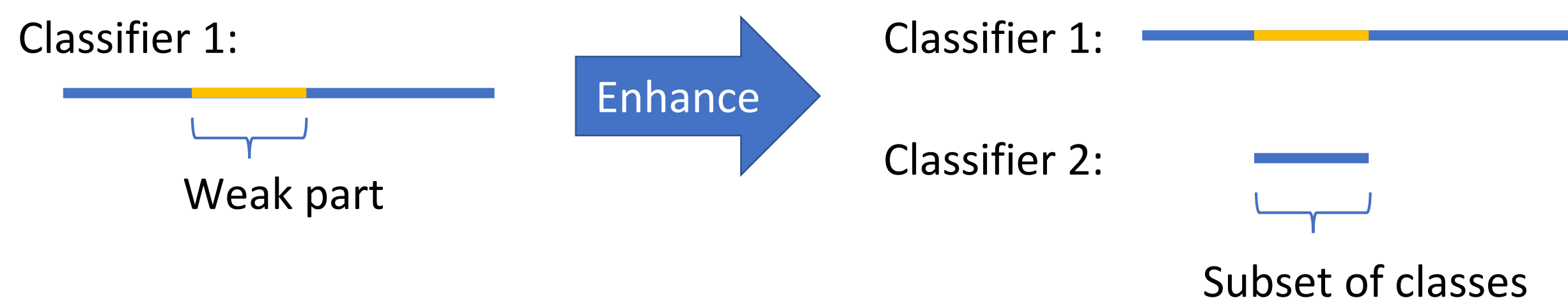
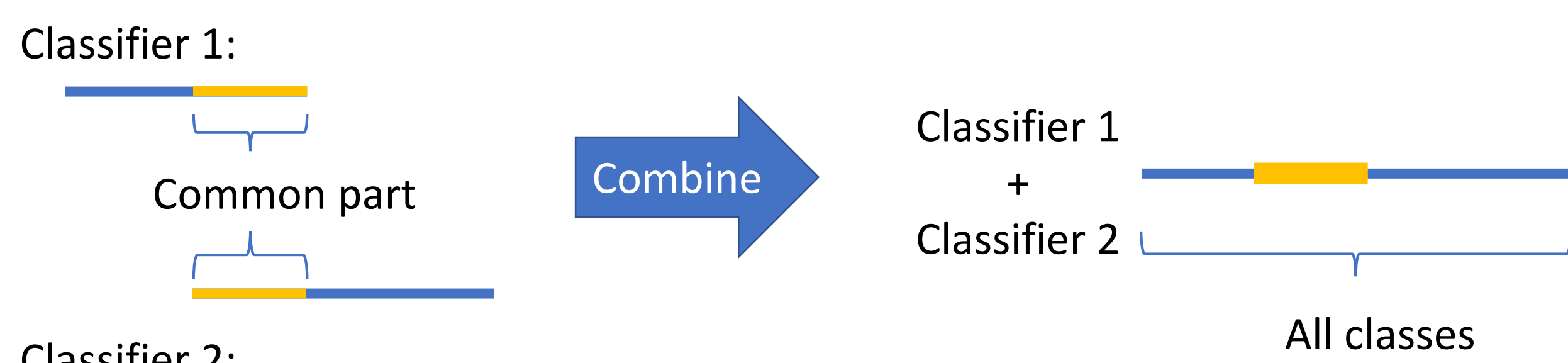


Ensemble of Specialized Classifiers

- We consider ensemble in the presence of Specialized Classifiers
- Specialized classifiers** are trained using a subset of classes
- Scenario 1: Enhance the weak part



