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O.R. Applications

A Steiner arborescence model for the feeder reconfiguration in electric distribution networks

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Abstract

In this paper we address the problem of finding the radial configuration of an electric distribution network that minimizes the total losses due to the Joule effect. We propose an interpretation of the feeder reconfiguration problem as a Steiner arborescence problem, formulated through a model with a separable quadratic objective function. The problem is then solved by a mixed-integer quadratic programming solver. Computational experience on test networks is reported, showing the effectiveness of the formulation.

Keywords: Energy; Feeder reconfiguration; Distribution network; Steiner arborescence

1. Introduction

The diffusion in Europe of open markets for energy has increased the need for new technologies in power systems management and control. The new situation has created a market where the electric companies compete over the *price* and *reliability* of the energy they supply. To pursue these aims, energy management systems need to be enhanced, with new automated control functions able to operate in real-time.

One of the most important new control functions is *feeder reconfiguration*, which consists of altering the topological structure of distribution feeders by remote control of the on/off status of the switches under both normal and abnormal operating conditions. The benefits of feeder reconfiguration include, among others: (i) restoring power to any outage partitions of a feeder, (ii) relieving overloads on feeders by shifting the load in real time to adjacent feeders, and (iii) reducing resistive line losses.

The *feeder reconfiguration problem for loss reduction* has been widely addressed in literature. For a complete and detailed survey we refer the reader to the M.S. thesis of Krishnan (1998), where solution approaches are classified into four classes of methods: (i) iterative, (ii) successive load flows, (iii) artificial intelligence, and (iv) mathematical programming.

Here we briefly summarise some of main approaches based on mathematical programming

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methods. In Glamocanin (1990) feeder reconfiguration is formulated as a quadratic transhipment problem. Huddleston et al. (1990) propose a reconfiguration algorithm based on a linearly constrained quadratic programming problem. Cherkaoui et al. (1993) formulate the problem as an integer linear programming problem and solve the problem by Tabu Search. Other formulations of feeder reconfiguration as an integer linear programming problem are proposed in Chen and Cho (1993) and Sarma and Prakasa Rao (1995).

In this paper we formulate loss minimisation feeder reconfiguration as a Steiner arborescence problem with a separable quadratic objective function introducing a basic class of valid inequalities (Section 2). The quadratic Steiner arborescence problem is then efficiently solved to optimality by a commercial mixed-integer quadratic programming package. In Section 3 we report computational experience showing that the proposed formulation can operate efficiently in real-time energy management systems.

2. Problem formulation

An electric distribution network can be represented by a directed graph $G(N, A)$ (Fig. 1) where N is the set of nodes and A is the set of arcs, which may be either *feeders*, transferring the energy between two adjacent nodes, or *transformers*, modifying the voltage levels between different sections of the network.

The nodes can be classified as *supply nodes*, belonging to the set S, if they supply energy to the network, *load nodes*, belonging to the set L, if they draw energy from the network to the loads (clients or secondary networks), and *interconnection nodes*, denoted by I, if the balance between the energy entering and leaving the node is zero. A voltage level is associated to each node. We will assume that a single virtual supply node exists, i.e. $S = \{s\}$ (the root node 1 in Fig. 1) representing exogenous supply.

Given a subset of arcs $H \subseteq A$, we refer to a *network configuration* as a sub-graph $G(V, H)$ of the distribution network, obtained by remote control of the on/off status of the switches on the network which enable/disable a connection between the nodes.

A configuration of the distribution network is said to be *radial* if no cycles are allowed and if each load node is supplied through only one arc. Distribution networks are operated in a radial configuration mainly for the purpose of an easy restoration in case of faults.

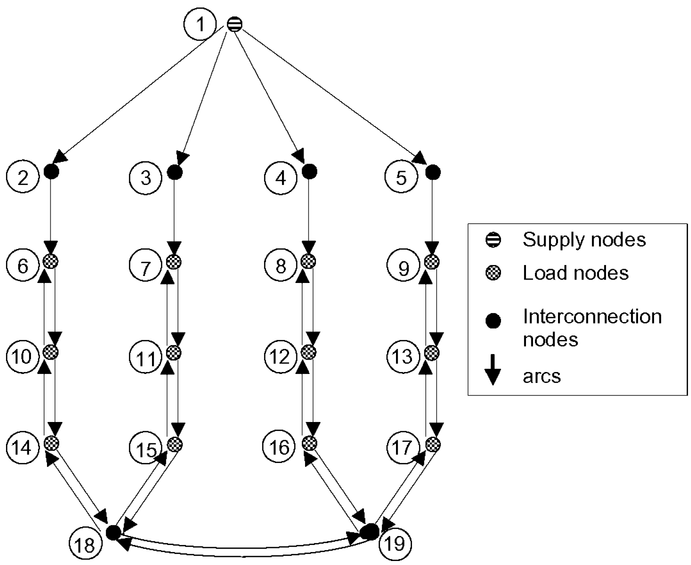

Fig. 1. Graph of the distribution system.

