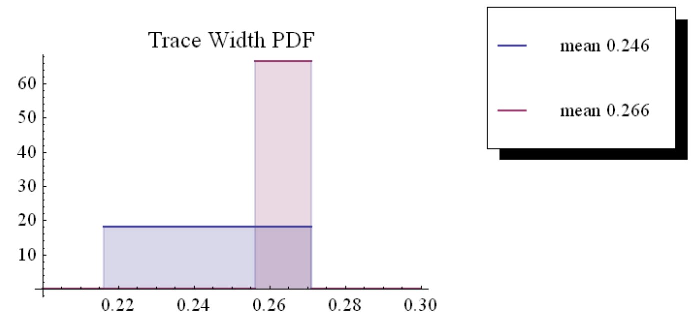


Figure 3.2

At the same tolerance of characteristic impedance, the passing rate becomes 85%.

But does smaller standard deviation of variables always mean higher passing rate? The answer is no. Only when high accuracy and high precision happen at same time for every variable, high passing rate can be achieved. So when discussing the passing rate, we should never think mean, standard deviation and the tolerance interval separately. Because of variables' tolerance, the standard deviation may also change when the mean value changes. The first example in this section should be modified:



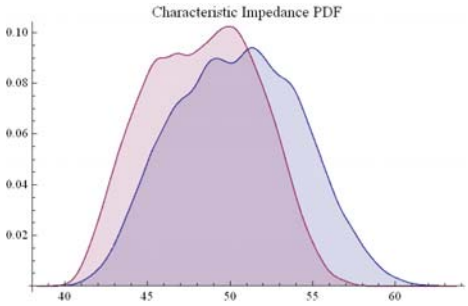


Figure 3.3

The passing rate is now 80.5%. It is almost the same as the reference group with 0.246 as the mean. In more extreme case, the passing rate of result with highly concentrated incomes could be lower than the passing rate of these with uniformly distributed incomes. In this particular case, the passing rate would stop here, if we did not change the mean of trace width and any value of other variables, no matter how hard we were trying to narrow down the variation of trace width.

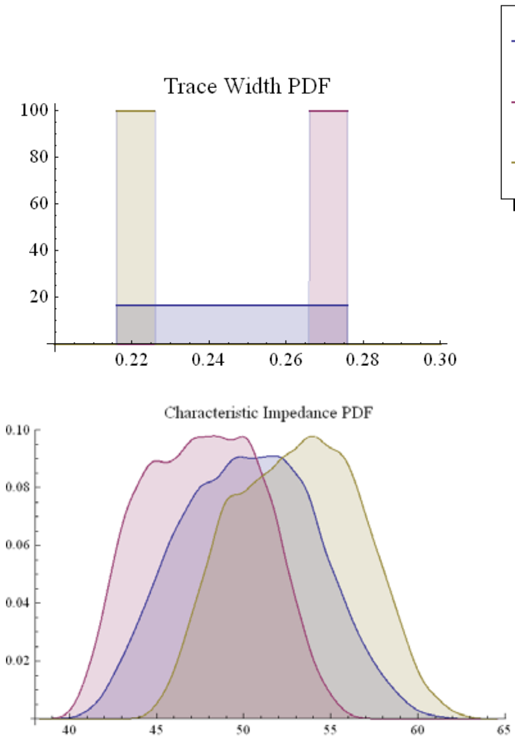


Figure 3.4

Let us see a more complex case (figure 3.4). The blue area is still for the reference set. The red and yellow areas are for two sets of outcomes with different means but same variation. The simulation result is in table below:

Group	Mean	Standard Deviation	(50+/- 5 ohm) Passing Rate
Blue (ref.)	50.3	3.82	81%
Red	47.6	3.31	77.4%
Yellow	53.0	3.57	69.7%

This table shows that the mean of variable affects also result's variation. How does this happen? The diagram below may help us understand.

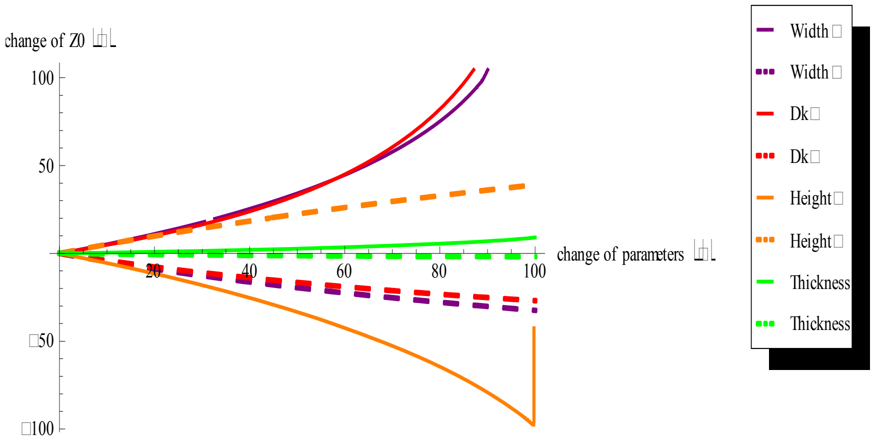


Figure 3.5

This diagram shows how many percents the result changes when every single variable increases or decreases from a particular point, there width = 0.246, Dk =4.53, Height= 0.150 and Thickness=0.050. We see the characteristic impedance changes much more quickly when trace width decreases than when it increases. With same size of variation, the set of trace width that has larger average value, gives larger variation of result. That is the real reason why mean of variable can affect the variation of the result.

3.1.2 Distribution form

We notice that we can get smaller standard deviation of characteristic impedance by adjusting the distribution form of every variable, not simply just their standard deviation or mean. Different distributions may give same results, or oppositely, similar distributions give totally different results. We can imagine how big improvement could be when we use normal distribution in stead of uniform distribution, but considering the fact shown in figure 3.5, a skew normal distribution may give even better result. Therefore the data that how every single variable is distributed is important for PCB variations analysis.

But in reality, it could be hard to gather enough data to find out the actual distribution of every variable. We usually assume they follow normal distribution. Process capability index[5] and other values provided by supplier are all based on this assumption. The commonly-accepted process capability index includes C_p, $C_{p,\,lower}$, $C_{p,\,upper}$, C_{pk} and so on. Suppliers provide Cpk value to indicate process yield of every single variable. As long as Cpk value is larger than 1, we can say the process yield is 100%. How close Cpk is to Cp tells us how far the mean of the set is from the center of the interval. If Cpk appears alone without Cp or mean, it is meaningless for PCB variations analysis. Suppliers usually provide only C_p and C_{pk}. In fact, the distribution shifted up or down is also important. Figure 3.7 shows what happens

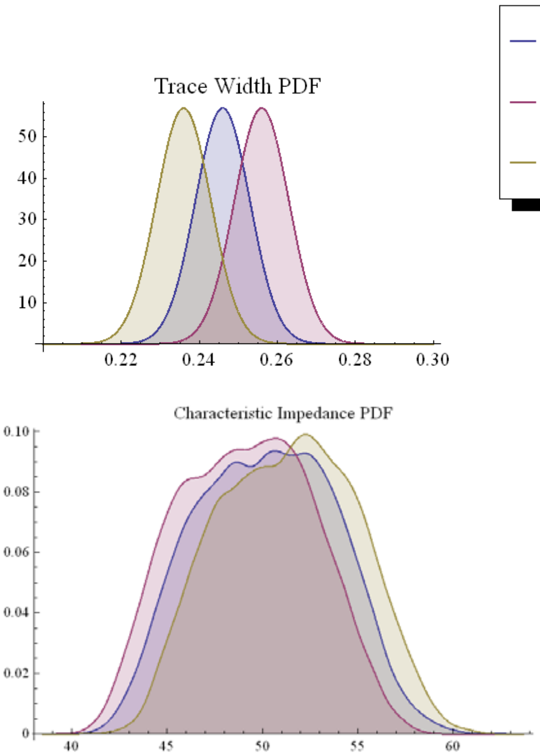


Figure 3.7

Results:

Group	Mean	Standard Deviation	(50+/- 5 ohm) Passing Rate	0.99 Confidence Level
Blue (ref.)	50.2	3.51	84.5%	(41.2, 59.3)
Red (up)	49.1	3.45	84.4%	(40.4, 58.0)
Yellow (down)	51.3	3.55	80.8%	(42.1, 60.6)

The passing rate is barely changed when we compare blue and red areas, but yellow area gives a lower passing rate. We can also see the difference of their 0.99 confidence intervals. The red set has the smallest size while the yellow set has the largest.

[5]Process capability index: http://en.wikipedia.org/wiki/Process_capability_index, 20120620

3.2 Sensitivity for variables

3.2.1 Introduction

Sensitivity is how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input.[6] The most popular methodology to study sensitivity is called local methods, such as the simple derivative of the output Y with respect to an input factor X_i:

$$\left|\frac{\partial Y}{\partial X_i}\right|_{\mathbf{x}^0},$$

Where the subscript $\mathbf{x}^0$ indicates that the derivative is taken at some fixed point in the space of the input (hence the 'local' in the name of the class).[7]

According to the definition, figure 3.5 shows indirectly the sensitivity of impedance for every variable. We can compute the partial derivatives and plot it more clearly:

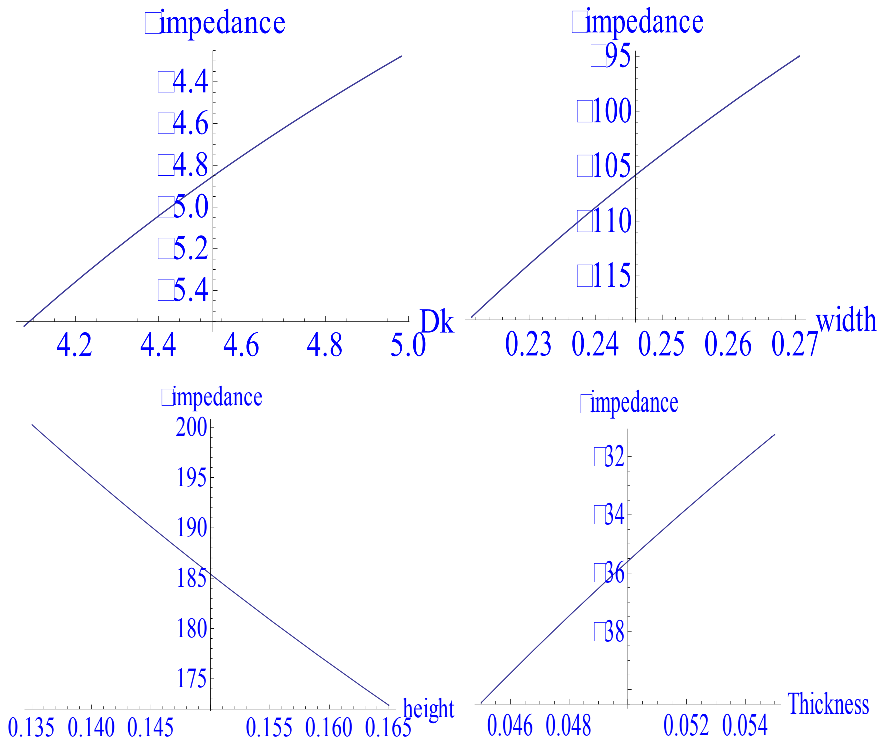


Figure 3.8

We already see how the sensitivity of result for variables affects the distribution in the last chapter. Less sensitive means smaller change for the variable and the result is more concentrated. Sensitivity varies from point to point. It is hard to derive a general conclusion for sensitivity and makes no sense without considering specific requirements, for instance, value of characteristic

[6] Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D. Saisana, M., and Tarantola, S., 2008, *Global Sensitivity Analysis. The Primer*, John Wiley & Sons.

