Sugihara Causality Brain Region Analysis

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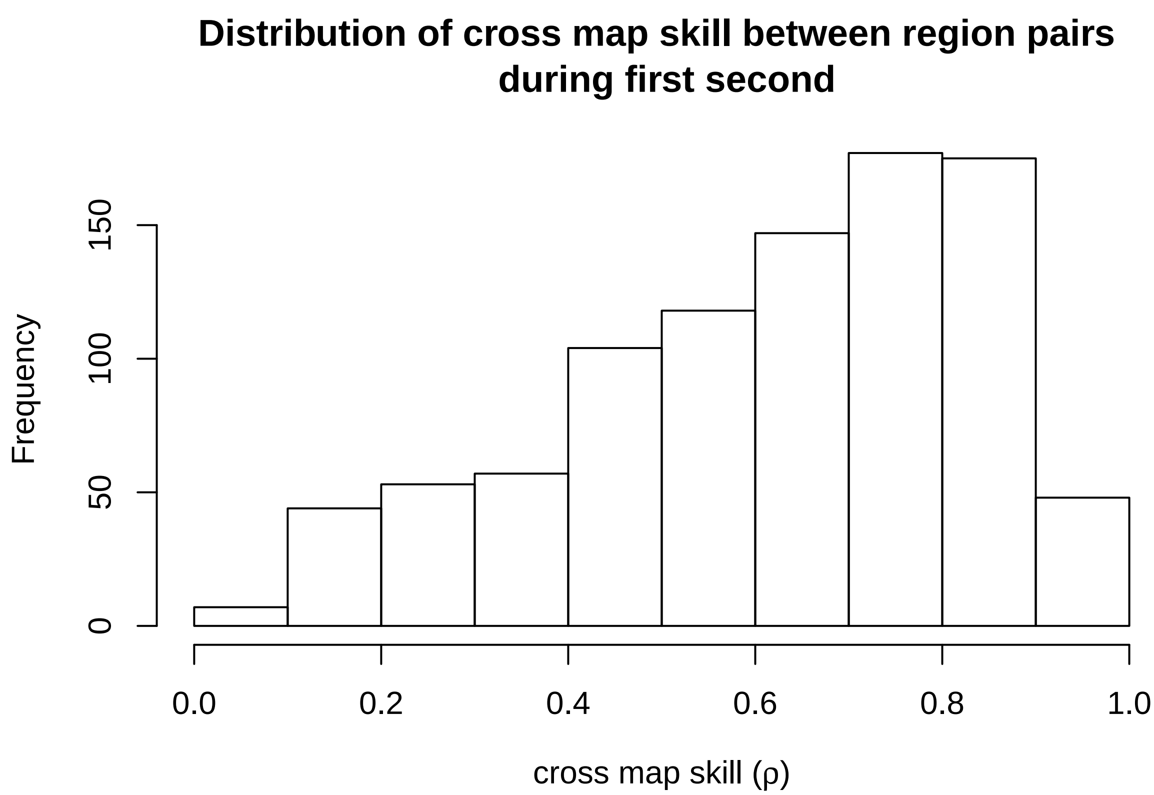
Note Unless explicitly stated otherwise, the data analysis in this document pertain only to the first second of 500 during the experiment.

Sugihara Cross Map Skill (ρ) analysis

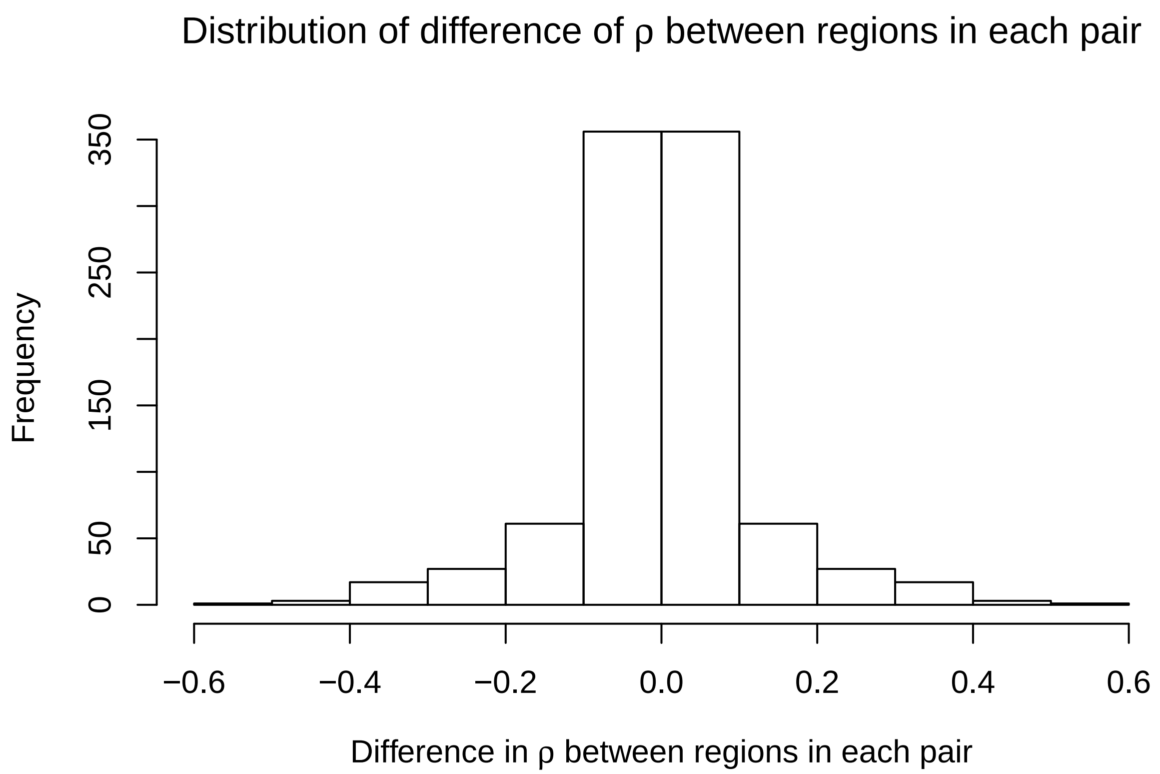
First we measured pairwise Sugihara cross map skill (ρ) measure. Most of the pairs showed high causality measures as shown by the histogram.

```
hist(netw$strength, main="Distribution of cross map skill between region pairs \n during first second",
```



