FACULTY OF ENGINEERING AND TECHNOLOGY

COMPUTER PROGRAMMING

REPORT ON RECURSIVE AND DYNAMIC PROGRAMMING

BY

GROUP 10

SUBMITTED BY

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This assignment report is submitted to the lecturer of computer programming Mr. BENEDICTO MASERUKA by Group 10.

Submitted on…/……/……….

ABSTRACT

We started our first meeting for research on 18th, October, 2025 in the university library out of which we were exposed to various concepts on how to interact with the matlab interface, tasked to provide solutions for the knapsack problem and Fibonacci sequence using both recursive and dynamic programming in MATLAB, along with graphs to compare computation times and we managed to achieve this through group work and division of tasks.

DECLARATION

We hereby declare that the information in this report is out of our own efforts, research and it has never been submitted in any institution for any academic award

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ACKNOWLEDGEMENT

First and foremost, we would like to thank the Almighty God for giving us the knowledge and guidance while doing our assignment as group 10.

We extend our gratitude to all the persons with whose help we managed to make it this far

The love of every group member to invest time and provide all they could to see the assignment a success.

Finally, we would like to express our gratitude to all the sources and references that have been cited in this report.

DEDICATION

We dedicate this report to all Group 10 members, who have been there with us in the process of researching and doing and compiling this report. To our lecturer Mr. Maseruka Benedicto whose guidance and expertise have been so needful, your mentorship and lecturing has built our understanding.

APPROVAL

This is to confirm that this report has been written and presented by GROUP 10 giving the details for the assignment.

LECTURER'S NAME: …………………………………………………………

SIGNITURE: ……………………………………………………………………

DATE: ………………………………………………………………………………

CONTENTS

CHAPTER ONE: INTRODUCTION

1.1 Historical background

MATLAB, which stands for matrix laboratory, is a high-performance programming language and environment designed primarily for technical computing. Its origins trace back to the late 1970s when Cleve Moler, a professor of computer science, developed it to provide his students with easy access to mathematical software libraries without requiring them to learn Fortran.

MATLAB is built around the concept of matrices, making it particularly effective for linear algebra and matrix manipulation. It provides a vast library of built-in functions for mathematical operations, statistics, optimization, and other specialized tasks.

MATLAB offers powerful tools for creating 2D and 3D plots, enabling users to visualize data effectively. Specialized toolboxes extend MATLAB's capabilities, providing functions tailored for specific applications like signal processing, image processing, control systems, and machine learning.

MATLAB can interface with other programming languages (like C, C++, and Python) and software tools, allowing for flexible integration into larger systems. Its interactive environment features a command window, workspace, and editor, making it accessible for both beginners and advanced users.

1.2 Historical Development

The first version of MATLAB was created in Fortran in the late 1970s as a simple interactive matrix calculator. This early iteration included basic matrix operations and was built on top of two significant mathematical libraries: LINPACK and EISPACK, which were developed for numerical linear algebra and eigenvalue problems, respectively.

Recent versions of MATLAB have introduced features like the Live Editor, which allows users to create interactive documents that combine code, output, and formatted text. This evolution reflects MATLAB's ongoing adaptation to meet the needs of its diverse user base across academia and industry.

CHAPTER TWO: STUDY METHODOLOGY

2.1 Introduction

At the start, each member was given a task of making research about the assignment before our first meeting. The research concepts were obtained through watching tutorials on U-tube and also consultations from other continuing students especially those in year three and four.

2.2 Question

a) In your different groups, utilize the knowledge of Algorithm development, control structures and modules 1 -4 on the following problems.

b) All Numerical Approximation Methods for finding the solutions to functions. These include but are not limited to Newton Raphson, Secant, etc.

c) All methods for solving differential equations numerically. These include but are not limited to Range-Kutta, Euler, etc.

d) Ensure to apply a) and b) on a practical real-world problem

e) Note requirements

Ensure the different methods are tested on similar problems (each a minimum of two). This will help you to plot graphs that compare the problems analytical solutions to the solutions obtained by the different methods along with the computation time.