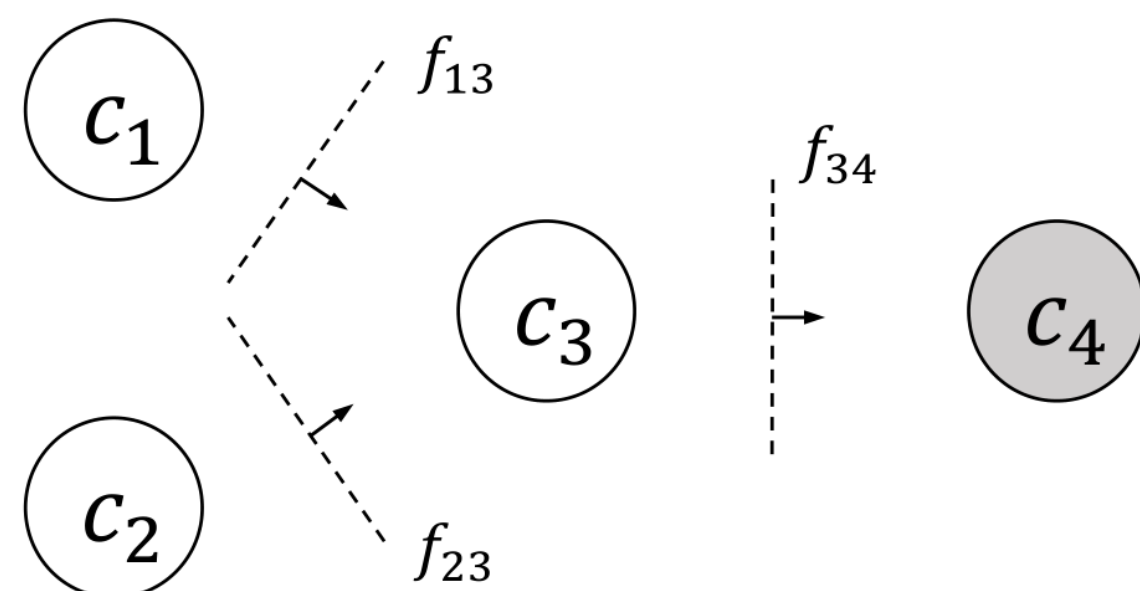
- Scenario 2: Integrate classifiers from different domain



- Challenges**



- Motivation:** tracing the preferences



4 classes in 2-dim plane. 3 specialized classifiers are drawn with decision boundaries. If we trace along the arrows, we will arrive the correct class.

- Idea:** tracing -> random walk
If the configuration of specialized classifiers are complex, the simple tracing algorithm faces problems like multiple solutions, or trapped in loops. Random walk is a better replacement.

Tool: Continuous Time Markov Chain

- Homogeneous CTMC is a stochastic process characterized by the transition rates between states
- If the chain is ergodic, there exists an equilibrium distribution

