Exercise 3

Solution/Scratch:



date _____

$R_{school} = +5$    $R_{stay} = -5$    $\gamma = 0.9$
$\phantom{R_{school} =} +3$    $\phantom{R_{stay} =} +1$

$P_{school} = \begin{matrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{matrix}$    $P_{stay} = \begin{matrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{matrix}$

1. Sunny $r_\pi = 0.5 \times 5 + 0.5 \times (-5) = 2.5 - 2.5$
$\phantom{1. Sunny r_\pi} = 0$

2. Cloudy $r_\pi = 0.5 \times 3 + 0.5 \times 1 = 1.5 + 0.5$
$\phantom{2. Cloudy r_\pi} = 2$

3. $r_\pi \begin{matrix} 0 \\ 2 \end{matrix}$

Row 1 Sunny
$P_\pi(1,1) = 0.5 \times 0.8 + 0.5 \times 0.9 = 0.4 + 0.45$
$\phantom{P_\pi(1,1)} = 0.85$
$P_\pi(1,4) = 0.5 \times 0.2 + 0.5 \times 0.1 = 0.1 + 0.05$
$\phantom{P_\pi(1,4)} = 0.15$

Row 2 Cloudy
$P_\pi(4,1) = 0.5 \times 0.4 + 0.5 \times 0.3 = 0.2 + 0.15$
$\phantom{P_\pi(4,1)} = 0.35$
$P_\pi(2,4) = 0.5 \times 0.6 + 0.5 \times 0.7 = 0.3 + 0.35$
$\phantom{P_\pi(2,4)} = 0.65$

$1^{\sim} 2_{\pi}$   7.  $P_{\pi}$    0.85    0.15
                              0.35    0.65

$V_1 = 0 + 0.9 (0.85 v_1 + 0.15 v_2)$
$= 0 + 0.765 v_1 + 0.135 v_2$
$= 0 + 0.765 v_1 - 0.135 v_2 = 0$
$V_1 - 0.765 v_1 - 0.135 v_2 = 0$
$0.235 v_1 - 0.135 v_2 = 0$

5.  $V_1 = \dfrac{0.135 v_2}{0.235}$

$V_2 = 2^2 + 0.9 (0.35 v_1 + 0.65 v_2)$
$2 + 0.315 v_1 + 0.585 v_2$
$V_2 - 0.585 v_2 - 0.315 v_1 = 2$

~~$V_2 + 0.315 v_2$~~
~~$0.315$~~

~~$0.9 \times 15 \sqrt{2}$~~

G.  $- 0.315 v_1 + 0.415 v_2 = 2$

$-0.315 \left( \dfrac{0.135 v_2}{0.235} \right) + 0.415 v_2 = 2$

$\dfrac{-0.042525 v_2 + 0.415 v_2 = 2}{0.235}$

$-0.180957 7748 v_2 + 0.415 v_2 = 2$

$0.234042553 v_2 = 2$

$V_\pi (cloudy)$

4.  7.    $V_\pi = 8.54594545$

$$V_1 = \frac{0.135 \, V_2}{0.235}$$

$$V_1 = \frac{0.135 \, (8.5454545454)}{0.235}$$

$V_{II} (sunny)$

6. $= 4.909090909$

$V_*(sunny) = 5\{ + 0.9 (0.8 V_1 + 0.2 V_2)$
$= 5 + 0.72 V_1 + 0.18 V_2 \}$

$V_*(cloudy) = 3 + 0.9 (0.4 V_1 + 0.6 V_2)$
$3 + 0.36 V_1 + 0.54 V_2$

$V_1 = 5 + 0.72 V_1 + 0.18 V_2$ | $V_1 = \frac{5 + 0.18 V_2}{0.28}$

$V_1 - 0.72 V_1 - 0.18 V_2 = 5$

9. $0.28 V_1 - 0.18 V_2 = 5$

$V_2 = 3 + 0.36 V_1 + 0.54 V_2$
$V_2 - 0.54 V_2 - 0.36 V_1 = 3$

10. $-0.36 V_1 + 0.46 V_2 = 3$

$-0.36 \left( \frac{5 + 0.18 V_2}{0.28} \right) + 0.46 V_2 = 3$

$-0.36 \left( \frac{5}{0.28} \right) - 0.36 \left( \frac{0.18 V_2}{0.28} \right) + 0.46 V_2 = 3$

