ELEG 5491 HW1

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1 Problem 1

By definition of Cross Entropy, we have

$$\text{CrossEntropy}(\mathcal{D}) = -\frac{1}{N}\sum_{i=1}^{N}\sum_{k=1}^{c}\mathbb{I}(k = y_i)\log z_{i,k} = -\frac{1}{N}\sum_{i=1}^{N}\log z_{i,y_i}. \tag{1}$$

Move the sum inside $\log$, we have

$$\text{CrossEntropy}(\mathcal{D}) = -\frac{1}{N}\log\prod_{i=1}^{N} z_{i,y_i}, \tag{2}$$

which can be interpreted as the negative log-likelihood on the training set (with a constant factor).

2 Problem 2

2.1

$$f_{11}(x_1) = \begin{cases} -1, & \text{when } x_1 \leq 0.5, \\ 1, & \text{otherwise.} \end{cases} \tag{3}$$

$$f_{12}(x_2) = \begin{cases} -1, & \text{when } x_2 \leq 0.5, \\ 1, & \text{otherwise.} \end{cases} \tag{4}$$

$$g(h_{11}, h_{12}) = h_{11}h_{12}. \tag{5}$$

2.2

Like in Figure 1, we insert a layer above the input, which contains two neurons $h_{21} = f_{21}(x_1)$, $h_{22} = f_{22}(x_2)$, and

$$f_{21}(x_1) = \begin{cases} x_1, & \text{when } x_1 \leq 1, \\ 2 - x_1, & \text{otherwise.} \end{cases} \tag{6}$$

$$f_{22}(x_2) := f_{21}(x_2). \tag{7}$$

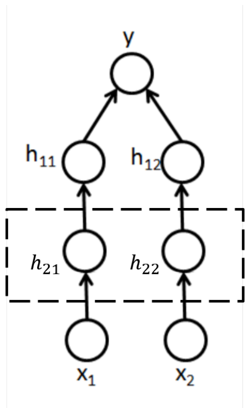

Figure 1: new layer.

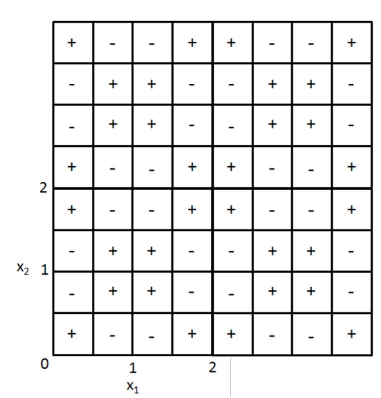

Figure 2: 4×4 pattern.

2.3

Image the input region as a piece of paper in squared shape. Fold them horizontally and vertically along the center line, we will reduce a 2×2 square to a 1×1 square. Folding like this multiple times will be able to create more complex patterns. So such regularity and global structure is a complex symmetry pattern that resulted from this folding process. An example of 4×4 square is shown if Figure 2.

2.4

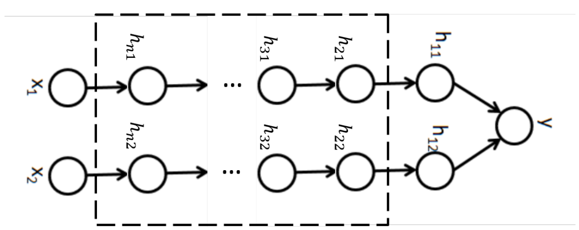

Figure 3: deep net.

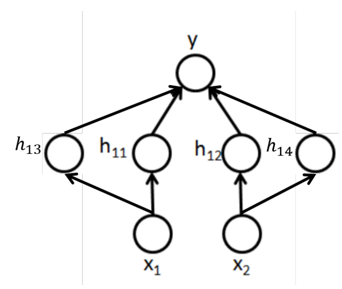

Figure 4: shallow net.

The network structure is illustrated in Figure 3. The inserted neurons are $h_{ij} = f_{ij}(x), i = 2, 3, \ldots, n, j = 1, 2$, where

$$f_{ij}(x) = \begin{cases} x, & \text{when } x \leq 2^{i-1}, \\ 2^i - x, & \text{otherwise.} \end{cases} \tag{8}$$