Then we observe that many of the pairs have very similar ρ measure between them. This indicates that if region a causes region b, then it is likely that region b also causes region a. And since there are many causal relationships in this network, then it can be deduced the majority of the regions cause one another.

```r
#create an empty list to store the data
rho_diff <- c()
for (i in 1:31)
{
  for (j in 1:31)
  {
    if (i != j)
    {
      #get the difference of causality between i -> j and i <- j
      rho_diff <- c(rho_diff, netw$strength[netw$from==i & netw$to==j] -
                      netw$strength[netw$from==j & netw$to==i])
    }
  }
}
hist(rho_diff, main = expression(paste("Distribution of difference of ", rho,
                                       " between regions in each pair")),
     xlab=expression(paste("Difference in ", rho, " between regions in each pair")))
```

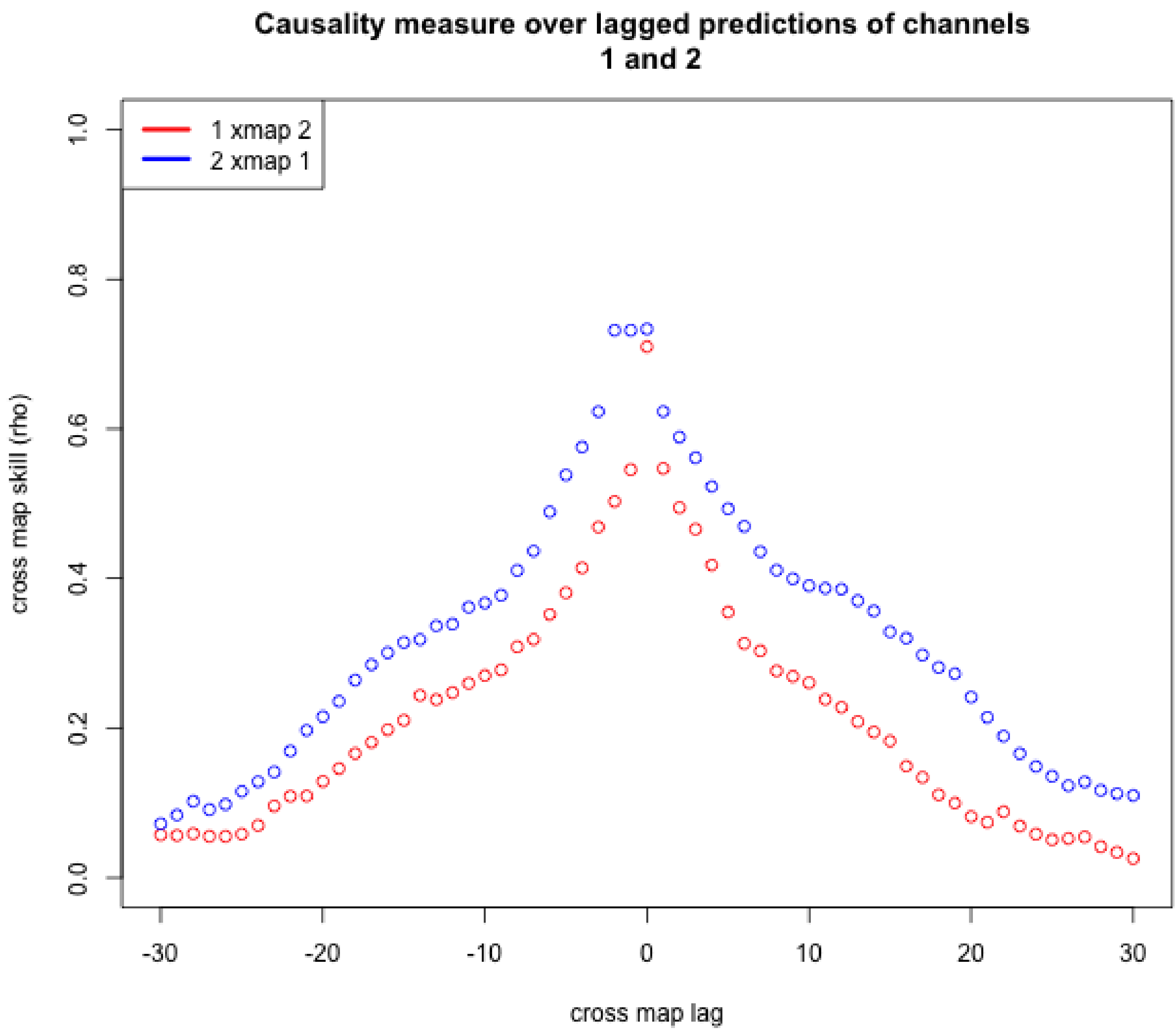


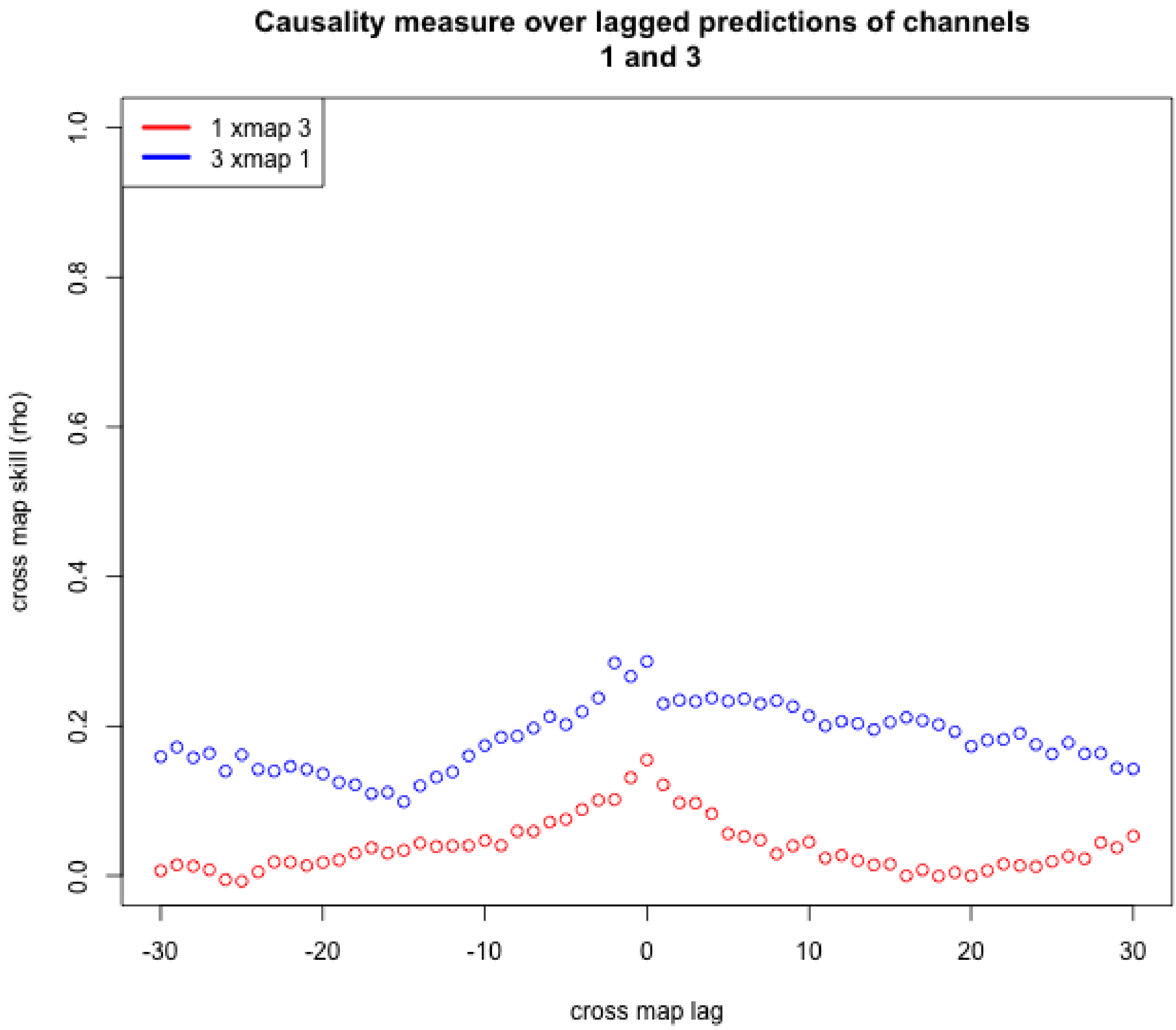
Examining unidirectional forcing

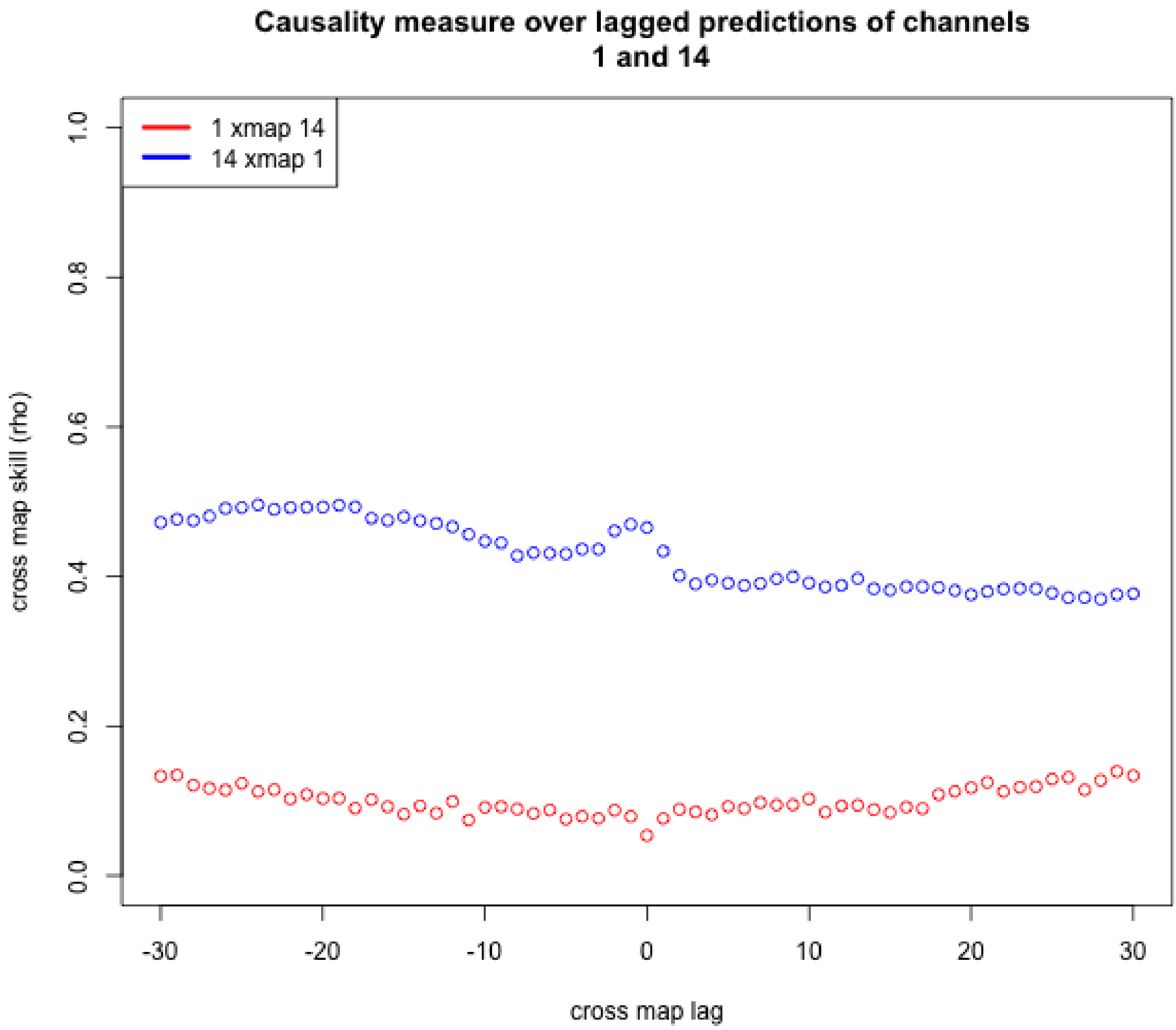
However, as Ye pointed out recently, this could be due to unidirectional forcing of one region on another. This means if region a strongly dictates region b's behavior, then naturally the state of b can be deduced from that of a, but also that the state of a can also be deduced form the state of b. This is because the dynamic of b become "enslaved" to the dynamics of a. This is the problem of "synchrony". Although