[7] Sensitivity analysis: http://en.wikipedia.org/wiki/Sensitivity_analysis#cite_note-Primer-0, 20120620

impedance and its tolerance. Our main purpose is to control characteristic impedance's variations. Finding out sensitivity for variables is one of the major methods to reach the aim. It can only achieve with notice of certain condition.

To find a less sensitive combination of variables is very meaningful to variation control. Hundreds of combinations can give same characteristic impedance, but sensitivity for every individual combination is definitely not same. Some times we could not get the variation we want, even though every variable is quite precise. High sensitivity for one or several of these variables is usually behind this strange phenomenon.

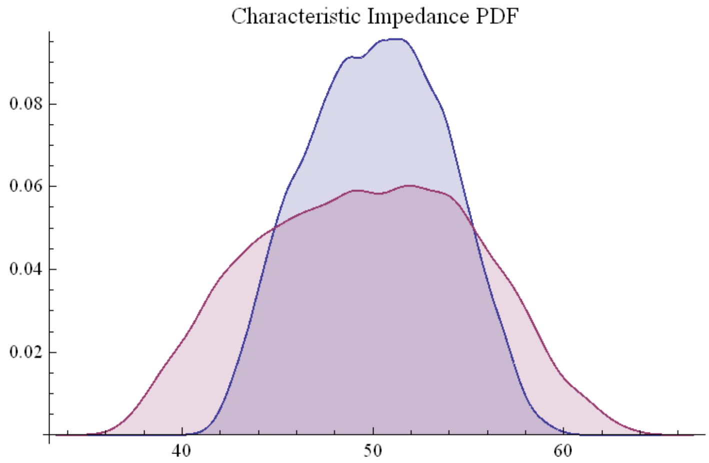


Figure 3.9

Figure 3.9 shows distributions of two sets of characteristic impedance. They have the same mean, 50 ohm, but different variations. The standard deviations for the blue area (reference set) and the red area are respectively 3.6 and 5.6. The 0.99 confidence level is extended for extra 5 ohm on each side for the red. The distribution for every variable is as below:

	Width	Height	Thickness	Dk
Blue (Ref.)	$\mathcal{U}(0.216, 0.276)$	$\mathcal{U}(0.12, 0.18)$	$\mathcal{U}(0.043, 0.053)$	$\mathcal{U}(4.2, 4.8)$
Red	$\mathcal{U}(0.170, 0.230)$	$\mathcal{U}(0.07, 0.13)$	$\mathcal{U}(0.010, 0.020)$	$\mathcal{U}(3.5, 4.1)$

Although the corresponding variables have same variation, the results have big differences. We take a look at the sensitivity for variables of the red set at the expected values (figure 3.10). The partial derivatives get higher absolute values comparing with in figure 3.8, which shows the sensitivity in the blue area.

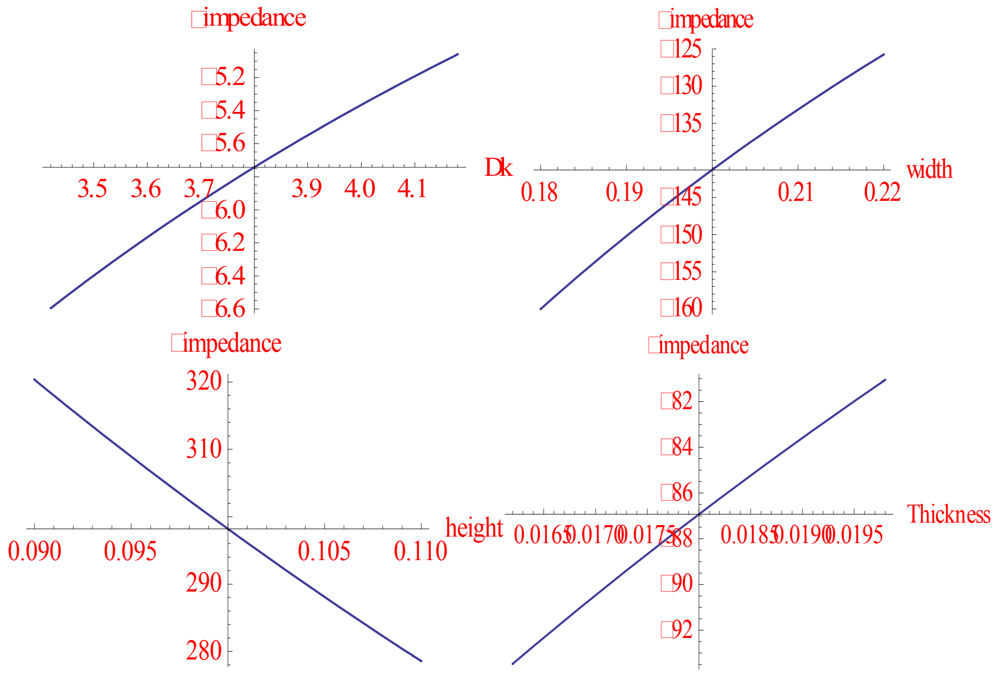


Figure 3.10

3.2.2 Relatively stable point

Impedance is given by many signals, and usually has been decided before designing the layout. For Microstrip line, we can get the most stable combination which gives particular value of characteristic impedance by setting values of dielectric constant, trace width and conductor thickness as large as possible and then adapting the dielectric substrate height to the result. For strip line, we in stead adapt the trace width to the particular value. For coupled lines, distance between lines usually has small impact on characteristic impedance, but sensitivities for other variables increase slightly with the distance's growing. The major impact still comes from trace width. This is all under ideal condition. In reality we have no many choices for these variables. Conductor thickness is usually 17 um for inner layer and 50 um for outer layer. Dielectric thickness is 0.15 mm for 14-20 layers PCB and ≥ 0.20 mm for ≤ 14 layers. This means trace width for single ended line has been also decided. For coupled lines, the most stable point is determined by both space distance and trace width. The table below gives the relatively stable combinations for coupled strip lines:

Diff. impedance	Dielectric constant	Dielectric thickness 0.15mm		Dielectric thickness 0.20mm	
		Trace width(mm)	Space distance(mm)	Trace width(mm)	Space distance(mm)
100 ohm diff	4.1	0.10	0.20	0.14	0.25
	4.2	0.10	0.22	0.14	0.28
	4.3	0.10	0.25	0.13	0.24
	4.4	0.10	0.29	0.13	0.26
	4.5	0.09	0.21	0.13	0.29
	4.6	0.09	0.25	0.12	0.25
	4.7	0.09	0.27	0.12	0.27
	4.8	0.09	0.3	0.12	0.3
50 ohm diff	4.1	0.37	0.26	0.5	0.27
	4.2	0.37	0.41	0.5	0.32
	4.3	0.36	0.29	0.5	0.45
	4.4	0.36	0.43	0.49	0.45
	4.5	0.35	0.32	0.48	0.40
	4.6	0.34	0.25	0.47	0.37
	4.7	0.34	0.31	0.47	0.50
	4.8	0.33	0.26	0.46	0.44

Assuming characteristic impedance has not been decided, we can find the comparatively most stable point by setting trace as wide as possible. The narrower the trace is, the more sensitive the characteristic impedance is for it. All of the partial derivatives become smaller with growing of trace width.

3.2.3 Variation estimation for outcomes

Another thing sensitivity can help us is to find the critical variables. “Critical” here means affecting the variation of result most. In order to get the variation we can accept, comparatively stricter requirement may be set for them. We can easily change the result’s variation by changing the critical variables’ variations. On the other side, if the critical variables have too large variation, the result's variation can hardly be changed, no matter how hard we try to narrow other variables' interval.

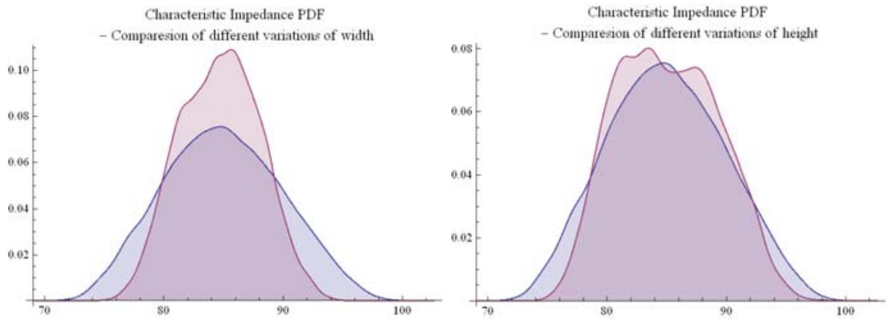


Figure 3.10

Figure 3.11 shows the effect after tolerance interval of a variable has been reduced. Both height and width was reduced by same size in millimeter. The sensitivities for them are respectively 255 and 145.

With notice of the limits for conductor thickness and dielectric height, we can find out the critical variables. For microstrip lines, dielectric substrate height is always one of the most important, the derivative for height is always over 100, even when the trace width is 1 mm; trace width can also be a very sensitive variable when its value is lower than 0.15mm (derivative's absolute value is over 150). When trace is wider than 0.4 mm, its derivative can be around -50. Comparing with microstrip, strip lines are more stable under the same condition. Dielectric substrate height is no longer an extremely sensitive variable. Its derivative is between 50 and 100. Trace width influences now conductor thickness quite much. The derivative's absolute value for thickness is over 100 when traces width smaller than 0.3mm; the derivative's absolute values for both thickness and width are over 100 when traces width smaller than 0.16 mm; the derivative's absolute values for thickness and width are over respectively 200 and 150 when traces width smaller than 0.1 mm.

How strict requirements for these critical variables should be depends on how precise and accurate we want the result to be, or the precision and the accuracy the product can reach. It can not be determined here. But at least we know conductor thickness and dielectric constant need not much attention.