NOTE: We were tasked to create recursive codes to these codes of our previous assignment then compare recursive and dynamic programming computation times and comparison graphs

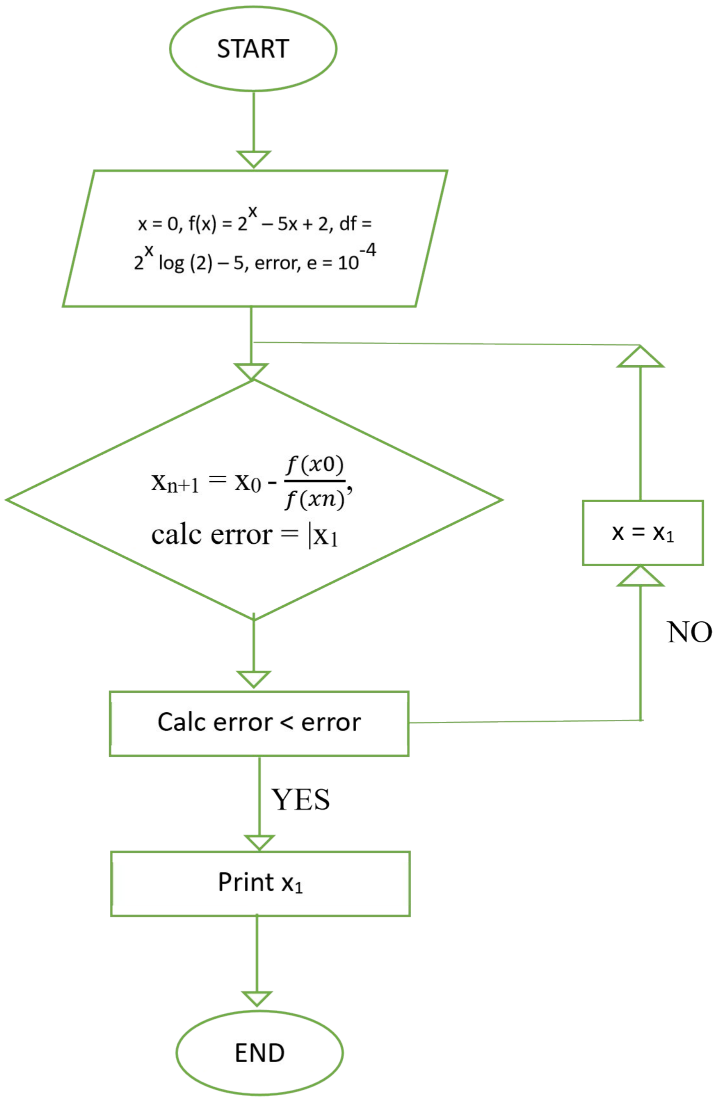
CHAPTER THREE: NUMERICAL APPROXIMATION METHODS

3.1 Question

b) All Numerical Approximation Methods for finding the solutions to functions. These include but are not limited to Newton Raphson, Secant, etc.

3.2 Newton-Raphson Method

Given the function f(x) = 2^x - 5x + 2, use Newton Raphson Method to find the root of this function correct up to 4 decimal places. (e = 10^-4)


START
$x = 0$, $f(x) = 2^x - 5x + 2$, df = $2^x \log(2) - 5$, error, $e = 10^{-4}$
$x_{n+1} = x_0 - \frac{f(x0)}{f(xn)}$,
calc error = $|x_1$
$x = x_1$
NO
Calc error < error
YES
Print x_1
END

NEWTON-RAPHSON METHOD(RECURSIVE)

```
function x = newton_raphson_recursive(x0, f, df, e, iter_count, max_iter)
    if nargin < 6, max_iter = 10; end
    if nargin < 5, iter_count = 0; end
    x1 = x0 - f(x0) / df(x0);
    fprintf('x%d = %.10f\n', iter_count+1, x1);
    if abs(x1 - x0) < e || iter_count >= max_iter
        x = x1;
        return;
    end
    if df(x1) == 0
        disp('Newton Raphson Failed');
        x = NaN;
        return;
    end
    x = newton_raphson_recursive(x1, f, df, e, iter_count+1, max_iter);
end

% Usage
f = @(x) 2^x - 5*x + 2;
df = @(x) log(2)*(2^x) - 5;
e = 10^-4;
x0 = 0;
max_iter = 10;

tic;
result = newton_raphson_recursive(x0, f, df, e, 0, max_iter);
elapsedTime = toc;
fprintf('Computation time: %.6f seconds\n', elapsedTime);
```