the Sugihara model would measure both a and b as causal to one another, the truth could be and in fact is that the only causal relationship is from a to b, and not the other way around.

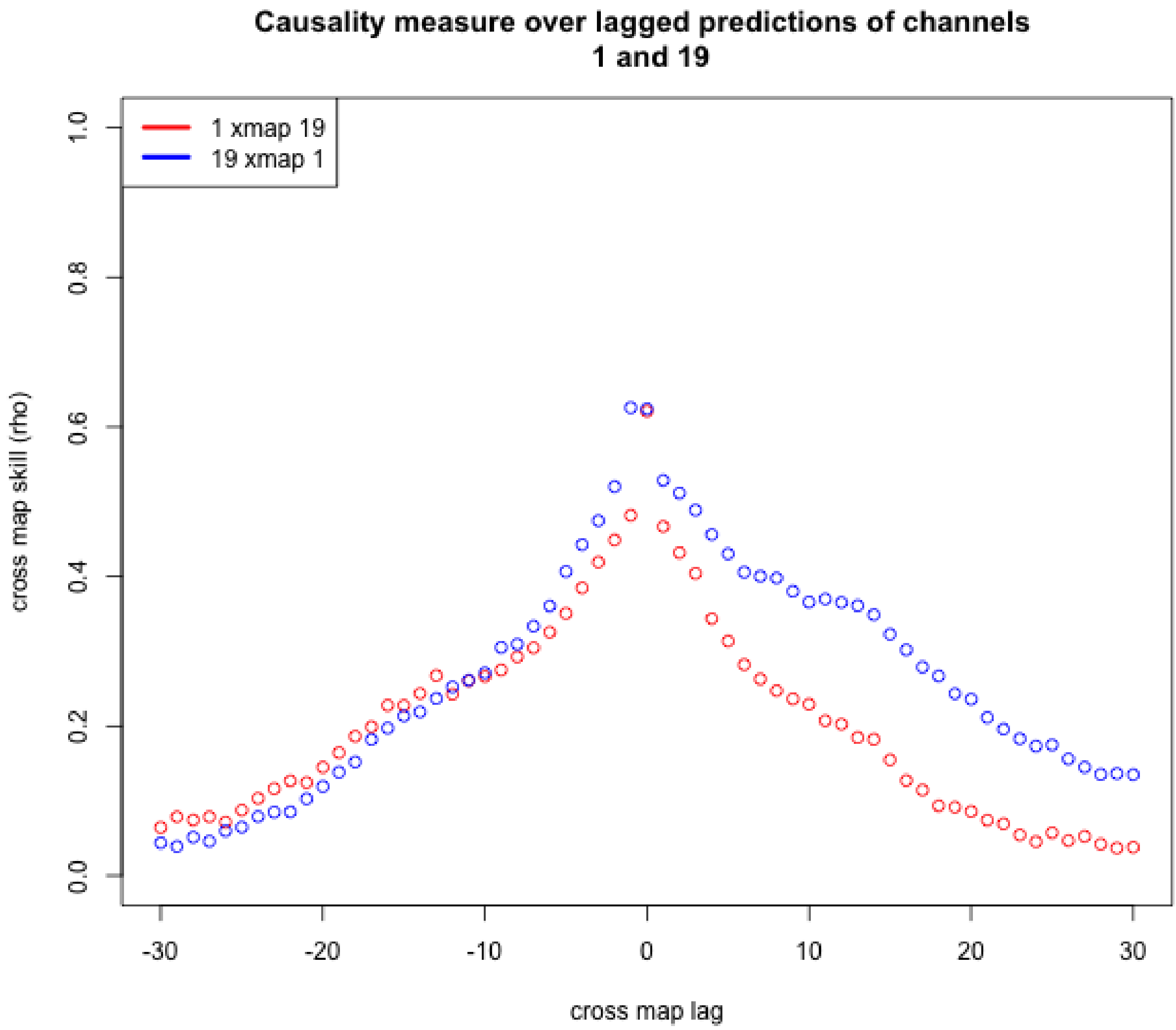
This problem can be allayed as Ye suggests by looking at the lags of each causal relationship in order to determine which variable best predicts the other in the past, and which best predicts the other in the future. If b is enslaved by a with a lag of 2 time periods, then b will be able to predict the past of a with a maximum ρ around negative 2 lag periods. Conversely, a is able to bet predict the future of b with a maximum ρ around positive 2 periods (This is shown well in in Figure 2 of Ye et al..