When variables' variations are small, we can estimate the size of result's possible deviation as below:

Upper-Side max deviation ≈

$$\left|(dZ/dh)\cdot\Delta h+(dZ/dw)\cdot\Delta w+(dZ/d\varepsilon)\cdot\Delta\varepsilon+(dZ/dt)\cdot\Delta t+(dZ/ds)\cdot\Delta s\right| \quad (3.1.1)$$

where $dh\cdot\Delta h>0,\ dw\cdot\Delta w>0,\ d\varepsilon\cdot\Delta\varepsilon>0,\ dt\cdot\Delta t>0,\ ds\cdot\Delta s>0$

Lower-Side max deviation ≈

$$\left|(dZ/dh)\cdot\Delta h+(dZ/dw)\cdot\Delta w+(dZ/d\varepsilon)\cdot\Delta\varepsilon+(dZ/dt)\cdot\Delta t+(dZ/ds)\cdot\Delta s\right| \quad (3.1.2)$$

where $dh \cdot \Delta h < 0$, $dw \cdot \Delta w < 0$, $d\varepsilon \cdot \Delta\varepsilon < 0$, $dt \cdot \Delta t < 0$, $ds \cdot \Delta s < 0$

- dZ/dh is the value of the partial derivative for dielectric height
- dZ/dw is the value of the partial derivative for trace width
- $dZ/d\varepsilon$ is the value of the partial derivative for dielectric constant
- dZ/dt is the value of the partial derivative for conductor thickness
- dZ/ds is the value of the partial derivative for space between traces. For single ended line, this value is Zero.
- Δh is one-side tolerance for dielectric height , positive or negative
- Δw is one-side tolerance for trace width, positive or negative
- $\Delta\varepsilon$ is one-side tolerance for dielectric constant, positive or negative
- Δt is one-side tolerance for conductor thickness, positive or negative
- Δs is one-side tolerance for space between traces, positive or negative. For single ended line, this value is Zero.

One-side max deviation here is the distance from 0.99 confidence level of the outcome's distribution to the expected value. In figure 3.11, one-side variation size is the distance between "expectation" and "V_{upper}", or between "expectation" and "V_{lower}". L_{upper} and L_{lower} here in the picture are the expected limit of the result. When L_{upper} is smaller than V_{upper} or L_{lower} is larger than V_{lower}, It can not ensure that 100% outcomes fall in the interval we expected.

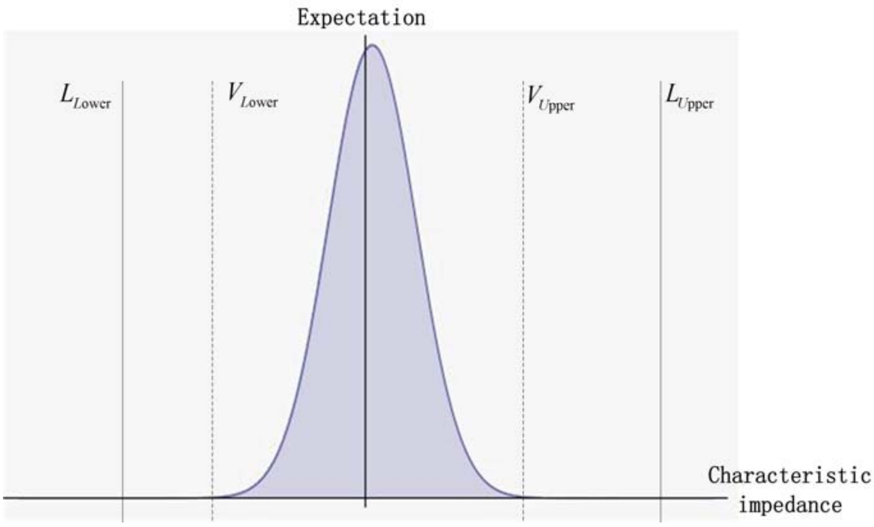


Figure 3.11

Take the case in Chapter 2.3 for example.

dz/dh=185, dz/dw= -105, dz/dε= -4.82, dz/dt= -40, dz/ds=0, Δh=±0.03, Δw=±0.03, Δε=±0.31, Δt=±0.02, Δs=0

Upper-Side variation size= Lower-Side variation size

= |dz/dh| ·|Δh|+ |dz/dw| ·|Δw|+ |dz/dε|·|Δε|+ |dz/dt| ·|Δt|+ |dz/ds| ·|Δs|

=5.55+3.15+1.55+0.8+0

=11.05

The estimated limits: 49.99+11.05=61.04; 49.99-11.05= 38.94

Comparing with the real limits 39.13 and 62.44, the estimation is pretty close to the simulation result.

However, the main purpose of using this equation is not to compute the minimal and maximal possible values of the outcomes. It can be done by setting every variable to its limit as I have done in chapter 2. What the equation can help us is to roughly get the tolerance for those critical variables that give a certain impedance interval. For instance, now we want the max deviation of the outcomes in chapter 2.3 to be 5, the equation can be created:

5-(|(dz/dε)·Δε+ (dz/dt) ·Δt|) = |(dz/dh)·Δh+ (dz/dw)·Δw| =>

2.65 = -105Δw +185Δh, where Δw<0, Δh >0

Or, -2.65 = -105Δw +185Δh, where Δw>0, Δh <0

Δw and Δh can be calculated according to other conditions, such as “Δh must be greater than 0.01mm”. Of course we can use mathematic tools to compute Δw and Δh. It is certainly that the result would be much more correct by using professional tools, but it needs also skills. My method is easier to apply.

When variable’s deviation is smaller than the distance from the limit to the expected value, we can be satisfied with all the outcomes. But we always want more outcomes to be close to the expected value .In order to achieve high accuracy in result, mean of critical variable discussed in the last section should be also considered. Sensitivity reveals not only how far every single event is shifted from the expectation, but also the tendency when most variables change their value towards a particular point. We see in earlier examples that the result is always more concentrated around the value that the variables’ means give. Therefore the result’s mean will also change, when a variable’s mean change. For those critical variables, the result’s mean changes more. It is actually quite important to set requirement for critical variables’ mean, but I did not see any in the documents I could access. According to my experience of studies, the following equation for can get pretty good result:

$$D_{a, mean, upper} = \frac{\mu_{Z0,upper} - V_{expected}}{n \cdot \left. \frac{\partial Z_0}{\partial a} \right|_{P0}} \quad (3.2.1)$$

$$D_{a,mean,lower} = \frac{V_{expected} - \mu_{Z0,lower}}{n \cdot \left.\frac{\partial Z_0}{\partial a}\right|_{P0}} \qquad (3.2.2)$$

Z0: characteristic impedance

P0: a particular point there all variables of Z0 are decided

a: one variable of Z0

n: the total number of critical variables

V expected: The expected value of outcome

μZ0, upper: the wished upper limit of outcomes' mean

μZ0, lower: the wished lower limit of outcomes' mean

D a, mean, upper: estimated upper side max deviation of the variable's mean

D a, mean, lower: estimated lower side max deviation of the variable's mean

3.3 Prepreg thickness after pressing

Laminate consists of glass fiber and epoxy. Dielectric constant of laminate can be expressed as such:

$$\mathrm{D}k_{laminate} = \frac{Dk_{glass} \cdot H_{glass} + Dk_{epoxy} \cdot H_{epoxy}}{H_{glass} + H_{epoxy}} \qquad (3.3)$$

Correlation of dielectric constant and dielectric substrate's height can also affect characteristic impedance.

Prepreg flows slightly during lamination process and fills the gap between traces. By this way copper traces are embedded in the prepreg and the thickness of prepreg is changed. The equation for prepreg thickness after pressing is as below:[8]

$$\text{Final thickness of prepreg} =$$
$$\text{finished thickness} + \frac{\text{remaining copper area} \times \text{copper thickness}}{\text{removed copper area}}$$

This leads to different height for dielectric substrate. The extra height comes all from epoxy. If we know laminate has dielectric constant Dk0 when height is H0, the dielectric constant can be computed when height is changed slightly by ΔH:

[8] Istvan Nagy, *How to calculate PCB trace width and differential pair separation, based on the impedance requirement and other variables*

$$H_0 = H_{glass} + H_{epoxy}$$

$$Dk_0 = \frac{Dk_{glass} \cdot H_{glass} + Dk_{epoxy} \cdot H_{epoxy}}{H_{glass} + H_{epoxy}} \Rightarrow$$

$$Dk_0 = \frac{Dk_{glass} \cdot (H_0 - H_{epoxy}) + Dk_{epoxy} \cdot H_{epoxy}}{H_0}$$

$$\Rightarrow$$

$$H_{epoxy} = \frac{Dk_0 - Dk_{glass}}{Dk_{epoxy} - Dk_{glass}} H_0$$

$$H_{glass} = \frac{Dk_{epoxy} - Dk_0}{Dk_{epoxy} - Dk_{glass}} H_0$$

$$\Rightarrow$$

$$Dk = \frac{Dk_{glass} \cdot H_{glass} + Dk_{epoxy} \cdot (H_{epoxy} + \Delta H)}{H_0 + \Delta H}$$

$$= \frac{Dk_{glass} H_0 (Dk_{epoxy} - Dk_0) + Dk_{epoxy} H_0 (Dk_0 - Dk_{glass}) + Dk_{epoxy} \Delta H (Dk_{epoxy} - Dk_{glass})}{(Dk_{epoxy} - Dk_{glass})(H_0 + \Delta H)}$$

Dielectric constant is 6 for glass and 3 for epoxy. If set the value in the equation, we can get:

$$Dk = Dk_0 + \frac{(3 - Dk_0)\Delta H}{H_0 + \Delta H} = 3 + \frac{(-3 + Dk_0) H_0}{H_0 + \Delta H} \tag{3.4}$$

Considering this fact, characteristic impedance's distribution must be modified from earlier estimation. Figure 3.13 below is comparison between the simulations with and without considering correlation. Data is from the example in Chapter 2.3. We see the distribution in the right picture has larger deviation.

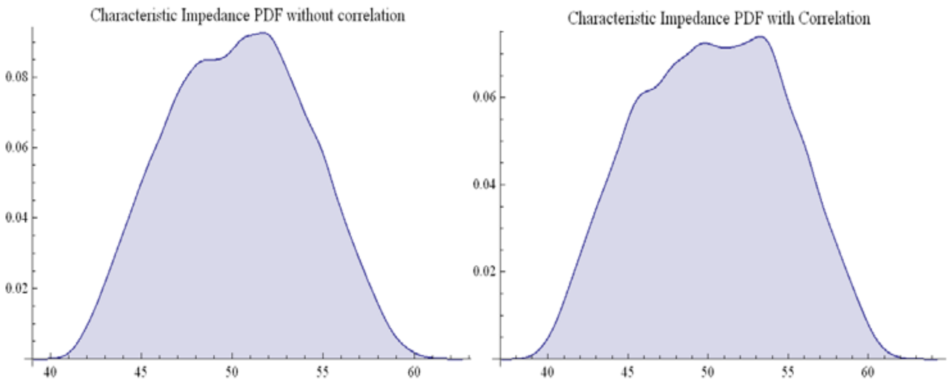


Figure 3.12

If check partial derivatives, we can see the result becomes more sensitive for dielectric substrate's height. According to equation 3.1, when value of a partial derivative changed, the deviation of result changed at the same time. Now the derivative for height is increased, the final derivation should also be increased. It meets the simulation result.