2.5

The network structure is shown in Figure 4. Definition of neurons are,

$$h_{11} = f_{11}(x_1) = \begin{cases} -1, & \text{when } (x_1 \mod 1) \leq 0.5, \\ 1, & \text{otherwise.} \end{cases} \tag{9}$$

$$h_{12} = f_{12}(x_2) = \begin{cases} -1, & \text{when } (x_2 \mod 1) \leq 0.5, \\ 1, & \text{otherwise.} \end{cases} \tag{10}$$

$$h_{13} = f_{13}(x_1) = \begin{cases} -1, & \text{when } (x_1 \mod 2) \leq 1, \\ 1, & \text{otherwise.} \end{cases} \tag{11}$$

$$h_{14} = f_{14}(x_2) = \begin{cases} -1, & \text{when } (x_2 \mod 2) \leq 1, \\ 1, & \text{otherwise.} \end{cases} \tag{12}$$

$$y = g(h_{11}, h_{12}, h_{13}, h_{14}) = h_{11}h_{12}h_{13}h_{14} \tag{13}$$

3 Problem 3

For one neuron at the output pooling layer of network (a), it correspond to 2×2 size of the second convolution layer, which correspond to 4×4 size of the first convolution layer, which correspond to 9×9 size of the input layer. So the receptive field is 9×9.

For one neuron at the output convolution layer of network (b), it correspond to 3×3 size of the pooling layer, which correspond to 9×9 size of the first convolution layer, which correspond to 13×13 size of the input layer. So the receptive field is 13×13.

4 Problem 4

4.1

$$\begin{aligned}
[[L_{\mathbf{t}} f] * w](\mathbf{x}) &= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} (L_{\mathbf{t}} f_k)(\mathbf{y}) w_k(\mathbf{y} - \mathbf{x}) \\
&= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} f_k(\mathbf{y} - \mathbf{t}) w_k(\mathbf{y} - \mathbf{x}) \\
&= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} f_k(\mathbf{y}) w_k(\mathbf{y} - (\mathbf{x} - \mathbf{t})) \\
&= [f * w](\mathbf{x} - \mathbf{t}) \\
&= [L_{\mathbf{t}}[f * w]](\mathbf{x}).
\end{aligned} \tag{14}$$

4.2

$$\begin{aligned}
[[L_{\mathbf{R}} f] * w](\mathbf{x}) &= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} (L_{\mathbf{R}} f_k)(\mathbf{y}) w_k(\mathbf{y} - \mathbf{x}) \\
&= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} f_k(\mathbf{R}^{-1}\mathbf{y}) w_k(\mathbf{y} - \mathbf{x}) \\
&= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} f_k(\mathbf{y}) w_k(\mathbf{R}\mathbf{y} - \mathbf{x})) \\
&= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} f_k(\mathbf{y}) w_k(\mathbf{R}(\mathbf{y} - \mathbf{R}^{-1}\mathbf{x})) \\
&= \sum_{\mathbf{y} \in \mathbb{Z}^2} \sum_{k=1}^{K} f_k(\mathbf{y}) [L_{\mathbf{R}^{-1}} w_k](\mathbf{y} - \mathbf{R}^{-1}\mathbf{x}) \\
&= [f * [L_{\mathbf{R}^{-1}} w]](\mathbf{R}^{-1}\mathbf{x}) \\
&= [L_{\mathbf{R}}[f * [L_{\mathbf{R}^{-1}} w]]](\mathbf{x}).
\end{aligned} \tag{15}$$

4.3

$$
\begin{aligned}
[[L_{\mathbf{u}} f] * w](\mathbf{g}) &= \sum_{\mathbf{h}\in G} \sum_{k=1}^{K} [L_{\mathbf{u}} f_k](\mathbf{h}) w_k(\mathbf{g}^{-1}\mathbf{h}) \\
&= \sum_{\mathbf{h}\in G} \sum_{k=1}^{K} f_k(\mathbf{u}^{-1}\mathbf{h}) w_k(\mathbf{g}^{-1}\mathbf{h}) \\
&= \sum_{\mathbf{h}\in G} \sum_{k=1}^{K} f_k(\mathbf{h}) w_k(\mathbf{g}^{-1}\mathbf{u}\mathbf{h}) \\
&= \sum_{\mathbf{h}\in G} \sum_{k=1}^{K} f_k(\mathbf{h}) w_k((\mathbf{u}^{-1}\mathbf{g})^{-1}\mathbf{h}) \\
&= (f * w)(\mathbf{u}^{-1}\mathbf{g}) \\
&= [L_{\mathbf{u}}[f * w]](\mathbf{g}).
\end{aligned}
\tag{16}
$$

View the feature map as four-fold images with rotation $0^\circ, 90^\circ, 180^\circ$ and 270°. For rotation d°, $d \in \{0, 90, 180, 270\}$, it is obtained by the following procedure: first rotate the original feature map by $0^\circ, 90^\circ, 180^\circ$ and 270°, and rotate the filter by d°. Then apply the rotated filter on the four rotated feature maps. Finally sum the four results together.

References

[1] X. Wang. Assignments, 2017. URL `http://dl.ee.cuhk.edu.hk/`.