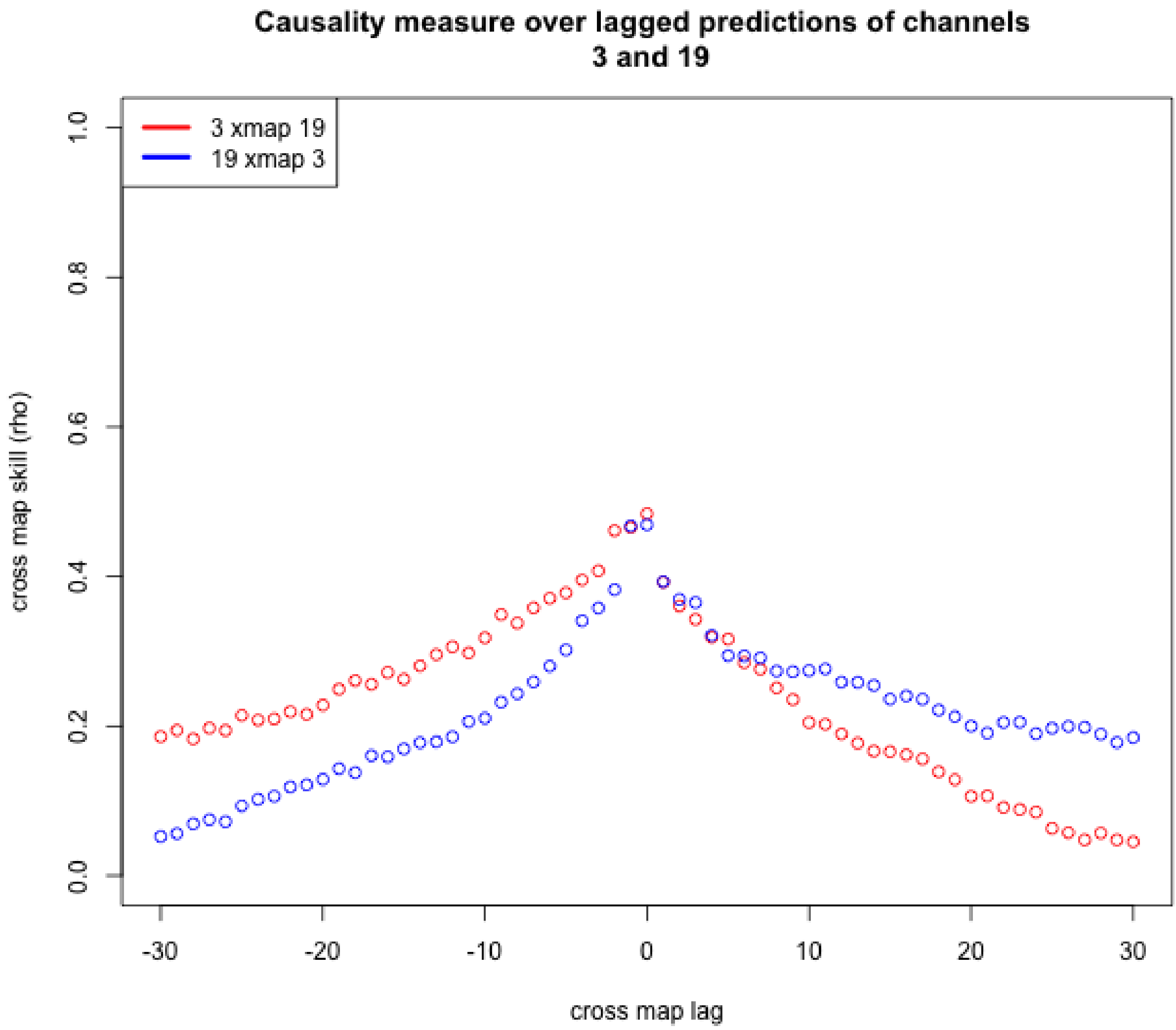
So in order to allay the problem of unidirectional forcing, we examine the lags of each pair. This is a tricky problem because there is no clear range for which to test the lag, since we are unsure of the time delay for neuronal activity, and how that translates to EEG data. As a simple measure, we test for lag in the 30 range.