Correlation of dk and height has no effect on other variables in single ended line. Their derivatives are exactly same with or without correlation. For coupled lines, effect on other variables is extremely small. The influence on height is more obvious in microstrip. The difference between the derivatives with and without correlation can be over 100 when the trace is very narrow, but it can be reduced with increasing of trace width. For strip line, the influence can be ignored when trace is wider than 0.25 mm.

Check appendix B partial derivatives if interested in the difference.

4 REAL CASE STUDY

4.1 General Analysis

Because of lack of data, it is almost impossible to compute the real distribution of characteristic impedance for every supplier. In order to reveal the result could happen according to the specification, this general analysis is necessary.

As discussed before, the mean value of every variable can affect the result, even if all test values are in the interval as the specification defined. Passing rate could change dramatically, especially when standard deviation of this variable is small. In this analysis, the mean values of variables are assumed to meet the expectation. The correlation between dielectric constant and dielectric thickness is considered.

Specification

Variable	Line Width	Dielectric thickness inner layer	Dielectric thickness outer layer	Copper thickness inner layer	Copper thickness outer layer
Requirement	100 ±30 um	200±30 um (core) 200±38 um (prepreg)	60 ± 20 um	17 +3/-7 um	40 +20/-15 um

Distribution of variables

Under real manufacturing, it is unlikely for every single variable to follow uniform distribution, which is worst case for both supplier and Ericsson. Most possible situation is that they follow normal distribution or some distribution form close to normal distribution. Uniform distribution may show the worst case we get, but also meaningless when the worst case is too far away from the expectation and has very low possibility to happen. Normal distribution is used here to show the most common result. For copper thickness, Skew normal distribution is used.

Define Standard deviation and Mean

Mean is simply set to the expected value.

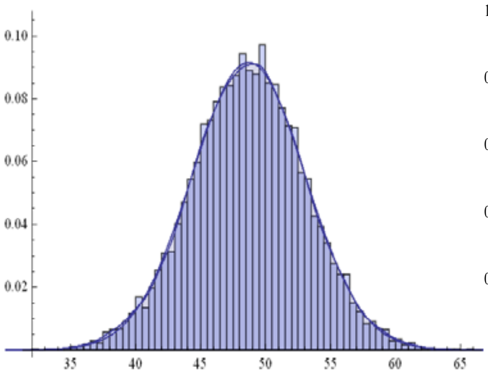
Standard deviation should let nearly all the samples fall in the tolerance interval. So I set

Standard deviation = (Upper limit- Lower limit)/6

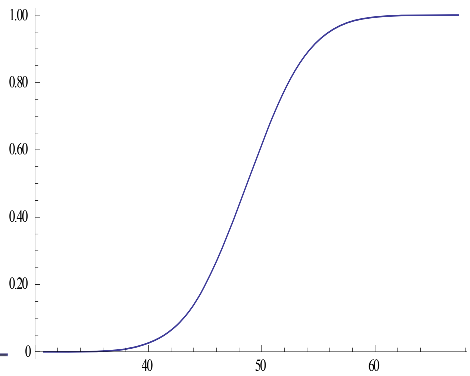
Variable	Line Width	Dielectric thickness inner layer	Dielectric thickness outer layer	Copper thickness inner layer	Copper thickness outer layer
Distribution	N(100,10)	N(200,10) (core) N(200,12.7) (prepreg)	N(60,6.7)	Skew Normal Distribution [19,2.6,-10]	Skew Normal Distribution [34.6,7.3,2.8]

Simulation plotting

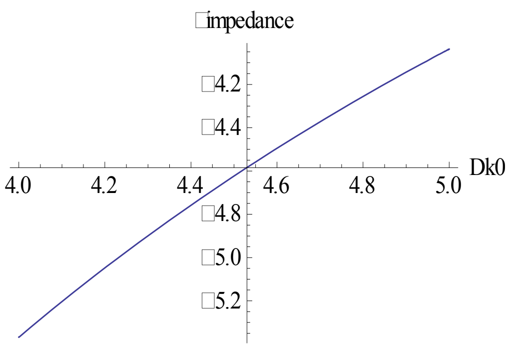
1) Microstrip line

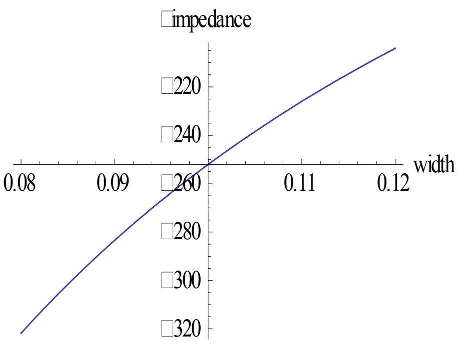


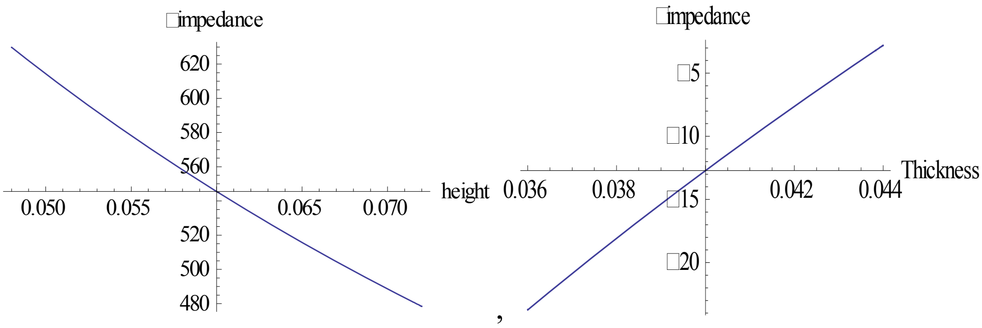
PDF



CDF



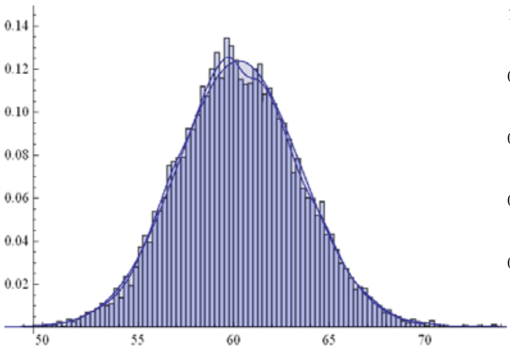




SENSITIVITY

Normality fit test: 0.938306. Characteristic impedance is normal distributed.
Mean value: 48.6915 Standard deviation: 4.35672 (worst case 7.7)
Min value: 31 Max value: 69
95% confidence interval (33.6, 66.8), which means 95% of simulation results are between 40.15 ohm and 57.23 ohm.

2) Strip line



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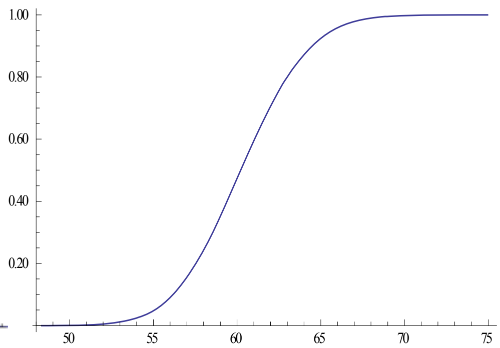
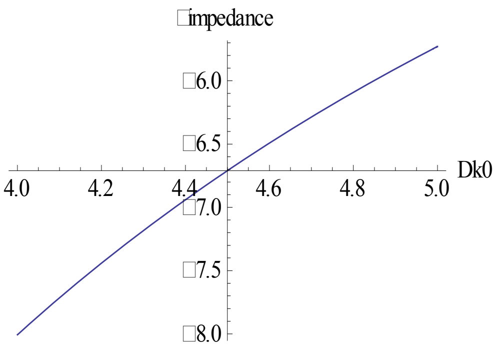
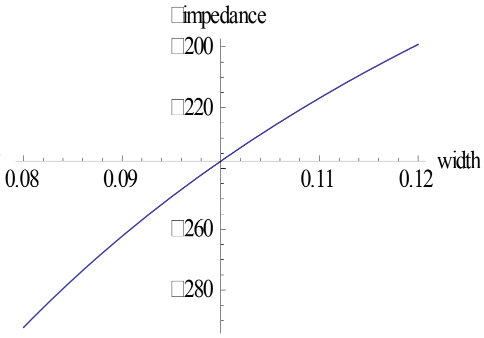
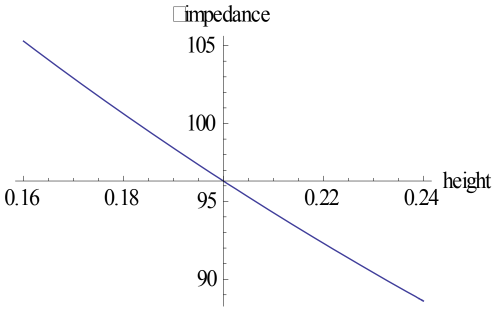


CDF







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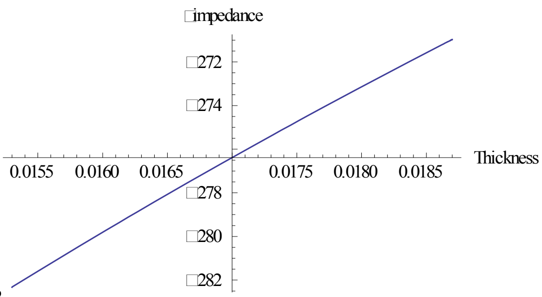
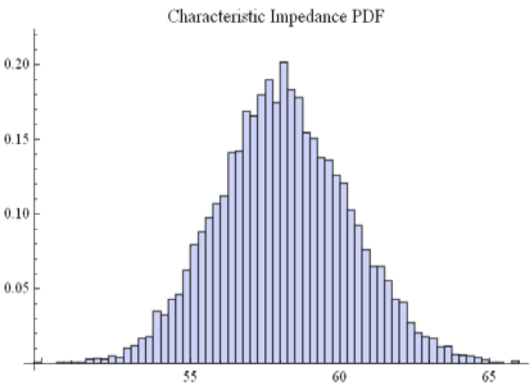


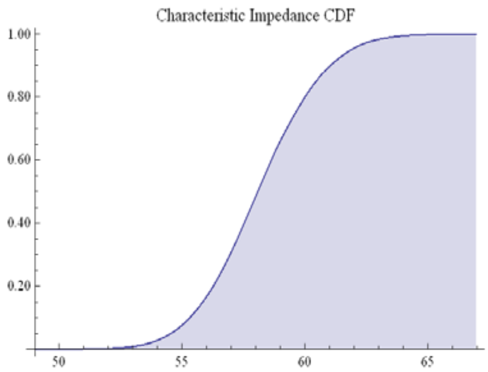
SENSITIVITY

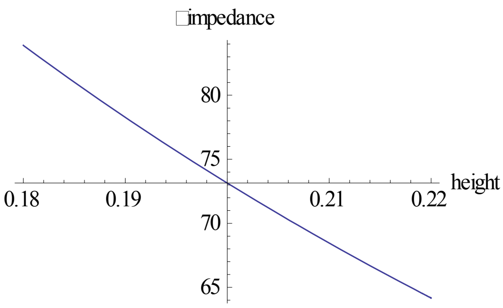
Normality fit test: 0.482. Characteristic impedance is normal distributed.
Mean value: 60.2803 Standard deviation: 3.22439 (worst case 5.8)
Min value: 46 Max value: 78
95% confidence interval (53.96, 66.60), which means 95% of simulation results are between 53.96 ohm and 66.60 ohm.