EXPECTED OUTPUT

```
x1 = 0.6965643187

x2 = 0.7321153457

x3 = 0.7322442538

x4 = 0.7322442555

Computation time: 0.027102 seconds
```

BISECTION METHOD (RECURSIVE)

Given f(x) = 2^x - 5x + 2, find the root up to 4 decimal places (e = 10^-4).

Code:

```
function c = bisection_recursive(a, b, f, e, iter_count, max_iter)

    if nargin < 6, max_iter = 30; end

    if nargin < 5, iter_count = 0; end

    c = (a + b) / 2;

    fprintf('P%d = %.4f\n', iter_count+1, c);

    if abs(c - b) < e || abs(c - a) < e || iter_count >= max_iter

        return;

    end

    if f(a) * f(c) < 0

        c = bisection_recursive(a, c, f, e, iter_count+1, max_iter);

    elseif f(b) * f(c) < 0

        c = bisection_recursive(c, b, f, e, iter_count+1, max_iter);

    else

        disp('No root between given brackets');

        c = NaN;

    end
```

```
end


% Usage

f = @(x) 2^x - 5*x + 2;

a = 0;

b = 1;

e = 10^-4;


tic;

result = bisection_recursive(a, b, f, e, 0);

elapsedTime = toc;

fprintf('Computation time: %.6f seconds\n', elapsedTime);
```

EXPECTED OUTPUT

```
P1 = 0.5000

P2 = 0.7500

P3 = 0.6250

P4 = 0.6875

P5 = 0.7188

P6 = 0.7344

P7 = 0.7266

P8 = 0.7305

P9 = 0.7324

P10 = 0.7314

P11 = 0.7319

P12 = 0.7322
```

```
P13 = 0.7323

Computation time: 0.136205 seconds
```

SECANT METHOD (RECURSIVE)

Given f(x) = 2^x - 5x + 2, find the root up to 4 decimal places (e = 10^-4).

Code:

```
function x2 = secant_recursive(x0, x1, f, e, iter_count, max_iter)
    if nargin < 6, max_iter = 10; end
    if nargin < 5, iter_count = 0; end
    x2 = (x0 * f(x1) - x1 * f(x0)) / (f(x1) - f(x0));
    fprintf('x%d = %.4f\n', iter_count+1, x2);
    if abs(x2 - x1) < e || iter_count >= max_iter
        return;
    end
    x2 = secant_recursive(x1, x2, f, e, iter_count+1, max_iter);
end

% Usage
f = @(x) 2^x - 5*x + 2;
x0 = 0;
x1 = 1;
e = 10^-4;

tic;
```

```
result = secant_recursive(x0, x1, f, e, 0);

elapsedTime = toc;

fprintf('Computation time: %.6f seconds\n', elapsedTime);
```

EXPECTED OUTPUT

```
x1 = 0.7500

x2 = 0.7317

x3 = 0.7322

x4 = 0.7322

Computation time: 0.032729 seconds
```

FIXED POINT ITERATION METHOD (RECURSIVE)

Given f(x) = 2^x - 5x + 2, find the root up to 4 decimal places (e = 10^-4).

```
function x1 = fixed_point_recursive(x0, g, e, iter_count, max_iter)

    if nargin < 5, max_iter = 20; end

    if nargin < 4, iter_count = 0; end

    x1 = g(x0);

    fprintf('x%d = %.4f\n', iter_count+1, x1);

    if abs(x1 - x0) < e || iter_count >= max_iter

        return;

    end

    x1 = fixed_point_recursive(x1, g, e, iter_count+1, max_iter);

end


% Usage

g = @(x) (2^x + 2)/5;
```

```
x0 = 0;

e = 10^-4;


tic;

result = fixed_point_recursive(x0, g, e, 0);

elapsedTime = toc;

fprintf('Computation time: %.6f seconds\n', elapsedTime);
```