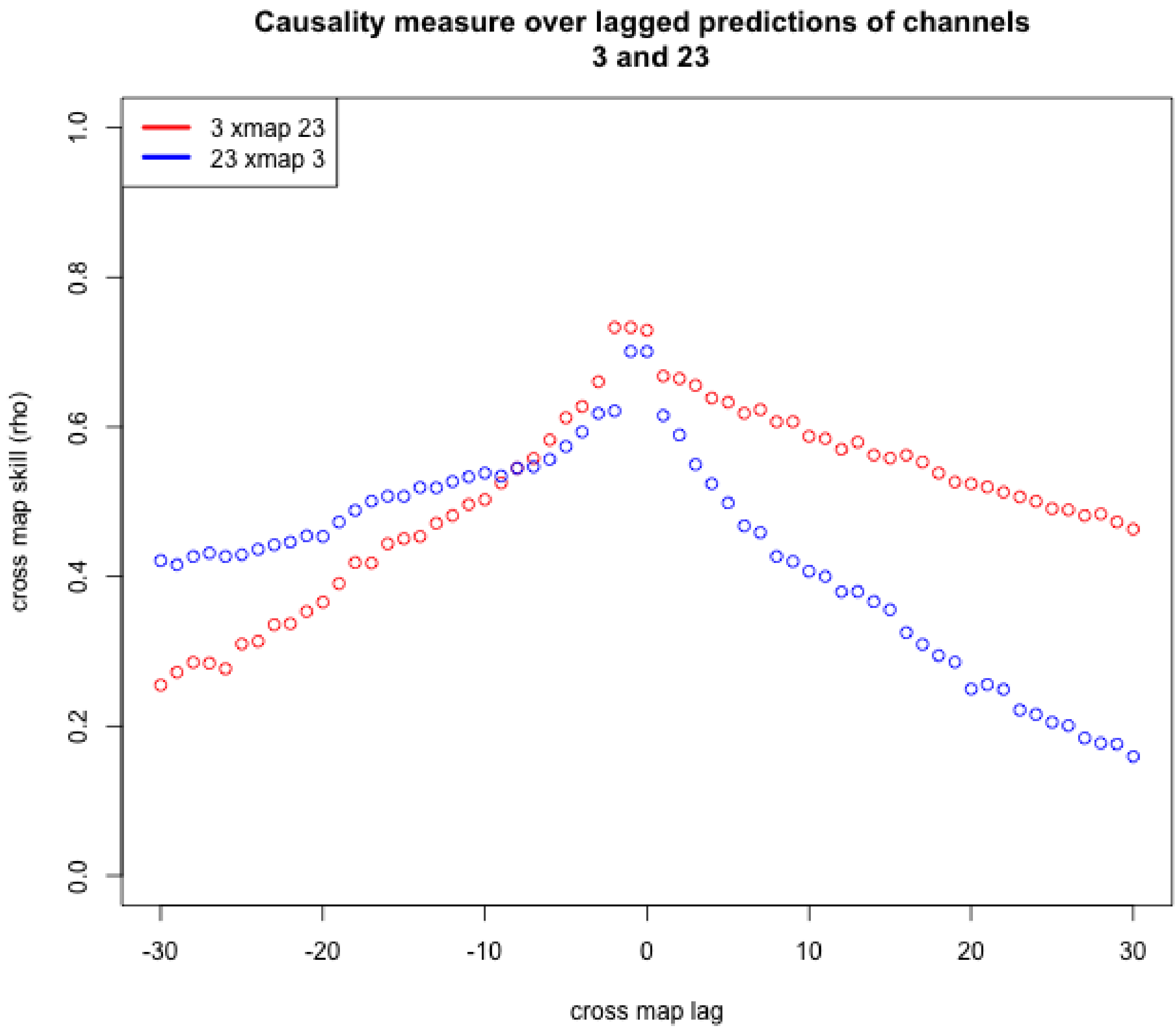



Causality measure over lagged predictions of channels
1 and 3
1 xmap 3
3 xmap 1
cross map skill (rho)
cross map lag
0.0
0.2
0.4
0.6
0.8
1.0
-30
-20
-10
0
10
20
30


Causality measure over lagged predictions of channels
1 and 14
1 xmap 14
14 xmap 1
cross map skill (rho)
cross map lag
0.0
0.2
0.4
0.6
0.8
1.0
-30
-20
-10
0
10
20
30


Causality measure over lagged predictions of channels
1 and 19
1 xmap 19
19 xmap 1
cross map skill (rho)
cross map lag
0.0
0.2
0.4
0.6
0.8
1.0
-30
-20
-10
0
10
20
30


Causality measure over lagged predictions of channels
3 and 19
3 xmap 19
19 xmap 3
cross map skill (rho)
0.0
0.2
0.4
0.6
0.8
1.0
-30
-20
-10
0
10
20
30
cross map lag



Seeing from the plots above, we can observe various behaviors, and since there are 465 combinations, it becomes unfeasible to analyze all the plots by hand, so we propose a heuristic measure we call the *left lag mean difference* (LLMD), which describes the difference between the mean of the left lag of a xmap b and b xmap a. If the LLMD of a xmap b vs. b xmap a is positive, then the unidirectional forcing relationship (*if* present) is b causes a because the lagged plot of a xmap b is skewed more to the left, so that is the more causal relationship.

There are serious weaknesses to the LLMD method, mainly its linear property and that it does not give intelligible answers when the relationship is unclear like when both causal lines cross each other multiple times like in channels 1 and 19, or when the relationship is almost symmetric like in channels 1 and 2. Therefore, a better measure that can properly discriminate between two unidirectional forcing possibilities is needed.

Defining a Causal Connection and Creating a Graph

It is still unclear, in the presence of a possibility of unidirectional forcing, how to clarify if the relationship is unidirectional or in truth simply bidirectional. Heuristic measures might be used, but a statistical approach is needed for mathematical rigor.