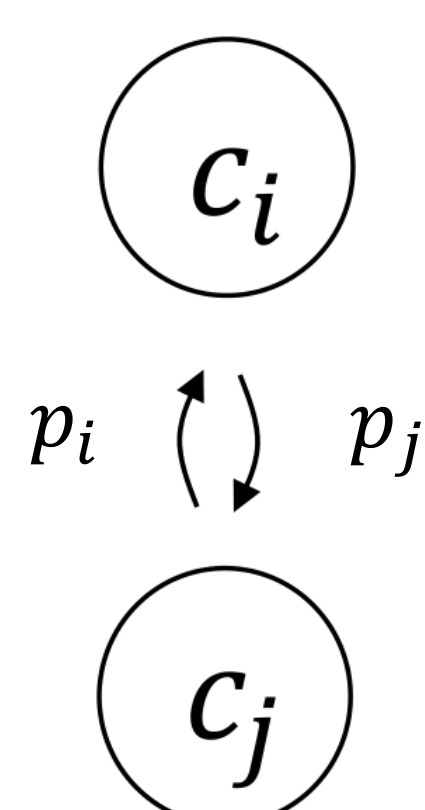
- Example**

Two states

Transition rate matrix

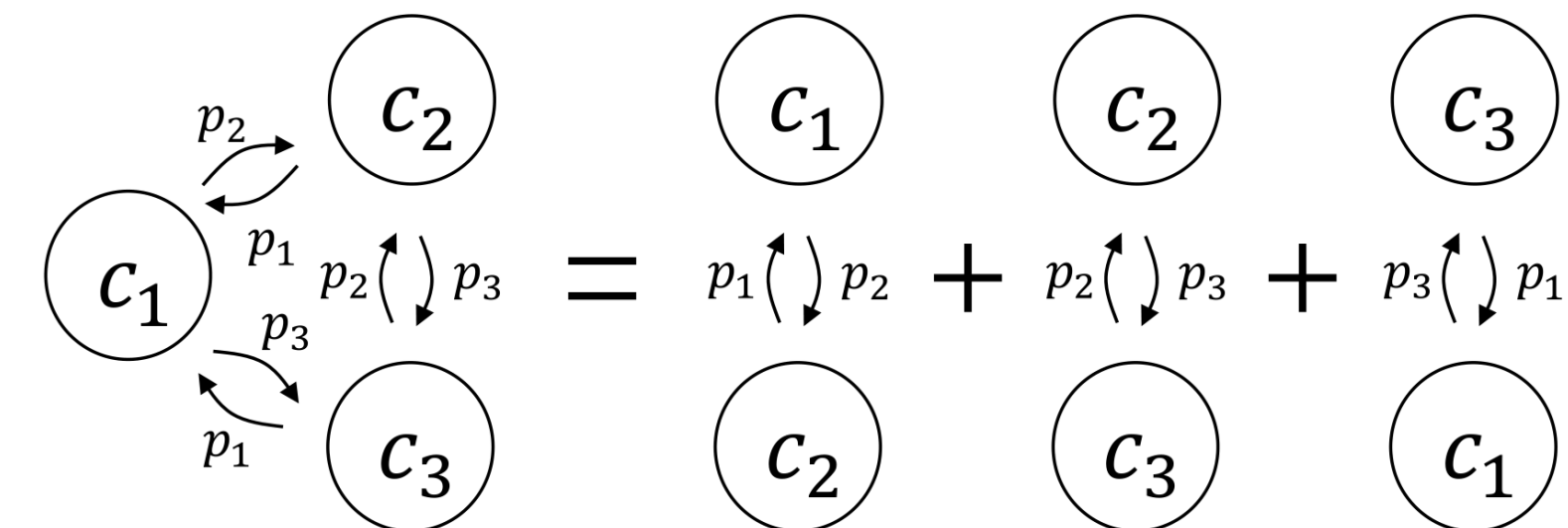
$$Q = \begin{bmatrix} -p_j & p_j \\ p_i & -p_i \end{bmatrix}$$

- Stationary distribution $\pi \propto (p_i, p_j)$
- A model of preference



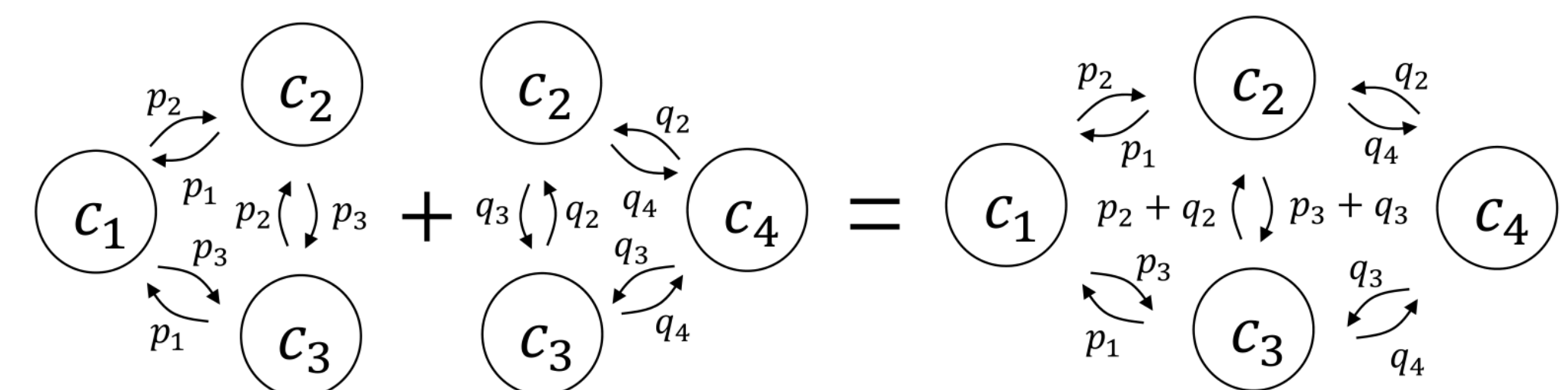
CTMC Ensemble

- Given a set of classifiers, assume they output probabilities
- Construct a continuous-time Markov chain as follows
- Classes as states, predictions as transition rates**
- Step 1:** Construct a chain for each of individual classifiers



Three classes with prediction (p_1, p_2, p_3) ,
Stationary distribution is also (p_1, p_2, p_3) .