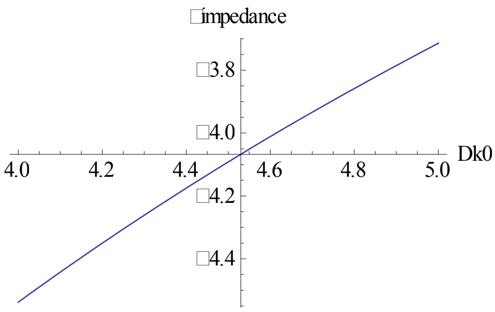
2) Strip Coupled Lines

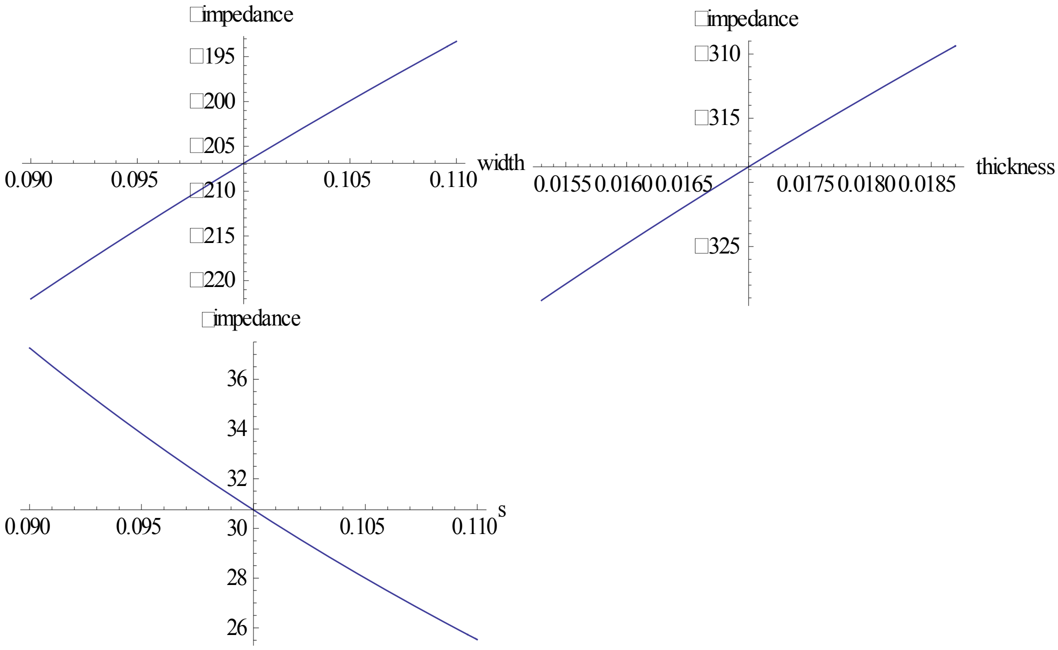
The requirement for spacing distance is Min 0.100mm. Here set spacing distance equal to 0.100mm.











Sensitivity

Mean value: 56.13 Standard deviation: 2.19 (worst case 4.0)
Min value: 44 Max value: 69

95% confidence interval (53.81, 62.43), which means 95% of simulation results are between 53.81ohm and 62.43 ohm.

Result

With the specification, the tolerances of characteristic impedance are expected:

Outer layer single line [31, 69] ohm
Inner layer single line [46, 78] ohm
Inner layer coupled lines [45, 69] ohm

The expected distributions are as above in this section. If supplier can provide high precision, the interval can be narrower. In order to get better accuracy, tolerance of critical variables' mean value should also be a part of requirement. Critical variable can be defined as the variable that impedance is sensitive for. For instance, if the mean dielectric thickness of the Microstrip line was shifted from 60 um to 68 um, 95% confidence interval would change to (46, 59). The tolerance depends on how precise the result should be.

4.2 Analysis based on Multek and GCE's data

The result can be seen in the intern report. The quantity of samples is not large enough to get real distributions. The result's reliability is based on the data supplier provided. If we set limits for the critical variables' mean, the variation from the expected value could be reduced.

5 CONCLUSION

Controlling impedance in PCB is a complicated task. It needs tons of data to ensure the reliability. Not only PCB supplier should be responsible for final product's quality, but also designer. Actually, Designer's role is more important. The maximal possible deviation of impedance depends on designer's requirement. As long as suppliers provide product with all variables falling in tolerance interval, we as designer can not reject the product. But the result could be terrible if the value of variable is very close to the tolerance interval's edge. The possibility of extreme cases happening may be small, but still could happen.

The actual quality of the final products depends on suppliers' accuracy and precision. Accuracy and precision are not bound to each other. Suppliers may provide high precision but low accuracy. When this happened, some extreme cases you thought should be would no longer be extreme. Suppliers will not be responsible for that; therefore designer must be very careful when set requirements to ensure their accuracy and precision.

If you are not satisfied with result's distribution, the quickest way for designer to improve it is to narrow its tolerance interval. This can be achieved by narrowing down the critical variables' interval. If you are satisfied with result's precision, but want to improve the accuracy, you can set stricter requirement for the critical variables' mean. Critical variables can be found through sensitivity study. Sensitivity study is enormously important to impedance control. It can not only tell us which variables are critical, but also tell us which combinations of variables can give relatively smaller deviation.

Suppliers should keep recording data during manufacturing. Too small amount of data can not show real distribution and be difficult for designer to analyze. It is better to provide CDF or PDF if possible. CPK value is based on normality assumption. The real distribution may not be normally distributed.

In this project, the targets are only these most basic types of transmission lines: surface single-ended microstripline, surface edge-coupled microstriplines, symmetric single-ended stripline, and edge-coupled Striplines. Other types are not included in the study, because suitable formulas were not found. Results of broadside-coupled striplines' formula in appendix A could not meet the results from Advanced Design System (ADS). All the design jobs are based on ADS; therefore we must assume the algorithms in ADS are always correct, and a wrong formula can not be used to analyze data. However, the method to analyze other type of transmission lines is same. In future studies, people can do similar analysis if can access to the formulas that give more close results to ADS.

6 REFERENCE LIST

[1] Clayton R.Paul, (2010), Transmission Lines In Digital and Analog Electronic Systems: Signal Integrity and Crosstalk, Wiley, ISBN 978-0-470-59230-4

[2] Sawilowsky, Shlomo S; Fahoome, Gail C. (2003), Statistics via Monte Carlo Simulation with FORTRAN, Rochester Hills, ISBN 0-9740236-0-4

[3] Saltelli A., Ratto M., Andres T., Campolongo F., Cariboni J., Gatelli D., Saisana M., and Tarantola S., (2008), Global Sensitivity Analysis, The Primer, John Wiley & Sons.

[4] Istvan Nagy, How to calculate PCB trace width and differential pair separation, based on the impedance requirement and other variables.

[5] Inder Bahl and Prakash Bhartia, Microwave Solid State Circuit Design, Second Edition

7 APPENDIX A – CHARACTERISTIC IMPEDANCE EXPRESSIONS[9]

The tables are copied from *Microwave Solid State Circuit Design, Second Edition.*

Table 2.6 Microstrip-Line Characteristics

Parameters	Expressions
Characteristic impedance (Ω)	$Z_0 = \begin{cases} \dfrac{\eta_0}{2\pi\sqrt{\epsilon_e}} \ln\left(\dfrac{8h}{W'} + 0.25\dfrac{W'}{h}\right), & \dfrac{W}{h} \le 1 \\ \dfrac{\eta_0}{\sqrt{\epsilon_e}}\left[\dfrac{W'}{h} + 1.393 + 0.667 \ln\left(\dfrac{W'}{h} + 1.444\right)\right]^{-1}, & \dfrac{W}{h} \ge 1 \end{cases}$
	$\eta_0 = 120\pi\Omega$
	$\dfrac{W'}{h} = \dfrac{W}{h} + \dfrac{1.25}{\pi}\dfrac{t}{h}\left(1 + \ln\dfrac{4\pi W}{t}\right), \quad \dfrac{W}{h} \le \dfrac{1}{2\pi}$
	$\dfrac{W'}{h} = \dfrac{W}{h} + \dfrac{1.25}{\pi}\dfrac{t}{h}\left(1 + \ln\dfrac{2h}{t}\right), \quad \dfrac{W}{h} \ge \dfrac{1}{2\pi}$
Effective dielectric constant	$\epsilon_e = \dfrac{\epsilon_r + 1}{2} + \dfrac{\epsilon_r - 1}{2} F\left(\dfrac{W}{h}\right) - \dfrac{\epsilon_r - 1}{4.6}\dfrac{t/h}{\sqrt{W/h}}$
	$F\left(\dfrac{W}{h}\right) = \begin{cases} \left(1 + 12\dfrac{h}{W}\right)^{-1/2} + 0.04\left(1 - \dfrac{W}{h}\right)^2, & \dfrac{W}{h} \le 1 \\ \left(1 + 12\dfrac{h}{W}\right)^{-1/2} & \dfrac{W}{h} \ge 1 \end{cases}$