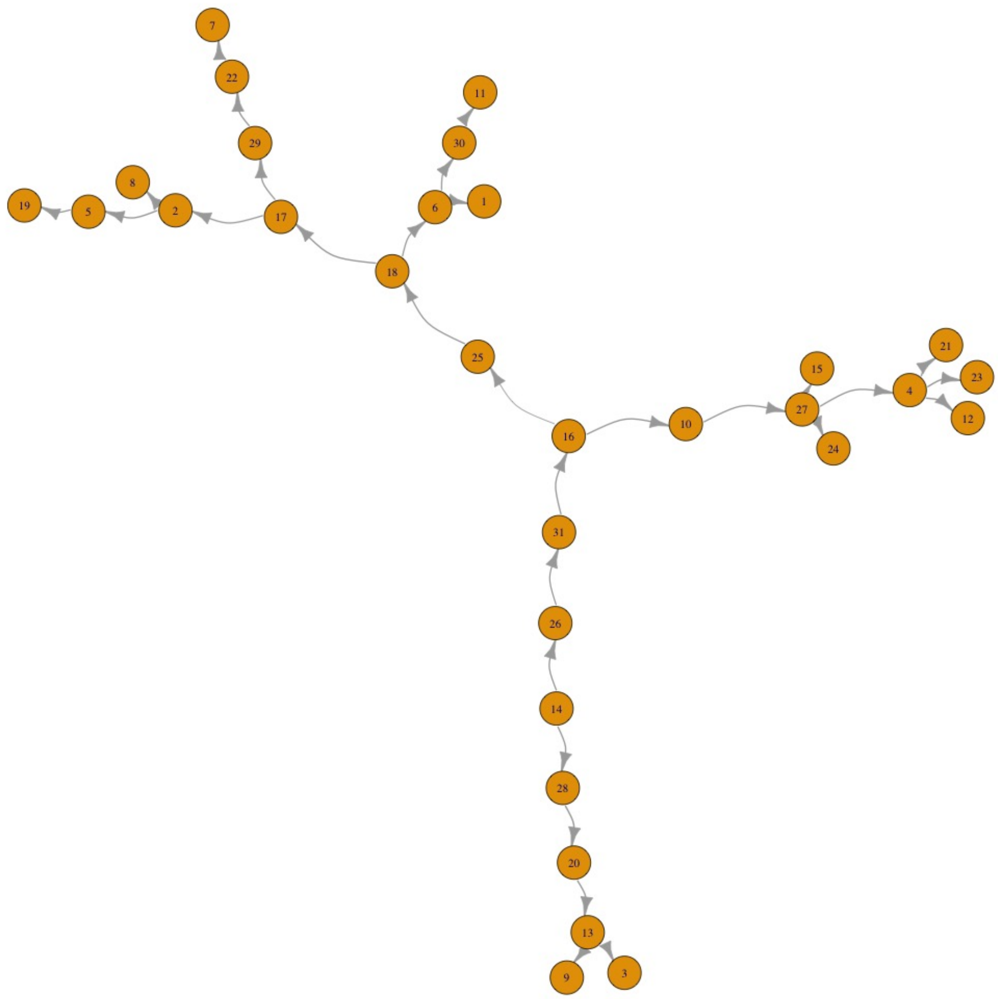
Another problem with lacking a clear mathematical definition of rejecting and accepting a Sugihara causality measure is that the network could be a complete graph because all the causality measures converge to a value. The reason for having what might seem like a complete graph is the transitivity property of Sugihara Causality. For example, if a causes b and b causes c, then convergent cross mapping will detect a causation from a to c, but to a lesser degree than the direct causation. Therefore, the presence of many connections could be the side effect of a cascading flow of information.Therefore, a measure to discriminate causality from cascading is also needed.

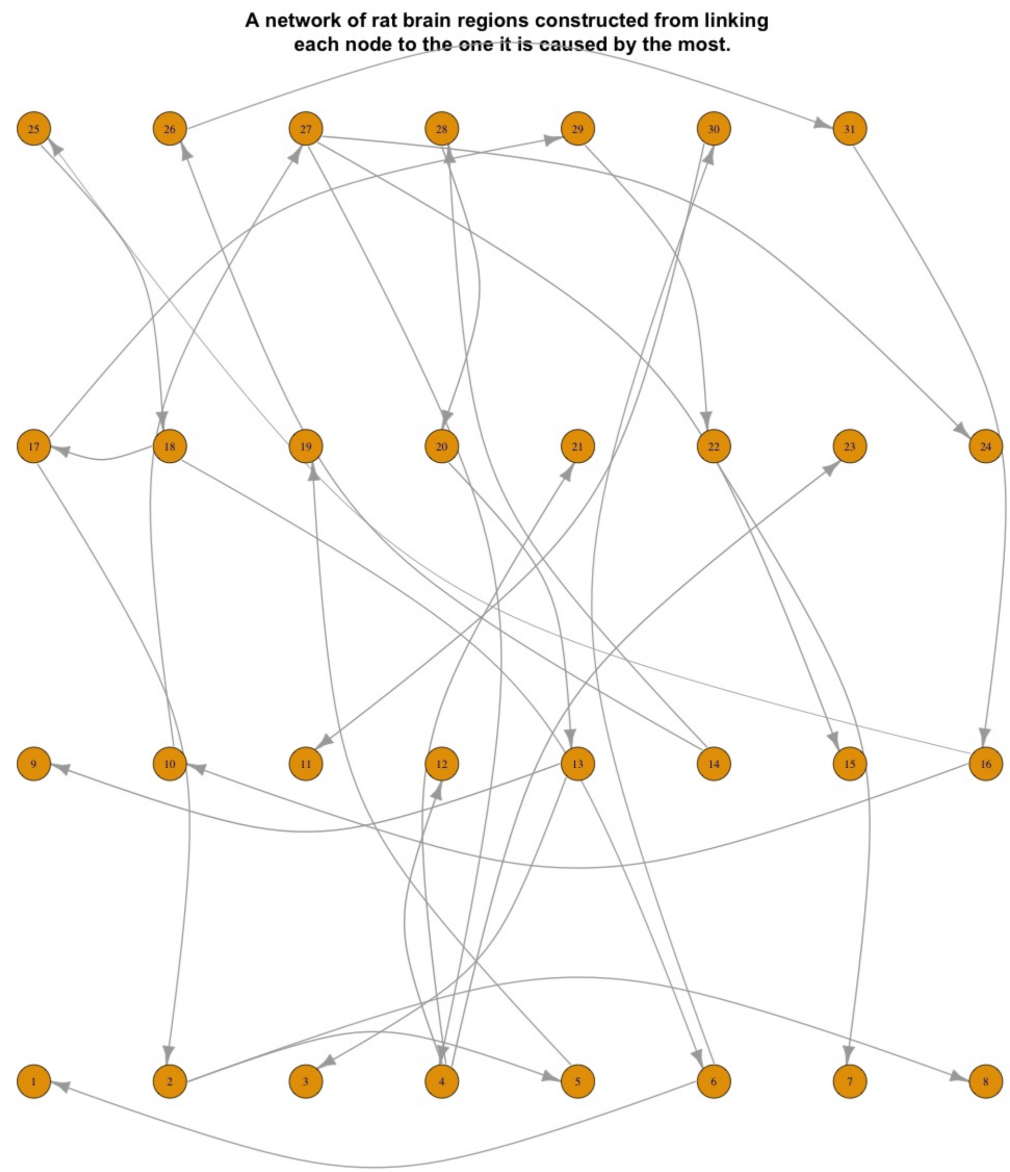
For now, we offer two simple heuristic measures to obtain an initial graph.