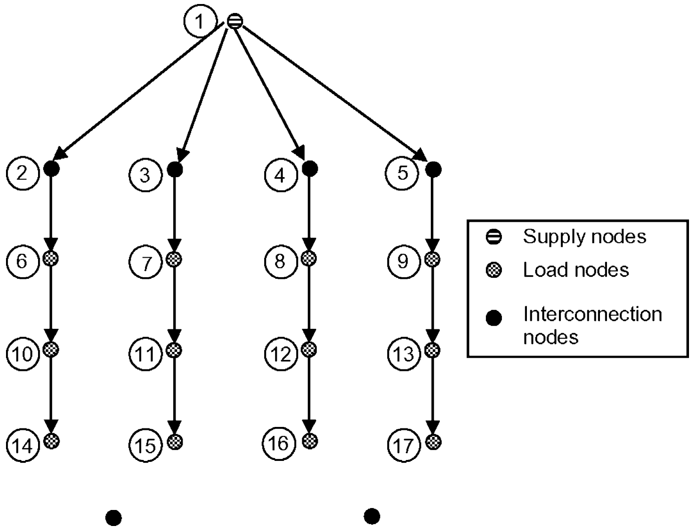

Fig. 2. A radial network.

A *radial* configuration (Fig. 2) corresponds on the graph G to an arborescence rooted in s, spanning every load node and a subset of the interconnection nodes, i.e. to a Steiner arborescence, where the load nodes are the *target nodes* of the Steiner problem and the interconnection nodes play the role of the *Steiner nodes*.

Let $s \in N$ be the root node (a supply node), let $L \subset N$ be the set of the load nodes and let $I \subset N$ be the set of the interconnection nodes. Let F_i be the demand of the load node $i \in L$. Let r_{ij}, z_{ij} and u_{ij} be, respectively, the resistance, the impedance and the capacity of the line ij. For each radial configuration we can compute a *total loss* $l(H)$, defined by

$$l(H) = \sum_{ij \in H} r_{ij} f_{ij}^2.$$

Hence we can define the *feeder reconfiguration problem (FRP)* as the problem of finding a Steiner arborescence H feasible with respect to voltage side constraints, minimising the total loss $l(H)$.

To provide a formulation for the feeder reconfiguration problem, we associate with each arc $ij \in A$ a variable x_{ij}, expressing the activation status of ij, and a variable f_{ij}, expressing the current flow on ij. Variables x_{ij} are binary since the line ij can be enabled ($x_{ij} = 1$) or disabled ($x_{ij} = 0$). A variable v_i, expressing the voltage in the node i, is associated with each node $i \in V$. Variables f_{ij} and v_i are expressed *per unit*, that is $f_{ij} \leqslant 1$ and $v_i \leqslant 1$. This scaling is usual for electrical network modelling.

A formulation of the problem could be:

$$\min \quad \sum_{ij \in A} r_{ij} f_{ij}^2$$

$$\sum_{j:ij \in A} f_{sj} = \sum_{i \in C} F_i, \tag{1}$$

$$\sum_{j:ji \in A} f_{ji} - \sum_{j:ij \in A} f_{ij} = 0, \quad i \in I, \tag{2}$$

$$\sum_{j:ji \in A} f_{ji} - \sum_{j:ij \in A} f_{ij} = F_i, \quad i \in L, \tag{3}$$