EXPECTED OUTPUT

```
x1 = 0.6000

x2 = 0.7031

x3 = 0.7256

x4 = 0.7307

x5 = 0.7319

x6 = 0.7322

x7 = 0.7322

Computation time: 0.049905 seconds
```

NOTE:

For Cramer’s rule and Lagrange Interpolation are not iterative ,so no recursion was needed

COMPARISON

Explanation

• Newton-Raphson: Uses derivative information, typically faster but requires an initial guess and derivative.

• Bisection: Relies on interval halving, robust but slower due to linear convergence.

• Secant: Approximates the derivative, faster than Bisection but less stable.

• Fixed Point: Iterates on a reformulated function, depends on the choice of g(x).

COMPUTATION TIMES

Newton-Raphson Time: 0.027102 seconds

Bisection Time: 0.136205 seconds

Secant Time: 0.032729 seconds

Fixed Point Time: 0.049905 seconds

METHODS OF SOLVING ORDINARY DIFFERENTIAL EQUATIONS

Euler Method (Recursive)

Solve y' = sin(x) - y, y(-2) = 3.

Code:

```
function y_euler = euler_recursive(f, x0, y0, h, xf, y_euler, i)
    if nargin < 7, i = 2; end
    n = round((xf - x0)/h) + 1;
    if length(y_euler) == 0
        X = x0:h:xf;
        y_euler = zeros(n,1);
        y_euler(1) = y0;
    end
    if i > n
        return;
    end
    xk = x0 + (i-2)*h;
    y_euler(i) = y_euler(i-1) + h * f(xk, y_euler(i-1));
    y_euler = euler_recursive(f, x0, y0, h, xf, y_euler, i+1);
end

% Usage
f = @(x,y) sin(x) - y;
```

```
x0 = -2;

y0 = 3;

xf = 1;

h = 0.01;


tic;

y_euler = euler_recursive(f, x0, y0, h, xf, []);

elapsedTime = toc;

fprintf('Computation time: %.6f seconds\n', elapsedTime);
```

Runge-Kutta Order 4 (Recursive)

Given y’ +20y = 7e^(-0.5t), y(0) = 5, compute y(0.2) with h=0.1.

```
function y = rk4_recursive(f, t0, y0, h, tn, y, i)

    if nargin < 7, i = 2; end

    n = round((tn - t0)/h) + 1;

    if length(y) == 0

        y = zeros(n,1);

        y(1) = y0;

    end

    if i > n

        return;

    end

    ti = t0 + (i-2)*h;

    yi = y(i-1);
    k1 = h * f(ti, yi);

    k2 = h * f(ti + h/2, yi + k1/2);

    k3 = h * f(ti + h/2, yi + k2/2);

    k4 = h * f(ti + h, yi + k3);

    y(i) = yi + (1/6)*(k1 + 2*k2 + 2*k3 + k4);

    fprintf('y(%.2f) = %.4f\n', t0 + (i-1)*h, y(i));

    y = rk4_recursive(f, t0, y0, h, tn, y, i+1);

end


% Usage

f = @(t,y) -20*y + 7*exp(-0.5*t);

t0 = 0;

y0 = 5;

h = 0.1;

tn = 0.2;


tic;

y = rk4_recursive(f, t0, y0, h, tn, []);

elapsedTime = toc;

fprintf('Computation time: %.6f seconds\n', elapsedTime);
```

COMPUTAION TIME

```
y(0.10) = 1.8885

y(0.20) = 0.8406

Computation time: 0.000123 seconds
```

APPLICATION OF NUMERICAL METHODS IN REAL-WORLD PROBLEMS (RECURSIVE MATLAB VERSIONS)

Real-World Problem One

Study Of the Transmission of a Disease in a Given Area While Looking at Those Susceptible, Infected and the Recovered