Table 2.5 Strip-Line Characteristics

Parameter	Expressions	Remarks
Characteristic impedance (Ω) for $t = 0$	$Z_0 = \dfrac{30\pi}{\sqrt{\epsilon_r}}\dfrac{K'(k)}{K(k)}, \qquad k = \tanh\dfrac{\pi W}{2b}$	Virtually exact [5]
Characteristic impedance (Ω) for $t \neq 0$	$Z_0 = \dfrac{30 \ln}{\sqrt{\epsilon_r}}\left\{1 + \dfrac{4}{\pi}\dfrac{b-t}{W'}\left[\dfrac{8}{\pi}\dfrac{b-t}{W'} + \sqrt{\left(\dfrac{8}{\pi}\dfrac{b-t}{W'}\right)^2 + 6.27}\right]\right\}$	For $W'/(b-t) < 10$ accuracy within 0.5% [11]
	$\dfrac{W'}{b-t} = \dfrac{W}{b-t} + \dfrac{\Delta W}{b-t}$	
	$\dfrac{\Delta W}{b-t} = \dfrac{x}{\pi(1-x)}\left\{1 - \dfrac{1}{2}\ln\left[\left(\dfrac{x}{2-x}\right)^2 + \left(\dfrac{0.0796x}{W/b + 1.1x}\right)^m\right]\right\}$	
	$m = 2\left[1 + \dfrac{2}{3}\dfrac{x}{1-x}\right]^{-1}, \qquad x = \dfrac{t}{b}$	
Attenuation constant (dB/unit length)	$\alpha_c = \dfrac{0.0231 R_s\sqrt{\epsilon_r}}{Z_0}\dfrac{\partial Z_0}{\partial W'}\left\{1 + \dfrac{2W'}{b-t} - \dfrac{1}{\pi}\left[\dfrac{3x}{2-x} + \ln\dfrac{x}{2-x}\right]\right\}$	[4]
	$\dfrac{\partial Z_0}{\partial W'} = \dfrac{30e^{-A}}{W'\sqrt{\epsilon_r}}\left[\dfrac{3.135}{Q} - \left(\dfrac{8}{\pi}\dfrac{b-t}{W'}\right)^2(1+Q)\right]$	
	$A = \dfrac{Z_0\sqrt{\epsilon_r}}{30\pi}, \qquad Q = \sqrt{1 + 6.27\left(\dfrac{\pi}{8}\dfrac{W'}{b-t}\right)^2}$	
	$\alpha_d = 27.3\sqrt{\epsilon_r}\dfrac{\tan\delta}{\lambda_0}$	
Cutoff for higher order mode (GHz)	$f_c = \dfrac{15}{b\sqrt{\epsilon_r}}\dfrac{1}{(W/b + \pi/4)}, \qquad W \text{ and } b \text{ in cm}$	TE is the lowest order mode [4]

9 *Inder Bahl and Prakash Bhartia*, *Microwave Solid State Circuit Design, Second Edition*

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Table 2.9 Characteristics for Coupled Structures

Characteristic Impedances (Ω)	Attenuation Constants (dB/unit length)	Remarks
	Strip Lines	
$Z_{0e} = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{K(k_e')}{K(k_e)}$		$t = 0$, virtually exact [28]
$Z_{0o} = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{K(k_o')}{K(k_o)}$		
$k_e = \tanh\left(\frac{\pi}{2}\frac{W}{b}\right) \tanh\left(\frac{\pi}{2}\frac{W+S}{b}\right)$		
$k_o = \tanh\left(\frac{\pi}{2}\frac{W}{b}\right) \coth\left(\frac{\pi}{2}\frac{W+S}{b}\right)$		
$Z_{0e} = \frac{30\pi(b-t)}{\sqrt{\epsilon_r}\left(W + \frac{bC_f}{2\pi}A_e\right)}$	$\alpha_c^e = \frac{0.0231 R_s \sqrt{\epsilon_r}}{30\pi(b-t)} \times \left\{60\pi + Z_{0e}\sqrt{\epsilon_r}\left[1 - \frac{A_e}{\pi}\left(\ln\frac{2b-t}{b-t} + \frac{1}{2}\ln\frac{t(2b-t)}{(b-t)^2}\right) + C_f\frac{(1+S/b)}{4\ln 2}\frac{\operatorname{sech}^2\theta}{1+\tanh\theta}\right]\right\}$	$\frac{t}{b} < 0.1$; $\frac{W}{b} \geq 0.35$ [28]
$Z_{0o} = \frac{30\pi(b-t)}{\sqrt{\epsilon_r}\left(W + \frac{bC_f}{2\pi}A_o\right)}$		
$A_e = 1 + \frac{\ln(1+\tanh\theta)}{\ln 2}$	$\alpha_c^o = \frac{0.0231 R_s \sqrt{\epsilon_r}}{30\pi(b-t)} \times \left\{60\pi + Z_{0o}\sqrt{\epsilon_r}\left[1 - \frac{A_o}{\pi}\left(\ln\frac{2b-t}{b-t} + \frac{1}{2}\frac{t(2b-t)}{(b-t)^2}\right) - C_f\frac{(1+S/b)}{4\ln 2}\frac{\operatorname{cosech}^2\theta}{1+\coth\theta}\right]\right\}$	
$A_o = 1 + \frac{\ln(1+\coth\theta)}{\ln 2}$		
$\theta = \frac{\pi S}{2b}$		
$C_f = 2\ln\left(\frac{2b-t}{b-t}\right) - \frac{t}{b}\ln\left[\frac{t(2b-t)}{(b-t)^2}\right]$	$\alpha_d^e = \alpha_d^o = 27.3\sqrt{\epsilon_r}\frac{\tan\delta}{\lambda_0}$	
	Microstrip Lines	
$Z_{0i} = [c\sqrt{C_i C_i^a}]^{-1}, \quad i = e \text{ or } o$	$\alpha_c^i = \frac{8.686 R_s}{240\pi Z_{0i}}\frac{2}{h}\frac{1}{c(C_i^{at})^2} \times \left[\frac{\partial C_f^{at}}{\partial(W/h)}\left(1 + 2\frac{W}{2h}\right) - \frac{\partial C_i^{at}}{\partial(S/h)}\left(1 - 2\frac{S}{2h}\right) + \frac{\partial C_i^{at}}{\partial(t/h)}\left(1 + 2\frac{t}{2h}\right)\right]$	Accuracy better than 3% for $0.2 \leq \frac{W}{h} \leq 2$; $0.05 \leq \frac{S}{h} \leq 2$
$C_e = C_p + C_f + C_f'$		
$C_o = C_p + C_f + C_{ga} + C_{gd}$		
$C_p = \epsilon_0\epsilon_r\frac{W}{h}$	$\alpha_d^i = 27.3\frac{\epsilon_r}{\sqrt{\epsilon_e^i}}\frac{\epsilon_e^i - 1}{\epsilon_r - 1}\frac{\tan\delta}{\lambda_0}$	
$2C_f = \sqrt{\epsilon_e}/(cZ_0) - C_p, \quad c = 3 \times 10^8$ m/s	$\frac{W_t^e}{h} = \frac{W}{h} + \frac{\Delta W}{h}\left(1 - 0.5\exp\frac{-0.69\,\Delta W}{\Delta t}\right)$	
$C_f' = \frac{C_f\sqrt{\epsilon_r/\epsilon_e}}{1 + \exp[-0.1\exp(2.33 - 2.53W/h)](h/S)\tanh(10S/h)}$	$\frac{W_t^o}{h} = \frac{W_t^e}{h} + \frac{\Delta t}{h}, \quad \frac{\Delta t}{h} = \frac{1}{\epsilon_r}\frac{t/h}{S/h}$	
$C_{ga} = \epsilon_0\frac{K(k')}{K(k)}; \quad k = \frac{S/h}{S/h + 2W/h}$		
$C_{gd} = \frac{\epsilon_0\epsilon_r}{\pi}\ln\left[\coth\left(\frac{\pi S}{4h}\right)\right] + 0.65C_f\left[\frac{0.02}{S/h}\sqrt{\epsilon_r} + 1 - \epsilon_r^{-2}\right]$		
$\epsilon_e^i = \frac{C_i}{C_i^a}$		
	Broadside-Coupled Strip Lines	
$Z_{0o} = \frac{60\pi/\sqrt{\epsilon_r}}{\frac{W}{b-S} + C_{f0} + 2\{(1+t/S)\ln(1+t/S) - (t/S)\ln(t/S)\}/\pi}$	$\alpha_c^i = \frac{0.0231 R_s\sqrt{\epsilon_r}}{Z_{0i}}\left(\frac{\partial Z_{0i}}{\partial b} + \frac{\partial Z_{0i}}{\partial S} - \frac{\partial Z_{0i}}{\partial W} - \frac{\partial Z_{0i}}{\partial t}\right)$	[31–33]
$Z_{0e} = \frac{60\pi/\sqrt{\epsilon_r}}{\frac{W}{b-S} + 0.443 + \left(\ln\frac{b+2t}{b-S} + \frac{S+2t}{b-S}\ln[(b+2t)/(S+2t)]\right)/\pi}$	$\alpha_d = 27.3\sqrt{\epsilon_r}\frac{\tan\delta}{\lambda_0}$	$\frac{W/b'}{1-S/b} \geq 0.35$; $\frac{W}{S} \geq 0.35$
$C_{f0} = \frac{b}{S\pi}\left(\ln\frac{1}{1-S/b} + \frac{S/b}{1-S/b}\ln\frac{b}{S}\right)$		

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Note: Superscripts a and t denote air as dielectric and strip thickness effect, respectively.

8 APPEDIX B – PARTIAL DERIVATIVES (dk=4.5)

Microstrip line without correlation (t=0.05mm)

	Height	Width	ddk	dw	dh	dt
1	0.15	0.075	-7.31533	-324.352	189.617	-82.3239
2	0.15	0.1	-6.78593	-253.839	193.034	-71.4245
3	0.15	0.125	-6.3396	-207.718	193.606	-61.5214
4	0.15	0.15	-5.95817	-174.708	192.383	-53.0223
5	0.15	0.175	-5.61728	-151.246	192.46	-48.0162
6	0.15	0.2	-5.32049	-132.163	190.583	-43.0979
7	0.15	0.225	-5.05513	-116.693	187.969	-38.7863
8	0.15	0.25	-4.8164	-103.929	184.89	-35.0235
9	0.15	0.275	-4.60039	-93.246	181.531	-31.74
10	0.15	0.3	-4.40394	-84.1963	178.015	-28.8684
11	0.15	0.325	-4.22446	-76.4518	174.429	-26.349
12	0.15	0.35	-4.05977	-69.7653	170.829	-24.1303
13	0.2	0.075	-8.087	-314.296	145.832	-111.885
14	0.2	0.1	-7.56579	-249.732	149.095	-96.9134
15	0.2	0.125	-7.12057	-207.122	150.511	-84.2375
16	0.2	0.15	-6.73481	-176.456	150.774	-73.7281
17	0.2	0.175	-6.39652	-153.145	150.241	-64.9577
18	0.2	0.2	-6.09698	-134.731	149.117	-57.5456
19	0.2	0.225	-5.82205	-120.107	148.179	-52.2347
20	0.2	0.25	-5.5778	-108.185	147.171	-47.7593
21	0.2	0.275	-5.35446	-98.0579	145.776	-43.787
22	0.2	0.3	-5.1494	-89.3632	144.11	-40.2587
23	0.2	0.325	-4.96041	-81.8301	142.254	-37.1187
24	0.2	0.35	-4.78562	-75.2519	140.27	-34.317

Microstrip line with correlation (t=0.05mm)