$$\sum_{i:ij \in A} x_{ij} = 1, \quad j \in L, \tag{4}$$

$$\sum_{i:ij \in A} x_{ij} \leqslant 1, \quad j \in I, \tag{5}$$

$$0 \leqslant f_{ij} \leqslant u_{ij} x_{ij}, \quad ij \in A, \tag{6}$$

$$x_{ij}(v_i - v_j) = z_{ij} f_{ij}, \quad ij \in A, \tag{7}$$

$$V_i^{\min} \leqslant v_i \leqslant V_i^{\max}, \quad i \in N, \tag{8}$$

$$x_{ij} \in \{0, 1\}, \quad ij \in A.$$

Constraint (1) requires that the flow leaving the root node s is equal to the sum of the flows required by the load nodes. Constraints (2) require that for each interconnection node $i \in I$ the total flow entering i is equal to the total flow leaving the same node. Constraint (3) requires that the flow entering every load node $i \in L$ is equal to the demand of i. Constraints (4) require that every load node $i \in L$ (target node) has exactly one entering arc. Constraints (5) require that every interconnection node $i \in I$ (Steiner node) has at most one entering arc. Constraints (6) are *variable upper bounds*: they require that no flow f_{ij} can traverse the arc ij if it is not enabled ($x_{ij} = 0$), while if the arc is enabled ($x_{ij} = 1$) the flow on arc ij is bounded by the arc capacity u_{ij}. The quadratic constraints (7) require that if $x_{ij} = 0$ then $f_{ij} = 0$, otherwise, if $x_{ij} = 1$, Ohm's law $(v_i - v_j) = z_{ij}f_{ij}$ applies. Constraints (8) impose *voltage upper and lower bounds*, since any feasible solution must guarantee reliability bounds for node voltage.

Since commercial quadratic programming packages only allow us to solve problems with linear constraints, it is useful to replace the quadratic constraints (7) with linear inequalities. For each $ij \in A$, every Eq. (7) can be split into two linear inequalities, (7a) and (7b), where M is a constant with a suitable value:

$$\frac{v_i - v_j}{z_{ij}} \leqslant f_{ij} + (1 - x_{ij})M, \tag{7a}$$

$$\frac{v_i - v_j}{z_{ij}} \geqslant f_{ij} - (1 - x_{ij})M. \tag{7b}$$

If $x_{ij} = 1$ (i.e. the arc ij is enabled), we have two inequalities:

$$\frac{v_i - v_j}{z_{ij}} \leqslant f_{ij},$$

$$\frac{v_i - v_j}{z_{ij}} \geqslant f_{ij}$$

that can be shrunk to a single equation expressing Ohm's law.

If the variable x_{ij} is zero (i.e. if the arc ij is disabled), the current f_{ij} is zero too (due to constraints (4)), and we have two inequalities:

$$\frac{v_i - v_j}{z_{ij}} \leqslant M,$$

$$\frac{v_i - v_j}{z_{ij}} \geqslant -M$$

where the M constant must be set to a suitable value in order to make the new linear constraints redundant. On the basis of computational experience we set $M = 1/z_{ij}$. In fact, since the v_i are expressed per unit, we have $-1 \leqslant v_i - v_j \leqslant 1$ and so the inequalities (7a) and (7b) are always satisfied when $x_{ij} = 0$.

The MIQ formulation (1)–(8) can be strengthened by adding valid inequalities which exploit the combinatorial nature of the problem as already experienced by Bienstock (1996). Since every feasible solution of our problem defines a Steiner Arborescence on the graph G, valid inequalities can be introduced to ensure connectivity between the root node s and each of the load (target) nodes of the set L (Magnanti and Wolsey, 1995).

3. Computational experience

The feeder reconfiguration problem (1)–(8) has been solved to optimality by the *branch-and-bound* algorithm provided by the XPRESS-MP mixed-integer quadratic programming solver (Dash Associates, 2000). The test-bed consists of five networks from the M.S. thesis of Krishnan (1998), who collected the main test instances used in several papers, and on a real test network, "Napoli", kindly provided by ENEL, the Italian Agency for Electrical Energy. All the tests have been performed on a Pentium II 300 MHz Personal Computer with 256 Mbytes RAM.

In Table 1 we report computational results. Columns "Name", "$|V|$" and "$|A|$" contain respectively the problem name, the number of nodes and the number of arcs in the graph G. Columns "Lb" and "Opt" report the lower bound at the root node and the value of the optimal solution, respectively. Finally columns "B & B time" and "B & B nodes" report the CPU time (seconds) and number of B & B nodes spent in solving the problem to optimality. We observe that computation times obtained by using the MIQP solver are fully compatible with the time restrictions on the output of real-world applications.

Table 1
Experimental results

Name	$\|V\|$	$\|A\|$	Lb	Opt	B & B nodes	B & B time
Glamocanin	10	26	0.012822	0.014078	99	2
Grainger	16	32	0.006826	0.00767	34	1
Whei Min	17	30	0.005458	0.00595	22	0.5
Napoli	35	76	0.075866	0.085478	678	67
Chiang	69	146	0.04288	0.05236	140	21
Baran and Wu	69	144	0.022202	0.02621	211	29

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