Appendix A Noise Figure

Any signal other than the target returns in the radar receiver is considered as noise. This includes interfering signals from outside the radar and thermal noise generated within the receiver itself. Thermal noise (thermal agitation of electrons) and shot noise (variation in carrier density of a semiconductor) are the two main internal noise sources within a radar receiver.

The power spectral density of thermal noise is given by

$$S_n(\omega) = \frac{|\omega|h}{\pi\left[\exp\left(\frac{|\omega|h}{2\pi kT}\right) - 1\right]} \tag{A.1}$$

where $|\omega|$ is the absolute value of the frequency in radians per second, T is temperature of the conducting medium in degrees Kelvin, k is Boltzman's constant, and h is Plank's constant ($h = 6.625 \times 10^{-34}$ $joule$ $seconds$). When the condition $|\omega| \ll 2\pi kT/h$ is true, it can be shown that Eq. (A.1) is approximated by

$$S_n(\omega) \approx 2kT \tag{A.2}$$

This approximation is widely accepted, since, in practice, radar systems operate at frequencies less than $100 \ GHz$; and, for example, if $T = 290K$, then $2\pi kT/h \approx 6000 \ GHz$.

The mean square noise voltage (noise power) generated across a $1 \ ohm$ resistance is then

$$\langle n^2 \rangle = \frac{1}{2\pi} \int_{-2\pi B}^{2\pi B} 2kT \quad d\omega = 4kTB \tag{A.3}$$

where B is the system bandwidth in hertz.

Any electrical system containing thermal noise and having input resistance R_{in} can be replaced by an equivalent noiseless system with a series combination of a noise equivalent voltage source and a noiseless input resistor R_{in} added at its input. This is illustrated in Fig. A.1.

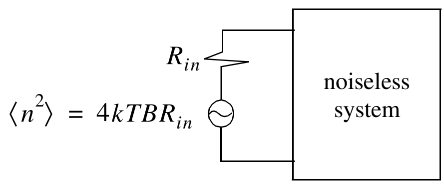

Figure A.1. Noiseless system with an input noise voltage source.

The amount of noise power that can physically be extracted from $\langle n^2 \rangle$ is one fourth the value computed in Eq. (A.3). The proof is left as an exercise.

Consider a noisy system with power gain A_P, as shown in Fig. A.2.

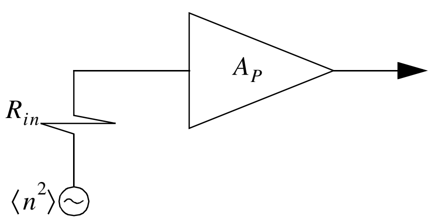

Figure A.2. Noisy amplifier replaced by its noiseless equivalent and an input voltage source in series with a resistor.

The noise figure is defined by

$$F_{dB} = 10 \log \frac{total\ noise\ power\ out}{noise\ power\ out\ due\ to\ R_{in}\ alone} \tag{A.4}$$

More precisely,

$$F_{dB} = 10 \log \frac{N_o}{N_i\ A_p} \tag{A.5}$$

where N_o and N_i are, respectively, the noise power at the output and input of the system.

If we define the input and output signal power by S_i and S_o, respectively, then the power gain is

$$A_P = \frac{S_o}{S_i} \tag{A.6}$$

It follows that

$$F_{dB} = 10\log\left(\frac{S_i/Ni}{S_o/N_o}\right) = \left(\frac{S_i}{N_i}\right)_{dB} - \left(\frac{S_o}{N_o}\right)_{dB} \tag{A.7}$$

where

$$\left(\frac{S_i}{N_i}\right)_{dB} > \left(\frac{S_o}{N_o}\right)_{dB} \tag{A.8}$$

Thus, it can be said that the noise figure is the loss in the signal-to-noise ratio due to the added thermal noise of the amplifier $((SNR)_o = (SNR)_i - F \;\; in \;\; dB)$.

We can also express the noise figure in terms of the system's effective temperature T_e. Consider the amplifier shown in Fig. A.2, and let its effective temperature be T_e. Assume the input noise temperature is T_o. Thus, the input noise power is

$$N_i = kT_oB \tag{A.9}$$

and the output noise power is

$$N_o = kT_oB \;\; A_p + kT_eB \;\; A_p \tag{A.10}$$

where the first term on the right-hand side of Eq. (A.10) corresponds to the input noise, and the latter term is due to thermal noise generated inside the system. It follows that the noise figure can be expressed as

$$F = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i}{kT_oB} \;\; kBA_p \;\; \frac{T_o + T_e}{S_o} = 1 + \frac{T_e}{T_o} \tag{A.11}$$

Equivalently, we can write

$$T_e = (F-1)T_o \tag{A.12}$$

Example A.1: An amplifier has a 4dB noise figure; the bandwidth is $B = 500 \;\; KHz$. Calculate the input signal power that yields a unity SNR at the output. Assume $T_o = 290$ degree Kelvin and an input resistance of one ohm.