	Height	Width	ddk	dw	dh	dt
1	0.15	0.075	-7.31533	-324.352	264.234	-82.3239
2	0.15	0.1	-6.78593	-253.839	262.251	-71.4245
3	0.15	0.125	-6.3396	-207.718	258.27	-61.5214
4	0.15	0.15	-5.95817	-174.708	253.156	-53.0223
5	0.15	0.175	-5.61728	-151.246	249.756	-48.0162
6	0.15	0.2	-5.32049	-132.163	244.852	-43.0979
7	0.15	0.225	-5.05513	-116.693	239.531	-38.7863
8	0.15	0.25	-4.8164	-103.929	234.017	-35.0235
9	0.15	0.275	-4.60039	-93.246	228.455	-31.74
10	0.15	0.3	-4.40394	-84.1963	222.936	-28.8684
11	0.15	0.325	-4.22446	-76.4518	217.518	-26.349
12	0.15	0.35	-4.05977	-69.7653	212.239	-24.1303
13	0.2	0.075	-8.087	-314.296	207.698	-111.885
14	0.2	0.1	-7.56579	-249.732	206.973	-96.9134
15	0.2	0.125	-7.12057	-207.122	204.983	-84.2375
16	0.2	0.15	-6.73481	-176.456	202.295	-73.7281
17	0.2	0.175	-6.39652	-153.145	199.175	-64.9577
18	0.2	0.2	-6.09698	-134.731	195.759	-57.5456
19	0.2	0.225	-5.82205	-120.107	192.717	-52.2347
20	0.2	0.25	-5.5778	-108.185	189.841	-47.7593
21	0.2	0.275	-5.35446	-98.0579	186.738	-43.787
22	0.2	0.3	-5.1494	-89.3632	183.502	-40.2587
23	0.2	0.325	-4.96041	-81.8301	180.201	-37.1187
24	0.2	0.35	-4.78562	-75.2519	176.88	-34.317

Strip line without correlation (t=0.017mm)

	Height	Width	ddk	dw	dh	dt
1	0.15	0.075	-6.47658	-303.477	95.6957	-345.414
2	0.15	0.1	-5.74826	-230.43	95.1191	-290.176
3	0.15	0.125	-5.18132	-183.547	94.4211	-252.159
4	0.15	0.15	-4.72247	-150.808	93.6119	-223.764
5	0.15	0.175	-4.34123	-126.646	92.7024	-201.413
6	0.15	0.2	-4.0184	-108.113	91.704	-183.187
7	0.15	0.225	-3.741	-93.4917	90.628	-167.952
8	0.15	0.25	-3.49984	-81.7058	89.4859	-154.983
9	0.15	0.275	-3.28814	-72.0428	88.2885	-143.792
10	0.15	0.3	-3.10076	-64.0096	87.0465	-134.03
11	0.15	0.325	-2.93371	-57.2535	85.7697	-125.439
12	0.15	0.35	-2.78386	-51.5146	84.467	-117.824
13	0.2	0.075	-7.38801	-308.8	71.5845	-328.534
14	0.2	0.1	-6.64351	-236.787	71.3416	-275.467
15	0.2	0.125	-6.05788	-190.7	71.0445	-239.572
16	0.2	0.15	-5.57845	-158.531	70.6959	-213.166
17	0.2	0.175	-5.17535	-134.737	70.2988	-192.624
18	0.2	0.2	-4.82989	-116.401	69.8561	-176.003
19	0.2	0.225	-4.52952	-101.835	69.3713	-162.165
20	0.2	0.25	-4.26539	-89.9938	68.8476	-150.395
21	0.2	0.275	-4.03099	-80.1911	68.2886	-140.218
22	0.2	0.3	-3.82139	-71.957	67.6977	-131.307
23	0.2	0.325	-3.63275	-64.9574	67.0784	-123.424
24	0.2	0.35	-3.462	-58.9475	66.4341	-116.393

Strip line with correlation (t=0.017mm)

	Height	Width	ddk	dw	dh	dt
1	0.15	0.075	-6.47658	-303.477	128.473	-345.414
2	0.15	0.1	-5.74826	-230.43	122.569	-290.176
3	0.15	0.125	-5.18132	-183.547	117.192	-252.159
4	0.15	0.15	-4.72247	-150.808	112.168	-223.764
5	0.15	0.175	-4.34123	-126.646	107.43	-201.413
6	0.15	0.2	-4.0184	-108.113	102.95	-183.187
7	0.15	0.225	-3.741	-93.4917	98.7151	-167.952
8	0.15	0.25	-3.49984	-81.7058	94.7198	-154.983
9	0.15	0.275	-3.28814	-72.0428	90.957	-143.792
10	0.15	0.3	-3.10076	-64.0096	87.4185	-134.03
11	0.15	0.325	-2.93371	-57.2535	84.0947	-125.439
12	0.15	0.35	-2.78386	-51.5146	80.9747	-117.824
13	0.2	0.075	-7.38801	-308.8	100.122	-328.534
14	0.2	0.1	-6.64351	-236.787	96.3156	-275.467
15	0.2	0.125	-6.05788	-190.7	92.9471	-239.572
16	0.2	0.15	-5.57845	-158.531	89.8463	-213.166
17	0.2	0.175	-5.17535	-134.737	86.9298	-192.624
18	0.2	0.2	-4.82989	-116.401	84.1549	-176.003
19	0.2	0.225	-4.52952	-101.835	81.4997	-162.165
20	0.2	0.25	-4.26539	-89.9938	78.953	-150.395
21	0.2	0.275	-4.03099	-80.1911	76.5092	-140.218
22	0.2	0.3	-3.82139	-71.957	74.1651	-131.307
23	0.2	0.325	-3.63275	-64.9574	71.9187	-123.424
24	0.2	0.35	-3.462	-58.9475	69.768	-116.393

Strip coupled lines without correlation (s=0.1mm, t=0.017mm)

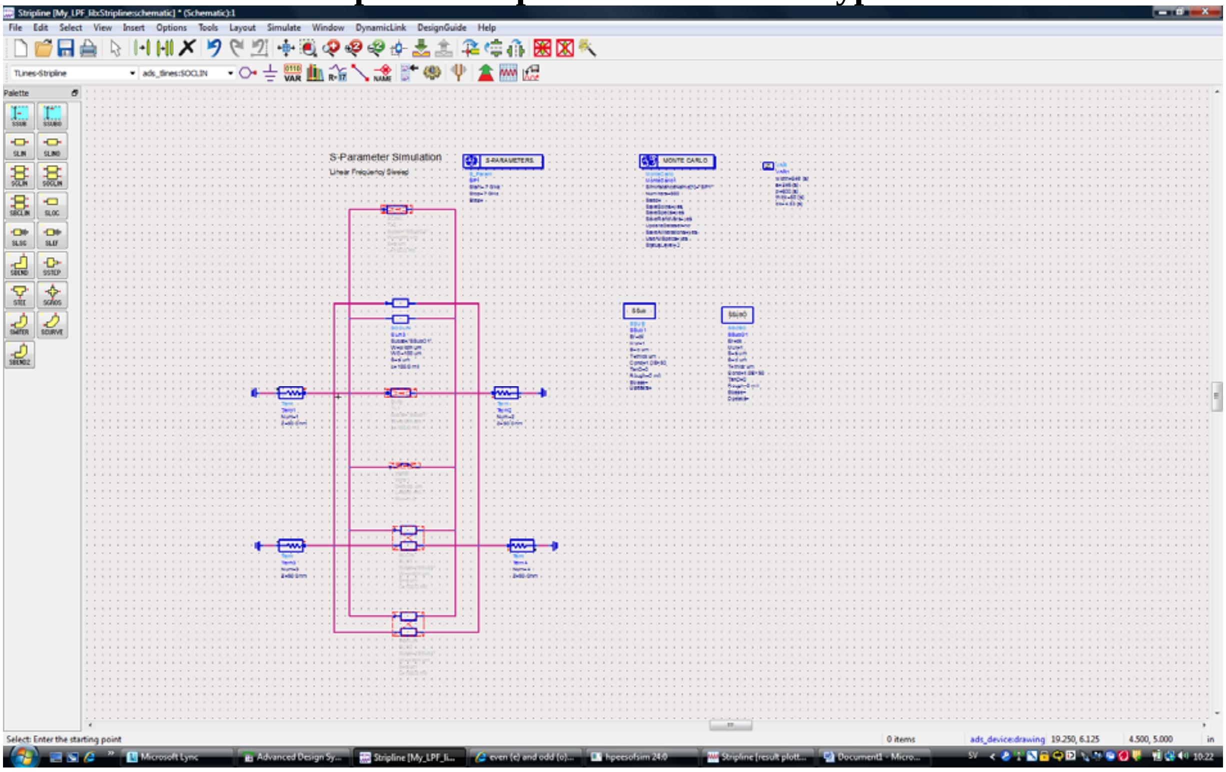
	Height	Width	ddk	dw	dh	dt	ds
1	0.15	0.075	-6.12668	-245.413	75.0055	-358.442	19.9785
2	0.15	0.1	-5.51404	-198.236	77.6615	-308.088	17.6265
3	0.15	0.125	-5.01387	-163.567	78.2322	-269.363	15.554
4	0.15	0.15	-4.59758	-137.318	77.5996	-238.762	13.7675
5	0.15	0.175	-4.24557	-116.952	76.2716	-214.041	12.2388
6	0.15	0.2	-3.94393	-100.826	74.55	-193.7	10.9316
7	0.15	0.225	-3.68253	-87.8346	72.6187	-176.702	9.81095
8	0.15	0.25	-3.45378	-77.2114	70.5914	-162.306	8.84631
9	0.15	0.275	-3.25189	-68.4121	68.5389	-149.974	8.01202
10	0.15	0.3	-3.07239	-61.0405	66.5053	-139.303	7.28684
11	0.15	0.325	-2.91173	-54.8027	64.5177	-129.986	6.65333
12	0.15	0.35	-2.76709	-49.4771	62.5922	-121.788	6.0972
13	0.2	0.075	-6.80148	-226.937	48.9674	-332.865	30.9207
14	0.2	0.1	-6.22588	-189.249	52.6747	-291.597	28.1217
15	0.2	0.125	-5.74198	-160.378	54.737	-258.779	25.5172
16	0.2	0.15	-5.32914	-137.739	55.727	-232.127	23.1626
17	0.2	0.175	-4.97257	-119.639	56.0058	-210.103	21.0621
18	0.2	0.2	-4.66136	-104.927	55.8081	-191.632	19.1989
19	0.2	0.225	-4.38726	-92.7995	55.2901	-175.945	17.549
20	0.2	0.25	-4.14395	-82.6794	54.558	-162.476	16.0874
21	0.2	0.275	-3.92647	-74.1433	53.6849	-150.799	14.7902
22	0.2	0.3	-3.73087	-66.8745	52.7219	-140.591	13.6363
23	0.2	0.325	-3.55399	-60.6323	51.705	-131.599	12.6068
24	0.2	0.35	-3.39325	-55.2309	50.6596	-123.624	11.6857

