IFT6390
Fondements de l’apprentissage machine

Beyond Linear Classifiers

First solution:

Explicit non-linear mappings

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How can we build a *non-linear* classifier using a linear classifier?

An old trick...

- First, apply a non-linear transform to x (projection into a new feature space) $$\tilde{\mathbf{x}} = \phi(\mathbf{x})$$

  non-linear feature map

- Learn a linear discriminant function in this new space
  (using one of the many algorithms we saw for learning linear classifiers)

- The separating hyperplane in this new space corresponds to a non-linear decision boundary in the original space!

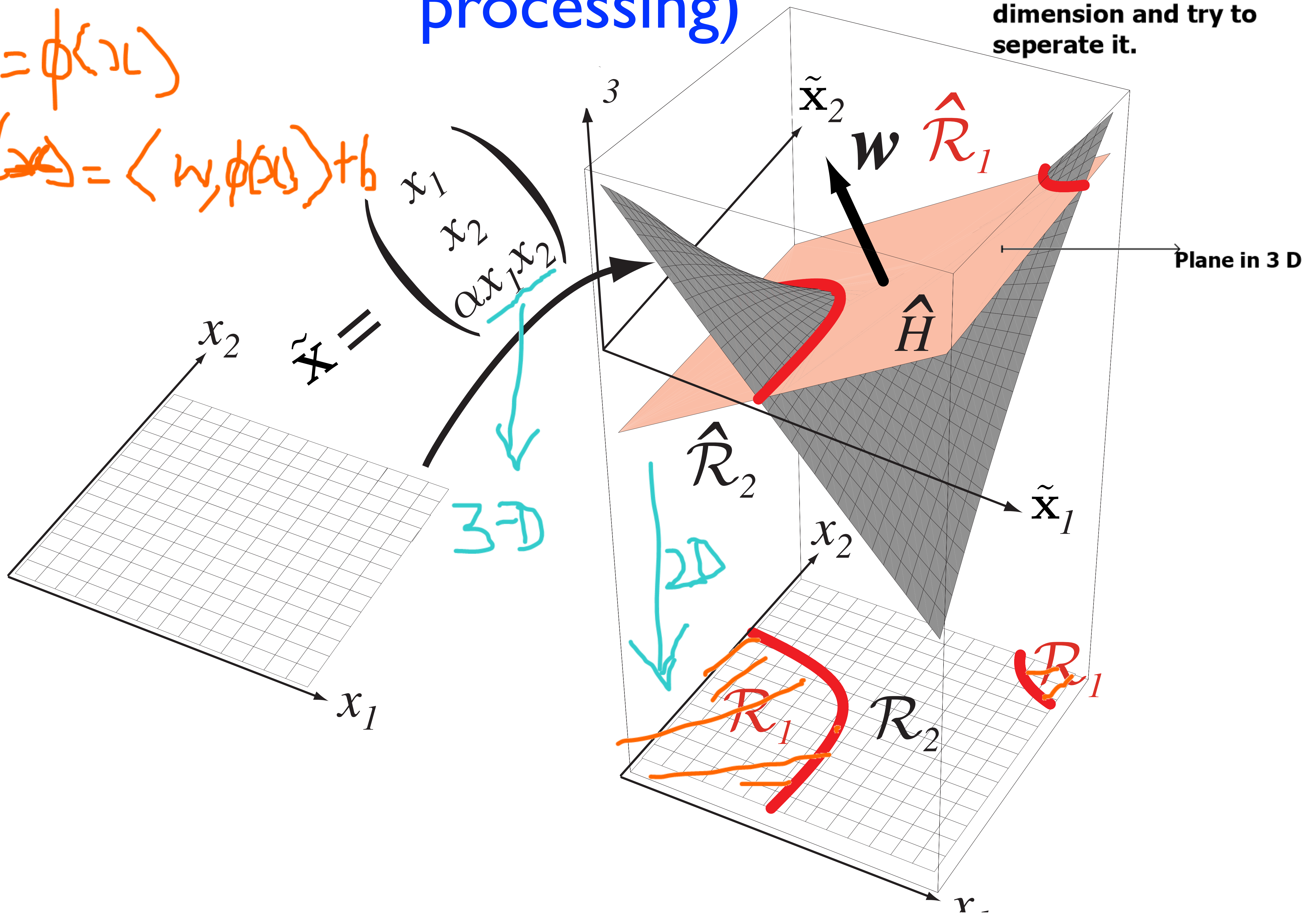
- And voilà! ⇨ A non-linear classifier (with increased capacity).

Ex. *a priori* linear transform (pre-processing)

Move to higher dimension and try to seperate it.

$\tilde{x} = \phi(x)$

$g(x) = \langle w, \phi(x) \rangle + b$



Ex. *a priori* linear transform (pre-processing)

How to build a *polynomial* classifier (or regressor) using a linear classifier (or regressor)?

What is a *linear (affine)* function of *x* (e.g. in 2D)?

2 coefficients(scalars)

$$y = f_\theta([x_1, x_2]) = w_1 x_1 + w_2 x_2 + b$$

What is a *degree 2 polynomial* of *x* (e.g. in 2D)?

5 coeff.

$$poly_2([x_1, x_2]) = a_{11} x_1^2 + a_{22} x_2^2 + a_{12} x_1 x_2 + a_1 x_1 + a_2 x_2 + b$$

$\phi(x)$

What is a *linear (affine)* function of *x* (e.g. in 2D)?

...

Ex. *a priori* linear transform (pre-processing)

How to build a *polynomial* classifier/regressor using a linear classifier/regressor?

- First apply some non-linear feature map to *x* (projection into a new feature space)

$$\tilde{x} = \varphi(x) = \varphi([x_1, x_2]) = [x_1^2, x_2^2, x_1 x_2, x_1, x_2]$$

- Then, learn a linear discriminant function in this new space (using one of the many algorithms we saw for learning linear classifiers)

$$f(x) = w^T \tilde{x} + b = w_1 \underbrace{\tilde{x}_1}_{x_1^2} + w_2 \underbrace{\tilde{x}_2}_{x_2^2} + w_3 \underbrace{\tilde{x}_3}_{x_1 x_2} + w_4 \underbrace{\tilde{x}_4}_{x_1} + w_5 \underbrace{\tilde{x}_5}_{x_2} + b$$

- The separating hyperplane in this new space corresponds to a non-linear decision boundary in the original space!

- And voilà! ⇨ A non-linear classifier (with increased capacity).

But which transformation?

Three way to obtain a non-linear feature map

- Explicitly choose a fixed transformation *a priori*
  ➠ Previous example

- Implicitly choose a fixed transformation *a priori*
  ➠ Kernel method (kernelized SVM, kernelized logistic regression, ...)

- Learn the parameters of a parametrized feature map:
  ➠ Neural networks
  Multilayer perceptron (MLP)