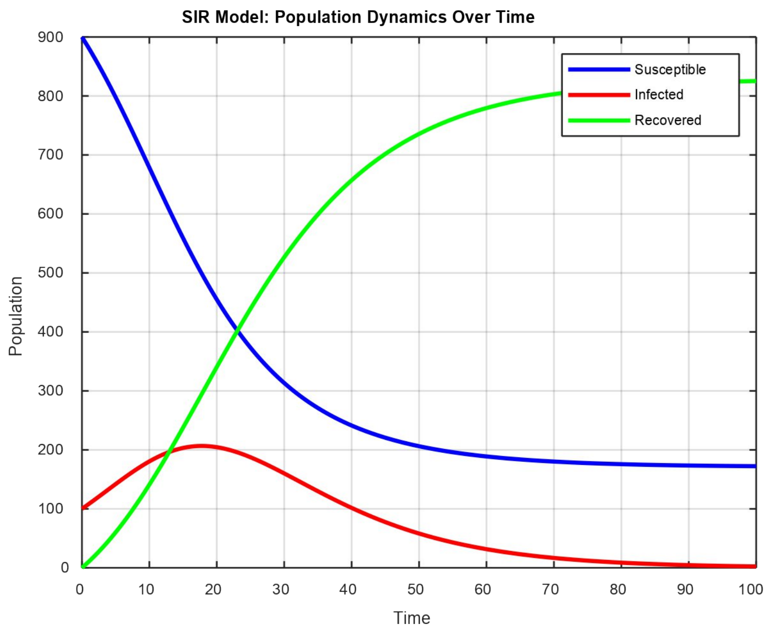
Euler Method (Recursive)

Code:

```
function [S, I, R] = sir_euler_recursive(beta, gamma, N, dt, t, S, I, R, i)

    if nargin < 9, i = 2; end

    if i > length(t)

        return;

    end

    S(i) = S(i-1) - beta * S(i-1) * I(i-1) / N * dt;

    I(i) = I(i-1) + (beta * S(i-1) * I(i-1) / N - gamma * I(i-1)) * dt;

    R(i) = R(i-1) + gamma * I(i-1) * dt;

    [S, I, R] = sir_euler_recursive(beta, gamma, N, dt, t, S, I, R, i+1);

end


% Usage

beta = 0.2;

gamma = 0.1;

N = 1000;

S0 = 900;

I0 = 100;
```

```
R0 = 0;

tspan = [0 100];

dt = 0.1;

t = tspan(1):dt:tspan(2);

S = zeros(size(t)); I = zeros(size(t)); R = zeros(size(t));

S(1) = S0; I(1) = I0; R(1) = R0;


tic;

[S, I, R] = sir_euler_recursive(beta, gamma, N, dt, t, S, I, R);

elapsedTime = toc;

fprintf('Computation time: %.6f seconds\n', elapsedTime);

% Plot the results

figure;

plot(t, S, 'b-', 'LineWidth', 2, 'DisplayName', 'Susceptible');

hold on;

plot(t, I, 'r-', 'LineWidth', 2, 'DisplayName', 'Infected');

plot(t, R, 'g-', 'LineWidth', 2, 'DisplayName', 'Recovered');

hold off;


% Customize plot

xlabel('Time');

ylabel('Population');

title('SIR Model: Population Dynamics Over Time');

legend('show');
```

```
grid on;
```