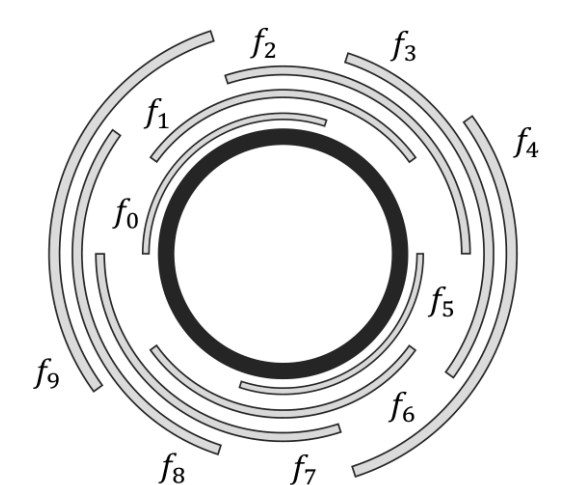
- Step 2:** Combine the by superposition



- Step 3:** Output the equilibrium distribution as prediction

Experiments

- On ImageNet dataset
1000 classes, 1.2M images
Train deep neural networks as classifiers



- From local to global

Classifier Set	Top-5 Error (%)					
	vote	mean	m-mean	prod	m-prod	CTMC
$\{f_0, f_2, f_4, f_6, f_8\}$	15.962	12.332	11.734	13.376	11.144	7.952
$\{f_1, f_3, f_5, f_7, f_9\}$	16.052	12.742	12.078	12.600	10.686	7.718
$\{f_0, f_2, f_4, f_6, f_8\} \cup \{f_1\}$	19.706	12.148	11.508	12.964	10.536	7.696
$\{f_0, f_2, f_4, f_6, f_8\} \cup \{f_1, f_3\}$	23.898	11.806	11.046	12.924	10.050	7.528
$\{f_0, f_2, f_4, f_6, f_8\} \cup \{f_1, f_3, f_5\}$	22.020	11.096	10.620	12.326	9.634	7.206
$\{f_0, f_2, f_4, f_6, f_8\} \cup \{f_1, f_3, f_5, f_7\}$	14.856	10.334	10.136	9.954	9.206	6.886
$\{f_0, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9\}$	14.172	10.014	10.014	8.548	8.548	6.676

- Unbalanced coverage

Classifier Set	Top-5 Error (%)					
	vote	mean	m-mean	prod	m-prod	CTMC
$\{h_0\}$	21.998	6.146	6.146	6.146	6.146	6.146
$\{h_0\} \cup R$	16.540	8.356	8.356	6.404	6.404	5.972
$\{h_0\} \cup S$	22.600	15.292	15.292	7.670	7.670	6.108
$\{h_0\} \cup U$	25.912	16.532	15.196	9.948	8.794	5.802

h_0 : general classifier. R, S, U : three sets of specialized classifiers.
Classes are covered by 1-4 times unevenly.

Properties

- Proposition 1.** If the coverage is connected, and the prediction probabilities are positive, then the solution **exists** and is **unique**
- Proposition 2.** The output π satisfies

$$\pi(i) = \frac{1}{|\mathcal{F}_i|} \sum_{k \in \mathcal{F}_i} \omega_k \mathbf{p}_k(i), \text{ with } \omega_k = \sum_{j \in I_k} \pi(j).$$

where \mathcal{F}_i is the set of all classifiers that covers class c_i , and $\mathbf{p}_k(i)$ is the prediction of k -th classifier on class c_i .