Solution: The input noise power is

$$kT_oB = 1.38 \times 10^{-23} \times 290 \times 500 \times 10^3 = 2.0 \times 10^{-15} w$$

Assuming a voltage signal, then the input noise mean squared voltage is

$$\langle n_i^2 \rangle = kT_oB = 2.0 \times 10^{-15} \quad v^2$$

$$F = 10^{0.4} = 2.51$$

From the noise figure definition we get

$$\frac{S_i}{N_i} = F\left(\frac{S_o}{N_o}\right) = F$$

and

$$\langle s_i^2 \rangle = F \langle n_i^2 \rangle = 2.51 \times 2.0 \times 10^{-15} = 5.02 \times 10^{-15} \quad v^2$$

Finally,

$$\sqrt{\langle s_i^2 \rangle} = 70.852 nv$$

Consider a cascaded system as in Fig. A.3. Network 1 is defined by noise figure F_1, power gain G_1, bandwidth B, and temperature T_{e1}. Similarly, network 2 is defined by F_2, G_2, B, and T_{e2}. Assume the input noise has temperature T_0.

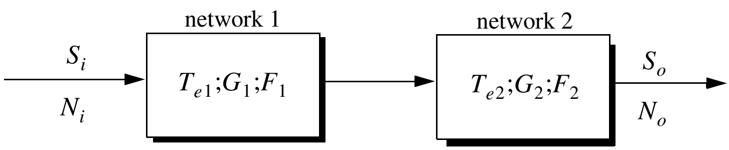

Figure A.3. Cascaded linear system.

The output signal power is

$$S_o = S_i G_1 G_2 \tag{A.13}$$

The input and output noise powers are, respectively, given by

$$N_i = kT_oB \tag{A.14}$$

$$N_o = kT_0BG_1G_2 + kT_{e1}BG_1G_2 + kT_{e2}BG_2 \tag{A.15}$$

where the three terms on the right-hand side of Eq. (A.15), respectively, correspond to the input noise power, thermal noise generated inside network 1, and thermal noise generated inside network 2.

Now if we use the relation $T_e = (F-1)T_0$ along with Eq. (A.13) and Eq. (A.14), we can express the overall output noise power as

$$N_o = F_1N_iG_1G_2 + (F_2-1)NiG_2 \tag{A.16}$$

It follows that the overall noise figure for the cascaded system is

$$F = \frac{(S_i/N_i)}{(S_o/N_o)} = F_1 + \frac{F_2-1}{G_1} \tag{A.17}$$

In general, for an n-stage system we get

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \cdot \cdot \cdot + \frac{F_n-1}{G_1G_2G_3 \cdot \cdot \cdot G_{n-1}} \tag{A.18}$$

Also, the n-stage system effective temperatures can be computed as

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \cdot \cdot \cdot + \frac{T_{en}}{G_1G_2G_3 \cdot \cdot \cdot G_{n-1}} \tag{A.19}$$

As suggested by Eq. (A.18) and Eq. (A.19), the overall noise figure is mainly dominated by the first stage. Thus, radar receivers employ low noise power amplifiers in the first stage in order to minimize the overall receiver noise figure. However, for radar systems that are built for low RCS operations every stage should be included in the analysis.

Example A.2: A radar receiver consists of an antenna with cable loss $L = 1dB = F_1$, an RF amplifier with $F_2 = 6dB$, and gain $G_2 = 20dB$, followed by a mixer whose noise figure is $F_3 = 10dB$ and conversion loss $L = 8dB$, and finally, an integrated circuit IF amplifier with $F_4 = 6dB$ and gain $G_4 = 60dB$. Find the overall noise figure.

Solution:

From Eq. (A.18) we have

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \frac{F_4-1}{G_1G_2G_3}$$

G_1	G_2	G_3	G_4	F_1	F_2	F_3	F_4
$-1dB$	$20dB$	$-8dB$	$60dB$	$1dB$	$6dB$	$10dB$	$6dB$
0.7943	100	0.1585	10^6	1.2589	3.9811	10	3.9811

It follows that

$$F = 1.2589 + \frac{3.9811 - 1}{0.7943} + \frac{10 - 1}{100 \times 0.7943} + \frac{3.9811 - 1}{0.158 \times 1000.7943} = 5.3628$$

$$F = 10\log(5.3628) = 7.294dB$$

Problems

A.1. A source with equivalent temperature $T_e = 500K$ is followed by three amplifiers with specifications shown in the table below.

Amplifier	**F, *dB***	**G, *dB***	**T_e**
1	You must compute	12	350
2	10	22	
3	15	35	

Assume a bandwidth of $150KHz$. (a) Compute the noise figure for the three cascaded amplifiers. (b) Compute the effective temperature for the three cascaded amplifiers. (c) Compute the overall system noise figure.

A.2. Derive Eq. (A.19).