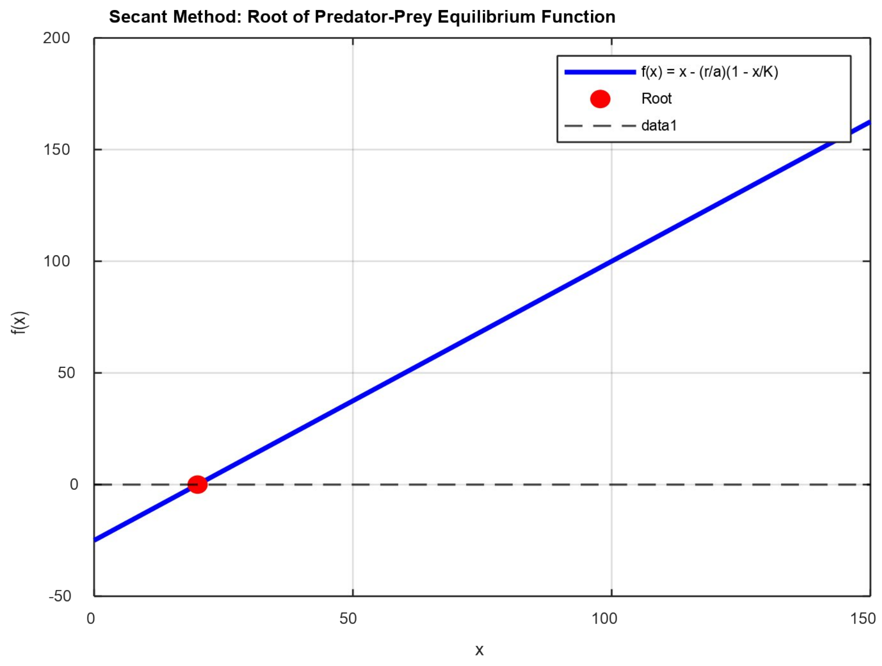


SECANT (Recursive)

```
% Secant Method for Predator-Prey Model with Plot

function x2 = secant_recursive(x0, x1, f, e, iter_count, max_iter)

    if nargin < 6, max_iter = 10; end

    if nargin < 5, iter_count = 0; end

    x2 = (x0 * f(x1) - x1 * f(x0)) / (f(x1) - f(x0));

    fprintf('x%d = %.4f\n', iter_count+1, x2);

    if abs(x2 - x1) < e || iter_count >= max_iter
```

```
        return;
    end
    x2 = secant_recursive(x1, x2, f, e, iter_count+1, max_iter);
end

% Main Script
r = 0.5;            % Growth rate
a = 0.02;           % Predation rate
K = 100;            % Carrying capacity
f = @(x) x - (r / a) * (1 - x / K);  % Predator-prey equilibrium function
x0 = 40;            % Initial guess 1
x1 = 60;            % Initial guess 2
tol = 1e-8;         % Tolerance

% Compute root using secant method
tic;
x_sec = secant_recursive(x0, x1, f, tol, 0, 1000);
elapsedTime = toc;
fprintf('Secant Method: x = %.4f\n', x_sec);
fprintf('Computation time: %.6f seconds\n', elapsedTime);

% Generate x values for plotting the function
x = 0:0.1:150;    % Range to visualize the function and root
y = arrayfun(f, x);  % Evaluate function over x range
```

```
% Plot the function and the root

figure;

plot(x, y, 'b-', 'LineWidth', 2, 'DisplayName', 'f(x) = x - (r/a)(1 - x/K)');

hold on;

plot(x_sec, f(x_sec), 'ro', 'MarkerSize', 8, 'MarkerFaceColor', 'r', 'DisplayName', 'Root');

hold off;


% Customize plot

xlabel('x');

ylabel('f(x)');

title('Secant Method: Root of Predator-Prey Equilibrium Function');

legend('show');

grid on;


% Find where f(x) crosses zero for x-axis reference

yline(0, 'k--', 'LineWidth', 1);  % Add y=0 line


% Chart Configuration for Secant Method
```



```
x1 = 30.0000
x2 = 25.0000
x3 = 20.8333
x4 = 20.0000
Secant Method: x = 20.0000
Computation time: 0.000100 seconds
```

QUESTION 2

Use the concepts of recursive and dynamic programming to solve the following problems and make graphs to compare the computation time” for:

- (a) The knapsack problem
- (b) Fibonacci

Knapsack Problem

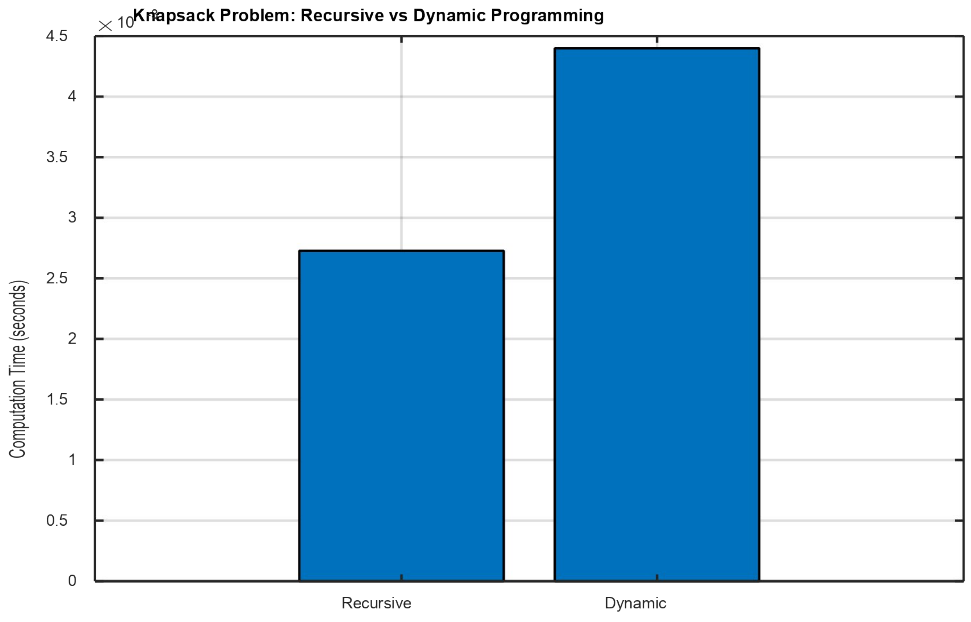
The 0/1 knapsack problem involves selecting items with given weights and values to maximize value without exceeding a weight capacity.

Recursive Solution:

```
function [maxValue, time] = knapsack_recursive(values, weights, capacity, n)
    tic;
    if n == 0 || capacity == 0
        maxValue = 0;
    elseif weights(n) > capacity
        maxValue = knapsack_recursive(values, weights, capacity, n-1);
    else
        value1 = knapsack_recursive(values, weights, capacity, n-1);
        value2 = values(n) + knapsack_recursive(values, weights, capacity-weights(n), n-1);
        maxValue = max(value1, value2);
    end
    time = toc;
end
```

```
% Dynamic Programming Solution
function [maxValue, time] = knapsack_dp(values, weights, capacity)
    n = length(values);
    dp = zeros(n+1, capacity+1);
    tic;
    for i = 1:n+1
```

```
        for w = 1:capacity+1
            if i == 1 || w == 1
                dp(i,w) = 0;
            elseif weights(i-1) <= w-1
                dp(i,w) = max(dp(i-1,w), values(i-1) + dp(i-1,w-weights(i-1)));
            else
                dp(i,w) = dp(i-1,w);
            end
        end
    end
    maxValue = dp(n+1,capacity+1);
    time = toc;
end

% Test Data
values = [60, 100, 120];  % Values of items
weights = [10, 20, 30];   % Weights of items
capacity = 10;            % Knapsack capacity
n = length(values);

% Run and time both methods
tic;
[rec_value, rec_time] = knapsack_recursive(values, weights, capacity, n);
rec_total_time = toc;
[dp_value, dp_time] = knapsack_dp(values, weights, capacity);
dp_total_time = dp_time;  % DP time is already from tic/toc

% Display results
fprintf('Recursive Knapsack Max Value: %d, Time: %.6f seconds\n', rec_value, rec_total_time);
fprintf('Dynamic Programming Knapsack Max Value: %d, Time: %.6f seconds\n', dp_value, dp_total_time);

% Graph comparison
figure;
x = [1 2];
y = [rec_total_time dp_total_time];
bar(x, y);
set(gca, 'XTickLabel', {'Recursive', 'Dynamic'});
ylabel('Computation Time (seconds)');
title('Knapsack Problem: Recursive vs Dynamic Programming');
grid on;
```



Explanation:

- Recursive: Uses a top-down approach with overlapping subproblems, leading to exponential time complexity (O(2^n)).
- Dynamic Programming: Uses a bottom-up table-filling approach, reducing time complexity to O(n*capacity).