$$\frac{-1.8}{0.28} - \frac{0.048\,v_2}{0.28} + 0.4\,v_2 = 3$$

$$\frac{-1.8}{0.28} + 0.2285714284\,v_2 = 3$$

$$0.2285714284\,v_2 = 3 + \frac{1.8}{0.28}$$

$$0.2285714286\,v_2 = 9.428571429$$

(cloud)

11.      $v_2 = 41.25$

$$V_1 = \frac{5 + 0.18\,v_2}{0.28}$$

$$V_1 = \frac{5 + 0.18(41.25)}{0.28}$$

(sunny)

14.  $v_x = 49.375$

15.  $q(1, school) = 5 + 0.9(0.8\,v_1 + 0.2\,v_2$

$$=$$

## Step 7

$$q(1, \text{school}) = 5 + 0.9(0.8 * 44.375 + 0.2 * 41.25)$$
$$= 44.375 \quad 44.375$$

$$q(1, \text{stay}) = -5 + 0.9(0.9 * 44.375 + 0.1 * 41.25)$$
$$= 34.645625$$

$$q(2, \text{school}) = 3 + 0.9(0.4 * 44.375 + 0.6 * 41.25)$$
$$= 41.25$$

$$q(2, \text{stay}) = 1 + 0.9(0.3 * 44.375 + 0.7 * 41.25)$$
$$= 38.94875$$

### Formulas

$$q(1, \text{school}) = 5 + 0.9(0.8 v_1 + 0.2 v_2)$$
$$q(1, \text{stay}) = -5 + 0.9(0.9 v_1 + 0.1 v_2)$$
$$q(2, \text{school}) = 3 + 0.9(0.4 v_1 + 0.6 v_2)$$
$$q(2, \text{stay}) = 1 + 0.9(0.3 v_1 + 0.7 v_2)$$

Final Answers

Answers:

1. $V_{\pi}(sunny) = 0$
2. $r_{\pi}(cloudy) = 2$
3. $V_{\pi} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

row 1 Sunny:
$P_{\pi}(1,1) = 0.85$      $P_{\pi}(2,1) = 0.35$
$P_{\pi}(1,2) = 0.15$      $P_{\pi}(2,2) = 0.65$

4. $P_{\pi} = \begin{bmatrix} 0.85 & 0.15 \\ 0.35 & 0.65 \end{bmatrix}$

5. $V_1 = \dfrac{0.135 \, V_2}{0.235}$

6. $V_2 = \dfrac{2 + 0.315 \, V_1}{0.415}$

or $-0.315 \, V_1 + 0.415 \, V_2 = 2$

7. $V_{\pi}(cloudy) = 8.575757575754$
8. $V_{\pi}(sunny) = 9.090909090696$

Go to school

9. ~~V~~ Go to school    $V_1 = \dfrac{5 + 0.18 \, V_2}{0.28}$

10. ✗ $V_2 = -0.34 \, V_1 + 0.74 \, V_2 = 3$

or $V_2 = \dfrac{3 + 0.34 \, V_1}{0.74}$

11. $V_{\pi}(cloudy) = 41.15$      12. $V_R(sunny) = 44.375$

13. $q(1, \text{school}) = 44.375$

14. $q(1, \text{stay}) = 34.65625$

15. $q(2, \text{school}) = 41.25$

16. $q(2, \text{stay}) = ~~38.98~~ 38.96875$