1. Top causal relationship
   For each node n_i in the node set $N = n_1, n_2, n_3, \ldots, n_{31}$, we find the node n_j such that the convergent cross mapping skill from n_i to n_j, $\rho_{i,j}$, is the maximum, and LLMD is positive, and we create an directed edge from n_j to n_i with weight $\rho_{i,j}$. Another constraint is that $n_i \neq n_j$. This translates to finding for each node the node that causes it the most, and forming an edge with the causation strength from it.

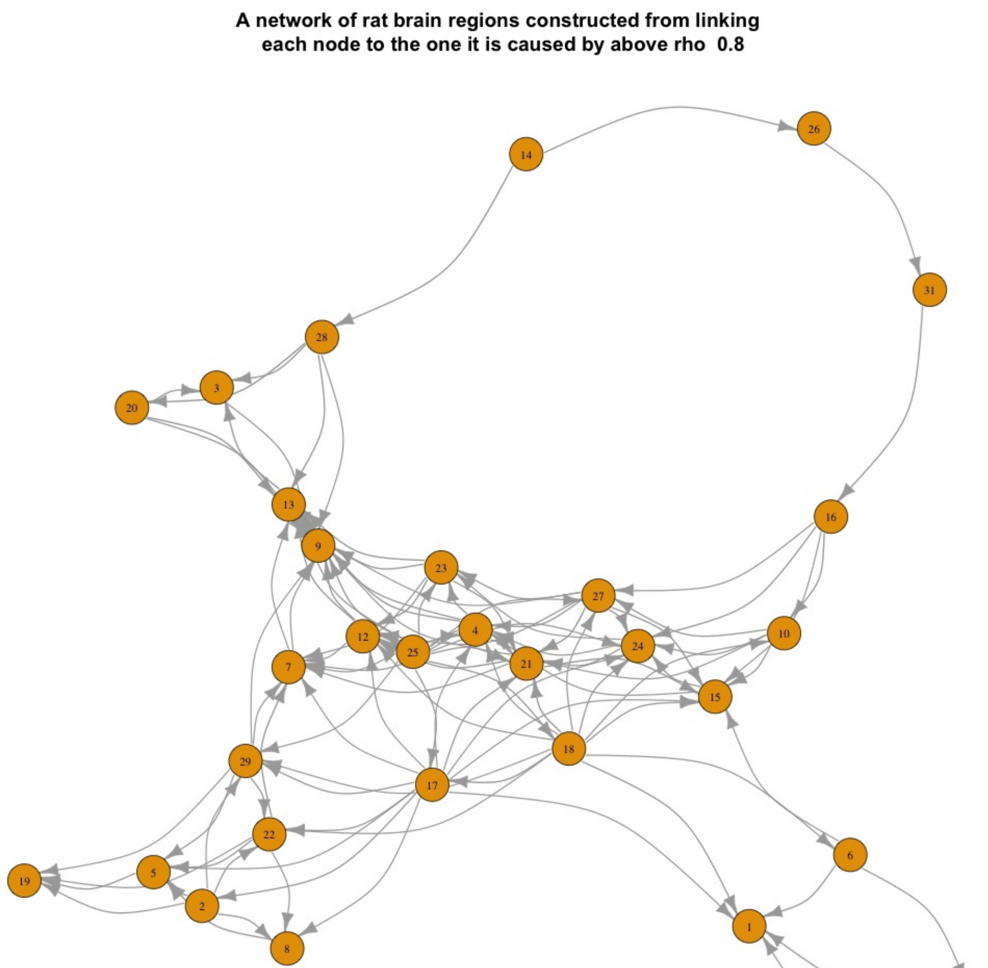
This produces the graph below

A network of rat brain regions constructed from linking
each node to the one it is caused by the most.





2. Since it is obvious that each brain region is not necessarily caused by only one other brain region, we use another heuristic: ρ values above a certain threshold ρ_t, and with a positive LLMD, will be accepted as being causal.


A network of rat brain regions constructed from linking
each node to the one it is caused by above rho 0.8
14
26
31
28
3
20
13
9
23
27
16
10
12
4
24
25
7
21
15
18
29
17
22
6
19
5
2
1
8
30
11

A network of rat brain regions constructed from linking
each node to the one it is caused by above rho 0.8