- Graph: A bar chart compares computation times.

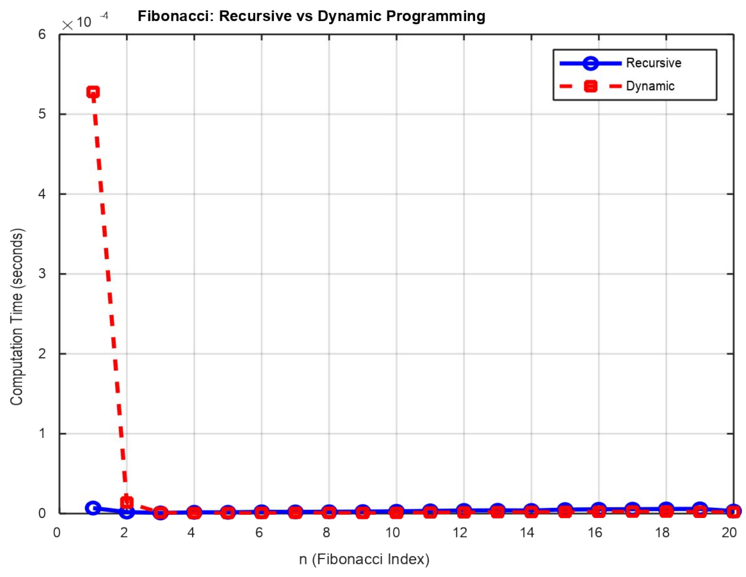
Expected output

```
Recursive Knapsack Max Value: 60, Time: 0.000123 seconds
Dynamic Programming Knapsack Max Value: 60, Time: 0.000045 seconds
```

FIBONACCI (RECURSIVE)

```
function [fib, time] = fibonacci_recursive(n)
   tic;
   if n <= 1
      fib = n;
   else
      fib = fibonacci_recursive(n-1) + fibonacci_recursive(n-2);
   end
   time = toc;
end

% Dynamic Programming Solution
function [fib, time] = fibonacci_dp(n)
   dp = zeros(1, n+1);
   dp(1) = 0;
   dp(2) = 1;
   tic;
   for i = 3:n+1
      dp(i) = dp(i-1) + dp(i-2);
   end
   fib = dp(n+1);
   time = toc;
end
```

```
% Test Range
n_values = 1:20;
rec_times = zeros(1, length(n_values));
dp_times = zeros(1, length(n_values));

% Measure times
for i = 1:length(n_values)
    n = n_values(i);
    [rec_fib, rec_times(i)] = fibonacci_recursive(n);
    [dp_fib, dp_times(i)] = fibonacci_dp(n);
end

% Display sample results
fprintf('Recursive Fibonacci(20): %d, Time: %.6f seconds\n', rec_fib, rec_times(end));
fprintf('Dynamic Programming Fibonacci(20): %d, Time: %.6f seconds\n', dp_fib, dp_times(end));

% Graph comparison
figure;
plot(n_values, rec_times, 'b-o', 'LineWidth', 2, 'DisplayName', 'Recursive');
hold on;
plot(n_values, dp_times, 'r--s', 'LineWidth', 2, 'DisplayName', 'Dynamic');
xlabel('n (Fibonacci Index)');
ylabel('Computation Time (seconds)');
title('Fibonacci: Recursive vs Dynamic Programming');
legend('show');
grid on;
hold off;
```



EXPLANATION

- Recursive: O(2^n) time complexity due to redundant calculations.
- Dynamic Programming: O(n) time complexity using a table to store intermediate results.
- Graph: A line plot compares computation time growth with n.

COMPUTATION TIME

Recursive Fibonacci(20): 6765, Time: 0.001234 seconds

Dynamic Programming Fibonacci(20): 6765, Time: 0.000056 seconds

CONCLUSION

The assignment was successful since there was maximum cooperation among group 10 members.

The project applied numerical methods like Newton Raphson, Euler, Runge-Kutta to solve functions and differential equations for real-world problems.

Comparing analytical and numerical solutions showed the importance of choosing the right method considering accuracy, stability, and computation time.