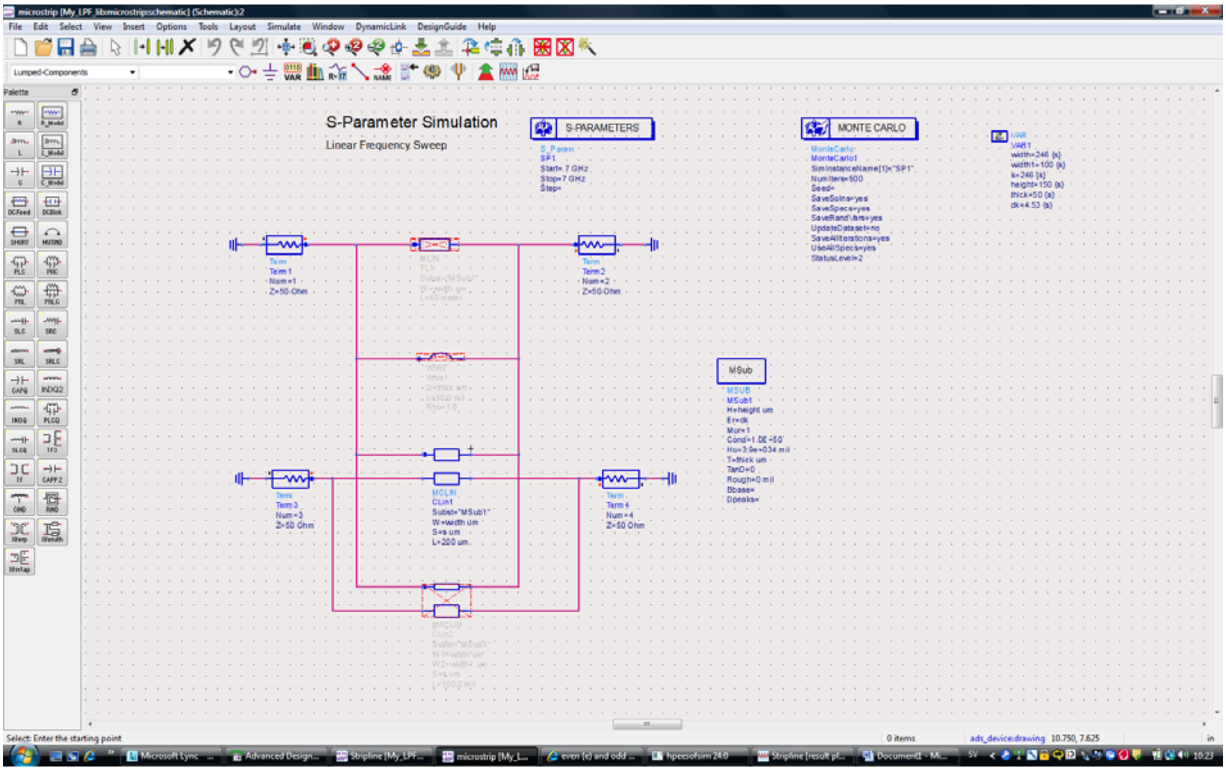
Strip coupled lines with correlation (s=0.1mm, t=0.017mm)

	Height	Width	ddk	dw	dh	dt	ds
1	0.15	0.075	-6.06592	-244.599	105.693	-357.253	19.9122
2	0.15	0.1	-5.45935	-197.579	105.247	-307.067	17.5681
3	0.15	0.125	-4.96414	-163.025	103.29	-268.469	15.5024
4	0.15	0.15	-4.55198	-136.862	100.557	-237.97	13.7218
5	0.15	0.175	-4.20346	-116.564	97.4563	-213.331	12.1982
6	0.15	0.2	-3.90482	-100.492	94.2174	-193.058	10.8953
7	0.15	0.225	-3.64601	-87.5433	90.9725	-176.116	9.77841
8	0.15	0.25	-3.41953	-76.9553	87.7969	-161.768	8.81697
9	0.15	0.275	-3.21964	-68.1852	84.7318	-149.477	7.98544
10	0.15	0.3	-3.04192	-60.838	81.7986	-138.841	7.26267
11	0.15	0.325	-2.88286	-54.621	79.0063	-129.555	6.63127
12	0.15	0.35	-2.73965	-49.313	76.3568	-121.384	6.07698
13	0.2	0.075	-6.73403	-226.185	74.5626	-331.761	30.8181
14	0.2	0.1	-6.16413	-188.621	76.0778	-290.63	28.0284
15	0.2	0.125	-5.68503	-159.846	76.3007	-257.921	25.4325
16	0.2	0.15	-5.27629	-137.282	75.724	-231.357	23.0858
17	0.2	0.175	-4.92326	-119.242	74.6515	-209.406	20.9922
18	0.2	0.2	-4.61513	-104.579	73.2759	-190.996	19.1352
19	0.2	0.225	-4.34375	-92.4917	71.7216	-175.362	17.4908
20	0.2	0.25	-4.10285	-82.4052	70.0705	-161.937	16.034
21	0.2	0.275	-3.88753	-73.8974	68.3767	-150.299	14.7412
22	0.2	0.3	-3.69387	-66.6527	66.6761	-140.125	13.5911
23	0.2	0.325	-3.51875	-60.4312	64.9927	-131.162	12.565
24	0.2	0.35	-3.3596	-55.0477	63.342	-123.214	11.647

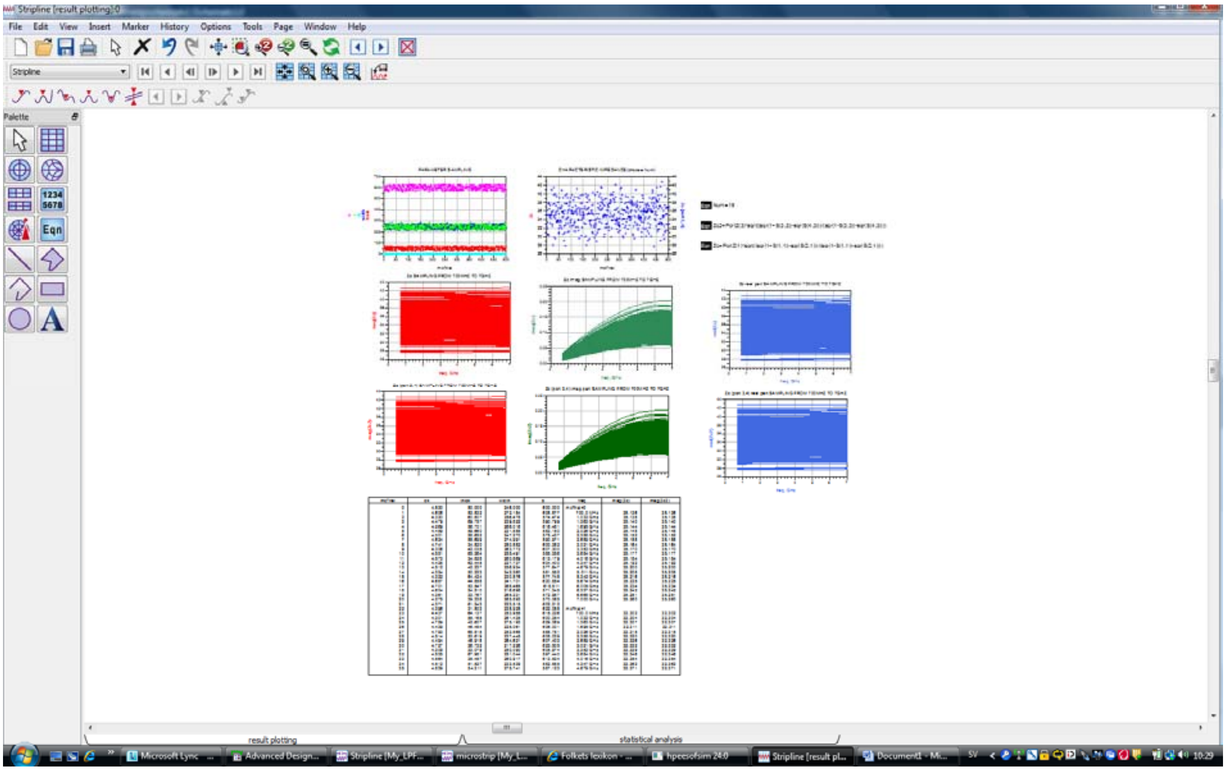
9 APPENDIX C- A SIMPLE ADS EXAMPLE FOR STATISTIC ANALYSIS

Schematic of microstrip- and stripline with different types of conductor

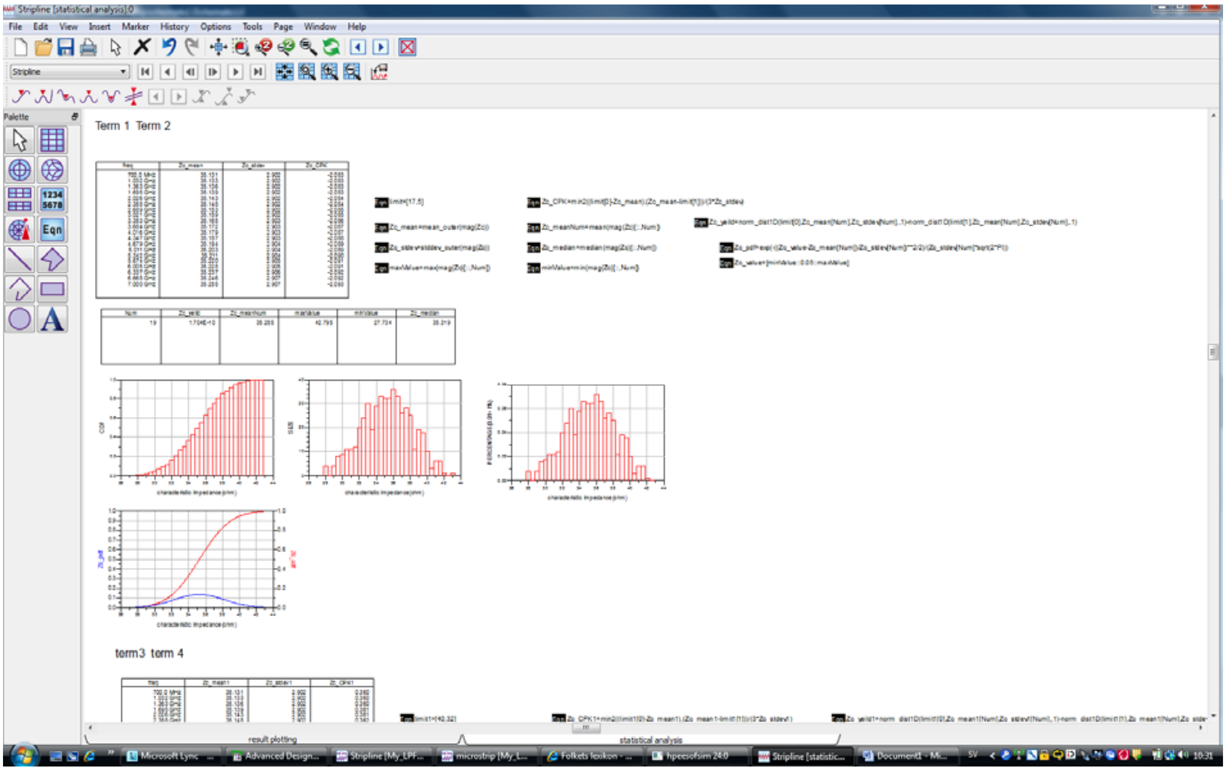




Result



Statistic analysis



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